LIGO-T960007-00-D

HYTEC-TN-LIGO-01
01/23/96

Isolation Stack Modeling

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January 23, 1996

Abstract

A specialized MATLAB based rigid body modeling code has been developed to simulate the behavior of the LIGO seismic isolation stacks. This code is capable of modeling multistage, 3-dimensional stacks of rigid bodies and springs with damping and frequency-dependent properties. It enables convenient remodeling of stacks with varying design parameters and allows us to perform inexpensive trend and optimization studies.

The code has been validated by simulations of the MIT prototype stack and in-house tests conducted on a single-stage isolation platform, both using VITON springs. Fine mesh finite element (NASTRAN) models of the springs are used to evaluate the axial and shear stiffnesses of the VITON springs that are used in the MATLAB simulations. The analytical-experimental match is satisfactory and represents an improvement over previous ABAQUS simulations.

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LIGO PROJECT

1. MATLAB Rigid Body Model

The code is capable of representing any arrangement of any number of 3D rigid bodies, connected together with linear springs (Figure 1). Each rigid body is given 6 degrees of freedom (d.o.f., 3 translations and 3 rotations) and is defined by the coordinates of its center of mass (CM) and its mass and inertia tensor at the CM. Each spring connects one body to another and is defined by the location of its attachment point on each body (with respect to the CM of that body) and its stiffnesses, viscous damping and loss factor (structural damping) in 3 directions parallel to the axes of the reference system. Bending and torsional stiffnesses are neglected. If these properties are frequency dependent, they can be provided as look-up tables (linear interpolation is used within the code).

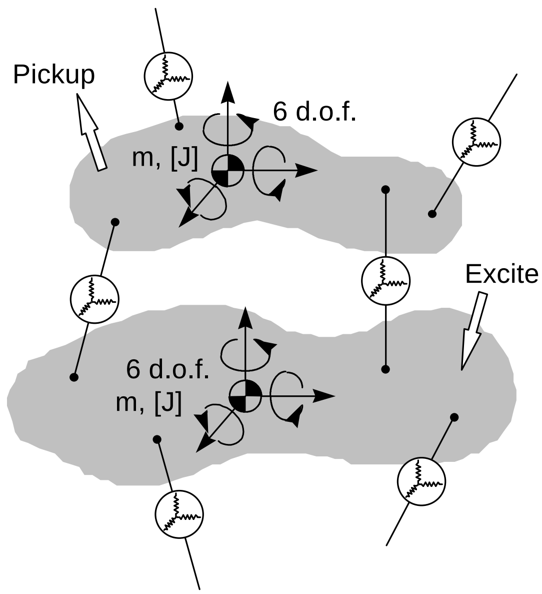

Figure 1: 3D rigid body & springs model.

The code can calculate the stack's frequency response functions from force and/or displacement excitations to angular or linear displacements. The excitation and response pickup points can be defined either in the space of degrees of freedom or as physical points on any of the rigid bodies. In addition, the code can evaluate natural mode shapes, frequencies and damping (using DC properties for the springs) of the stack. Mode shapes can be visualized and animated in 3 dimensions.

The code is organized as a set of MATLAB M-files that can be used either in an interactive mode or as building blocks for specific analysis routines.

2. VITON Spring Model

The prototype stacks created and tested at MIT make use of conical VITON (a type of rubber, also known as *Fluorel*) springs as the elastic elements. Because VITON springs will be one of the options for the LIGO stacks, and because MIT test results on the prototype stacks are essential for validating the simulations, we must be capable of evaluating the shear and axial stiffnesses of those springs for use in the MATLAB simulations.

A table of frequency dependent, complex Young's moduli for VITON was obtained from MIT. Those values were initially based on simple tests (for frequency dependence) and later scaled to achieve analytical-experimental match for the response of the prototype stack. However, the scaling is specific to the model used by MIT for the springs: different modeling assumptions/approximations would lead to different scalings. In the MIT models (ABAQUS), each VITON spring is modeled using two quadratic (20-node) solid brick elements. Because of that very coarse mesh and of the quasi incompressibility of VITON (ν=0.499) this leads to overestimation of the spring constants by a factor of more than two. Figure 2 shows a comparison of NASTRAN results for axial stiffness of the VITON spring, using either a 2-element pyramidal model (as in MIT simulations) or a more refined mesh with the actual conical geometry.

Because the match on vertical-vertical transmissibility (*Tzz*) achieved by the MIT model is excellent (see [1]), the DC axial stiffness $K_{ax}(f=0)$ =1709. lb/in given by the 2-brick model appears realistic. However, the lateral tests performed on the stacks were "polluted" by rocking of the stack support plate so that these should not be used for identifying the shear stiffness K_{sh}. Instead, we rescale the material properties such that the fine mesh model gives a DC axial stiffness of 1709. lb/in. This brings the DC modulus to 1972.3 lb/in (13.59 MPa). The same model is then used to evaluate the DC shear stiffness $K_{sh}(f=0)$ (Figure 3).

The frequency dependence of both the axial and shear stiffnesses was kept unchanged from MIT data. The stiffnesses input into the MATLAB stack simulation are then complex, frequency dependent values given by

$$K_{ax}(f) = 1709.1\,(k_1 + \mathrm{i}\, k_2),$$

$$K_{sh}(f) = 284.9\,(k_1 + \mathrm{i}\, k_2),$$

where i is the imaginary unit and the coefficients k_1 and k_2 are given in Table 1.

f (Hz)	k_2	$1-k_1$
0	0	0
1	.132	.00892
2	.176	-.0615
3	.220	-.101
5	.300	-.157
10	.469	-.277
20	.695	-.496
50	1.164	-.874
100	1.633	-1.254
300	1.633	-3.726
1000	1.633	-3.726

Table 1: frequency dependence of axial and shear stiffnesses of a VITON spring[1].

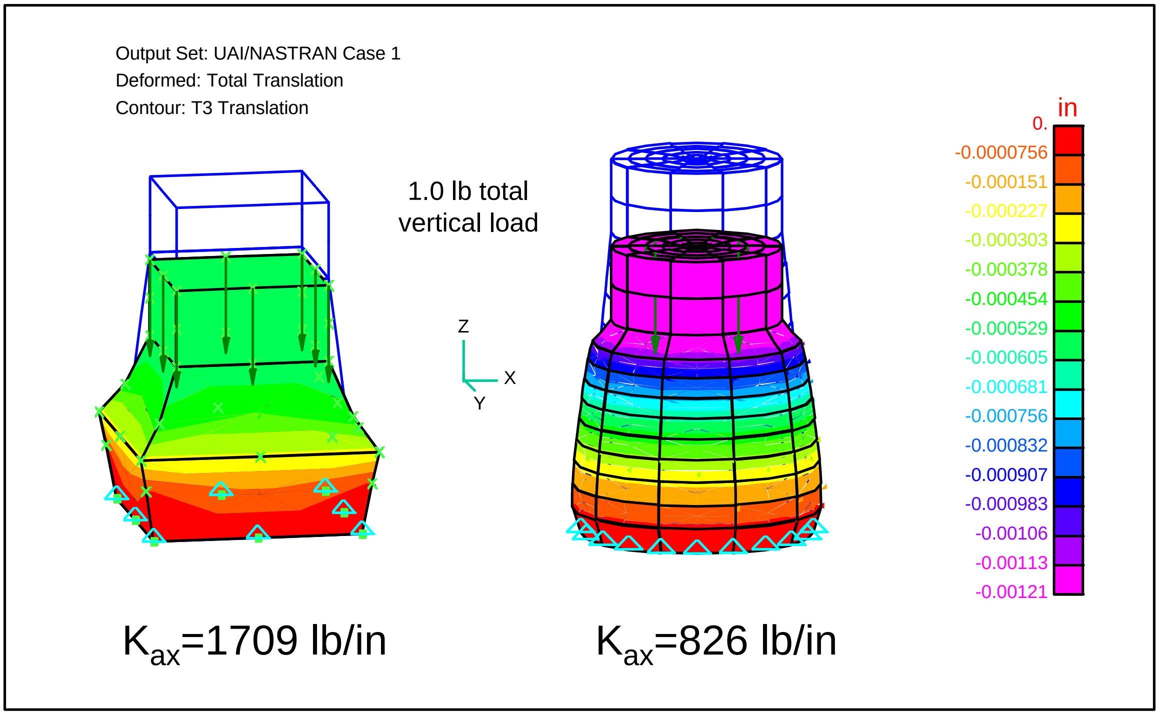

Figure 2: Comparison of 2-brick and fine NASTRAN models for axial stiffness of a VITON spring; elements in both models are quadratic (20-node) bricks; the rubber is compressed between the ground (fixed nodes) and a steel top plate.

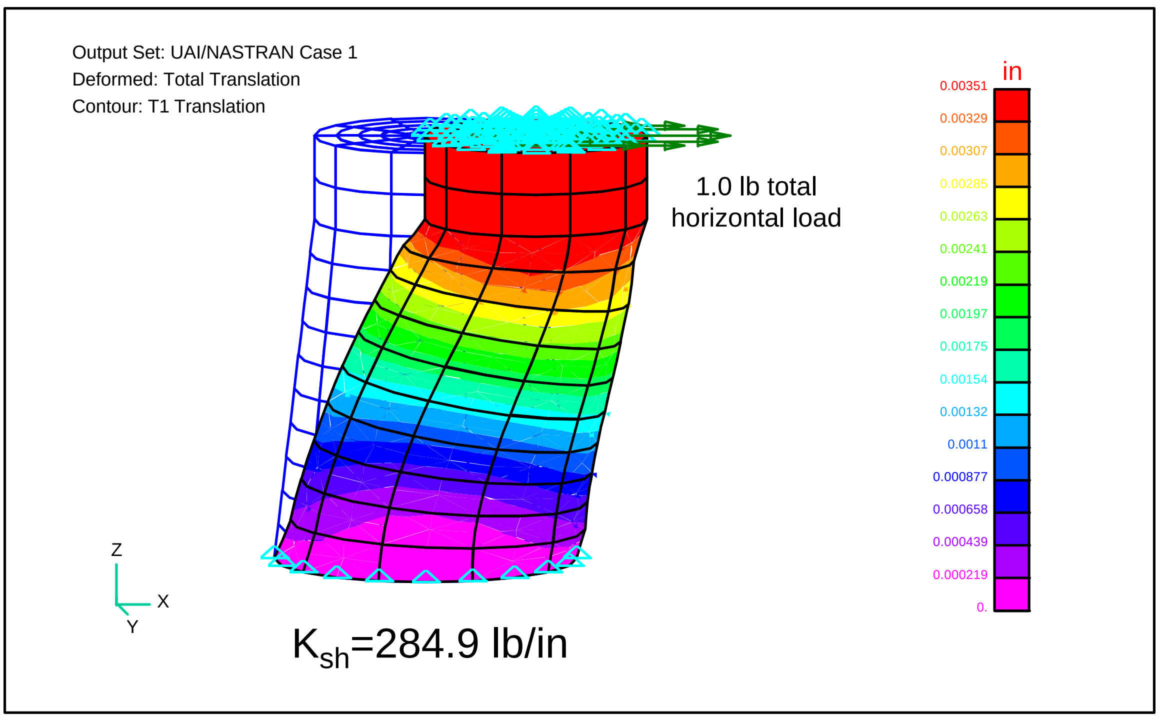

Figure 3: VITON spring, shear stiffness calculation; the steel top plate is not allowed to move up/down or to rock; the nodes of the base are all fixed.

3. Validation

3.1 MIT Prototype Stack

The MIT prototype stack A[1] is composed of 3 legs with 3 stages each, supporting an upper plate that constitutes the 4th stage. All elastic elements are conical VITON springs.

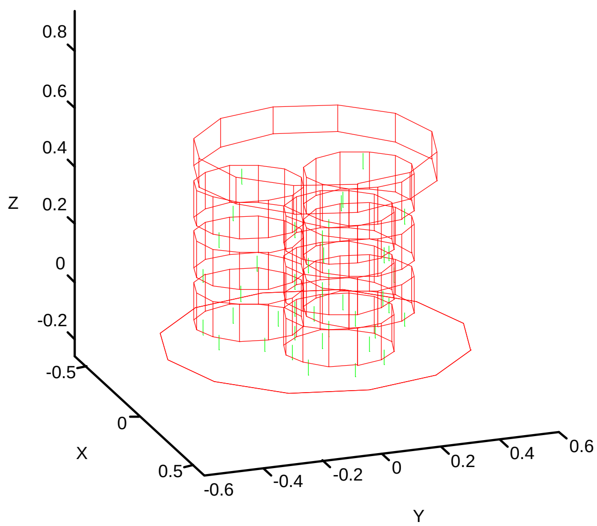

Figure 4: MATLAB model of prototpe isolation stack (MIT, stack A)

A 3-dimensional model of that stack was created in MATLAB using the spring properties defined in Section 2. The model is shown in Figure 4.

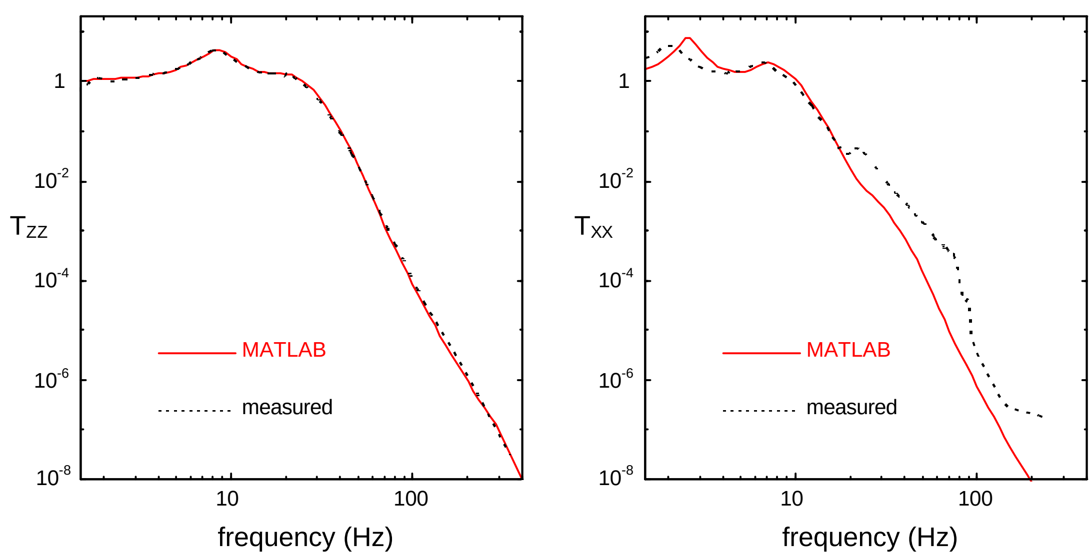

Figure 5: MIT prototype stack A; measured transmissions T_{XX} and T_{ZZ} compared with MATLAB predictions.

Horizontal and vertical transmissibilities were measured at MIT[1] and are compared with MATLAB predictions in Figure 5. Excellent match is observed in the vertical direction. In the horizontal direction (T_{XX}), the match is not as good; note however that the tests in the horizontal direction were affected by unmodeled rocking of the base.

3.2 Single Stage Isolation Platform

To avoid relying exclusively on one set of results (MIT stack measurements) for validation, we performed a series of tests on a single stage isolation platform (Figure 6). The platform is an off-the-shelf rectangular (47.5 in. by 36 in., 8.5 in. thick) optical table weighing approximately 815 lbs and resting on 4 VITON springs, themselves resting on a massive and stiff base attached to the lab floor. The springs are arranged symmetrically around the center of gravity so that all 4 springs support about the same portion of the weight (204 lbs/spring) and shear and axial modes are decoupled as much as possible. The table is excited near one of its corners by an electromagnetic shaker; the excitation load is picked up by a piezoelectric load cell. Accelerometers measure the motion at locations collocated with the excitations.

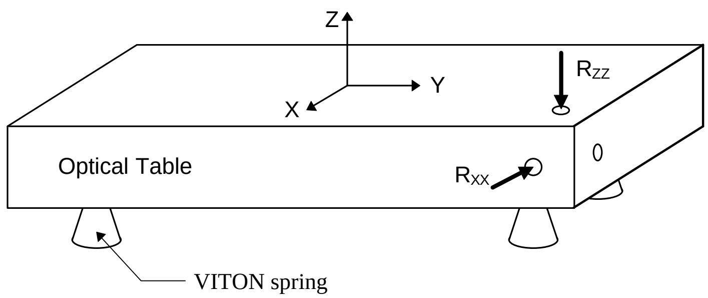

Figure 6: single stage isolation platform (not to scale).

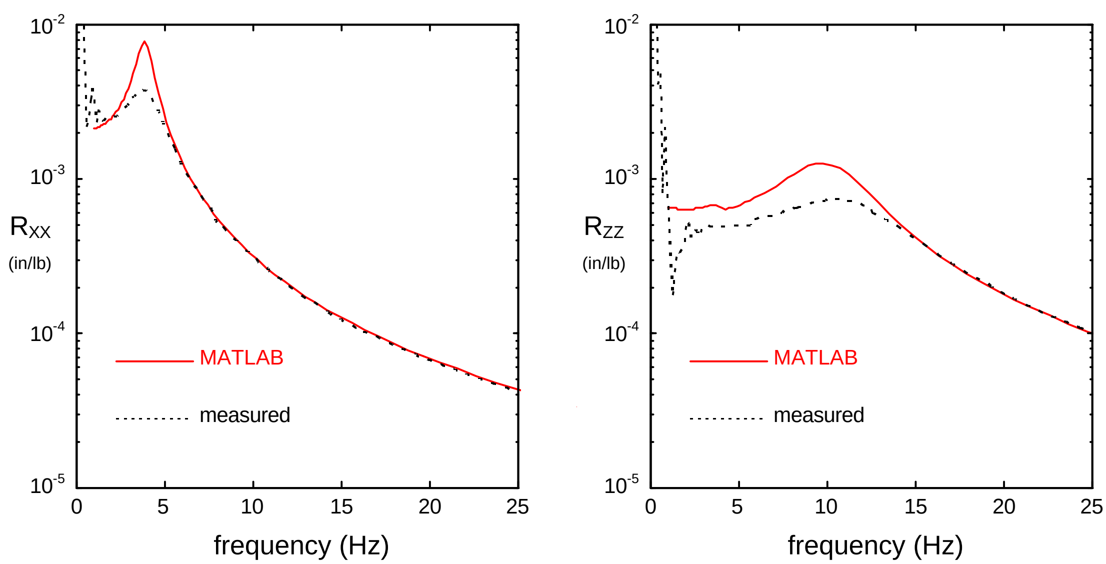

Figure 7: single stage isolation, comparison of measured and predicted receptance; R_{XX} and R_{ZZ} are shown; R_{YY} is similar to R_{XX}.

This system is again modeled in MATLAB and predicted receptances R_{XX} and R_{ZZ} (displacement per unit load vs. frequency) are compared to measurements (Figure 7). The match is excellent in the isolation range (above 15 Hz). At lower frequencies, the resonances due to suspension modes are observed. Their frequencies are predicted well by the model. The damping of those modes however, is underestimated. This could be due to orders of magnitude difference in vibration amplitude between this and MIT tests (rubber has amplitude dependent damping[2]). Our tests were straining the springs by about 4.10^{-3} in. over their 2 in. length (or 2000 µstrain) while MIT were using much smaller amplitudes.

4. References

1. Giaime, J., Saha, P., Shoemaker, D., and Sievers, L., “A Passive Vibration Isolation Stack for LIGO: Design, Modeling and Testing,” submitted to *Review of Scientific Instruments*, 1995.
2. Harris, C., M., *Shock & Vibration Handbook*, 3rd Ed., McGraw-Hill.

Note 1, Linda Turner, 09/03/99 11:24:10 AM
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