Laser Noise Coupling in Frontally Modulated RSE

James Mason

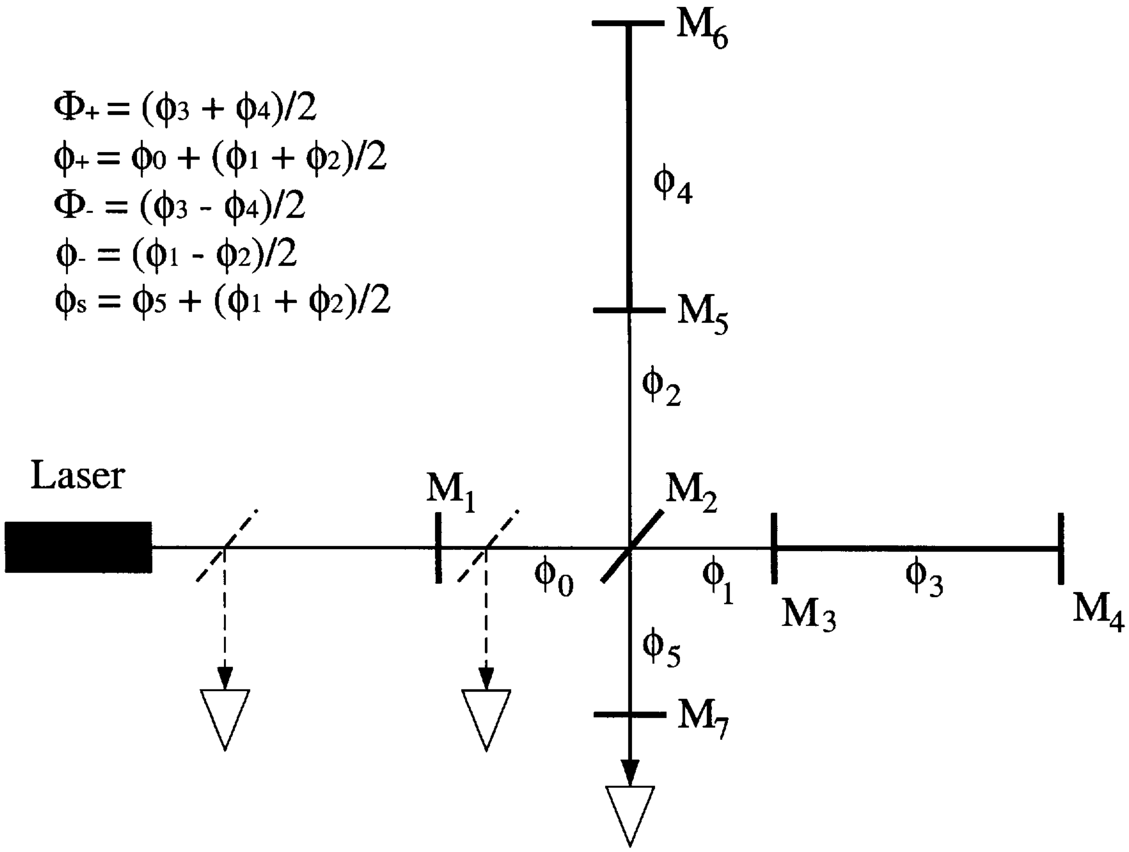
California Institute of Technology

LSC-LLO meeting

March 16-18, 2000

Resonant Sideband Extraction

$\Phi_+ = (\phi_3 + \phi_4)/2$
$\phi_+ = \phi_0 + (\phi_1 + \phi_2)/2$
$\Phi_- = (\phi_3 - \phi_4)/2$
$\phi_- = (\phi_1 - \phi_2)/2$
$\phi_s = \phi_5 + (\phi_1 + \phi_2)/2$



- Add'l mirror at dark port

  ››Allows tailoring of frequency response beyond simply setting the arm cavity pole.

  – **Detunable, increase bandwidth (decrease power recycling)**

  ››Mode cleaning at the dark port

GW Readout for RSE

- Broadband

››Same as initial LIGO.. 2 phase modulated sidebands acting as LO for GW signal in optical heterodyne signal extraction scheme.

- Detuned

››Signal cavity is tuned off-resonant from the carrier frequency.

– **This precludes the possibility that the 2 phase modulated sidebands can appear at the dark port with the same magnitude and phase.**

››For maximum signal strength, the parameters are adjusted such that one of the sidebands transmits efficiently.

– **The other is significantly attenuated.**

– **Optimum demodulation phase changes.**

››The upshot is that it is unclear how laser phase and amplitude noise will couple to the GW signal now.

– **It might be expected that some noise terms, which previously cancelled due to the balanced nature of the sidebands, might now appear.**

The calculation

- Analysis follows the approach taken by Camp, et al

  ››"Analysis of light noise sources in a recycled Michelson interferometer with Fabry-Perot arms", Camp, Yamamoto, Whitcomb, McClelland / J. Opt. Soc. Am. A / Vol 17 No. 1 / p.120

- Readout scheme is modeled

  ››Formula for optically heterodyned signal, with electronic demodulation and low-pass filtering.

- Noise is modeled

  ››Fourier components of noise terms derived

- Transmission of all frequency terms is derived

  ››9 terms : Carrier and 2 GW LO sidebands, each with two "audio noise" sidebands.

  ››Imperfections in optics and servo control built into the transmission function.

- The figure of merit

  ››Compare the noise signal at the dark port to the shot noise

  – **Specifying this in terms of strain or displacement sensitivity is somewhat complicated, due to the complicated structure of the GW signal transfer function. (But I'm flexible on this one)**

Readout

- Assuming incident fields on photodiode have the form

$$
\begin{aligned}
E_{PD} &= E_{0+1}e^{i\omega t} + E_{00} + E_{0-1}e^{-i\omega t} \\
&+ E_{+1+1}e^{i(\Omega+\omega)t} + E_{+1\,0}e^{i\Omega t} + E_{+1-1}e^{i(\Omega-\omega)t} \\
&+ E_{-1+1}e^{-i(\Omega-\omega)t} + E_{-1\,0}e^{-i\Omega t} + E_{-1-1}e^{-i(\Omega+\omega)t}.
\end{aligned}
$$

››The power at the demodulation frequency Ω is

$$
\begin{aligned}
|E_{PD}|^2{}_{@\Omega} = 2Re\{ & E_{+1+1}E^*_{00}e^{i(\Omega+\omega)t} + E^*_{+1-1}E_{00}e^{-i(\Omega-\omega)t} \\
&+ E_{-1+1}E^*_{00}e^{-i(\Omega-\omega)t} + E_{-1-1}{}^*E_{00}e^{i(\Omega+\omega)t} \\
&+ E^*_{+1\,0}E_{0+1}e^{-i(\Omega-\omega)t} + E_{+1\,0}E^*_{0-1}e^{i(\Omega+\omega)t} \\
&+ E^*_{-1\,0}E_{0+1}e^{i(\Omega+\omega)t} + E_{-1\,0}E^*_{0-1}e^{-i(\Omega-\omega)t}\}
\end{aligned}
$$

››Modeling electronic demodulation and low-pass filtering as

$$
V_{DC} = \frac{1}{T}\int_{t-T}^{t} |E_{PD}|^2 cos(\Omega t' - \delta)dt'
$$

››The corresponding signal is given by

$$
\begin{aligned}
V = Re\{[& (E_{+1+1}E^*_{00} + E_{-1-1}{}^*E_{00} + E_{+1\,0}E^*_{0-1} + E^*_{-1\,0}E_{0+1})e^{i\delta} + \\
& (E^*_{+1-1}E_{00} + E_{-1+1}E^*_{00} + E^*_{+1\,0}E_{0+1} + E_{-1\,0}E^*_{0-1})e^{-i\delta}]e^{i\omega t}\}
\end{aligned}
$$

Input noise

- Frequency noise

$$E_{laser} = E_i e^{i\left(\omega_0 t + \Gamma cos(\Omega t) + \frac{\delta\nu}{\omega} cos(\omega t)\right)}$$

- Amplitude noise

$$E_i \rightarrow E_i\left(1 + \frac{\delta E}{E_i} cos(\omega t)\right)$$
$$= E_i\left(1 + \frac{\delta E}{2E_i} e^{i\omega t} + \frac{\delta E}{2E_i} e^{-i\omega t}\right)$$

- Table of noise

	E_{--}	E_{-0}	E_{-+}	E_{0-}	E_{00}	E_{0+}	E_{+-}	E_{+0}	E_{++}
Laser $\delta\nu$	$\frac{-J_1\pi\delta\nu}{\omega}$	iJ_1	$\frac{-J_1\pi\delta\nu}{\omega}$	$i\frac{J_0\pi\delta\nu}{\omega}$	J_0	$i\frac{J_0\pi\delta\nu}{\omega}$	$\frac{-J_1\pi\delta\nu}{\omega}$	iJ_1	$\frac{-J_1\pi\delta\nu}{\omega}$
Laser amplitude	$i\frac{J_1\delta E}{2E}$	iJ_1	$i\frac{J_1\delta E}{2E}$	$\frac{J_0\delta E}{2E}$	J_0	$\frac{J_0\delta E}{2E}$	$i\frac{J_1\delta E}{2E}$	iJ_1	$i\frac{J_1\delta E}{2E}$

Table 1: Frequency spectrum of noise sources

Transmission

›› Amplitude transmission from input (PRM) to dark port photodiode

$$t = \frac{t_1 t_F t_7 e^{-i(\phi_+ + \phi_s)}}{1 + r_1 r_F e^{-i2\phi_+} - r_7 r_B e^{-i2\phi_s} - r_1 r_7 r_c^2 A_{bs}^2 e^{-i2(\phi_+ + \phi_s)}}$$

$$t_F \equiv A_{bs} r_c (\delta r_c cos(2\phi_-) + i sin(2\phi_-))$$

$$r_F \equiv r_c A_{bs} \{ cos(2\phi_-) + i(\Delta + \delta r_c) sin(2\phi_-) \}$$

$$r_B \equiv r_c A_{bs} \{ cos(2\phi_-) - i(\Delta - \delta r_c) sin(2\phi_-) \}$$

$$r_c = (r_{c2} + r_{c1})/2$$

$$\delta r_c = \frac{(r_{c2} - r_{c1})/2}{r_c}$$

$$A_{bs} = r_2^2 + t_2^2$$

$$\Delta = \frac{r_2^2 - t_2^2}{A}$$

››Note that frequency dependence is in the phases, as well as RMS offsets, and nominal values

$$\phi = \phi_{nominal} + d\phi_{RMS} + \frac{2\pi f \cdot l}{c}$$

››Frequency dependence dominated by arm cavity terms.

Carrier terms

- Inserting nominal values for the carrier (DC),

$$t = \frac{t_1 t_F t_7 e^{-i\phi_{dt}}}{\text{den}_p + \text{den}_{error}}(1 - i2(\theta_+ + \theta_s))$$

$$t_F = A_{bs} r_c(\delta r_c + i2\theta_-)$$

$$\text{den}_P = 1 + r_1 r_c A_{bs} - r_7 r_c A_{bs} e^{-i2\phi_{dt}} - r_1 r_7 r_c^2 A_{bs}^{\ 2} e^{-i2\phi_{dt}}$$

$$\text{den}_{error} = -i2 r_c A_{bs}(r_1\theta_+ - r_7\theta_s e^{-i2\phi_{dt}} - r_1 r_7 r_c A_{bs}(\theta_+ + \theta_s) e^{-i2\phi_{dt}})$$

››This reduces to the following transmission function

$$t_{0i} = t_1 A_{bs} r_{c0} t_7 G^c_{ifo} H(\omega)\left[\delta r_c(\omega) + i\left(\delta\Phi_- + 2\delta\phi_- + 2\frac{\omega\delta}{c}\right)\right]$$

$$G^c_{ifo} = \frac{e^{-i\phi_{dt}}}{1 + r_1 r_{c0} A_{bs} - r_7 r_{c0} A_{bs} e^{-i2\phi_{dt}} - r_1 r_7 r_{c0}^2 A_{bs}^{\ 2} e^{-i2\phi_{dt}}}$$

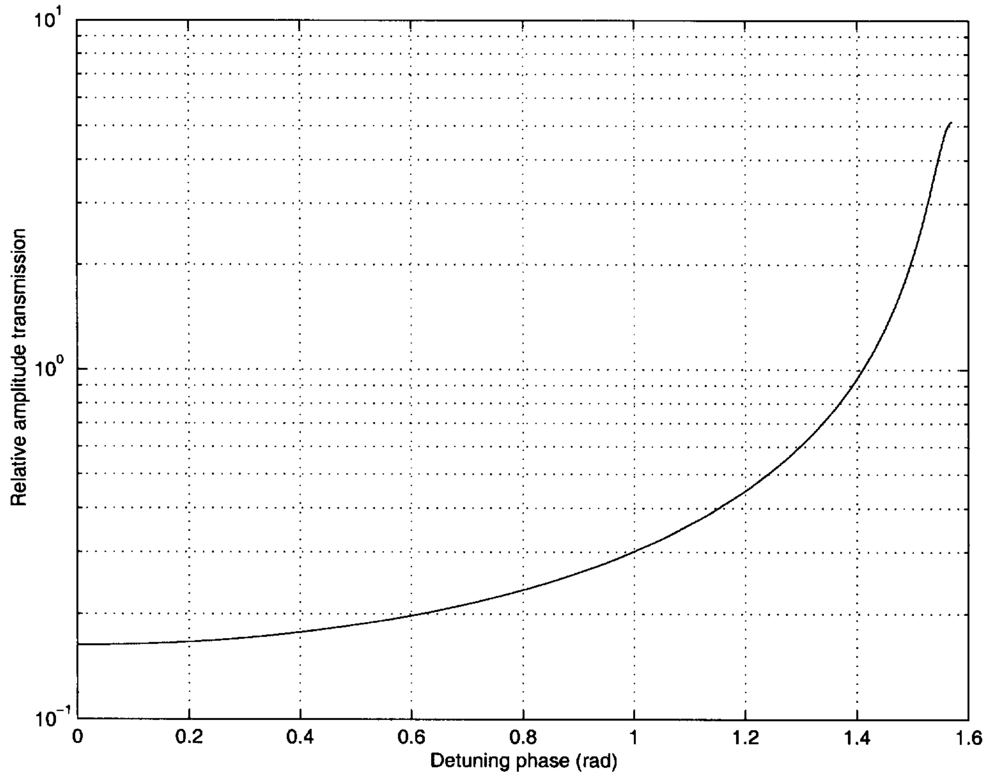
$$H(\omega) = \frac{\left(1 + i\frac{A_{itm}}{r_{c0}}\frac{\omega}{\omega_c}\right)\left(1 + i\frac{\omega}{\omega_c}\right)}{\left(1 + i\frac{\omega}{\omega_{cc}}\right)\left(1 + i\frac{\omega}{\omega_{sc}}\right)}$$

››I've recently discovered that the error part of the denominator may not be so negligible after all....

- **Overall effect is a gain term on the common mode cavity phases**

Carrier DC defect

- RSE compared to LIGO I

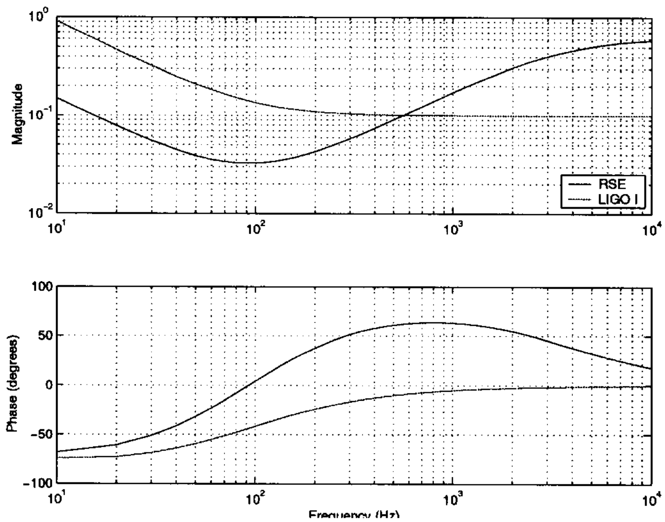


›› 2 factors in carrier defect suppression

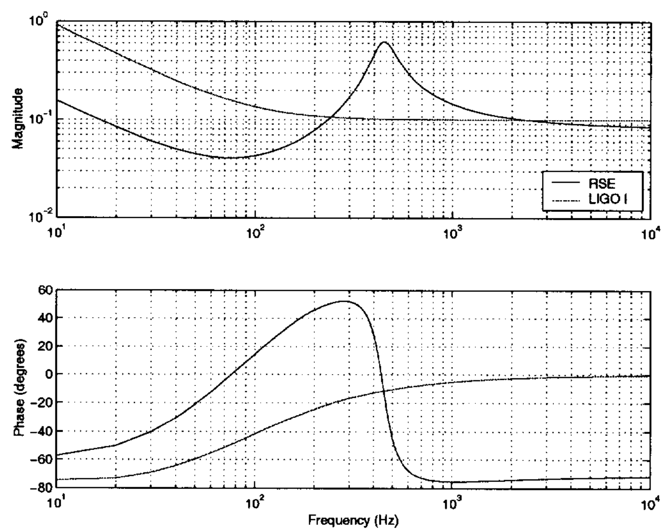
- **Transmittance of SEM (10% here)**
- **Anti-resonance of SEC (over-coupled arm cavities)**

Carrier noise terms

- Transfer functions
  - ››Broadband



  - ››Detuned



  - ››Increased carrier noise due to “signal extraction”

RF Sidebands

- Assumption made that arm cavities have unity reflectivity

$$t_{\pm i} = \pm t_1 A_{bs} t_7 G_{ifo}^{sb}\left(\pm sin\left(\frac{\Omega\delta}{c}\right) + d\phi_- \pm \frac{\omega\delta}{c} \pm i\,sin\left(\frac{\Omega\delta}{c}\right)\left(d\phi_+ + d\phi_s \pm \frac{\omega(l_+ + l_s)}{c}\right)\right)$$

$$G_{ifo}^{sb} = \frac{1}{1 - r_1 A\,cos\left(\frac{\Omega\delta}{c}\right) - r_7 A\,cos\left(\frac{\Omega\delta}{c}\right) + r_1 r_7 A^2}$$

››Sidebands and their noise terms, like LIGO I, do not experience any significant cavity filtering

››Again, it's (even more) likely that the denominator error terms will modify these formulas

  - **Predominant effect is to add a gain term to the cavity phases.**

- Modeling of the RF sideband transmission requires some knowledge about the specifics of the readout scheme

››IN PROGRESS

Left to do :

- Finish work on RF sideband transmission
- Complete the full noise calculation

››Preliminary expressions for laser frequency and amplitude noise couplings have been calculated for broadband RSE, and show similar expressions to those derived by Camp, et al.

– **Laser frequency noise : arm cavity mismatch**

$$|V_{lf}| = 4E_i^2 J_0 J_1 \frac{\pi\delta\nu}{\omega} T_1 T_7 G_{ifo}^c G_{ifo}^{sb} r_{c0} \sin\left(\frac{\Omega\delta}{c}\right)\delta |r_c|_0$$

– **Laser amplitude noise : phase errors in differential modes**

$$|V_{li}| = 4E_i^2 J_0 J_1 \frac{\delta E}{E_i} T_1 T_7 r_{c0} G_{ifo}^c G_{ifo}^{sb} \sin\left(\frac{\Omega\delta}{c}\right)(d\Phi_{c-} + d\phi_-)$$

– **Disclaimer : These results are PRELIMINARY and haven't undergone enough scrutiny to be believed yet.**

- "twiddle" can be modified to derive noise couplings exactly

››As a matter of fact, it already has been (but not "officially")

››twiddle seems to predict that the amplitude noise coupling has a second contribution, similar in magnitude, which couples carrier noise terms via a product of the arm cavity mismatch and the common mode phase errors.

Note 1, Linda Turner, 05/17/00 10:24:05 AM
LIGO-G000141-00-D