LIGO I: Bounding the Possible

Lee Samuel Finn

Center for Gravitational Physics and Geometry, Penn State University

Bounding the Possible: Upper Limits

- Source strength
  - » Stochastic signal
  - » Pulsars & all-sky, all frequency CW search
- Population
  - » Limit average wave strength over population: *e.g.*, γ-ray bursts, supernovae
- Event rates
  - » Unanticipated burst sources
    - – Waveform unknown
  - » Anticipated burst sources
    - – Waveform known, unknown
- Outline
  - » Confidence intervals, upper limits & credible sets
  - » Stochastic Signals
  - » Pulsars
  - » Burst sources
- Assumptions
  - » Best estimate $h(t)$ has all identifiable instrumental signatures removed or vetoed
    - – Relaxing assumption *ad hoc* until actual data is available: each can make their own from estimates here

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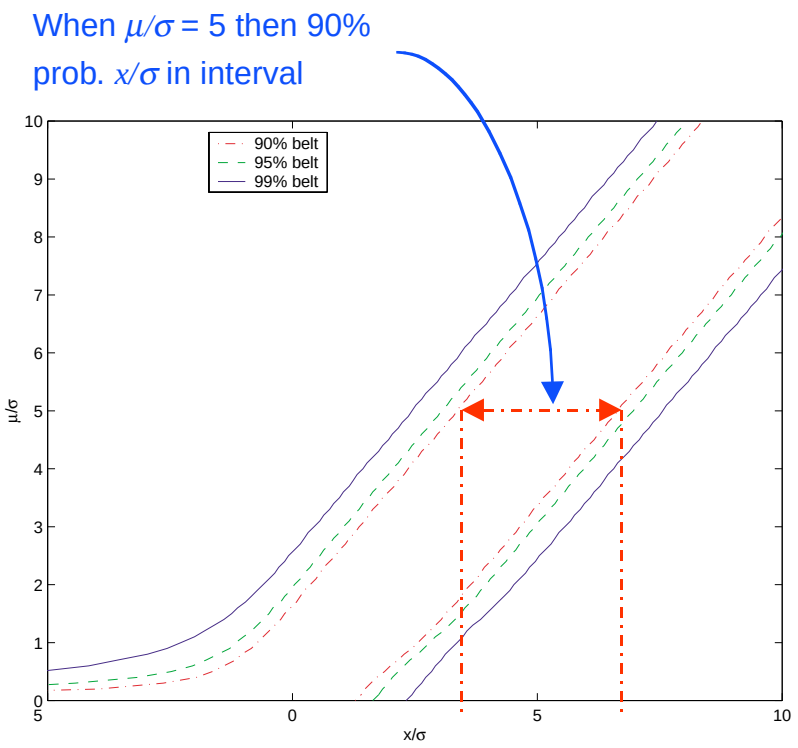
Confidence Intervals and Upper Limits

- Meaning
  - » Range of hypotheses μ for which prob. of observation x is p
- Choices
  - » Defining range: *e.g.*,
    - – Smallest interval: $P(x|\mu) > K$
    - – Lower (upper) limit: $P(x<x_0|\mu)$ $[or\ P(x>x_0|\mu)]$
    - – "Unified": $P(x|\mu)/P(x|\mu_{best}) > K$
  - » Different choices, different intervals
- Confidence Belt
  - » Range of probable observations as function of hypothesis



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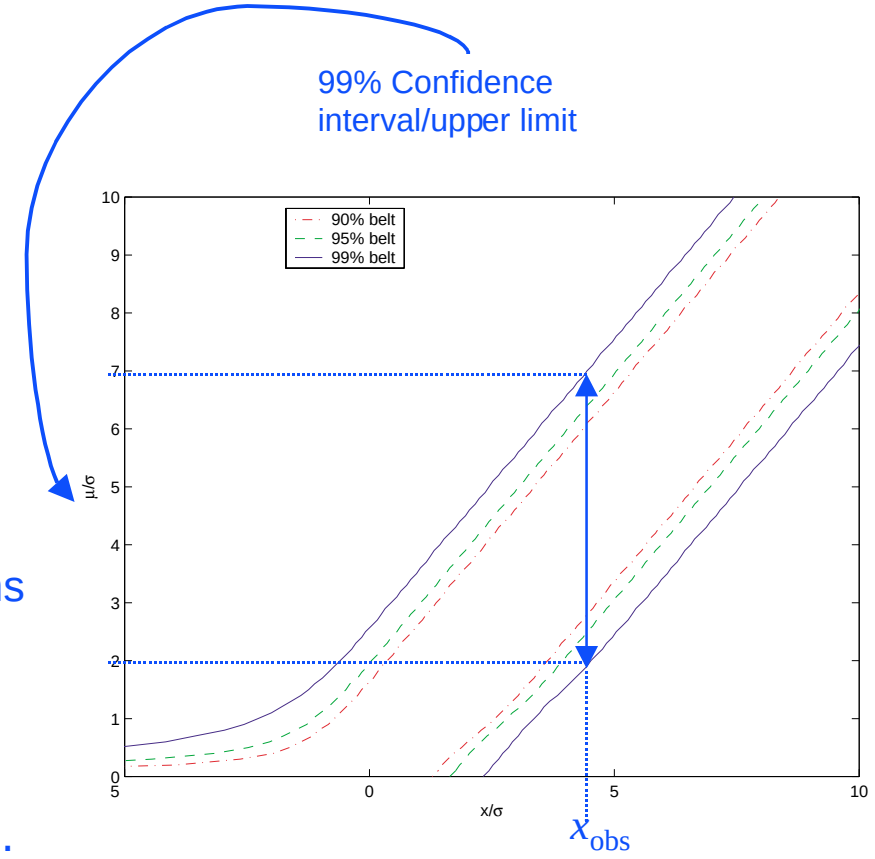
Confidence Intervals and Upper Limits

- Meaning
  - » Range of hypotheses θ for which prob. of observed x is p
- Choices
  - » Defining range
  - » Different choices, different intervals
- Confidence Belt
  - » Range of probable observations as function of hypothesis
- Interval
  - » Range μ for which x_{obs} likely
- Ref.: Feldman & Cousins, Phys. Rev. D., **57**:7, 3873 (1998)



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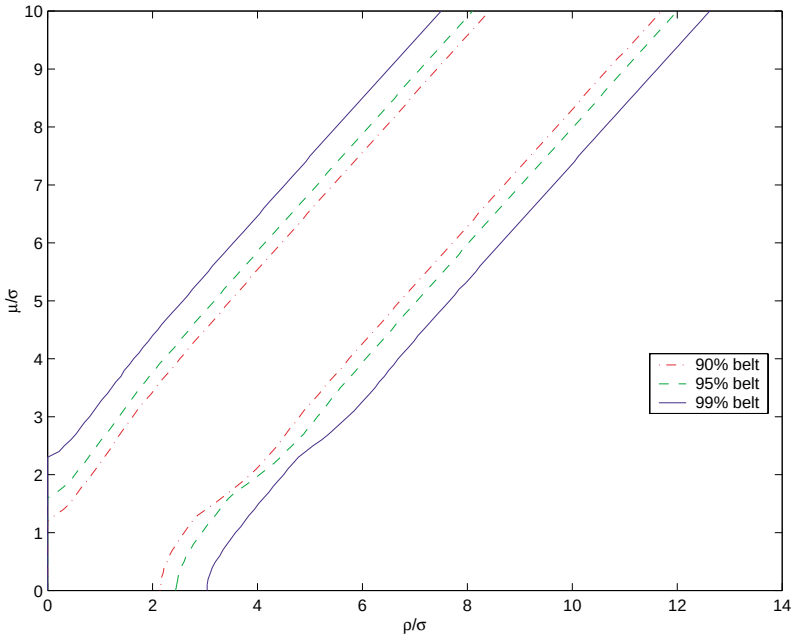
GW From Pulsars: Observational Limits, Theoretical Prejudices

- Wave amplitude (rms & angle-averaged):
  - » $h_{rms} = 10^{-26}\left(\frac{10\ \mathrm{Kpc}}{r}\right)\left(\frac{\varepsilon}{10^{-6}}\right)\left(\frac{I}{10^{45}\ \mathrm{g\ cm^{-3}}}\right)\left(\frac{f}{100\ \mathrm{Hz}}\right)^2$
- Observational Limits
  - » Pulsar timing of spindown shows dominant spin dissipation mechanism can't be GW
    - – $L \sim f^n$; $n_{\mathrm{obs}} \sim 4-5$, $n_{\mathrm{GW}} > 6$
  - » GW luminosity must be much less than implied by spindown
    - – $\varepsilon < 10^{-8}$ for ms (recycled) pulsars
- Theoretical Prejudices
  - » Radiation associated with non-axisymmetric deformation of crust; crust material strength limits ε less than several 10^{-6}

GW From Pulsars: Setting Upper Limits

- Beat output against known phase
  - » $\rho = \left| \int_0^T dt\, h(t) e^{-i\Phi(t)} \right|$
- Statistic: $|\rho|$
  - » Sampling distribution: Rice
    - – $P(\rho \mid \mu) = \frac{\rho}{\sigma^2} \exp\left[-\frac{\rho^2 + \mu^2}{2\sigma^2} \right] \mathrm{I}_0\left(\frac{\rho\mu}{\sigma^2} \right)$
    - – Variance σ^2: ms noise in band $1/T$ about f_{GW}
- For 1 year integrated observation …
  - » LIGO I sets limit $\varepsilon_{99\%} \sim 2\text{x}10^{-6}$ on 50 Hz pulsar at 10 Kpc



Unanticipated Transients: External Trigger

- GW Bursts correlated with other observational channels (*e.g.*, EM, neutrinos)
  - » Think SN, γ-ray bursts
- Test statistic: integrated cross-correlation of GW detectors
  - » $c = \iint dt\, dt'\, h_1(t) h_2(t') Q(t - t')$
  - » About, , in direction of, burst: on-source
  - » Other times: off-source
  - » Compare population means
    - – t statistic
  - » Limit on *in band population mean* h^2
    - – $h < 10^{-21}$ (in band; 99%) with 1000 observed γ-ray events and broadband GW-burst model

Unanticipated Transients: GW-Only

- New feature: internal event trigger
  - » Trigger distinguishes on, off source
  - » Possible triggers:
    - – "Energy" threshold
    - – Event classification (energy spectrum, time-frequency power distribution, etc.)
- Correlate between detectors on, off source
  - » $c = \iint dt\, dt'\, h_1(t) h_2(t') Q(t - t')$
  - » Use t-statistic to compare on, off source populations
- Weaker limit than with astrophysical trigger
  - » Unknown source direction, trigger efficiency weakens limit
  - » Background exclusion becomes critical

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Summary

- LIGO I will set physically significant upper limits
  - » Upper limits all improve with observation time
  - » LIGO engineeering run starts the ball rolling!
- Pulsars
  - » $\varepsilon_{99\%} \sim 10^{-4}$ in one week run at 1/10 sensitivity!
  - » Test theoretical prejudice based on crust strength upper limit
- Unanticipated GW bursts
  - » Model dependent, but $h_{UL} \sim 10^{-21}$ improves bar limits by factor 100 over broader band
- Supernovae
  - » Opportunistic constraint on SN efficiency
  - » Luck favors the prepared mind …

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