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Optical Paths Lengths Through a Wedged Optic at Non-normal Incidence: The aLIGO Mode Cleaner

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Abstract

This document provides a review of the geometry of thick, wedged, optics, using the aLIGO Mode Cleaner as an example. We obtain the optical path length between the AR surface and HR surface of MC1 (and MC3) to be $\ell_{eff} = 0.12259$ m as the carrier beam enters (exits) the mode cleaner from ISM2 (on its way to SM1).

1 Introduction

When dealing with transmissive interferometer optics of finite thickness, it is often necessary to compute the effective optical path between the two surfaces of the optic. If a beam enters the optic at normal incidence and the optic's surfaces are parallel, the effective optical path length, ℓ_{eff}, is simply the index of refraction of the optic material, n_O, multiplied by the thickness of the optic, t_O. In general, when the beam is incident at an arbitrary angle and the optic surfaces are not parallel, the geometry problem becomes less-than-trivial.

In this document, we walk through two such cases using the wedged, aLIGO mode cleaner mirrors as our examples. In the case of MC1, the incident beam enters on the wedged, AR side, which has wedge angle θ_W. In the case of MC3, the beam enters on the flat, HR side. For this document, we take "Surface 1" to be the "front," or HR side, "Surface 2" to be the "back," or AR side of the optic, and may use the terms interchangeably. In addition, n_V is the index of refraction of the environment surrounding the optic (vacuum, in these examples), and n_{FS} is the index of refraction inside the optic (fused silica for aLIGO).

2 Beam Entering Through the Wedged Side

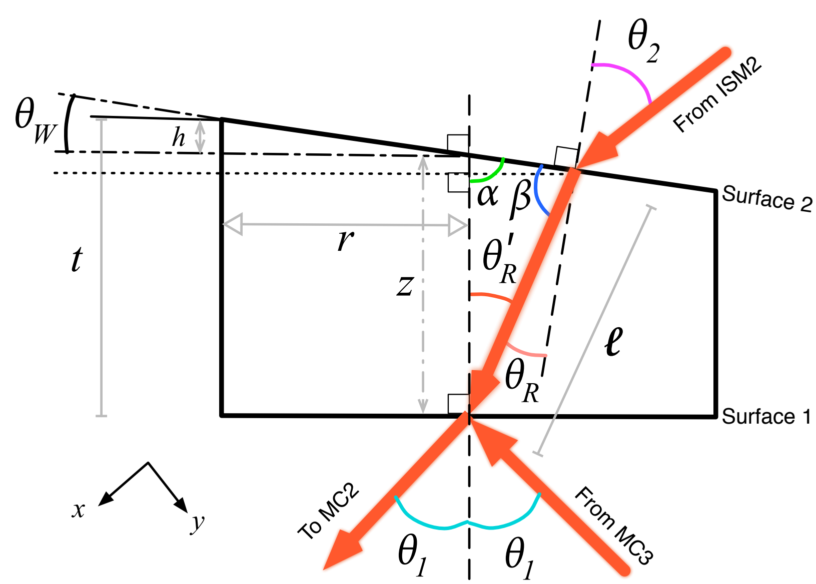

Figure 1: Depiction of the ray's optical path through MC1.

First, we assume that we want the carrier beam to come out in the center of the HR surface of the mode cleaner optic, a distance r (the radius of the optic) from the barrel edge. We also notice that, in this case, the relation between the two surfaces causes the refracted angles, θ_R and θ'_R, to be related by

$$\theta'_R = \theta_R + \theta_W. \tag{1}$$

instead of what is typical for an optic with parallel surfaces, $\theta'_R = \theta_R$.

When we invoke Snell's law to obtain the exiting angle from Surface 1 of the optic, θ_1; it is not equal to the angle of incidence, θ_2, as we would expect from an optic with two parallel surfaces, but

$$\begin{aligned} n_V \sin\theta_2 &= n_{FS} \sin\theta_R \\ \theta_R &= \arcsin\left(\frac{n_V}{n_{FS}} \sin\theta_2\right) \end{aligned} \tag{2}$$

$$\begin{aligned} n_{FS} \sin\theta'_R &= n_V \sin\theta_1 \\ \theta_1 &= \arcsin\left(\frac{n_{FS}}{n_V} \sin\theta'_R\right) \\ &= \arcsin\left(\frac{n_{FS}}{n_V} \sin\left[\theta_R + \theta_W\right]\right) \\ \theta_1 &= \arcsin\left(\frac{n_{FS}}{n_V} \sin\left[\left\{\arcsin\left(\frac{n_V}{n_{FS}} \sin\theta_2\right)\right\} + \theta_W\right]\right) \end{aligned} \tag{3}$$

To obtain the optical path length, we note that

$$\begin{aligned} h/r &= \tan\theta_W \\ \Rightarrow h &= r\ \tan\theta_W \end{aligned} \tag{4}$$

$$\begin{aligned} z &= t - h \\ \Rightarrow z &= t - r\ \tan\theta_W. \end{aligned} \tag{5}$$

By process of elimination we gather that the triangle created by Surface 2, the optical path length, ℓ, and the thickness of the optic at its center, z (see Figure 2), has the following angles,

$$\alpha = \pi/2 - \theta_W \tag{6}$$

$$\begin{aligned} \beta &= \pi - \left(\alpha + \theta'_R\right) \\ &= \pi - \left([\pi/2 - \theta_W] + \theta'_R\right) \\ \beta &= \pi/2 + \left(\theta_W - \theta'_R\right) \end{aligned} \tag{7}$$

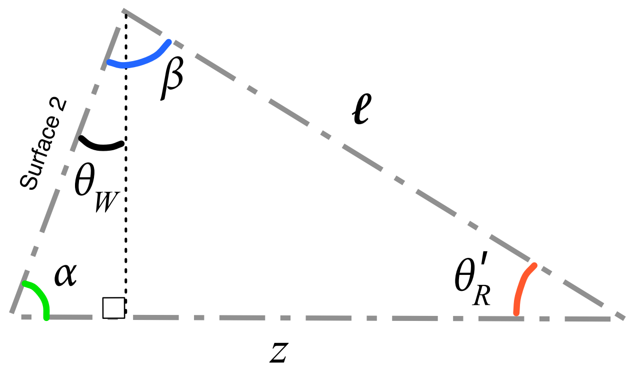

Figure 2: Zoom of the triangle created by Surface 2, the optical path, ℓ, and the thickness of the optic at its center, z as the carrier beam passes through MC1.

with which we can use the law of sines to find the path length through the optic, ℓ,

$$\begin{aligned}
\frac{z}{\sin\beta} &= \frac{\ell}{\sin\alpha} \\
\ell &= \frac{z\ \sin\alpha}{\sin\beta} \\
&= \frac{(t - r\ \tan\theta_W)\sin(\pi/2 - \theta_W)}{\sin\left[\pi/2 + \left(\theta_W - \theta_R'\right)\right]} \\
&\quad \left(\sin(\pi/2 \pm x) \;=\; \cos(\pm x) \;=\; \cos x\right) \\
&= \frac{(t - r\ \tan\theta_W)\cos\theta_W}{\cos\left(\theta_W - \theta_R'\right)} \\
&\quad \left(\begin{array}{rcl} \theta_R' &=& \theta_R - \theta_W \\ \theta_R &=& \theta_R' - \theta_W \\ -\theta_R &=& \theta_W - \theta_R' \end{array}\right) \\
&= \frac{(t - r\ \tan\theta_W)\cos\theta_W}{\cos(-\theta_R)} \\
&\quad \left(\cos(-x) \;=\; \cos(x)\right) \\
&= \frac{(t - r\ \tan\theta_W)\cos\theta_W}{\cos\theta_R} \\
\ell &= \frac{(t - r\ \tan\theta_W)\cos\theta_W}{\cos\left(\arcsin\left[\frac{n_V}{n_{FS}}\sin\theta_2\right]\right)} \qquad (8)
\end{aligned}$$

Because the beam is travelling through the optic medium, the effective optical path length, ℓ_{eff}, (if the ray were traveling through free space) is

$$\ell_{eff} \;=\; n_{FS}\,\ell \;=\; n_{FS}\,\frac{(t - r\ \tan\theta_W)\cos\theta_W}{\cos\left(\arcsin\left[\frac{n_V}{n_{FS}}\sin\theta_2\right]\right)} \qquad (9)$$

Putting numbers to Eq. 9, with $n_{FS} = 1.44963$, $n_V = 1.000$, $t = 0.075$ m [1], $r = 0.075$ m [1], $\theta_W = 0.5°$ [2], $\theta_2 = 43.7°$ [2], then we get

$$\ell_{eff} \;=\; 0.12259 \text{ m.} \tag{10}$$

3 Beam Entering Through the Flat Side

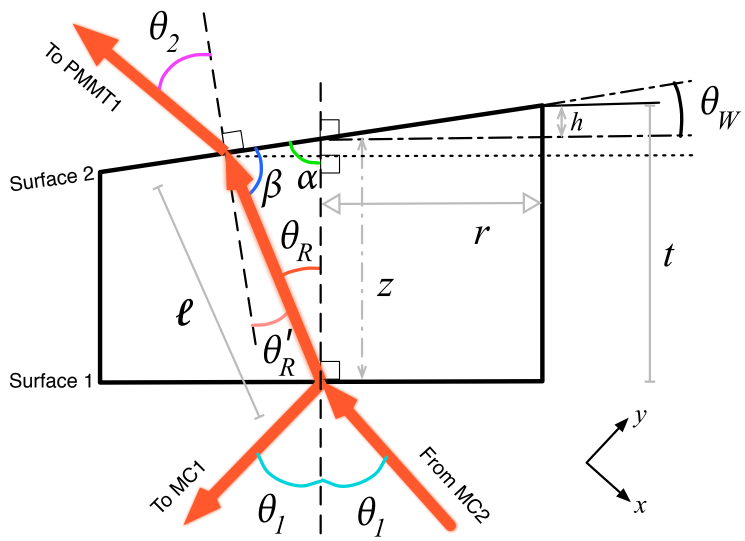

Figure 3: Depiction of the ray's optical path through MC3.

Here, we may use the same techniques as for entering through the wedged side, but the calculations will differ in that

$$\theta'_R \;=\; \theta_R - \theta_W \tag{11}$$

So, just because we're interested in a number (and it's easy to copy and paste in LaTeX), we'll walk through the derivation again:

$$\begin{aligned} n_V \sin\theta_1 &= n_{FS} \sin\theta_R \\ \theta_R &= \arcsin\left(\frac{n_V}{n_{FS}} \sin\theta_1\right) \end{aligned} \tag{12}$$

$$\begin{aligned} n_{FS} \sin\theta'_R &= n_V \sin\theta_2 \\ \theta_2 &= \arcsin\left(\frac{n_{FS}}{n_V} \sin\theta'_R\right) \\ &= \arcsin\left(\frac{n_{FS}}{n_V} \sin\left[\theta_R - \theta_W\right]\right) \\ \theta_2 &= \arcsin\left(\frac{n_{FS}}{n_V} \sin\left[\left\{\arcsin\left(\frac{n_V}{n_{FS}} \sin\theta_1\right)\right\} - \theta_W\right]\right) \end{aligned} \tag{13}$$

As before,

$$\begin{aligned} h/r &= \tan\theta_W \\ \Rightarrow h &= r\ \tan\theta_W && (14) \\ z &= t-h \\ \Rightarrow z &= t - r\ \tan\theta_W. && (15) \end{aligned}$$

but now the angles are subtly different,

$$\begin{aligned} \alpha &= \pi/2 - \theta_W && (16) \\ \beta &= \pi - (\alpha + \theta_R) \\ &= \pi - ([\pi/2 - \theta_W] + \theta_R) \\ \beta &= \pi/2 + (\theta_W - \theta_R) && (17) \end{aligned}$$

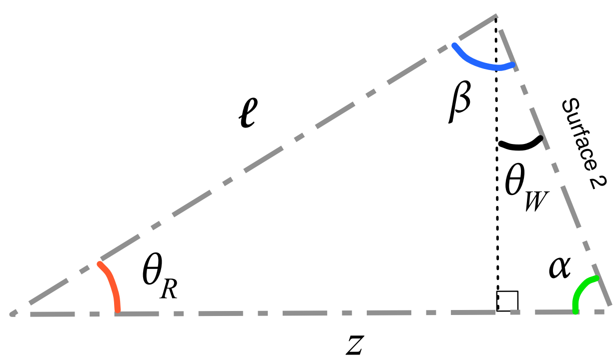

Figure 4: Zoom of the triangle created by Surface 2, the optical path, ℓ, and the thickness of the optic at its center, z as the carrier beam passes through MC3.

with which, we can again use the law of sines to find the path length through the optic, ℓ,

$$\begin{aligned} \frac{z}{\sin\beta} &= \frac{\ell}{\sin\alpha} \\ \ell &= \frac{z\ \sin\alpha}{\sin\beta} \\ &= \frac{(t - r\ \tan\theta_W)\sin(\pi/2 - \theta_W)}{\sin[\pi/2 + (\theta_W - \theta_R)]} \\ & \left(\sin(\pi/2 \pm x) \;=\; \cos(\pm x) \;=\; \cos x \right) \\ &= \frac{(t - r\ \tan\theta_W)\cos\theta_W}{\cos(\theta_W - \theta_R)} \\ \ell &= \frac{(t - r\ \tan\theta_W)\cos\theta_W}{\cos\left(\theta_W - \arcsin\left[\frac{n_V}{n_{FS}}\sin\theta_1\right]\right)} && (18) \end{aligned}$$

Of course, because MC1's wedge is thickest in the opposite direct as MC3's, and they have exactly the same physical properties, the effective optical path length works out to be identical when one plugs in the numbers, with the only difference being that $\theta_1 = 44.59°$ [2] (which checks out with Eq. 3),

$$\ell_{eff} = 0.12259 \text{ m.} \tag{19}$$

References

[1] L. Williams. “aLIGO Input Mode Cleaner Flat Mirror Substrate.” LIGO Internal Document. LIGO-D070091-v1. (2008)

[2] R. Martin, D. Reitze, D. Tanner. “Coated Substrate, aLIGO Input Mode Cleaner Flat Mirror.” LIGO Internal Document. LIGO-E070085-v1 (2009).