Hough Hierarchical Pulsar Search

Alicia M. Sintes
sintes@aei-potsdam.mpg.de

Albert Einstein Institut
Golm, Germany
http://www.aei-potsdam.mpg.de

Hanford LSC meeting. August 15-17, 2000

LIGO-G000287-00-Z

Hough transform

Input: Set of points in a time-frequency plane
(from DeFT according to a sky location & spin-down.)

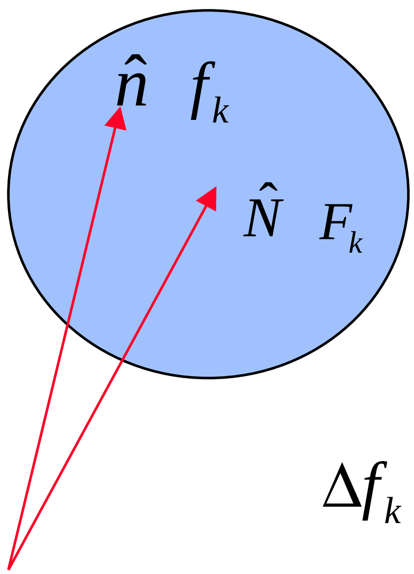
- A source located at the center of the patch with the same spin-down parameters will appear as a set of points forming an horizontal line at f_0.
- Due to the mismatch, the points will appear in patterns following the so called: *Hough transform master equation*.

Output: Histograms in parameter space

For every point in the t-f plane, one enhance the number count in the histogram in the pixels that are consistent.

Significant clustering in a pixel in parameter space indicates suspect consistency of data with a signal from a source with those parameters.

The Hough transform master equation



$$\nu - F_0 = \vec{\xi}(t)\cdot(\hat{n} - \hat{N})$$

$$F_0 \equiv f_0 + \sum_k \Delta f_k \left[T_{\hat{N}}(t) - T_{\hat{N}}(\hat{t}_0)\right]^k$$

ν Measured frequency of the signal at time t

f_0 Intrinsic frequency of the signal

$\Delta f_k \equiv f_k - F_k$ Residual spin-down parameter

$$T_{\hat{N}}(t) = t + \frac{\vec{x}(t)\cdot\hat{N}}{c} + \Lambda$$

Time at the solar system barycentre for a given sky position

$$\vec{\xi}(t) = \left(F_0 + \sum_k F_k \left[T_{\hat{N}}(t) - T_{\hat{N}}(\hat{t}_0)\right]^k\right)\frac{\vec{v}(t)}{c} + \left(\sum_k k F_k \left[T_{\hat{N}}(t) - T_{\hat{N}}(\hat{t}_0)\right]^{k-1}\right)\frac{\vec{x}(t) - \vec{x}(\hat{t}_0)}{c}$$

The issue:

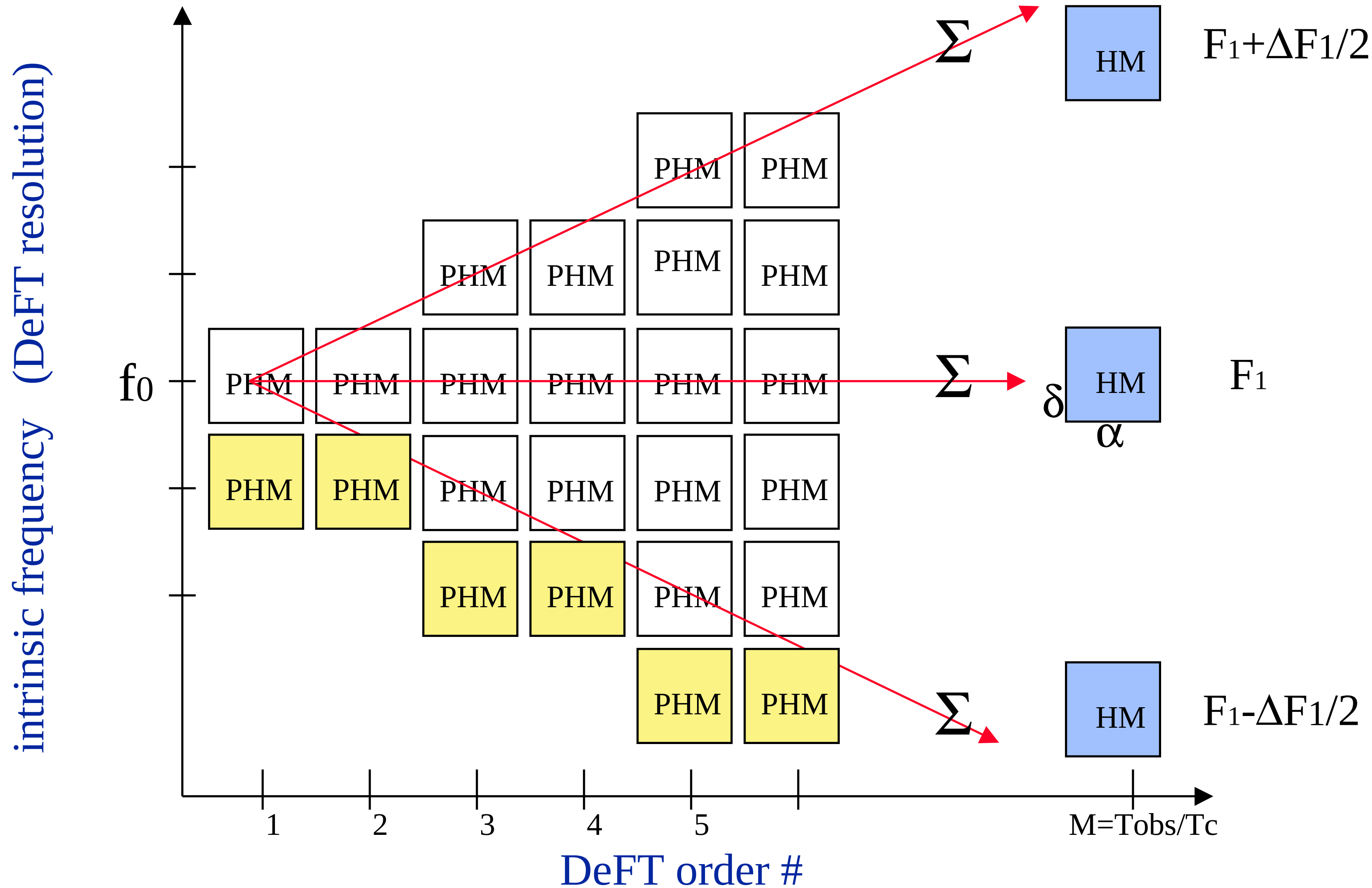
Build histograms, i.e., a Hough map (HM), in the parameter space: for each intrinsic frequency f_0, residual spin-down parameter Δf_k and refined sky location (inside the patch).

Note:

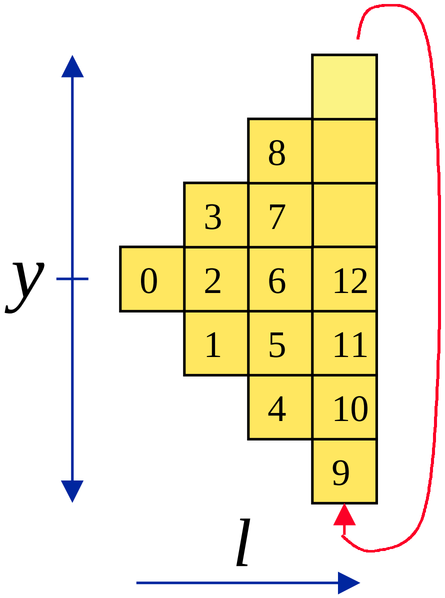
- The residual spin-down parameter produces only a change in F_0 and, at any given time, F_0 can be considered constant.
- The HM is a histogram, thus additive. It can be seen as the sum of several ***partial Hough maps (PHM)*** constructed using just one periodogram.

Can construct the HM for any f_0 and spin-down value by adding together at different times PHM corresponding to different F_0.

The cone of PHM



Sum of partial Hough maps



$$l \in [0, K, LMAX] \qquad y \in [-l, K, 0, K, l]$$

$$q(l, y) = l^2 + l + y$$

(index in a circular buffer)

Number of PHM (Σ_2) = $(LMAX + 1)^2$

Number of (Σ) HM = $2\,LMAX + 1$

Loop in spin-down `for(k=0;k<2*LMAX+1;++k){`

Copy PHM[0] into HM

Sum PHMs `for(l=1;l<=LMAX;++1) {`

Calculate `q(l,y)`

Add PMH[q] to HM `}}`

The partial Hough map

$$\nu - F_0 + \vec{\xi} \cdot \hat{N} = |\vec{\xi}| \cos\phi \quad \Rightarrow \quad (\nu, F_0) \rightarrow circle \begin{cases} \text{center pointed by } \vec{\xi}(t) \\ \phi = angle(\hat{n}, \vec{\xi}(t)) \end{cases}$$

discrete case: $[\nu_i - \delta f/2, \nu_i + \delta f/2]$ ⇒ annulus $\Delta\phi$

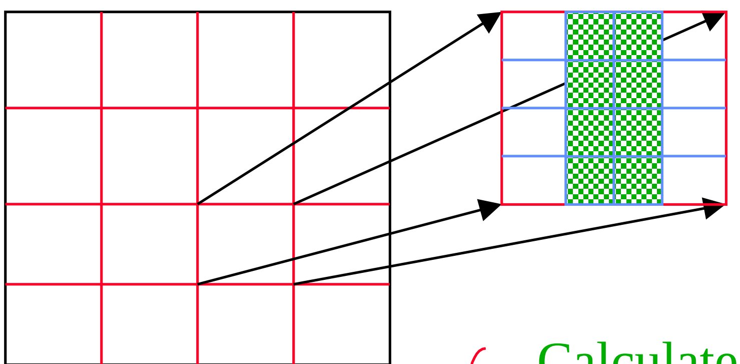
Set: $\Delta\phi_{pixel} \approx \frac{\Delta\phi_{min}}{2}$

$$\langle \Delta\phi \rangle \cong \begin{cases} \pi/2\, \Delta\phi_{min} & \text{all sky} \\ 5\, \Delta\phi_{min} & \text{near equator} \end{cases}$$

Mapping annuli on a discrete space:

- Algorithm optimized for thin annuli
- Annuli with uniform probability distribution (not weighted) to avoid discretization errors
- Biunivocal mapping to avoid border effects

The zooming algorithm



We study the whole patch, two zooming levels and the pixel size.

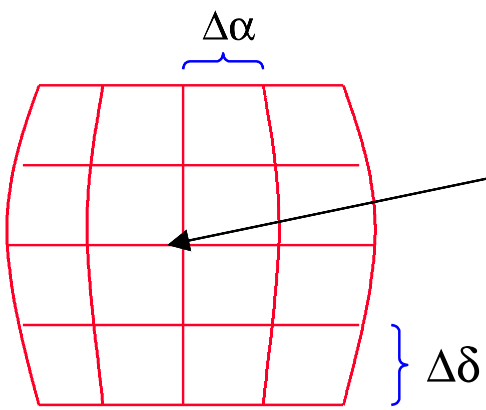
(sub)-patch level

- Calculate $\cos\phi_{min}$, $\cos\phi_{max}$, f_{min}, f_{max}, bin ν_{min}, ν_{max}.
- $\nu_{max} = \nu_{min}$,
  - no peak ⇒ no annuli intersection
  - peak ⇒ whole patch in the same annulus; enhance +1 all pixels
- $\nu_{max} > \nu_{min}$, count peaks:
  - $=0$ ⇒ no intersection
  - >0 ⇒ zoom

pixel size

- for the center of the pixel, calculate $\cos\phi$, f, ν_i.
- If there is a peak at ν_i, enhance the number count

Tiling the sky to produce efficiently PHM



- change of coordinates

$$\hat{N} = (1,0,0) \Rightarrow \alpha=0, \delta=0$$

- take parallels and meridians with constant

$$\Delta\alpha, \Delta\delta$$

Consequences:

- avoids distortions: the pixel size is almost constant independently of the sky location
- makes the zooming algorithm simpler
- patches do intersect

Performance

Hardware: DEC ALPHA 21264 , 667 MHz, 512 RAM

Compilation options:
```
-fast –tune host –speculate all
-assume restricted_pointers  –g3
```

Patch dimension: 256x256 pixels, 120 different periodograms

Bottle neck: Sum of PHM

not the construction of PHM