Estimates for Motions due to Sound Fields

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Useful reference: *Vibration and Sound* P.M. Morse, Mc Graw Hill, (1948) The method of handling the sound diffraction by cylinders is described on page 352 and in the hand written memo from RW dated Jan 28, 1985.

Ground noise spectrum used for comparison

LIGO standard spectrum

$$x(f) \ \le \ 10^{-7} \ \ \mathrm{cm}/\sqrt{(\mathrm{Hz})} \quad 1 \ \le \ f \le \ 10 \ \ \mathrm{Hz}$$

$$x(f) \ \le \ 10^{-5}/f^{2} \ \ \mathrm{cm}/\sqrt{(\mathrm{Hz})} \quad 10 \ \le \ f \ \ \mathrm{Hz}$$

Forces due to sound waves

The reference pressure for sound when using db is 2×10^{-4} dynes/cm^2 or 2×10^{-10} bar.

The sound pressure forces on a surface are easy to calculate in those cases where diffraction of the sound waves does not have to be considered such as the force on a surface of a sealed box. Under these circumstances the sound pressure force density is simply given as

$$F(f) \ = \ p(f)A$$

where p(f) is the sound pressure spectral density in dynes/cm$^2/\sqrt{\mathrm{Hz}}$ and A is the one sided exposed area of the object.

Diffraction of the sound wave around the object becomes important when the dimensions of the object are smaller than the sound wavelength. Then, the net force on the object is reduced relative to the simple case since the full sound pressure gradient is not developed across the object. Morse gives a series solution for the force on a cylinder of radius a per length of cylinder in the z direction when a plane wave of infinite extent encounters the the cylinder. The plane wave is propagating in the x direction with a peak pressure of P_0 and has an acoustic wavelength of λ. The important scale parameter is

$$\mu \ = \ \frac{2\pi \ a}{\lambda} \ = \frac{2\pi f \ a}{c_s}$$

where c_s is the sound velocity (3.3×10^4 cm/sec) and f the sound frequency.

In first order of the series expansion, the force per length on the cylinder is given by

$$\frac{F(f)}{L} \ \approx 2\pi \ a \ p(f) \ \mu \ \propto \ p(f)f \quad \mu \ \ll \ 1$$

and

$$\frac{F(f)}{L} \ \approx a \ p(f) \ (\frac{8\pi}{\mu})^{1/2} \propto \ p(f)/\sqrt{f} \quad \mu \ \gg \ 1$$

The peak sound pressure force per length on the cylinder occurs near $\mu = 1$ and is

$$\frac{F(f)}{L} \approx 1.6\pi \; a \; p(f)$$

Displacement estimates for various cases encountered in the LIGO buildings

Motion of the chambers:

Assume chambers as free masses, then the motion of the chamber in the sound field at frequency f_1 where $\mu = 1$ is given by

$$x(f_1) \;=\; \frac{0.4 \; a \; L \; p(f_1)}{\pi \; m \; f_1^2}$$

Chamber parameters

chamber	f_1 Hz	$p(f_1)$ dynes/cm^2/$\sqrt{\text{Hz}}$	a cm	L cm	m gm	notes
BS	37	5×10^{-3}	144	522	5×10^{6}	1 cm wall
HAM	53	6×10^{-3}	100	190	1.5×10^{6}	1 cm wall

The value of the pressure spectral density, $p(f_1)$ in the table produces a motion of the chamber equal to the standard LIGO ground noise displacement at f_1 assuming a free chamber with no resonances. The values correspond approximately to acoustic noise fields of 43 db for the BS chamber and 47 db for the HAM chamber at the respective frequencies.

Motion of the seismic isolation supports:

Motions longitudinal to the length of the support rods transmitted through the expansion joints are the most serious. The method of estimation assumes that the sound field is incident along the longitudinal dimension of the rods. In this case the plane wave exerts forces on both expansion joints normal to the acoustic wavefront. The expansion joints on the opposite side also experience a force so that the net force on the support rod is the gradient in the acoustic field times the separation of the expansion joints. The scale parameter is again μ with a being the longitudinal separation of the expansion joints. The net force on all four expansion joints becomes

$$F(f) \;=\; 2 \; A \; p(f) \; 2\pi \; f \; a/c_s \quad \mu \;\ll\; 1$$

and

$$F(f) \;=\; 2 \; A \; p(f) \quad \mu \;\gg\; 1$$

Sample parameters for the support assembly

a = length of the support rods = 280 cm

d = diameter of the support rods = 20 cm

m = mass of the support rods = 7×10^5 gms

A = normal area of expansion joints = 960 cm^2

f_1 = frequency for $\mu = 1$ = 24 Hz

For $f \geq f_1$, the permitted pressure spectral density is given

$$p(f) \leq \frac{x(f)\, \omega^2\, m}{4\, A}$$

where x(f) is the displacement noise permitted at the support point. Using the LIGO standard spectrum $x(f)$, $p(f)$ must be less than 7×10^{-2} dynes/$\text{cm}^2/\sqrt{\text{Hz}}$ or the sound field should be less than 64 db at 25Hz and 70 db at 100 Hz, as examples. The calculation is considerably different if one does not want to compromise the external active vibration isolation system. The expectation is that the seismic noise will be reduced at the support point by a factor of 30 in the band between 25 to 70 Hz. The allowed acoustic pressure fluctuations are correspondingly smaller by the same factor in this band, smaller than 2×10^{-3} dynes/$\text{cm}^2/\sqrt{\text{Hz}}$. The sound level at 25 Hz should then be less than 35db and at 70 Hz less than 38 db.

The acoustically driven noise in the direction transverse to the support rods is much smaller. The sound diffraction in this case is around the expansion joints giving f_1 as about 310 Hz.

Spectra of the acoustic noise in clean rooms and laboratories

Hal Amick of Acentech has sent us spectra of various clean rooms. The data is given in db in an octave band around a mean frequency.

Table of acoustic noise

center frequency	Class 100 clean room	Class 10000 Adv. Microstr. LSU
	12 ft x 40 ft	168 ft diameter x 35 ft height
Hz	dB	dB
8	77	64
16	75	64
32	74	65
63	71	74
125	69	72
250	67	61
500	63	58
1000	60	54
2000	56	52

The air turbulence from the circulation required to maintain the clean room conditions is clearly troublesome. The Advanced Microstructure facility at LSU is closer to the conditions in the LIGO vertex building although it has a cleanliness level higher than what we are asking for so there is some basis that it should be possible to reduce the sound levels below those measured at that facility for the LIGO. If there is no large improvement over these values it will be necessary to put acoustic shielding around the external active vibration isolation systems.