LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
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Modal Model Update 6

Mode Cleaner

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1 Abstract

This documents extends the modal model to a triangular cavity, i.e. the LIGO mode cleaner. Results for angular alignment sensitivities are shown. In particular, an alignment scheme using the non-resonant sidebands of the interferometer is developed. To give valid alignment signals these sidebands have to be slightly detuned in the mode cleaner, i.e. a fraction of the sideband power (e.g. 10%) is reflected by the mode cleaner for alignment. This scheme avoids the introduction of a new pair of sidebands which would be non-resonant in the mode cleaner and therefore not pass through. Using only 10% of the sideband power for alignment reduces the wavefront signals by about a factor of 3 compared to the non-resonant case.

2 Triangular Cavity Equations

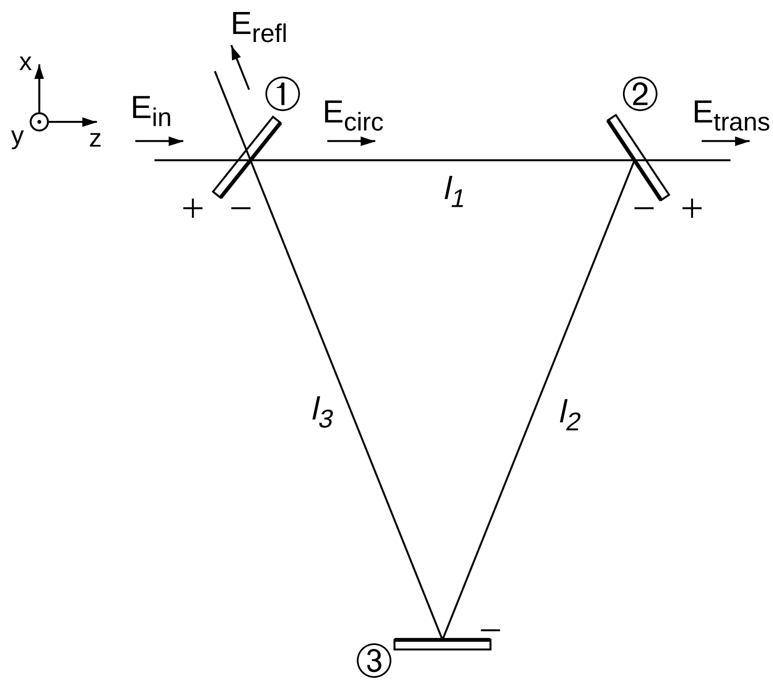

Figure 1: Triangular Cavity. Layout and parameters.

The layout of a triangular cavity is shown in Fig. 1. Even so a triangular cavity is very similar to a normal Fabry-Perrot interferometer, there is an important difference associated with the odd number of mirrors involved in the cavity. After one round-trip a spatial flip occurs for the x-direction (in the plane of the cavity), whereas no such flip occurs in the y-direction (perpendicular to the plane of the cavity). In the modal picture this "flip-in-x" operator X is easy to compute, since the Hermite-Gaussian modes are either symmetric or antisymmetric along a coordinate axis.

Symmetric modes are unaffected by the “flip-in-x”, whereas antisymmetric modes simply change their sign.

$$X_{mn,kl} = (-1)^m \delta_{mk} \delta_{nl} \tag{1}$$

The cavity equations can then readily be deduced:

$$E_{circ} = t_1 (\mathbf{1} - r_1 r_2 r_3 e^{-ikl} G_{rt})^{-1} E_{in} \tag{2}$$

$$E_{refl} = M_1 [r_1 - (r_1^2 + t_1^2) r_2 r_3 e^{-ikl} G_{rt}] (\mathbf{1} - r_1 r_2 r_3 e^{-ikl} G_{rt})^{-1} E_{in} \tag{3}$$

$$E_{trans} = t_1 t_2 e^{-ikl_1} P_{l_1} (\mathbf{1} - r_1 r_2 r_3 e^{-ikl} G_{rt})^{-1} E_{in} \tag{4}$$

with G_{rt} the round-trip operator and $l = l_1 + l_2 + l_3$ the round trip length:

$$G_{rt} = -X M_1^{\dagger} P_{l_3} M_3^{\dagger} P_{l_2} M_2^{\dagger} P_{l_1}. \tag{5}$$

We define differential and common misalignments as follows:

$$\begin{bmatrix} \Delta IO \\ \overline{IO} \\ 3^{rd} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \end{bmatrix} \tag{6}$$

3 Parameters

The parameters for the LIGO mode cleaner are listed in Table 1.

Table 1: Mode Cleaner parameters. LIGO 4km configuration.

Parameter	Unit	input (1)	output (2)	third (3)
length (round-trip)	m	25.1		
power transmission	%	0.2	0.2	0
power reflectivity	%	99.8	99.8	100
radius of curvature	m	∞	∞	18.15
modulation frequencies	MHz	23.8866		29.860
modulation depths	Γ	0.45		0.045
wave length	µm	1.064		
refractive index		1.44968		

4 Angular Sensitivity

The definition for the angular sensitivity of a wavefront sensor signal can be found in eqn. (1) of Ref. [6]:

$$WFS(t, \eta, \Theta) = 2J_0(\Gamma)J_1(\Gamma)Pf_{\mathrm{split}}\, k_{PD}^{10}\sum_i A_i\Theta_i\cos(\eta - \eta_{0i})\cos(\omega_m t + \phi_{0i}) \quad (7)$$

In Table 2 the alignment sensitivity coefficients for the light reflected from the mode cleaner are listed for non-resonant sidebands and for 'close-to-resonance' sidebands which are slightly detuned from resonance. Of course, no WFS signal would be produced, if the sidebands are exactly resonant. The frequency of the resonant sidebands is then chosen to allow 90% of the light to pass through the mode cleaner, or, in other words, 10% of the sideband power is 'lost' for the alignment of the mode cleaner. The non-resonant sidebands of the LIGO interferometer might serve as the close-to-resonance sidebands of the mode cleaner, leaving the resonant sidebands of the interferometer exactly resonant in the mode cleaner. This scheme has the advantage that no new sidebands are needed and that the resonant sidebands which are essential for the gravitational-wave detection don't increase their sensitivity to effects of misalignment in the mode cleaner.

Table 2: Wavefront sensor signals for the mode cleaner. Top entry in each cell is A_i, lower-left is rf-phase, and lower-right is the guoy phase η_{0i}. Values are given in units of the mode cleaner divergence angle which is 201 μrad.

		angular degree-of-freedom					
	port	ΔIO		$\overline{IO}$		Third	
horizontal	Reflected non-resonant sidebands	**3.38**		**–2.26**		**–5.17**	
		I	90°	I	0°	I	0°
	Reflected close-to-resonance sidebands	**1.07**		**–0.714**		**–1.64**	
		72°	90°	72°	0°	72°	0°
vertical	Reflected non-resonant sidebands	**–2.26**		**–1.51**		**–3.46**	
		I	0°	I	90°	I	90°
	Reflected close-to-resonance sidebands	**–0.714**		**–0.477**		**–1.09**	
		72°	0°	72°	90°	72°	90°

5 Longitudinal Sensitivity

Similar to eqn. (7) one can define the length sensitivity L_l as:

$$LS(t, \Delta l) = 2J_0(\Gamma)J_1(\Gamma)Pf_{\text{split}}L_l\,\Delta l\cos(\omega_m t + \phi_{0i}) \quad (8)$$

For the non-resonant sidebands the longitudinal sensitivity L_l is 2.95/nm and for the close-to-resonance sidebands it is 0.93/nm, indicating again that the sensitivity scales like the square root of the reflected sideband power.

Reference

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