LIGO-G000046-00-Z

PATRICK BRADY ASIS

The Power Filter for Unmodelled Sources

multiple detector analysis

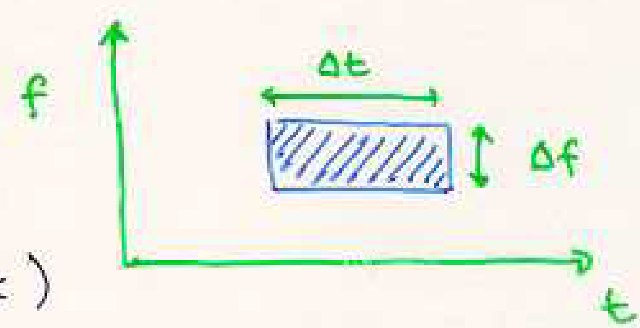
Warren Anderson, PRB, Jolien Creighton
and Eanna Flanagan.

e.g. black hole merger — e.g. binary inspiral

less — knowledge of the signal — more

power filter — matched filter

<u>Question</u>: what is the optimal filter when only time duration Δt and frequency band Δf of the signal is known in advance?



(adopt a frequentist viewpoint for this talk)

Single interferometer

Detector output $\quad \underbrace{\underline{h} = \{h_0, h_1, \ldots h_N\}}_{\text{vector of N samples}}$

$$\underbrace{\underline{h}}_{} = \underbrace{\underline{n}}_{\text{detector noise}} + \underline{s} \rightsquigarrow \text{possible signal}$$

Assume noise follows some N-dimensional probability distribution

$$p(\underline{n}) = \text{const} \; \exp\left[-\frac{1}{2}\sum_{i,j} n_i Q_{ij} n_j\right]$$

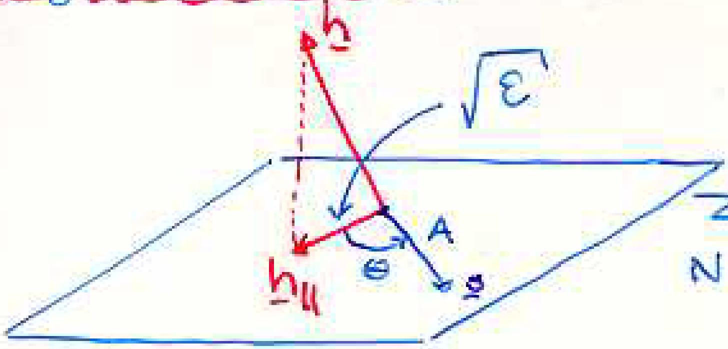
$$R_{ij} = (Q^{-1})_{ij} = \langle n_i n_j \rangle$$

Neyman - Pearson Criterion provides the answer to our question — the optimal detection criterion is a threshold decision rule based on the likelihood function

$$\Lambda[\underline{h}] = \int \underbrace{p[\underline{s}]\,d\underline{s}}_{} \; \frac{p(\underline{h}\,|\,\underline{s})}{p(\underline{h}\,|\,\underline{0})}$$

this distribution should represent our knowledge of signals

Signal Distribution



Σ_0 subspace of signals with N samples but specific Δt and Δf : Σ

- the vector $\underline{s}$ points in all directions within Σ with equal probability.

- distribution of A not needed

$$\Rightarrow \mathcal{E} = \sum_{i,j} h^{\parallel}_i Q_{ij} h^{\parallel}_j = \underbrace{2 \sum_{\Delta f} |\tilde{h}_k|^2 / S_k}_{\text{whitened power in time-frequency window.}}$$

<u>Aside</u> Cornell group is within ~ 1 week of providing LAL package which implements this [DRASCO, FLANAGAN]

Operating Characteristics

Use known statistical properties of Gaussian variables to get false alarm and false dismissal probabilities:

False Alarm: if there is no signal, then $\mathcal{E}$ is a sum of V random variables. It therefore has a χ^2 distribution with V degrees of freedom. False alarm probability for threshold $\mathcal{E}^*$ is

$$P(\mathcal{E} > \mathcal{E}^* \mid A = 0) = \frac{\Gamma(V, \mathcal{E}^*/2)}{\Gamma(V)}$$

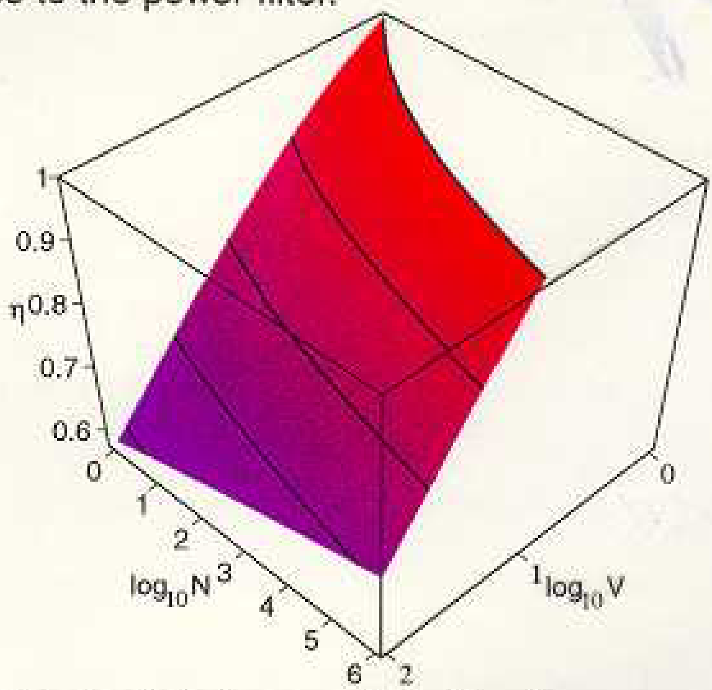
where $\Gamma(a, x)$ is incomplete Gamma function.

False Dismissal: if a signal of amplitude A is present, then $\mathcal{E}$ is distributed as a non-central χ^2 distribution with V degrees of freedom. False dismissal probabilities can be easily calculated numerically.

Comparison to Matched Filters

The matched filter is the optimal filter for a signal where the prior knowledge is the waveform. It is instructive to compare the effectiveness of the power filter. For a given time duration and frequency band we consider a bank of templates to search for N signals of that duration and band. We compare the performance of that bank of templates to the power filter.

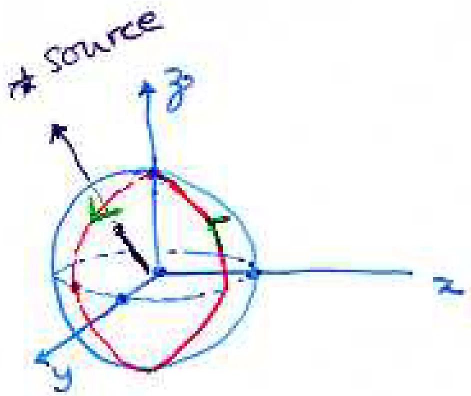
For fixed false alarm and false dismissal probabiliies, we obtain the required signal amplitude A for the power filter, and likewise for a bank of matched filters. η is the ratio of these amplitudes.



If the number of filters in the bank is large and the time-frequency volume is small, the power filter is almost as effective as a bank of matched filters.

Multiple Interferometers:

* different from known signal case *



$$\vec{h} = \{ \underline{h}^1, \underline{h}^2, \dots \underline{h}^D \} = \{ \underline{h}^A \}$$

$\tilde{h}^A_k \sim$ DFT of each time series

$$\tilde{h}^A_k = \underbrace{\tilde{n}^A_k}_{\text{noise}} + \underbrace{\tilde{s}^A_k}_{\text{possible signal.}}$$

The noise:

$$p[\vec{n}] = \exp\left[-\tfrac{1}{2}(\vec{n},\vec{n})\right] \times \text{const}$$

$$(\vec{p},\vec{q}) = 4\sum_{A,B=1}^{D} \mathrm{Re} \sum_{k=0}^{[N/2]} \tilde{p}^A_k \left(S^{-1}_k\right)^{AB} \tilde{q}^{B*}_k$$

$$S^{AB}_k \delta_{kk'} = \langle \tilde{n}^A_k \tilde{n}^{B*}_{k'} \rangle$$

The signal

time of flight for GW between origin & detecter.

$$\tilde{s}^A_k = e^{2\pi i \Delta_A k/N} \left[F^A_+ \tilde{s}^+_k + F^A_\times \tilde{s}^\times_k \right]$$

Depend on detecter and direction to source

$s^+, s^\times$: have some fixed Δt and Δf as before

Optimal detection : (Neyman - Pearson criterion)

Introduce "effective noise" correlation matrix

$$\boxed{\Theta^k_{\alpha\beta} = \sum_{A,B=1}^{D} e^{2\pi i (\Delta_A - \Delta_B) k/N} F_\alpha^A (\vec{S}_k)^{AB} F_\beta^B}$$

$\alpha, \beta \in \{+, \times\}$

depends on direction to the source

Effective strains:

$$\boxed{\tilde{h}^\alpha_k = \sum_{\beta=+,\times} \Theta_k^{\alpha\beta} \sum_{A,B=1}^{D} F_\alpha^A e^{-2\pi i \Delta_A k} (\vec{S}_k)^{AB} \tilde{h}^B_k}$$

two polarizations (plus & cross) for detector network.

Threshold on

$$\boxed{\mathcal{E} = \sum_{\alpha,\beta=+,\times} 4\,\mathrm{Re} \sum_{k=0}^{[N/2]} \tilde{h}^\alpha_{k\parallel}\, \Theta^k_{\alpha\beta}\, \tilde{h}^\beta_{k\parallel}}$$

where maximization over direction to source is understood.

Note 1, Linda Turner, 05/09/00 08:50:19 AM
LIGO-G000046-00-Z