# THERMOELASTIC NOISE IN LIGO-II

Kip Thorne

LSC Meeting

Livingston LA, 3/17/00

SWG / Optics Combo

LIGO-G000068-00-D

# Thermoelastic Noise

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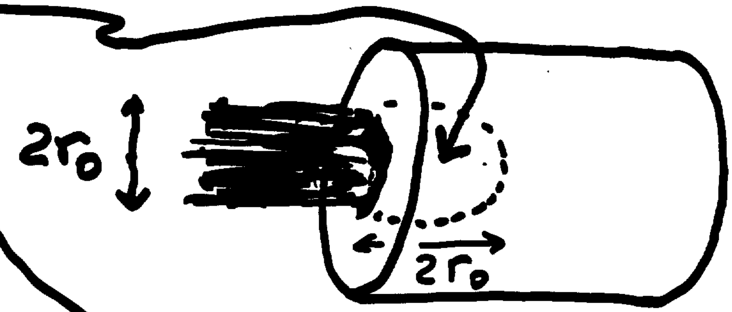
## I. Physical Mechanism & Order of Magnitude

$\delta T_\tau \equiv$ Temp fluct. avgd over $V \sim r_0^3$, on timescale $\tau \sim 1/f$




$$\delta x_\tau \sim \alpha\, \delta T_\tau \cdot r_0$$

$\alpha$: thermal expansion coefficient

$$\boxed{S_x \sim \frac{(\delta x_\tau)^2}{f} \sim \frac{\alpha^2 r_0^2\, \delta T_\tau^2}{f}}$$

What is $\delta T_\tau$?

- Time for heat to diffuse across $r_0$:

$$\boxed{\tau_0 \sim \frac{r_0^2}{D}} \qquad \boxed{D \equiv \frac{\kappa_{th}}{C\rho}} = \text{thermal diffusivity}$$

$\kappa_{th}$: thermal conductivity; $C$: specific heat; $\rho$: density

$$\tau_0 \sim 100\,\text{sec} \cdot (r_0/4\text{cm})^2 \gg \tau = \tfrac{1}{f} = .01\text{sec}$$

• rms Temp. fluct's on diffusion timescale $\frac{C^2}{\tau_c}$

$$\boxed{\delta T_0 = \sqrt{\frac{k_B T^2}{\rho C \underbrace{V}_{r_0^3}}}}$$

• random walk on shorter timescales

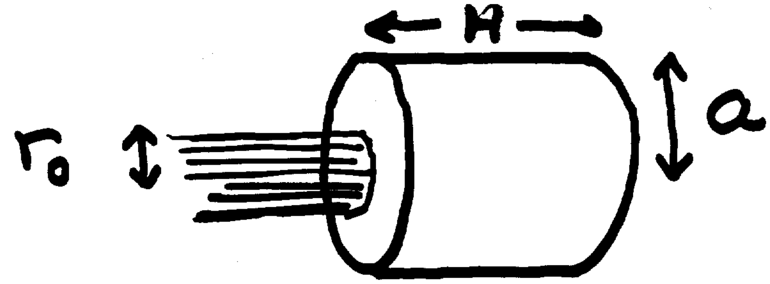
$$\boxed{\delta T_\tau \approx \delta T_0 \sqrt{\frac{\overbrace{\tau}^{1/f}}{\tau_0}}}$$

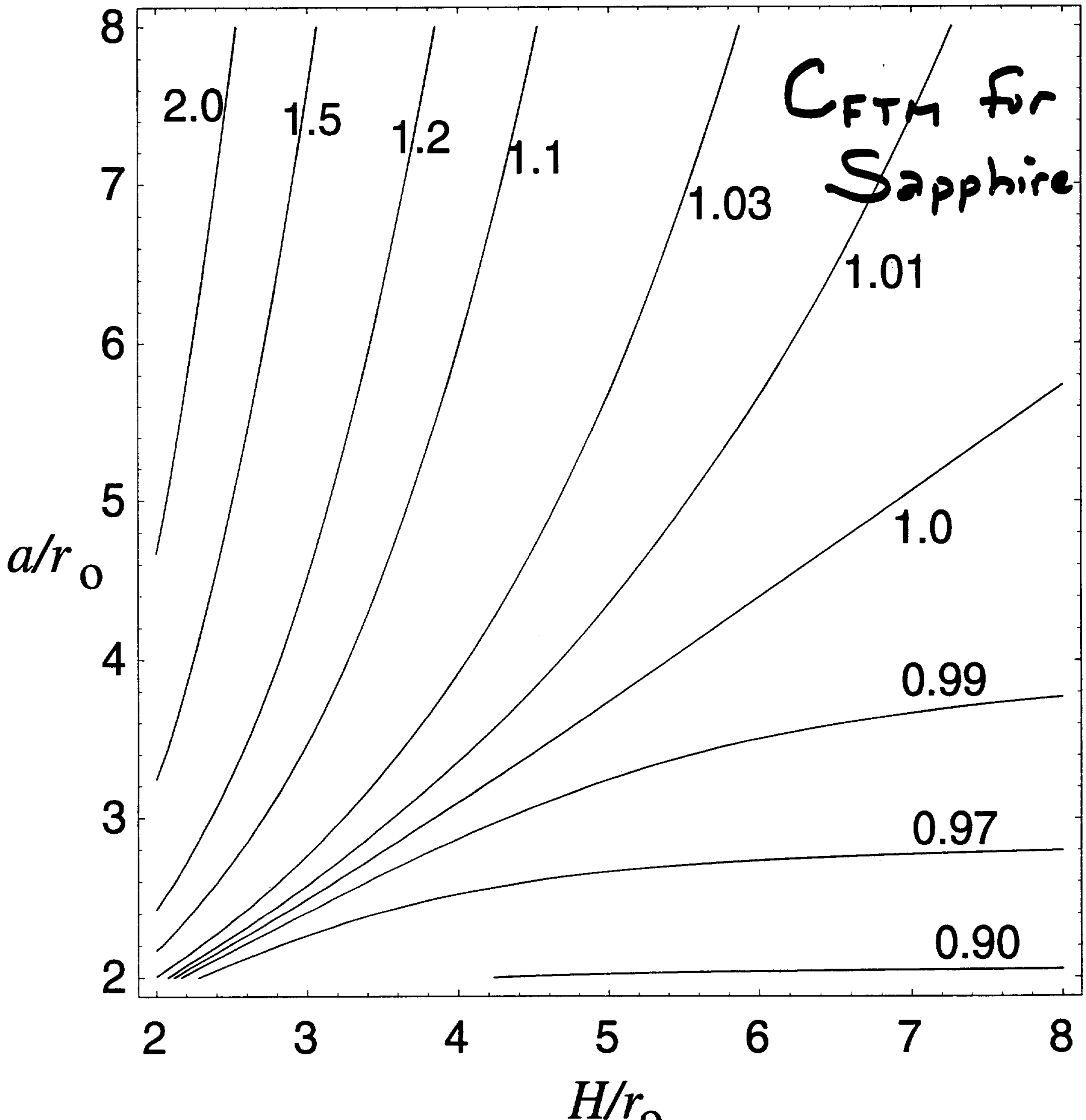
Net Result:

$$\boxed{S_x \sim \frac{\alpha^2 \kappa_{th} k_B T^2}{C^2 \rho^2 r_0^3 \underbrace{f^2}_{\text{same as SQL}}}}$$

# II. Result of "Exact" Calculation

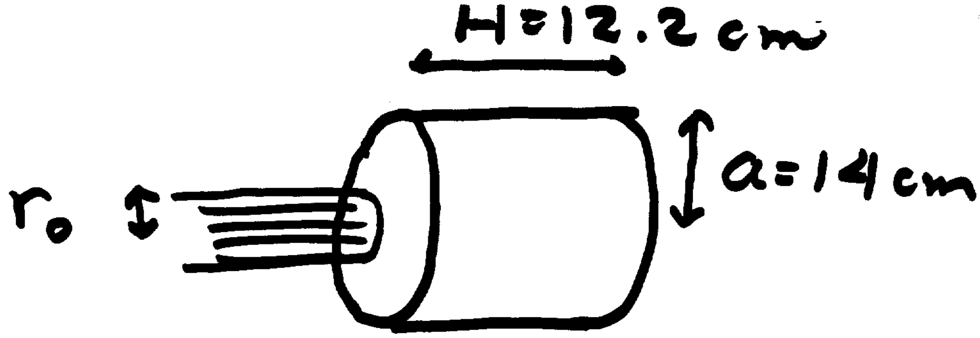
$$\boxed{S_x(f) = \underbrace{\frac{8(1+\overbrace{\sigma}^{\text{Poisson ratio}})^2}{\sqrt{2\pi}} \frac{\alpha^2 \kappa_{th} k_B T^2}{C^2 \rho^2 r_0^3 (2\pi f)^2}}_{\text{Braginsky, Gorodetsky, Vyatchanin}} \underbrace{C_{FTM}^2}_{\text{Liu \& Kip}}}$$


H
a
$r_0$

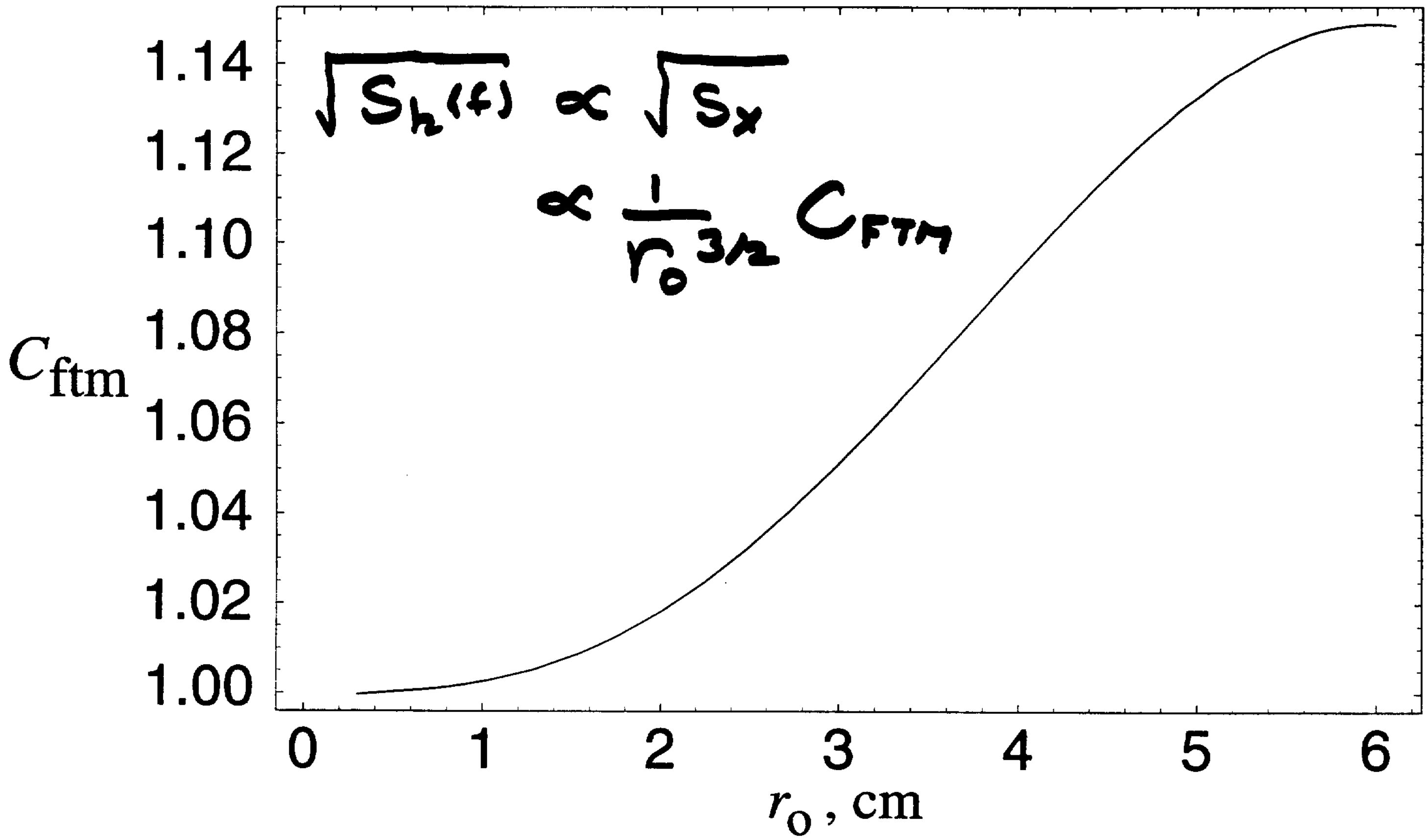

$C_{FTM}$ for Sapphire
2.0
1.5
1.2
1.1
1.03
1.01
1.0
0.99
0.97
0.90
$a/r_0$
$H/r_0$
2
3
4
5
6
7
8

4

Power $\sim e^{-r^2/r_0^2}$




Sapphire



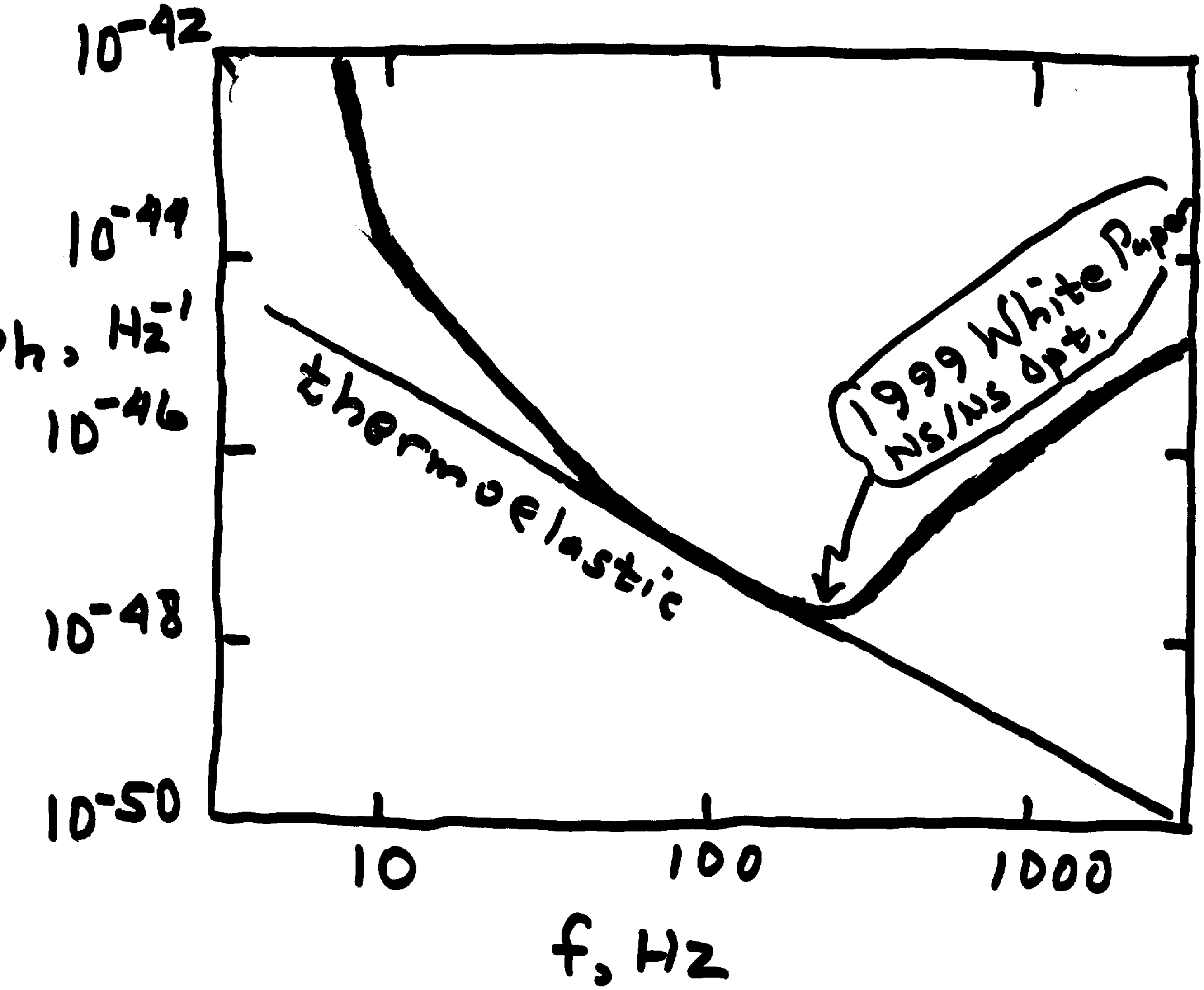

Want large $r_0$. - __But__:

Diffraction Losses $\lesssim \frac{10\,\text{ppm}}{\text{bounce}} \Rightarrow r_0 \lesssim 4\,\text{cm}$

$\left(\text{Radii of Curvature of Mirrors}\right) \lesssim 50\,\text{km} \Rightarrow r_0 \lesssim 4\,\text{cm}$

# III. IMPLICATIONS FOR LIGO-II

For $r_0 \simeq 4\,\text{cm}$ (almost certainly acceptable)




$\Rightarrow S_h$ doubled between 50 Hz & 250 Hz

Distance for NS/NS binaries reduced by $1/\sqrt{2}$ ... from 450 Mpc $\rightarrow$ 300 Mpc

CHALLENGE: HOW MUCH BETTER CAN WE DO BY OPTIMIZATION & NEW TRICKS?

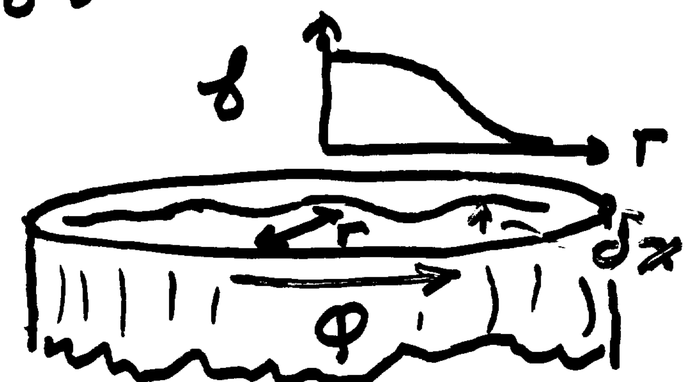
# V. Motivation for a New Trick:

## Levin's Direct Method to Compute Thermal Noise

(a variant of fluctuation-dissipation theorem)

- IFO measures test-mass generalized coordinate

$$g = \int f(r)\, \delta x(r,\varphi) \cdot r\, d\varphi\, dr$$




- For LIGO-II, as currently planned

$$f(r) = \underbrace{\frac{1}{\pi r_0^2} e^{-r^2/r_0^2}}_{\text{radial profile of light's power distribution}}$$

- Want to know $S_g(f)$ at some frequency $f$.

L'

- Thought Experiment:

Apply sinusoidal force to test-mass face

$$P = F_0 \underbrace{f(r)}_{} \cos(2\pi f t)$$

↑ pressure

↖ Same profile as appears in $g$

Compute (or measure) the power <u>dissipated</u> in the test mass, $W_{diss}$

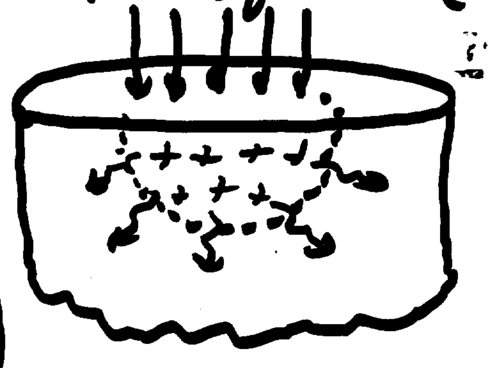
Then

$$\boxed{S_g(f) = \frac{8 k_B T\, W_{diss}}{F_0^2 (2\pi f)^2}}$$

- Classify Thermal Noises By Dissipation Mechanism &/or Location
  - Conventional thermal noise: (Saulson)
    - homogeneous dissipation – imaginary part of Young's modulus $S_g$ ~~[illegible]~~ $\propto r_0$
  - Surface noise (Levin)
    - dissipation in mirror coating & its attachment to substrate: $S_g \propto \frac{1}{r_0^2}$
  - Thermoelastic noise (Braginsky)
    - dissipation due to heat flow $S_g \propto \frac{1}{r_0^3}$

- Thermoelastic Dissipation:

$P = F_0 f(r) \cos(2\pi f t)$



$$\boxed{\begin{aligned} W_{diss} &= \left\langle T \frac{dS}{dt} \right\rangle \\ &= \left\langle \int \frac{\kappa_{th}}{T} (\nabla \delta T)^2 \, dvol \right\rangle \end{aligned}}$$

$\kappa_{th} \nabla T = (\text{heat flux}) = \frac{dQ}{dt}$

$\sim T \cdot \frac{dQ}{dt} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)$

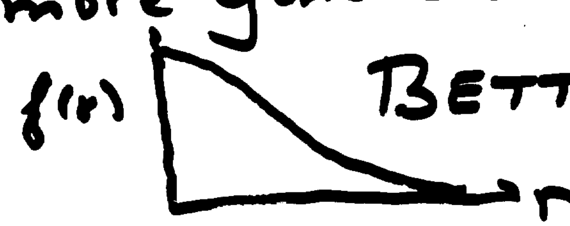
- Two Approximations in LIGO-II
  (may <u>not</u> be valid in expts to test thermoelastic noise!)

  - Adiabatic: $\tau = \frac{1}{f} \ll \tau_0 = \frac{r_0^2}{D}$

  - Quasistatic: $r_0 \ll C_s \cdot \tau$

- Foundation for "Trick":
  - Make $f(r)$ more gentle $\Rightarrow$ smaller $\nabla \delta T$

Gaussian: BAD


BETTER

# V. IMPLEMENTATIONS OF THE TRICK

- Actually reshape the laser beam
  - By adding some light in donut mode
  - By changing shape of mirrors (Hough)

- Expand beam so $r_0 \sim 4$-$5$ cm
  - $\Rightarrow$ in Fresnel diffraction regime
  - $\Rightarrow$ photons "conserve" their radii $r$
  - $\Rightarrow$ a photon has same radius $r$ on photodiode as on test mass

Read out photon flux $\frac{dN}{dt\,dA}(r)$ on photodiode.

In data analysis give photons a radius-dependent weight $w(r)$, so

$$f(r) = w(r) \cdot (\text{Intensity profile})$$

Optimize $w(r)$ to minimize Noise:
Thermoelastic + Shot + Radn Pressure

Optimize $w(r,f)$ ?

By How Much Can This Trick Reduce

$S_h^{T.E.}(f)$ ?

$\uparrow$ thermoelastic

$\boxed{\text{I do not know yet.}}$

<u>Guess</u>: some 10's of per cent

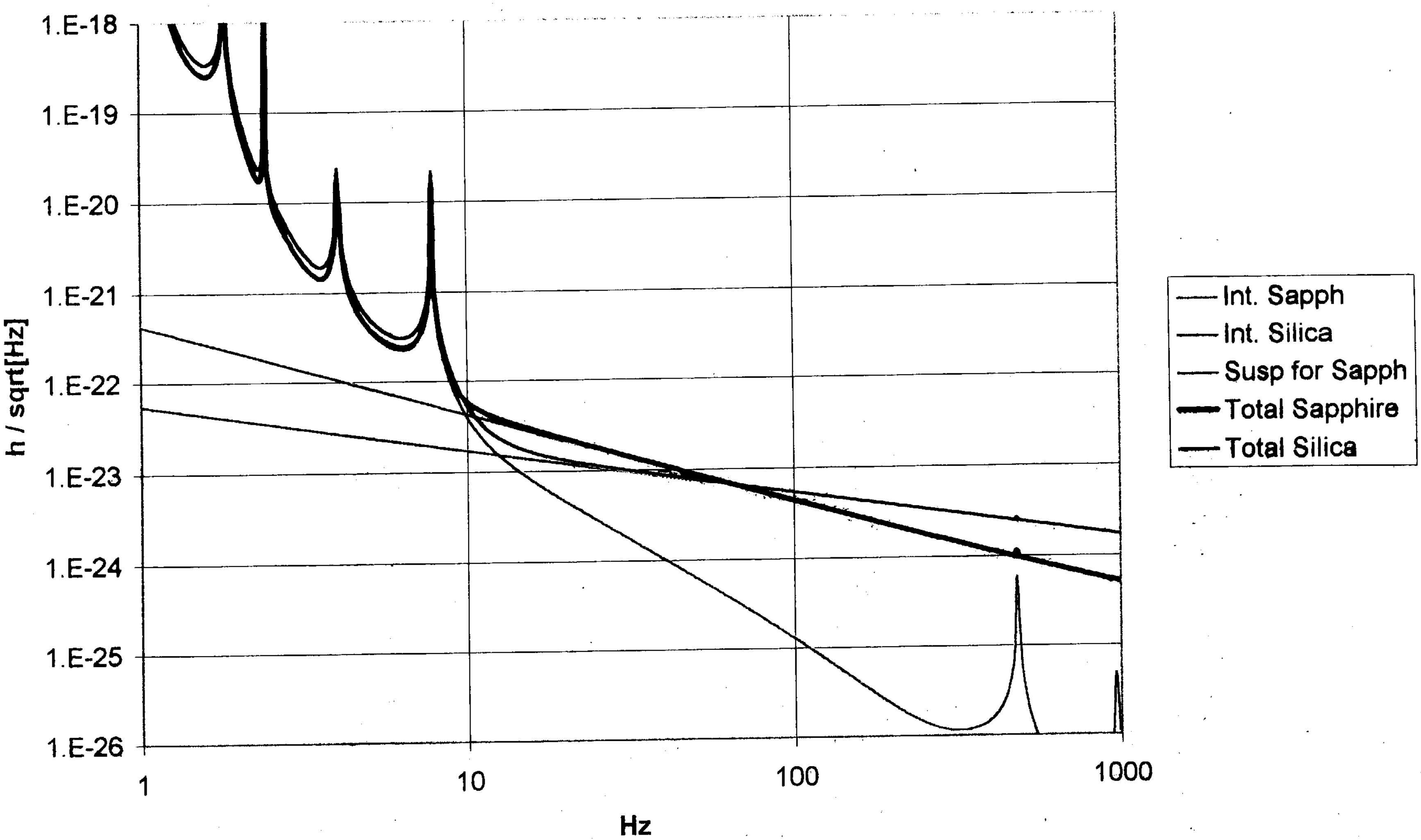
possibly a factor ~2.

$\Rightarrow$ Possibly a factor ~2 in event rate

if $S_h^{TE} \simeq (S_h)_{\text{shot + radn pressure}}$

Housh 1

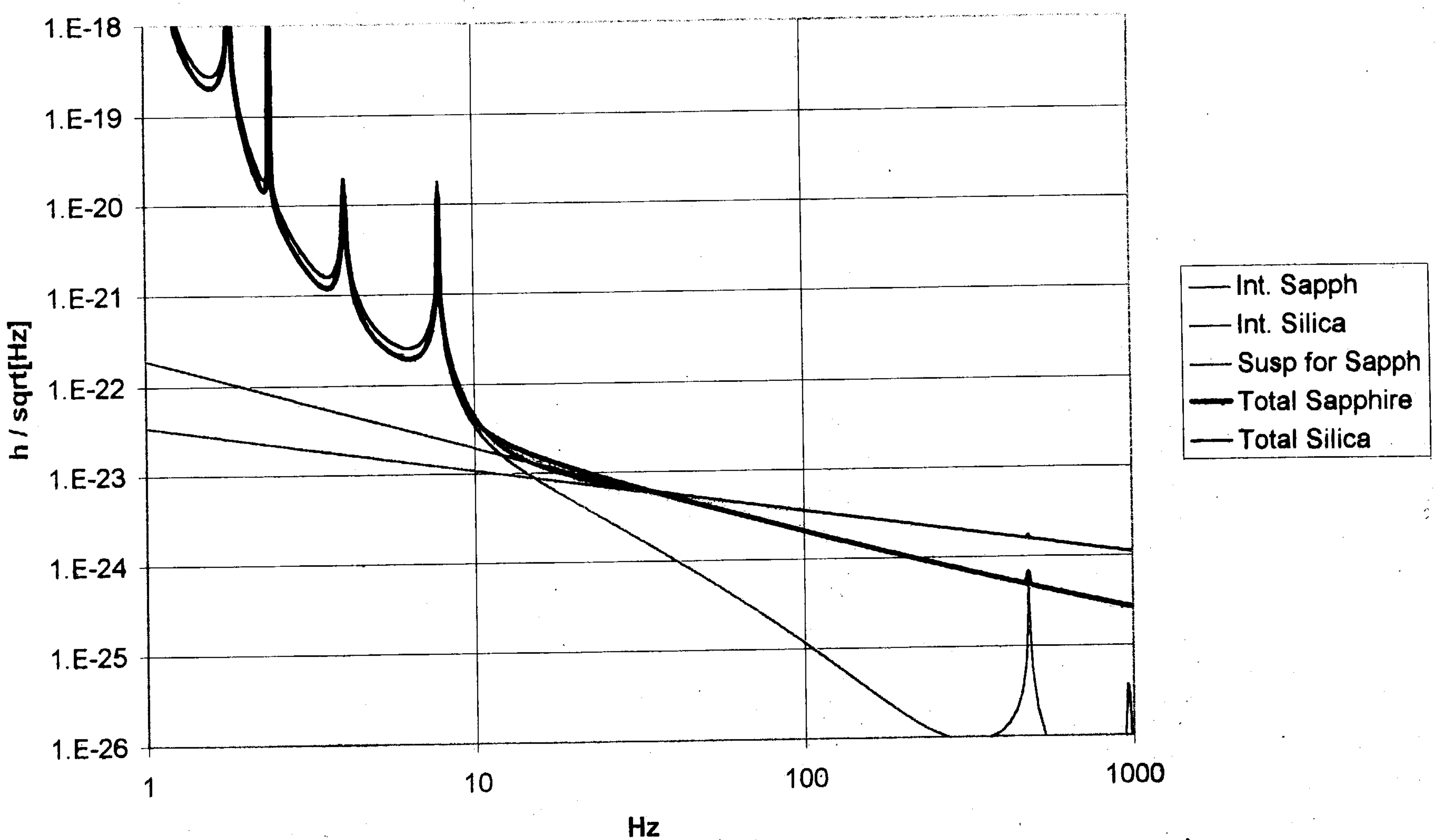
LIGO II - White Paper




G. Cagnoli

Hough-2

LIGO II - New Masses and Beam Radii (32cm diam x 13.7cm thick, 5cm beam




G. Cagnoli

Hough 7

# Impact on LIGO II sensitivity

| **Fused silica** test masses: | Beam radii (cm): | Range: |
|---|---|---|
| 28cm dia. x 12cm thick | 3.22, 2.57 | 120Mpc |
| 32cm x 13.7cm | 5, 5 | 160Mpc |

| **Sapphire** test masses: | | |
|---|---|---|
| 28cm dia. x 12cm thick | 3.22, 2.57 | 160Mpc |
| * 32cm x 13.7cm | 5, 5 | 260Mpc |

Previously, from Whitepaper

Sapphire test masses: 283Mpc

Thus for * source rate is ~ 77% of Whitepaper estimates

(K. Strain, Bench)

*Note 1, Linda Turner, 05/09/00 01:40:52 PM*

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