Proposal
Thermal and Thermoelastic Noise Research for Advanced LIGO Optics

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Content

- Objective and Approach
- Output to Date
  – Laser phase noise formulas for optical resonator and delay line
  – Coating noise estimation
- On-Going Activities
  – Coating noise estimation, II
  – Numerical Green's function calculation
- Work Statement
  – Coating noise studies
  – Circular membrane model
  – Cylindrical mirrors and other shapes
  – Studies of other possible noise sources
- Tasks & Time Lines
- Broader Impacts
- Summary

Objective and Approach

Objectives

- To advance the computation method for two-point phase-noise correlations to complex test mass objects.
- To estimate thermal noises for advanced LIGO optics designs.
- To prove a possible advantage of delay lines over resonators for realistic LIGO mirror geometry.
- To extend the two-point phase noise correlation formulas to include thermo-elastic noise.

Approaches

- Use the developed formula to compute two-point phase noise correlations.
- Use both analytically soluble and realistic mirror models.
- Use, for coating noise estimation, half-space mirror model with a lossy layer in a lossy host material
- Demonstrate the delay-line attribute by
  - Analytical membrane model
  - Numerical computation for cylindrical mirror model

Output to Date

- Prior NSF Support
  - PHY-9800976
  - 8/15/1998 – 7/31/ 2001
  - “Numerical Green’s Function Computation and its Application to Thermal Noise Estimation for Laser Interferometric Gravitational Wave Receivers”
- Paper I
  - N. Nakagawa, Eric Gustafson, P. Beyersdorf and M. M. Fejer “Estimating the off resonance thermal noise in mirrors, Fabry-Perot interferometers and delay-lines: the half infinite mirror with uniform loss”
- Paper II
  - N. Nakagawa, A. M. Gretarsson, E.K. Gustafson, and M. M. Fejer, “Thermal noise in half infinite mirrors with non-uniform loss: a slab of excess loss in a half infinite mirror.”

Output to Date

- Phase noise formulas for resonator and delay line
  - Developed formulas for two-point phase-noise correlations.
  - Noise formulas given for
    - single-reflection mirror
    - Fabry-Perot interferometer
    - delay-line interferometer
  - Shown that delay lines are potentially quieter than resonators
    - ← the spatial de-correlation.
    - Based on the half-space mirror model.

$$S_\varphi(\omega) \equiv \int_{-\infty}^{\infty} dt e^{i\omega t} \langle \delta\varphi(t) \delta\varphi(0) \rangle$$

$$S_\varphi(\omega, \vec{r}_1, \vec{r}_2) = 4k^2 \iint dS' \int dS'' \psi_{00}^w(\vec{r}\,' - \vec{r}_1) \psi_{00}^w(\vec{r}\,'' - \vec{r}_2) \langle u_n(\vec{r}\,') u_n(\vec{r}\,'') \rangle_\omega$$

$$\approx 4k^2 \frac{2k_B T}{\omega} \iint dS' \int dS'' \psi_{00}^w(\vec{r}\,' - \vec{r}_1) \psi_{00}^w(\vec{r}\,'' - \vec{r}_2)$$

$$\times \int_V d^3x \left[\partial_k \chi_{li}^\omega(\vec{x}, \vec{r}\,'; c') \right] c''_{klpq} \left[\partial_p \chi_{qj}^\omega(\vec{x}, \vec{r}\,''; c') \right]$$

$$c_{ijkl} = c'_{ijkl} - c''_{ijkl} \approx [1 - \phi(\omega)] c'_{ijkl}$$

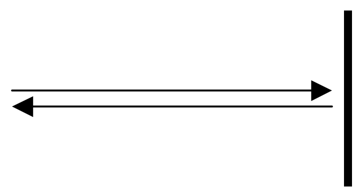
$S_\varphi(\omega)$	the laser-beam phase-noise power-spectrum correlation
$S_\varphi(\omega, \vec{r}_1, \vec{r}_2)$	the two-point phase-noise power-spectrum correlation
$\langle u_i(\vec{r}\,') u_j(\vec{r}\,'') \rangle_\omega$	the spectral displacement correlation
$\vec{r}_1, \vec{r}_2$	The laser beam reflection points (the beam centers)
χ_{ij}^ω	elastic Green's function
c_{ijkl} [c'_{ijkl}, c''_{ijkl}]	elastic constants [dispersive and absorptive parts]
$\phi(\omega)$	loss function
k_B , T	Boltzmann constant, and temperature
$\psi_{00}^w(\vec{r}) \left(\propto e^{-2\|\vec{r}\|^2 / w^2} \right)$	the Gaussian laser-beam profile function
w, k	the laser beam spot size (amplitude radius), and wave number

Output to Date

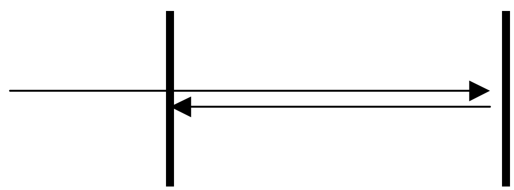
- Phase noise formulas

  Computed explicitly for

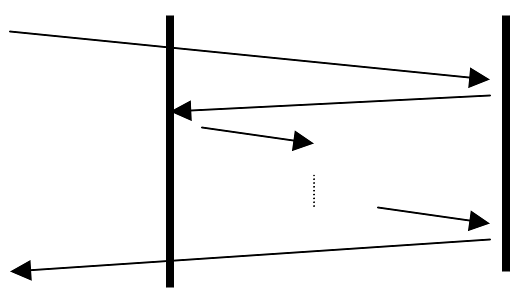
  – Single-reflection mirror



  – Fabry-Perot resonator



  – Optical delay line



$$S_\varphi^{Single}(\omega) = S_\varphi(\omega, \vec{r}, \vec{r})$$

$$S_\varphi^{FP}(\omega) = \left[\frac{(1+r_I)^2}{1+r_I^2}\right]\left[1 - \frac{2r_I}{1+r_I^2}\cos 2\omega\tau\right]^{-1}\left[S_\varphi^E(\omega) + r_I^2 S_\varphi^I(\omega)\right]$$

$$S_\varphi^{DL}(\omega) = \sum_{n=1}^{N} S_\varphi^E(\omega, \vec{r}_n, \vec{r}_n) + 2\sum_{n=2}^{N}\sum_{q=1}^{n-1}\cos[2(n-q)\tau\omega]\cdot S_\varphi^E(\omega, \vec{r}_n, \vec{r}_q)$$
$$+\sum_{n=1}^{N-1} S_\varphi^I(\omega, \vec{\rho}_n, \vec{\rho}_n) + 2\sum_{n=2}^{N-1}\sum_{q=1}^{n-1}\cos[2(n-q)\tau\omega]\cdot S_\varphi^I(\omega, \vec{\rho}_n, \vec{\rho}_q)$$

r_I	the input mirror reflection coefficient
τ	the transit time
$S_\varphi^E(\omega),\quad S_\varphi^I(\omega)$	the single-reflection phase noises of the input and end-point mirrors.
$\vec{r}_n$	the positions of the N-time reflections on the end-mirror surface
$\vec{\rho}_p$	the positions of the (N-1)-time reflections on the input-mirror surface
E, σ	Young's modulus, and Poisson ratio

Output to Date

- Fabry-Perot vs Delay lines
  - Analytical half-space mirror model
  - Fabry-Perot interferometer vs several delay lines
  - Storage time as proposed for LIGO II.
  - Delay line beam centers
    - evenly spaced
    - on a circle
- When the spots are not overlapping appreciably, the delay line is less noisy than the Fabry-Perot.
  - Noise levels are similar if
    - the spot circle radii comparable to the beam spot size
    - the spots are largely overlapping, and above several hundred Hertz.

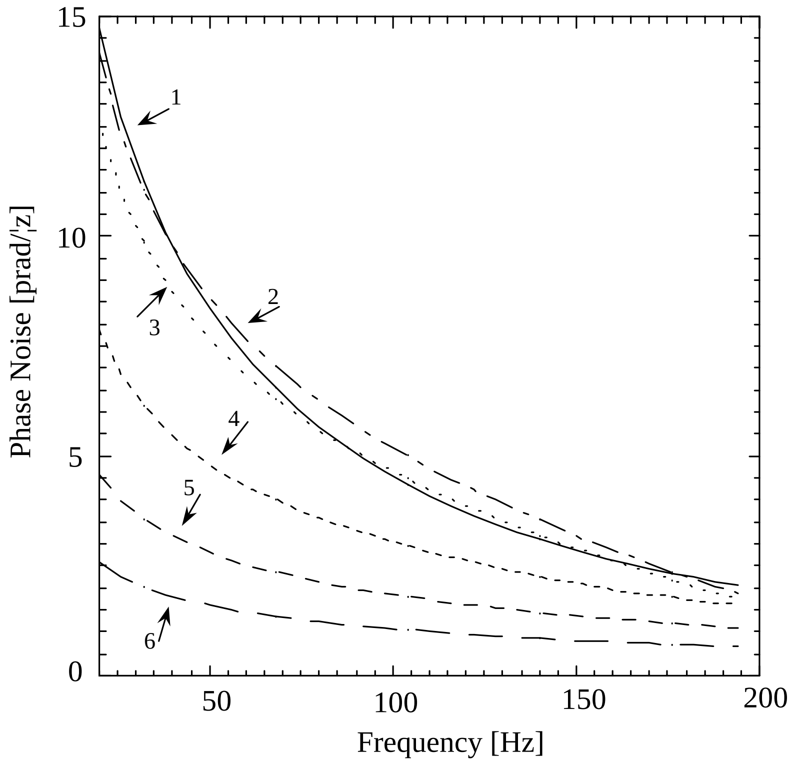


Figure 1: Comparison of the phase noise from a delay-line and a Fabry-Perot interferometer. The solid curve (1) is for a 4 km Fabry-Perot interferometer with power reflection coefficient $R_I=0.97$. The curves for delay lines all have 130 spots on the end mirror. The lower curves corresponds to spot circles of circumference (6) 130w, (5) 65w, (4)

Output to Date

- Coating noise estimation
  - an analytical model; half-infinite mirror with a coating layer
  - the layer material has excess loss compared to the substrate
  - Uniform elastic property
- Findings
  - For a thin lossy coating layer, the excess noise scales as the ratio of the coating loss to the substrate loss and as the ratio of the coating thickness to the laser beam spot size.
  - For a silica substrate with a loss function of 3×10^{-8}, the coating loss must be less than 3×10^{-5} for a 6 cm spot size and a 7 μm thick coating to avoid increasing the spectral density of displacement noise by more than 10%.

$$S_\varphi^{Total}(\omega) = S_\varphi^{Single}(f, \phi_{Substrate})\left\{1 + \frac{2}{\sqrt{\pi}} \frac{1-2\sigma}{1-\sigma} \frac{\phi_{coating}}{\phi_{substrate}} \left(\frac{d}{w}\right)\right\}$$

d	the coating layer thickness
w	the beam spot size (amplitude radius)
σ	the Poisson ratio (of the layer and substrate)
$\phi_{coating}$	the loss function of the coating
$\phi_{substrate}$	the loss function of the substrate
$S_\varphi^{Total}(\omega)$	the single-reflection noise spectral density for the coated mirror
$S_\varphi^{Single}(f, \phi_{Substrate})$	the single-reflection noise spectral density for the substrate mirror without any coating.

On-Going Activities

- Coating noise estimation, II
  - Elastic property mismatch
    - between the layer and substrate materials.
  - Preliminary conclusion
    - A significant correction for a large elastic disparity

$$\frac{\bar{E}}{E} \sim \frac{d}{w} \text{ or } \frac{w}{d}$$

    - Ex. the quoted parameters (spot size *w* = 6cm and coating thickness *d*=7μm)

PRELIMINARY

$$S_\varphi^{Total}(\omega) = S_\varphi^{Single}(\omega,\phi) \times \left\{ 1 + \frac{2}{\sqrt{\pi}} \frac{1-2\sigma}{1-\sigma} \frac{\bar{\phi} - \phi}{\phi} \frac{d}{w} + \frac{1}{\sqrt{\pi}} \frac{1}{(1-\bar{\sigma})(1-\sigma)} \left[-2\bar{\sigma}(1-2\sigma) - \frac{(1+\sigma)\bar{E}}{(1+\bar{\sigma})E}(1-2\sigma)^2 + \frac{(1+\bar{\sigma})E}{(1+\sigma)\bar{E}}(1-2\bar{\sigma}) \right] \frac{d}{w} \right\}$$

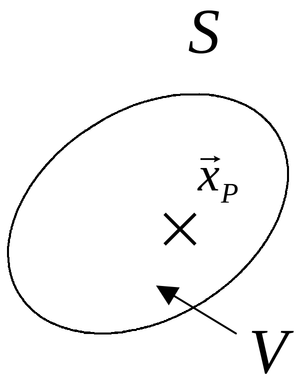
where

$$S_\varphi^{Single}(\omega,\phi) = 4k^2 \frac{2k_B T}{\omega} \frac{\phi}{\pi^{1/2} w} \frac{1-\sigma^2}{E}$$

$S_\varphi^{Total}(\omega)$	The phase noise correlation for a coated half-space mirror
$S_\varphi^{Single}(\omega,\phi)$	That of a uncoated mirror; of the substrate material
d	The coating thickness
w	The beam spot size (amplitude radius)
E, σ, ϕ	Young's modulus, Poisson ratio, and loss function of the substrate material
$\bar{E}, \bar{\sigma}, \bar{\phi}$	Those of the coating material

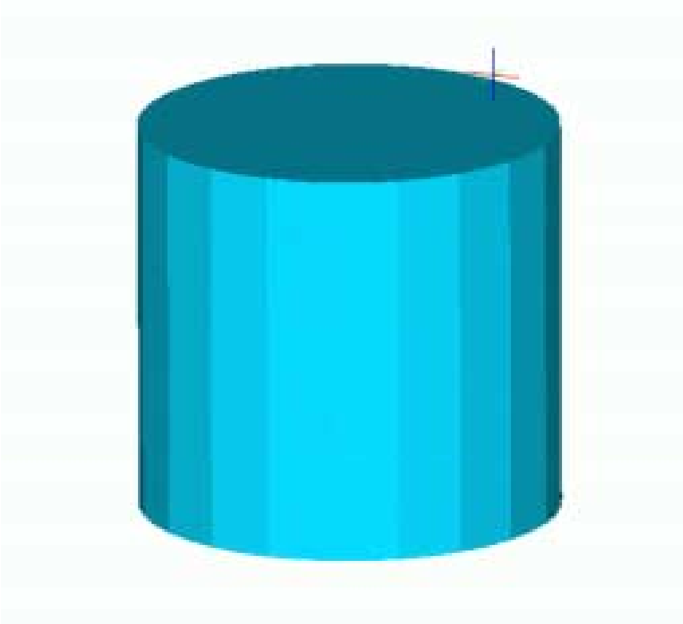
On-Going Activities

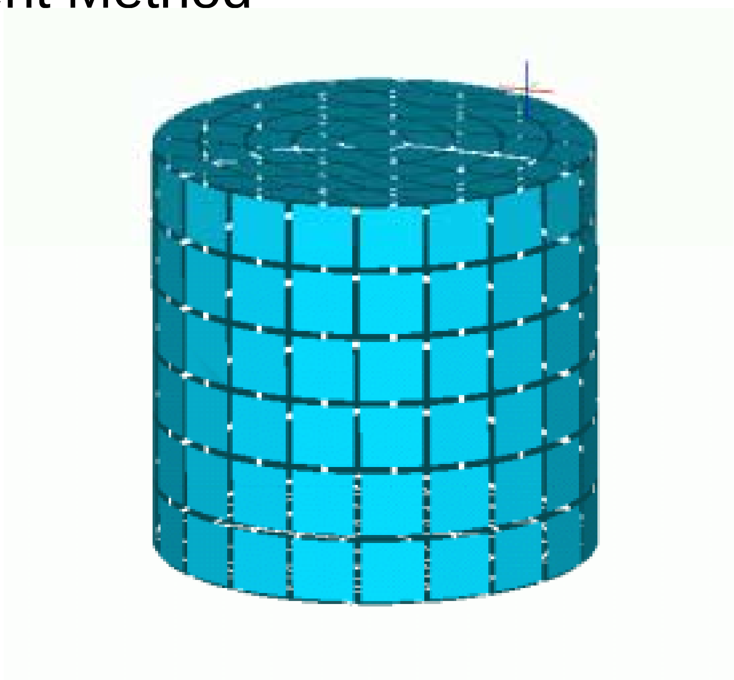
- Numerical Green's function calculation
  - Betti-Rayleigh-Somigliana formula



$$\chi^V(\vec{x}_P)u_k(\vec{x}_P) = \int_S dS_j\left[-u_i\Phi^{\omega}_{ijk} + T_{ij}\Gamma^{\omega}_{ik}\right]$$

  - Discretization by Boundary Element Method



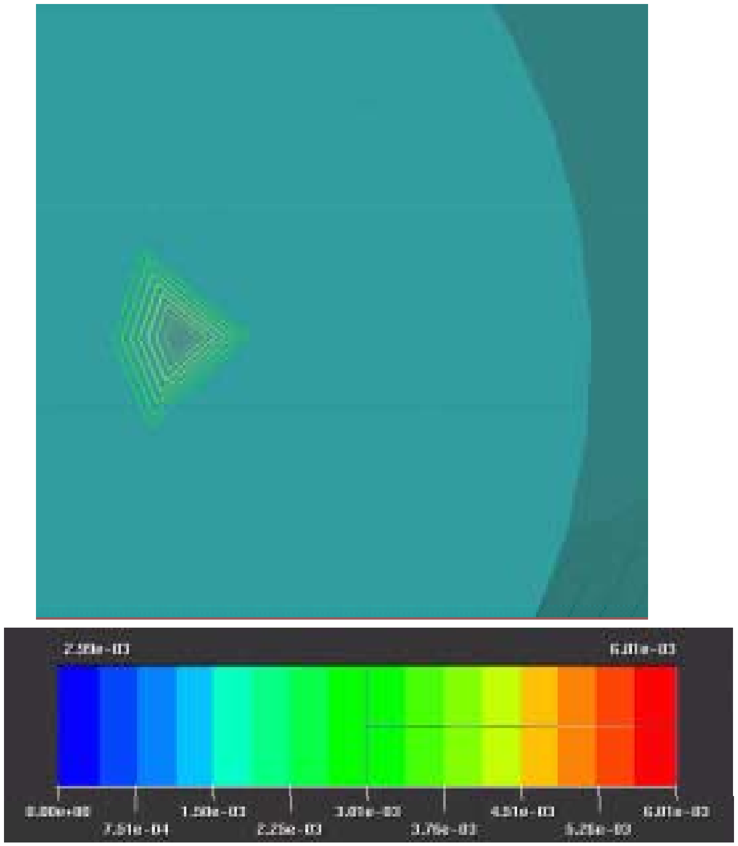


Ex. 32 cm diameter, 30 cm long

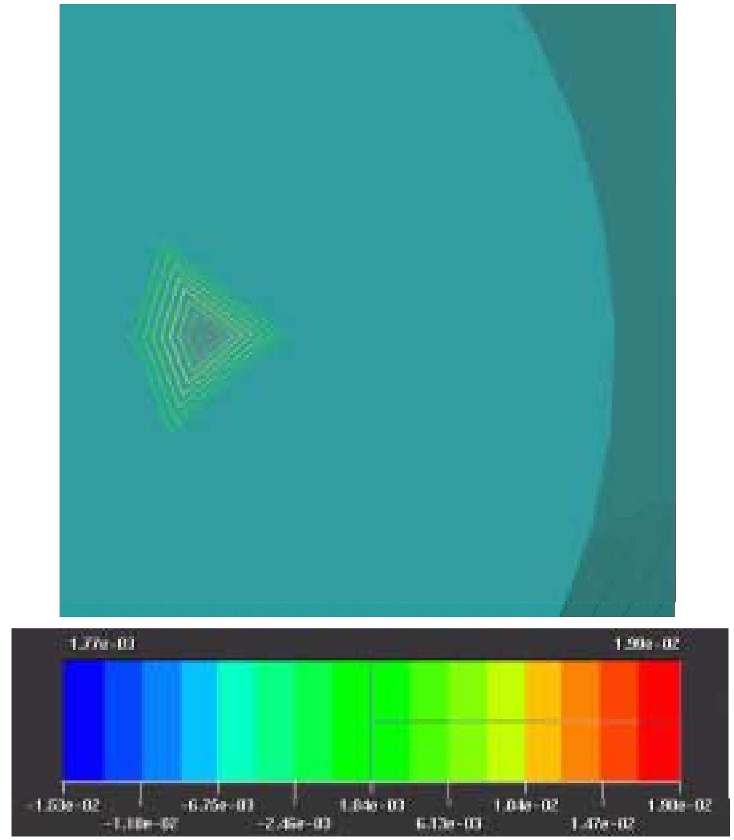
On-Going Activities

- Preliminary BEM result

Incident = force density



Response = displacement

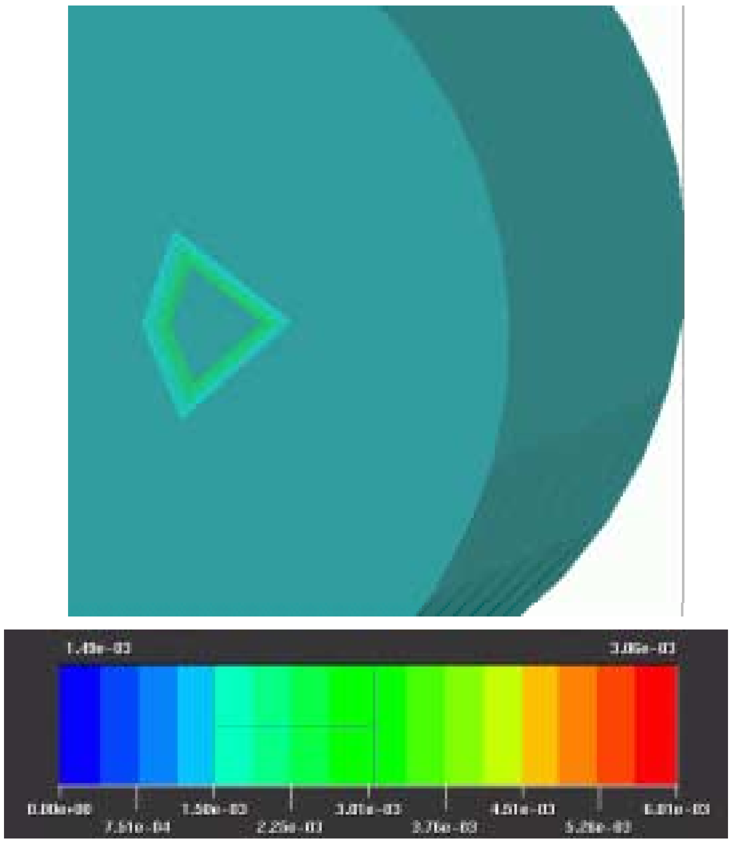


  - Field values between 50% &100% of the peak value

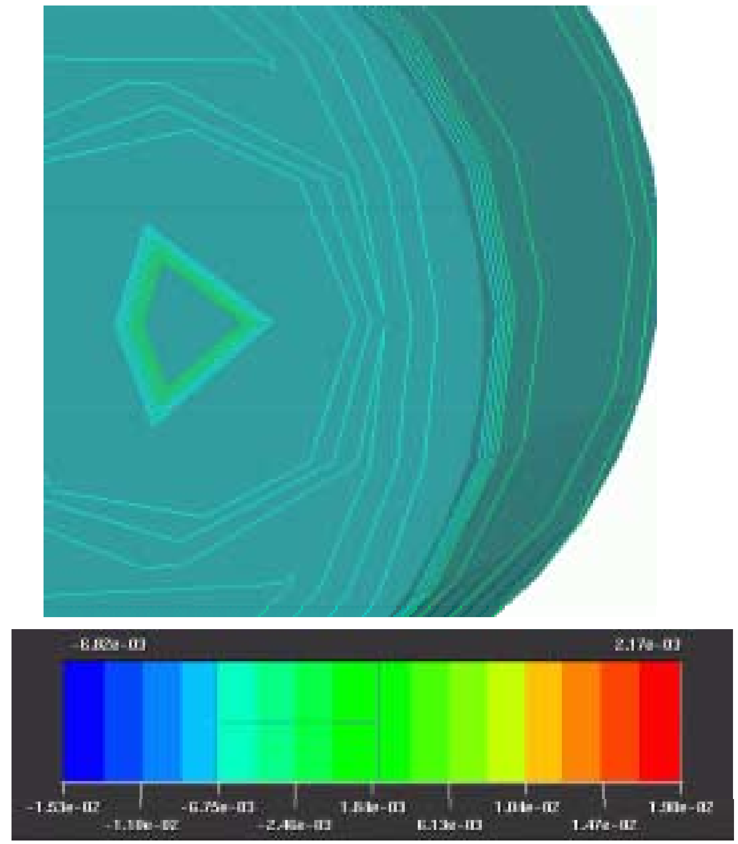
On-Going Activities

- Preliminary BEM result
  - Between 25% and 50% of the value ranges

Incident = force density



Response = displacement

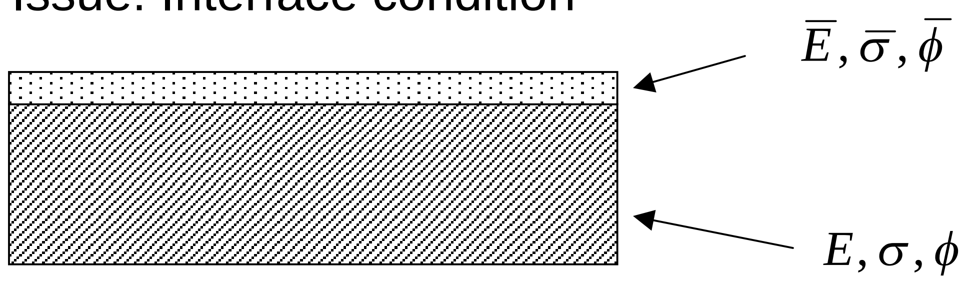


Work Statement

- List of Planned investigations
  - Coating noise studies
  - Noise study: an analytical membrane model
  - Noise study: Cylindrical mirror model
    - numerical computation
    - Other shapes
  - Studies of other possible noise sources
    - Thermoelastic damping
    - Dielectric relaxation

Work Statement

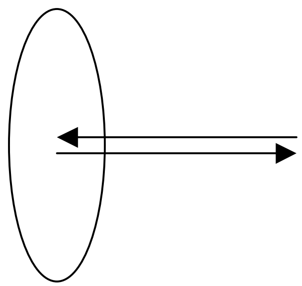
- Complete coating noise studies
  - Based on half-space mirror model
    - a lossy layer in a lossy host material
    - dissimilar elastic constants and loss functions
    - analytical solution
    - Issue: Interface condition



  - Examine experimental data
    - a fused silica coating on a silica substrate
    - a fused silica coating on a sapphire substrate

Work Statement

- Noise study: Membrane model
  - Stretched membrane as an analytical mirror model



$$g(\vec{x}, \vec{x}') = \frac{1}{2\pi} \log \frac{1}{|\vec{x} - \vec{x}'|} + (\text{image term})$$

  - Will demonstrate the delay-line vs. resonator assertion
    - Test the effect of the finite transverse dimensions on the noise de-correlation.
    - Use analytical Green's function.
  - Issue: Rigid-body motions
    - Establish a numerically amenable procedure to eliminate rigid-body translation and rotation.

Work Statement

- Noise study:
  Cylindrical mirrors
  - Numerical Green's function computation
    - Betti-Rayleigh-Somigliana formula
    - Nodal discretization by the boundary element method.
  - Cylindrical mirror model
    - Single reflection
    - Fabry-Perot
    - Delay-line
  - Demonstrate the delay-line vs. resonator assertion
  - Effects of mirror aspect ratios.

$$-\rho\omega^2 u_i - \partial_j T_{ij} = 0$$

$$-\rho\omega^2 \Gamma_{ik} - \partial_j \Phi_{ijk} = \delta_{ik}\delta(\vec{x} - \vec{x}_P)$$

$$\Rightarrow \chi^V(\vec{x}_P)u_k(\vec{x}_P) = \int_S dS_j \left[-u_i\Phi^{\omega}_{ijk} + T_{ij}\Gamma^{\omega}_{ik}\right]$$

$u_i(\vec{x})$	Displacement
$T_{ij}(\vec{x})$	stress tensor $(T_{ij} \equiv c_{ijkl}\partial_k u_l)$
$\Gamma^{\omega}_{ij}(\vec{x})$	the fundamental solution (Green's function)
$\Phi^{\omega}_{ijk}(\vec{x})$	$\equiv c_{ijlm}\partial_l \Gamma_{mk}$
c_{ijkl}	elastic constants
ρ	Density
V, S	volume of a region and its boundary surface
$\chi^V(\vec{x})$	the characteristic function of V (=1 inside V, =0 outside)

Work Statement

- Other noise source, I
  - Thermoelastic loss
    - Green's function-based approach
    - Hamiltonian

$$H' = -\int_V dV K(t)\alpha(T - T_0)\,\partial_l u_l$$

➔Obtain two-point correlations

➔ Various optical systems

– Single reflection

– Fabry-Perot resonator

– Delay line

- Other noise source, II
  - Dielectric relaxation
    - Consider the laser reflection as a wave reflection

n=index of refraction

$$n = n' - in''$$

then

$$\vec{E}^I e^{-ikz} - \vec{E}^R e^{ikz - i\theta}$$

$$\boxed{\tan\theta = \frac{2n''}{|n|^2 - 1}}$$

$$\langle \delta\theta\delta\theta \rangle_\omega \Leftarrow \langle \vec{D}(\vec{r})\vec{D}(\vec{r}\,') \rangle_\omega$$

- Magnitude of loss function
  - To be estimated

Tasks & Time Lines

T1. Coating noise estimation
- Dissimilar elastic constants and loss functions

T2. Sapphire mirror loss
- half-space mirror model.

T3. Circular membrane model
- Finite size effect

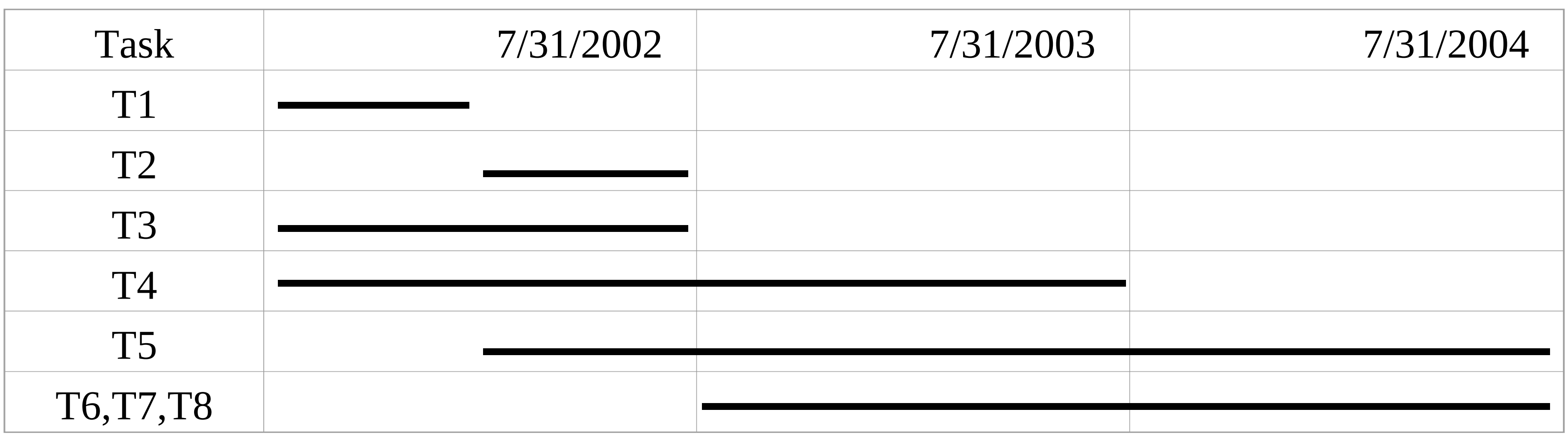
T4. Isotropic cylinder mirror
- delay-line vs. resonator

T5. Numerical code validation

T6. Thermoelastic noise estimation
- Multi-bounces

T7. Dielectric loss study

T8. Off-beam-axis scatterings



Broader Impacts

- Education
  - Student involvement through the collaboration with Stanford Group
- Contribution to LSC
  - The thermal noise estimation methodology
  - Impact on sorting out advanced optics designs
  - Impact on mirror material selection
- Impacts on other federally funded programs
  - Through advancement of computational physics methodology
    - NSF Industry/University Cooperative Research Center program
    - USAF program “Simulation studies of nondestructive evaluation”

Summary

Accomplishments

- Developed two-point phase noise correlation formulas for optical resonator and delay line
- Found that delay lines can be less noisy than resonators
- Estimated coating noise and predicted acceptable coating loss values
- Had the numerical Green's function calculation code operational

Proposed Activities

- Complete coating noise estimations
- Noise study by analytical model
  - Membrane model
  - Delay line vs Fabry-Perot
- Complete numerical code
  - Noise of cylindrical mirror
    - Delay line vs resonator
  - Realistic mirror shapes
- Inclusion of thermoelastic noise
- Examine other noise sources