# Linear Dynamic Model for Advanced LIGO Isolation System

Wensheng Hua, Brain Lantz, Dan Debra, Jonathan How, Corwin Hardham, Sam Richman, Rana Adhikari, Richard Mittleman

LIGO-G010135-00-Z

# Aim of modeling

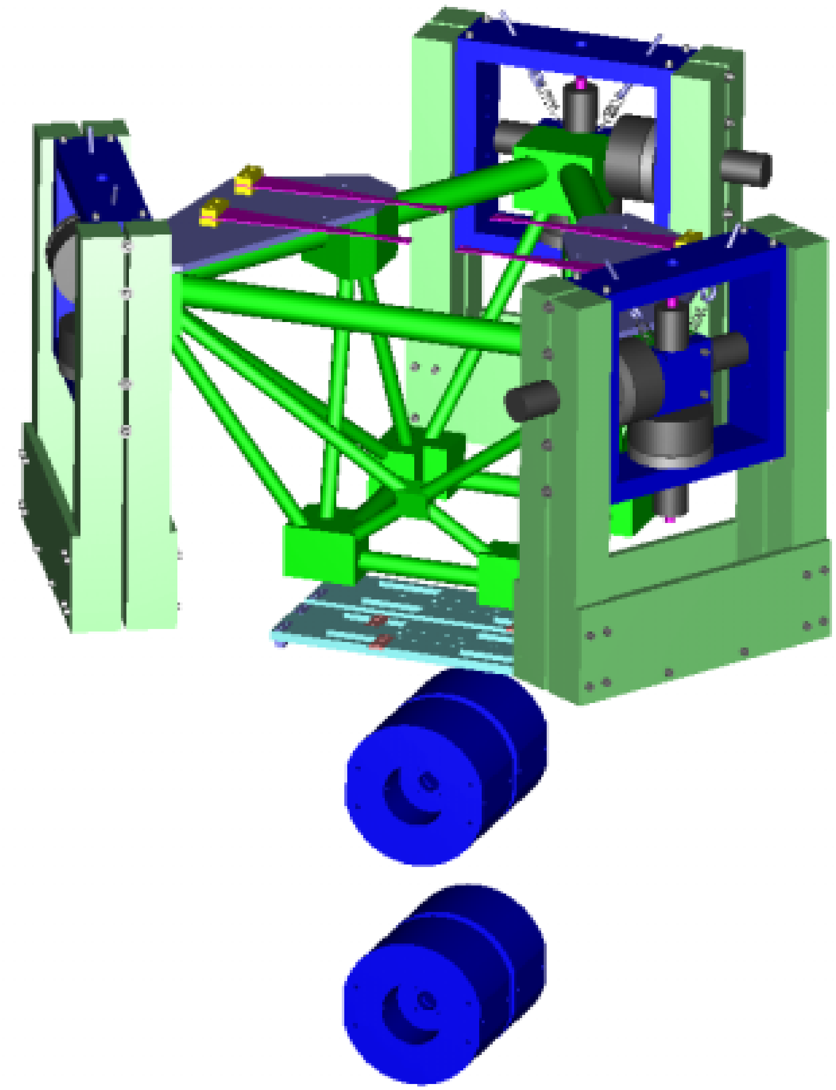
- Help to understand the dynamics of the system.
- Help to control the system.
- Help to design future system.

# What are we trying to model?

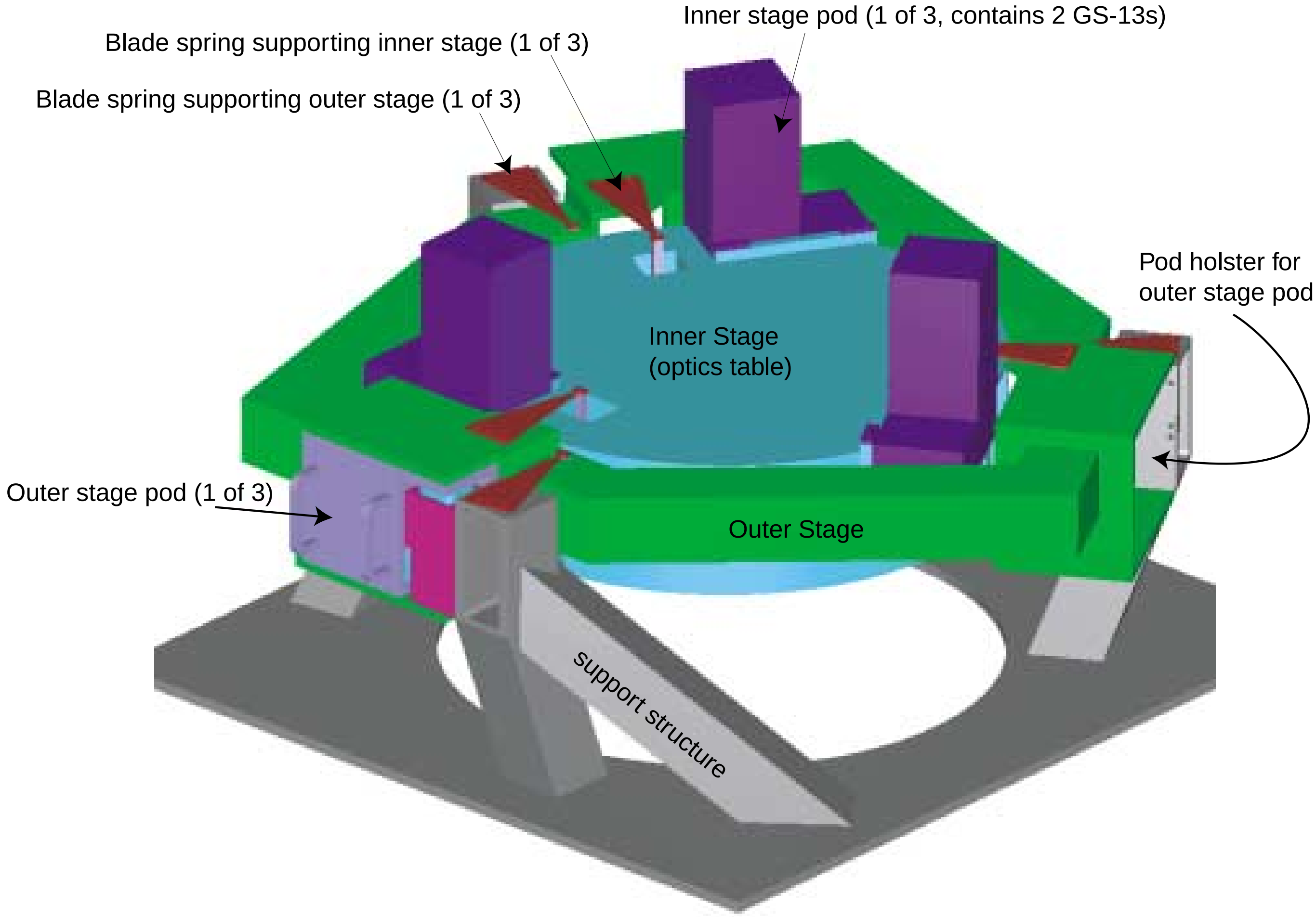
- LIGO suspension system is a complicated nonlinear dynamic mass spring system.

- We need to build a linear model of it in full degrees of freedom.

# Stanford Active Platform with Pendulums



# Two stage active platform in the BSC

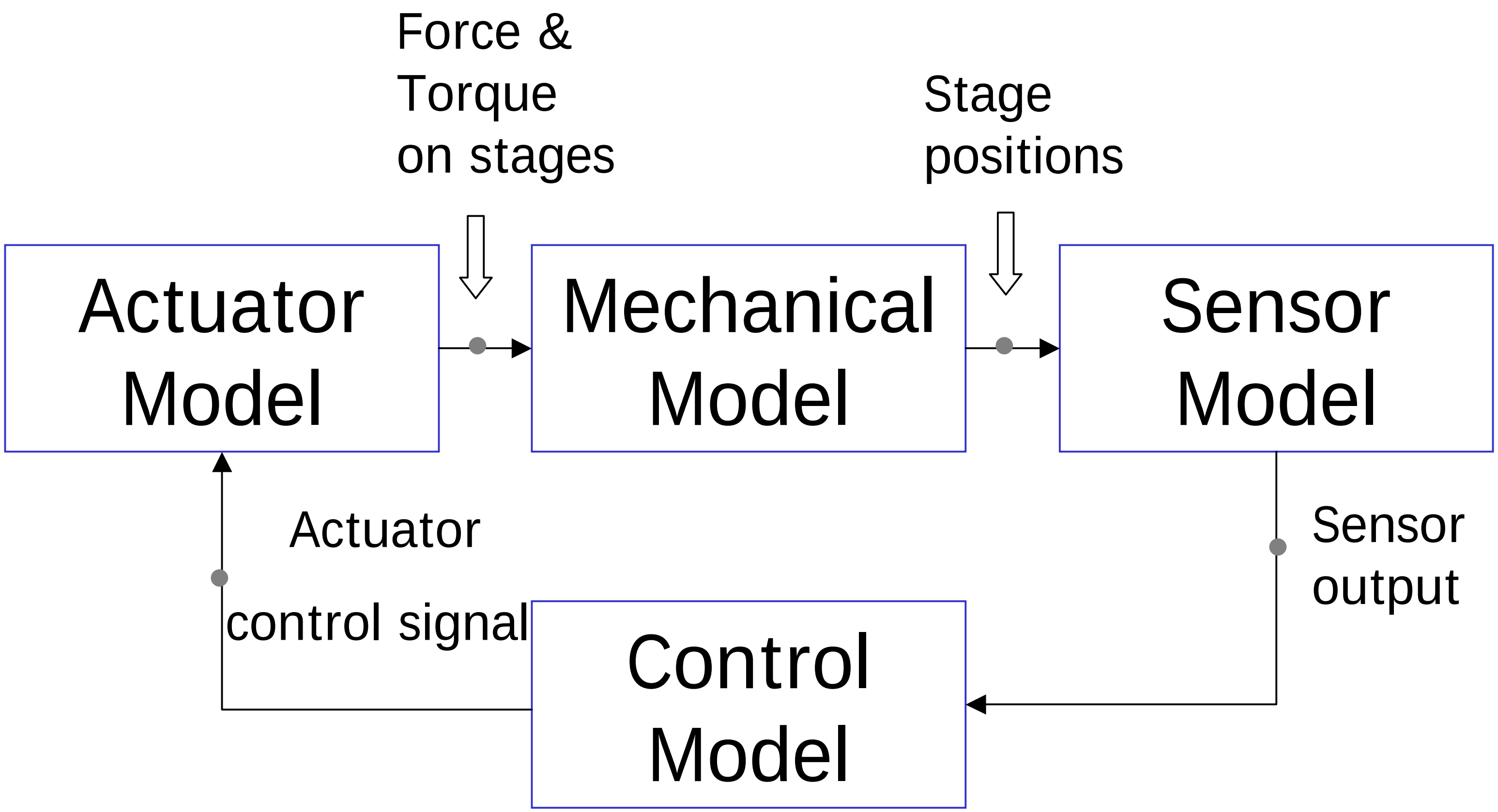


# Double Stage Prototype

# Elements of the systems.

- Stages (Masses)
- Springs
- Sensors
- Actuators

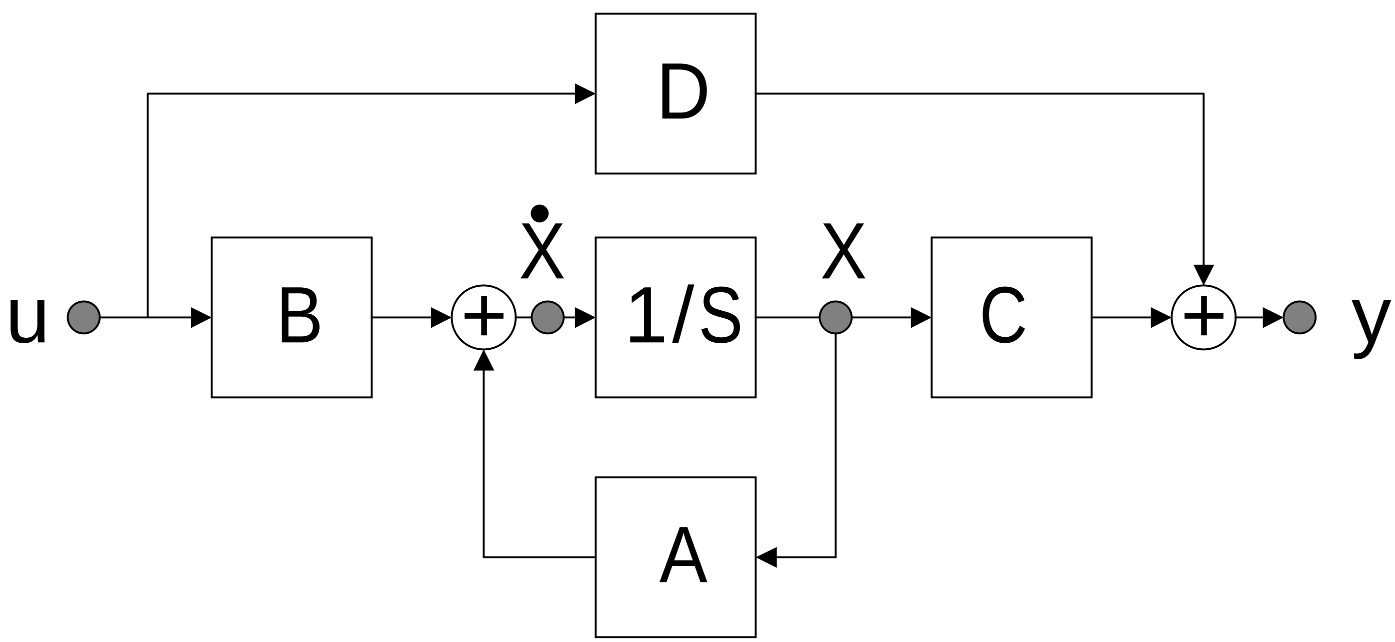
# Big picture of the model



# General Form of a Dynamic Model

$$\dot{X} = AX + Bu$$

$$Y = CX + Du$$



# Matrix A for a Suspension System

$$X = \begin{pmatrix} d \\ v \end{pmatrix} \qquad \dot{X} = AX$$

$$A = \begin{pmatrix} 0 & I \\ M^{-1}K & M^{-1}D \end{pmatrix}$$

$$\begin{pmatrix} \dot{d} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} 0 & I \\ M^{-1}K & M^{-1}D \end{pmatrix} \begin{pmatrix} d \\ v \end{pmatrix}$$

$v = \dot{d}$

$a = \dot{v}$

$F = Ma$

$F = Kd + Dv$

$a = M^{-1}Kd + M^{-1}Dv$

M: mass/inertia matrix

K: reaction force matrix

D: Damping force matrix

# Stiffness Matrix K

- The stiffness matrix is defined as the reaction forces and torques on stages due to small movement of the stages around equilibrium positions.

- In small range of motion, the changes of reaction forces and torques are linear to the perturbations of the positions of stages.

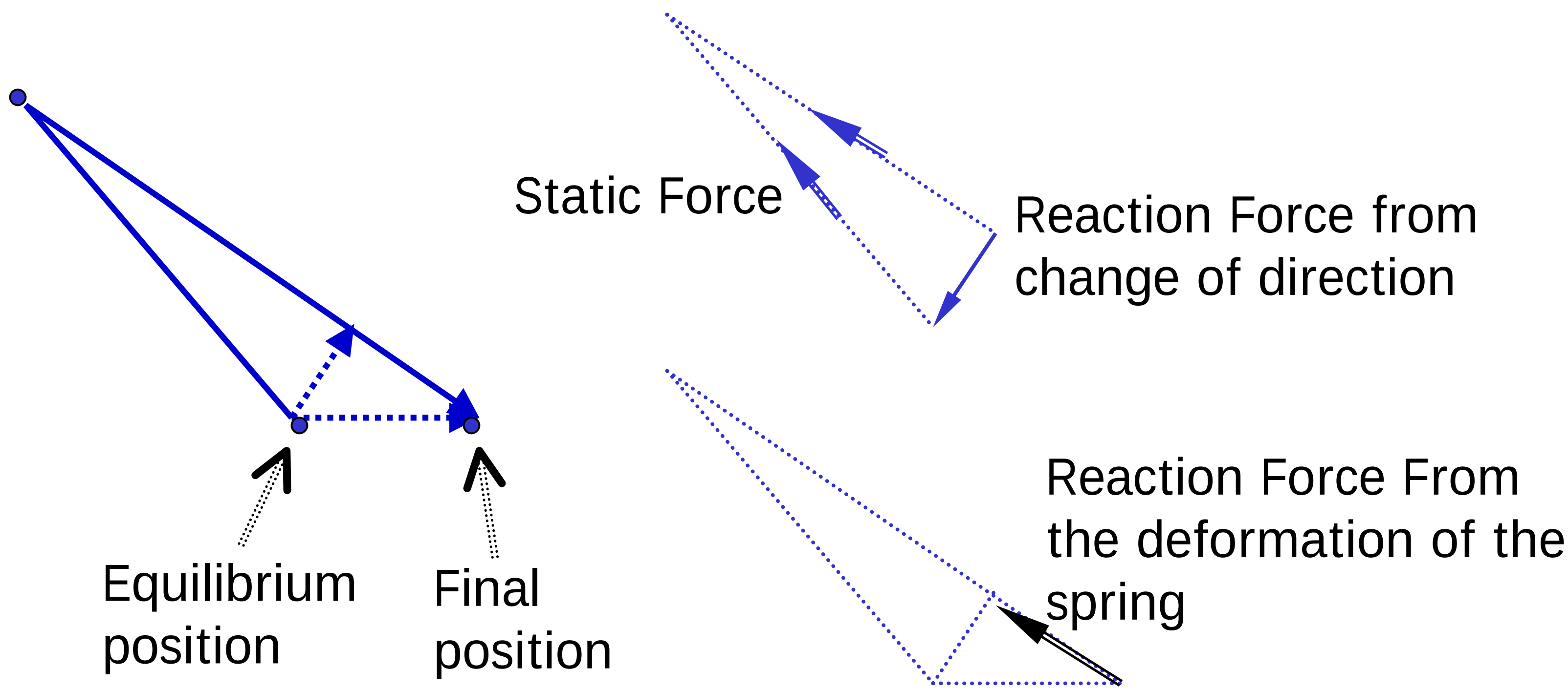
# 3 Steps of making stiffness matrix K

- Convert Stage motion into relative motion of two ends of springs around equilibrium positions.

- Calculate each spring’s reaction force and torque.

- Sum up forces and torques from all springs on stages.

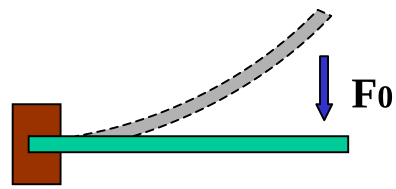
# Example: Simple Wire Spring



# Example: Loaded Blade

Free blade

Loaded blade



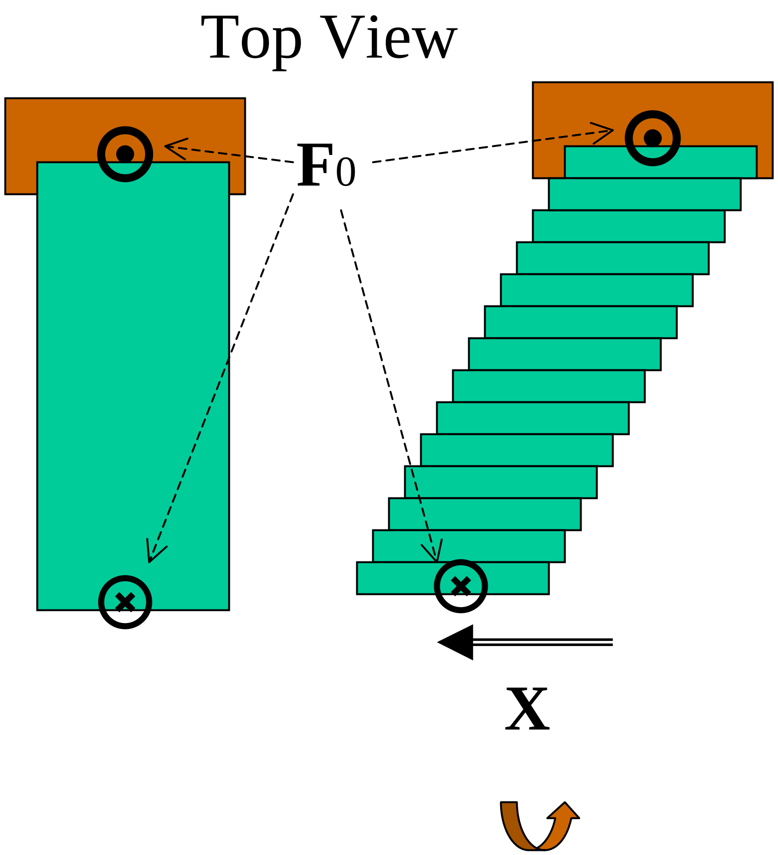

$$\begin{bmatrix} F \\ T \end{bmatrix} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix}$$

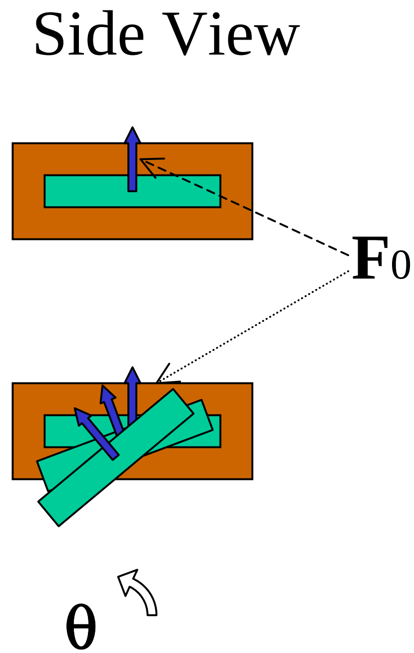
$$\begin{bmatrix} F \\ T \end{bmatrix} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} + \begin{pmatrix} 0 & -F_{0x} \\ F_{0x} & 0 \end{pmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix}$$

$F_{0x}$ is a matrix such that:

$$F_{0x} X = \vec{F_0} \otimes \vec{X}$$

# Reason




$$T = -F_0 \otimes x$$

$$F = F_0 \otimes \theta$$

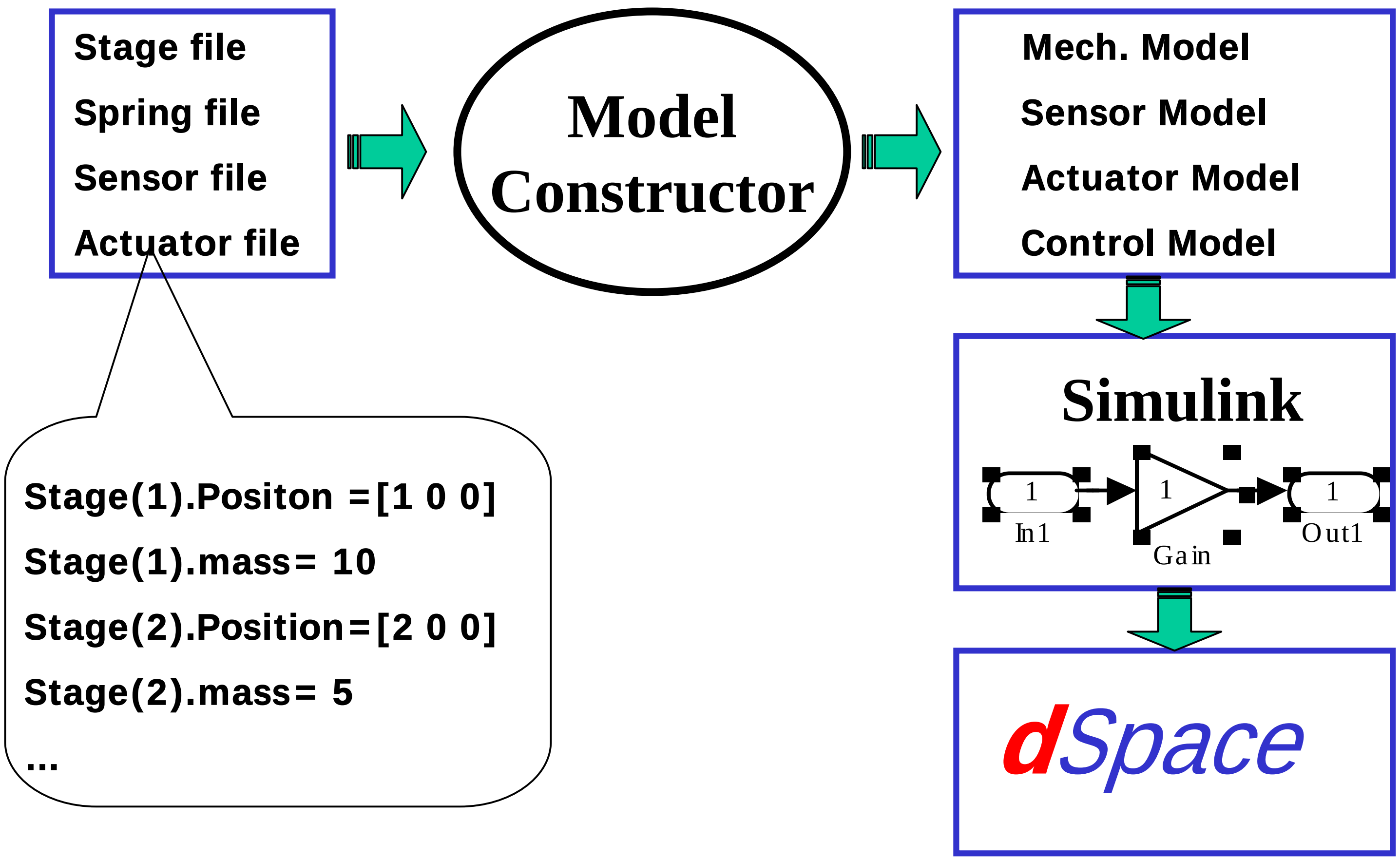
# Some features of the model

- The principle is simply based on F=Kx and F=Ma.
- To linearize each spring is simpler than to linearizie the whole system at ones.
- Make use of Simulink and toolboxes in Matlab.

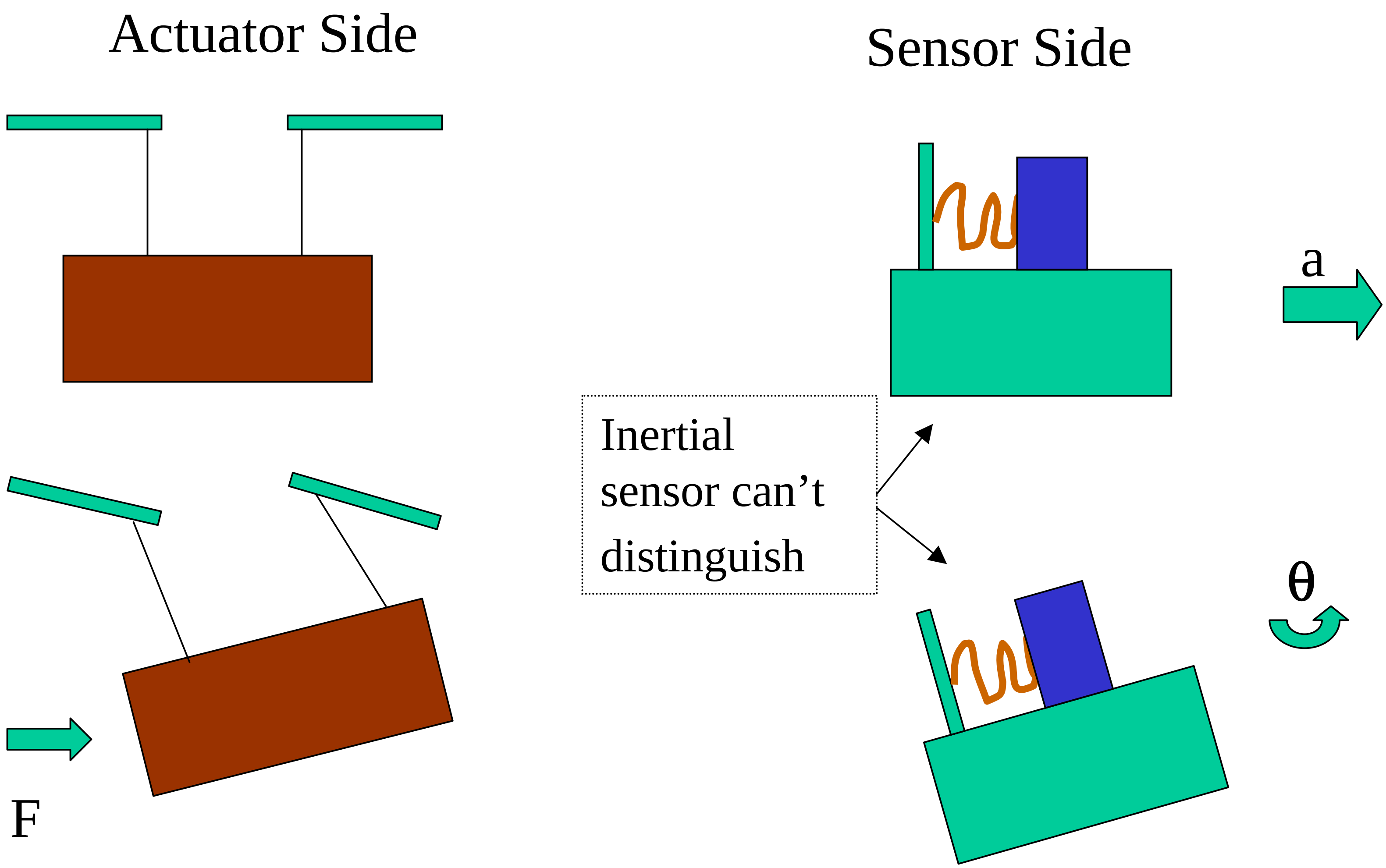
# Model constructor

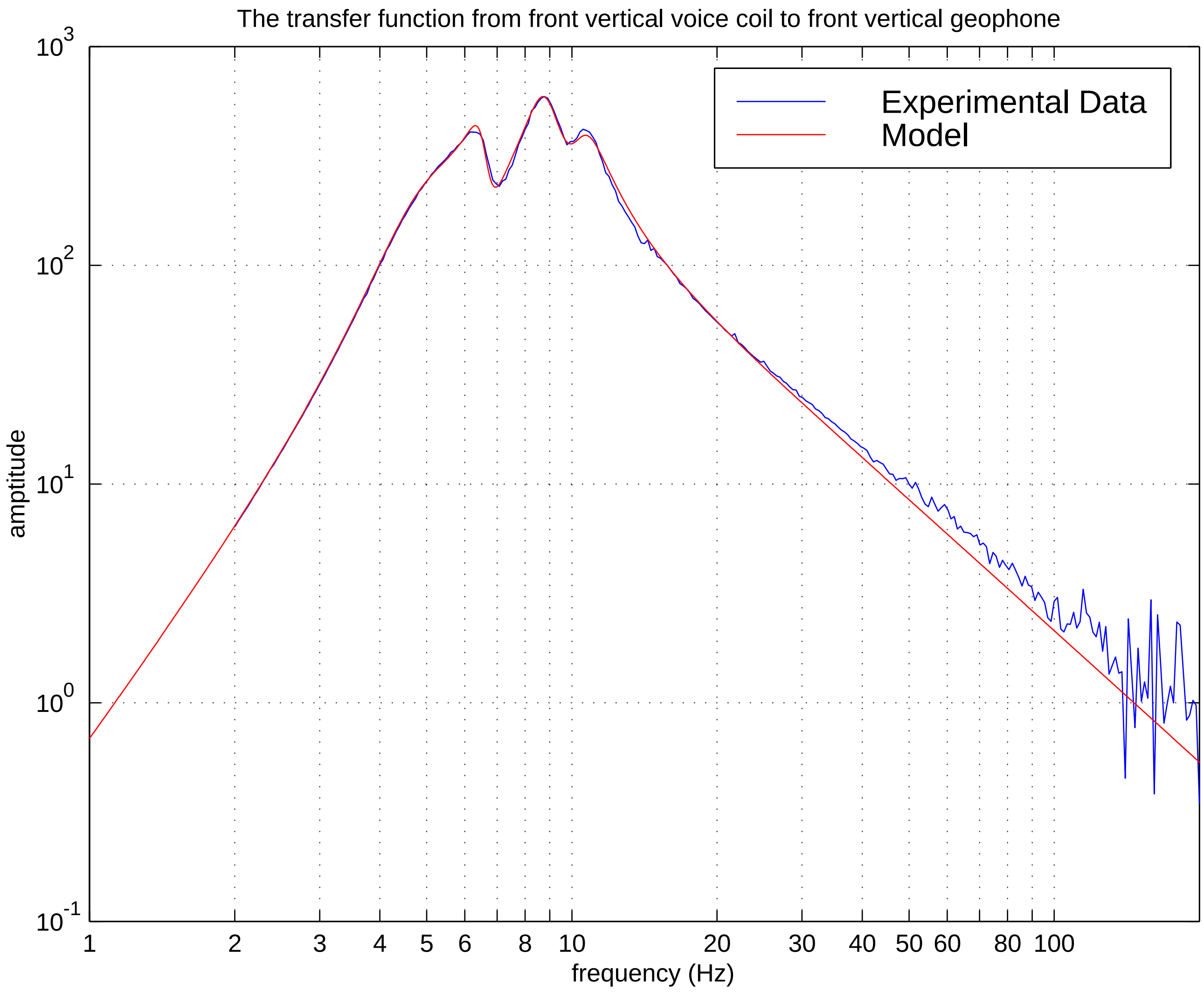
- There are many systems to be modeled.
- There are many different types of springs, actuators and sensors in the system.
  - Springs:  Stretchable Wire, Blade, more general spring.
  - Sensors: Optical Sensor, Geophone.
  - Actuators: Voice coil.
- For each system, we only need to feed its geometry and physical information to the model constructor.
- The information should be in the form of defined data structures.
  - Stage, spring, actuator, sensor, control law.

# Product line of modeling



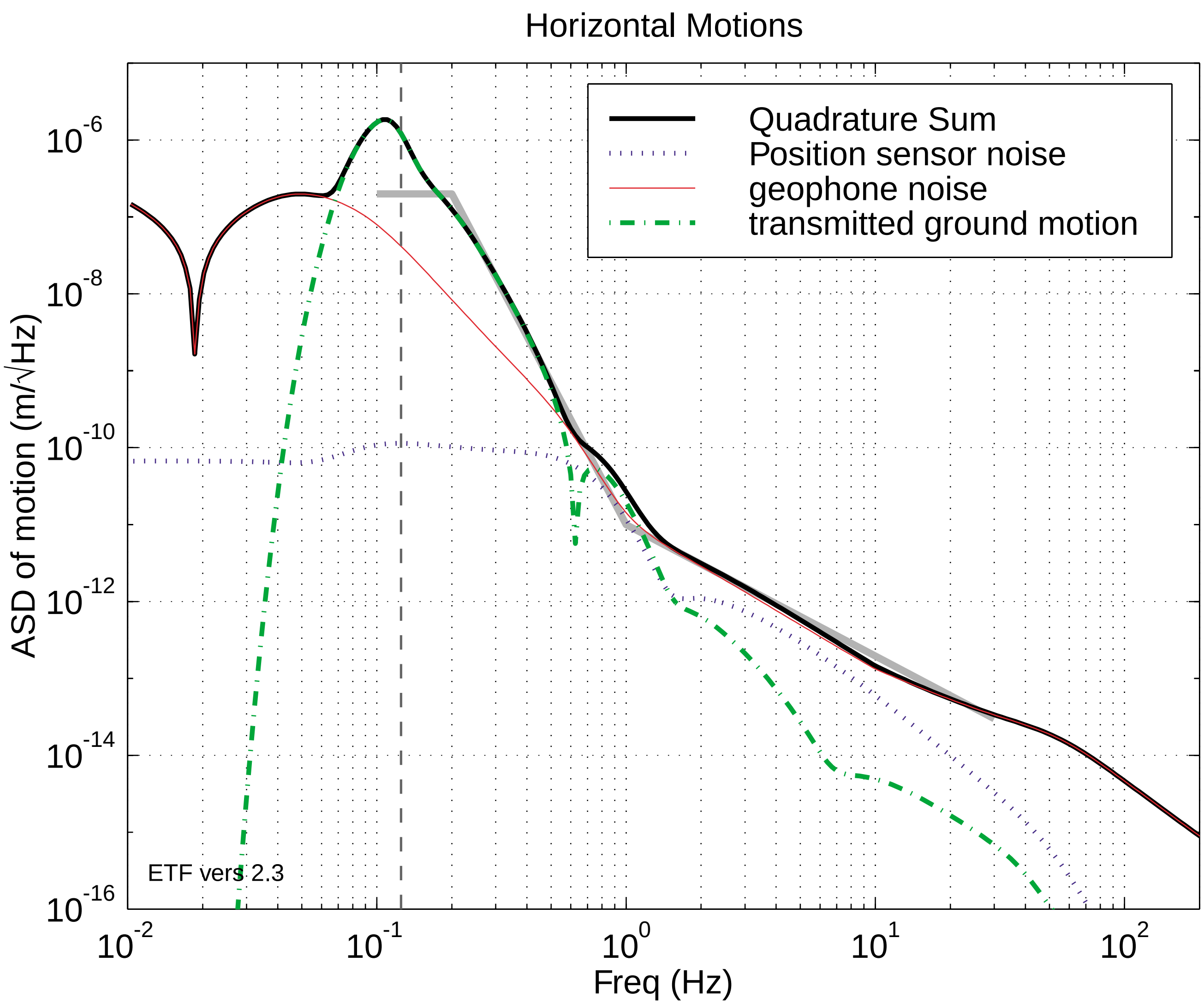
# Tilt horizontal coupling



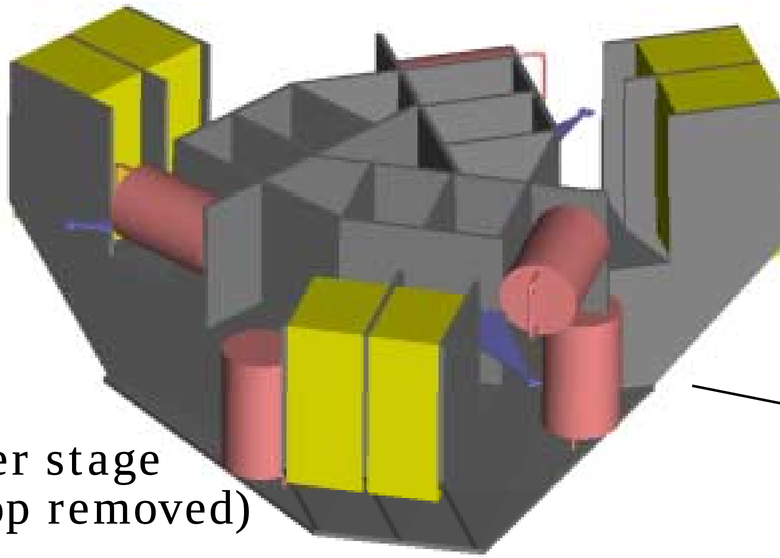


The transfer function from front vertical voice coil to front vertical geophone
Experimental Data
Model
amptitude
frequency (Hz)
10^3
10^2
10^1
10^0
10^-1
1
2
3
4
5
6
8
10
20
30
40
50
60
80
100

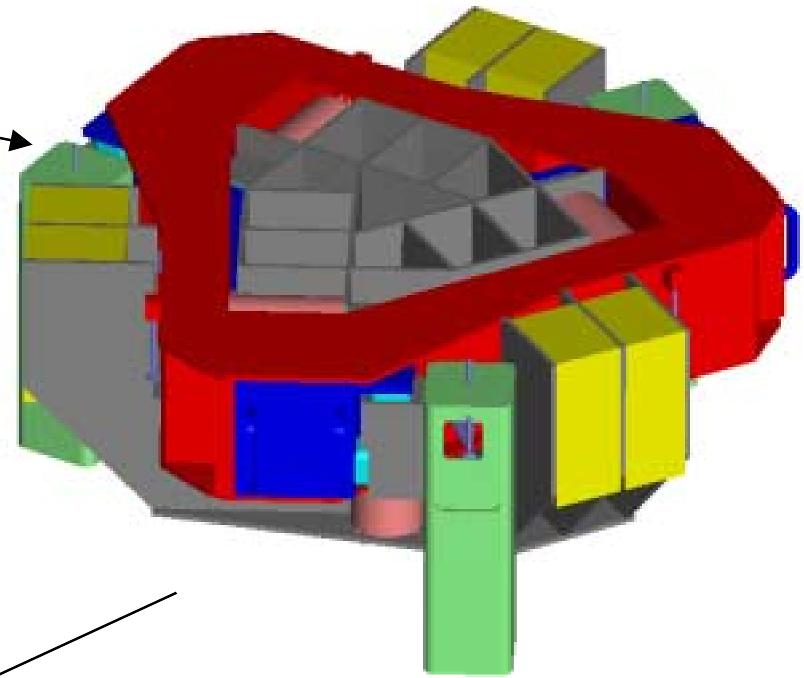
# Predicted Motion of Optics Table



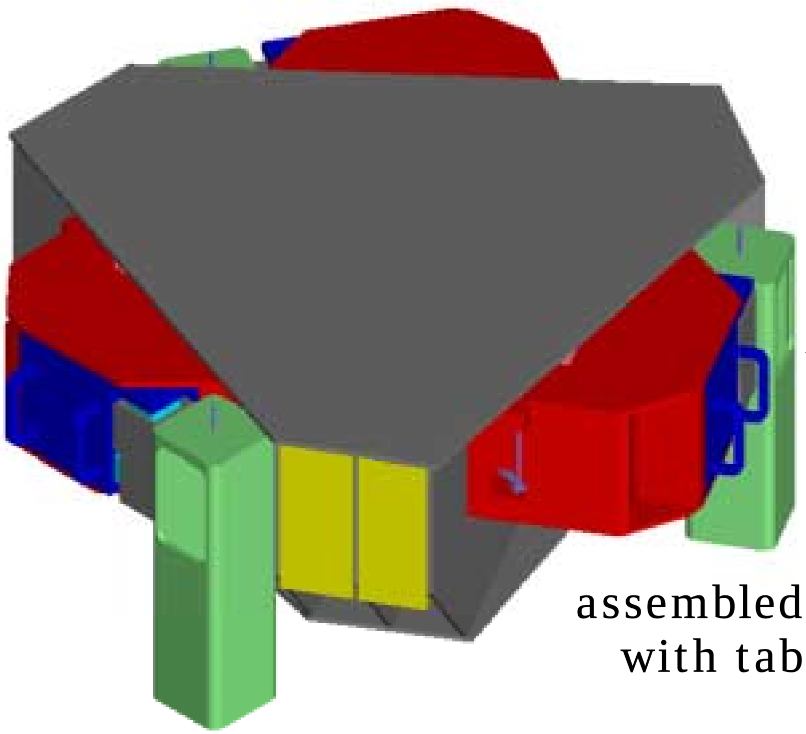
# Views of the Prototype



inner stage
(table top removed)

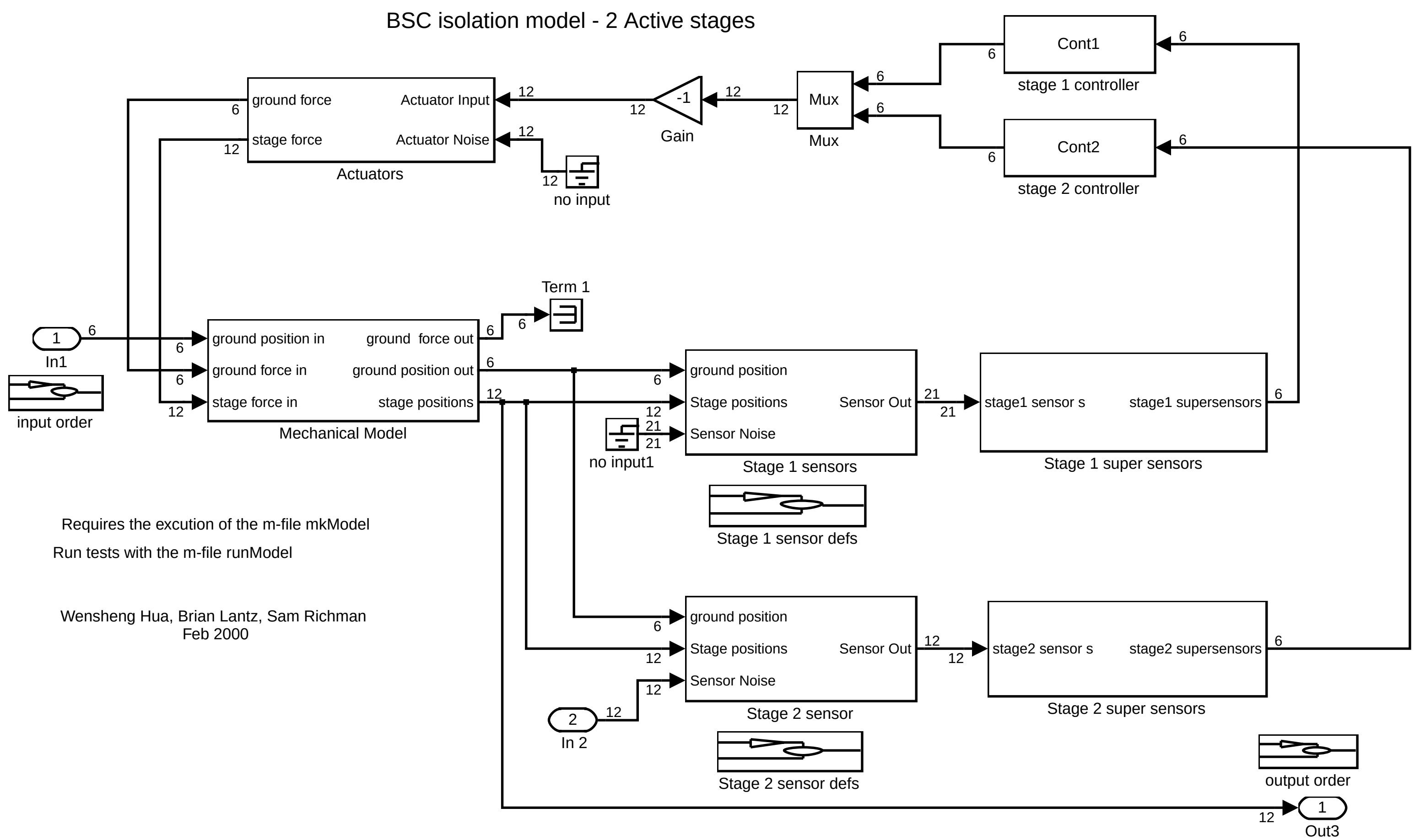


inner stage with outer stage
and supports



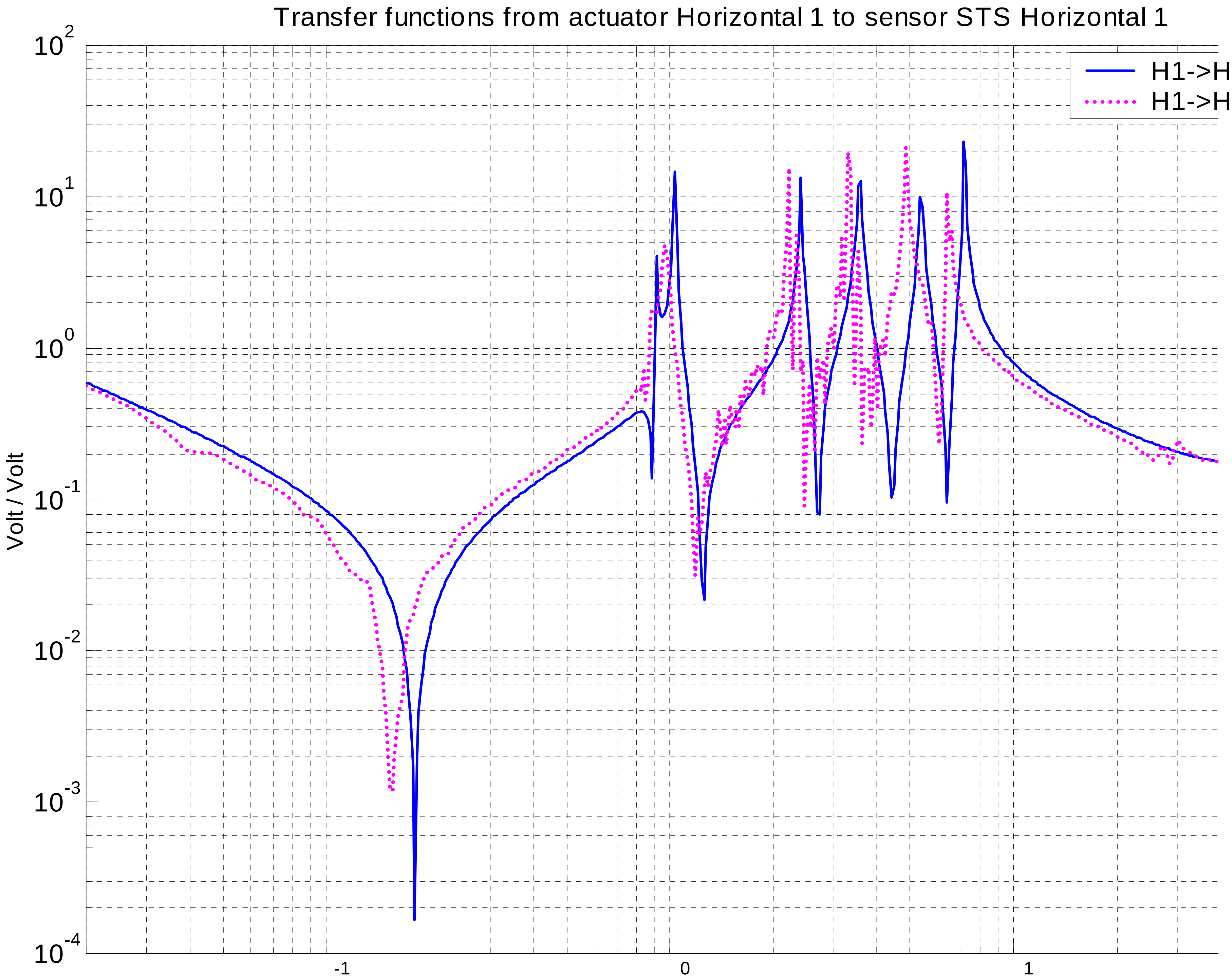
assembled system
with table top

# Simulink Model Diagram

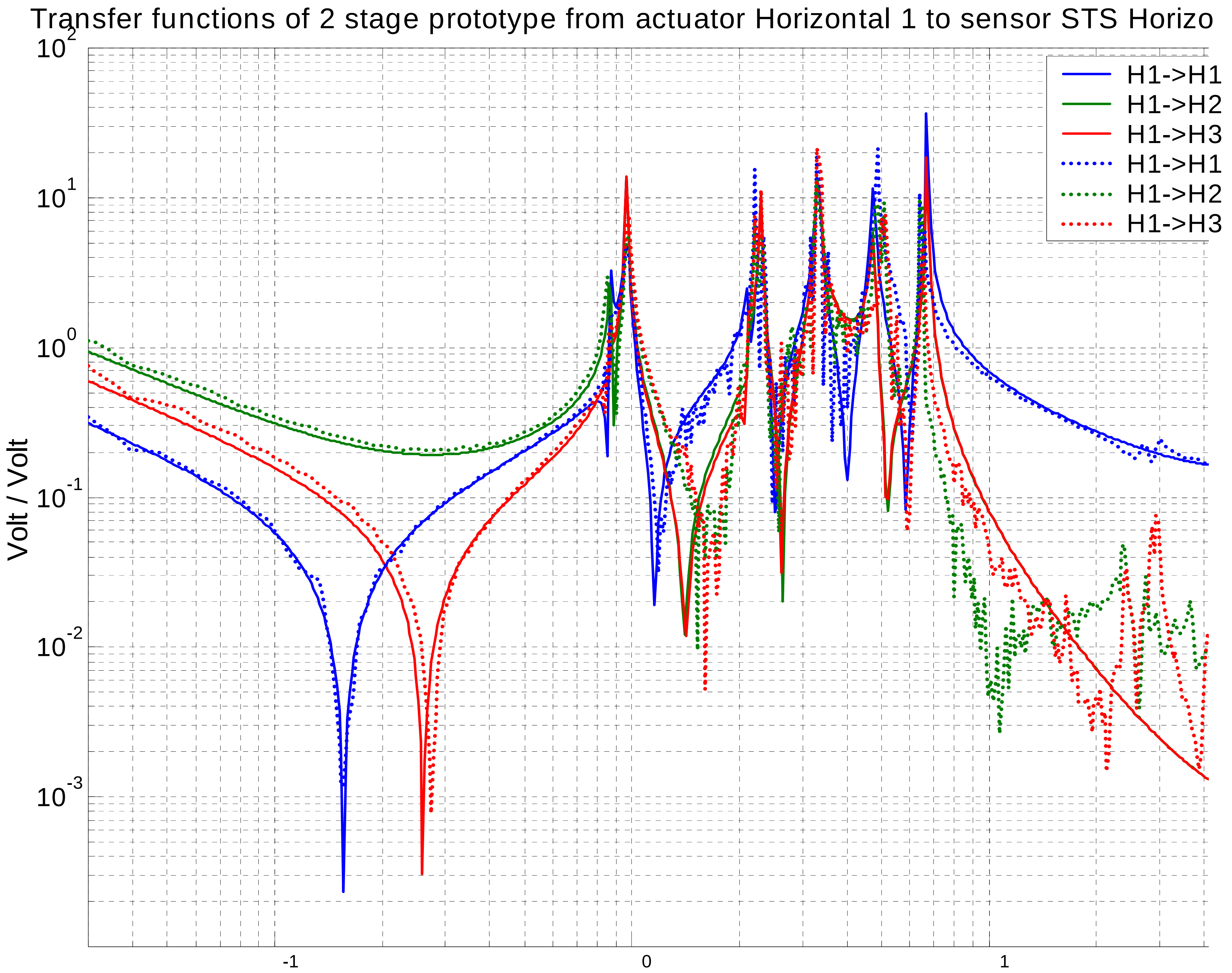



Model used to simulate the dynamics of the reference design.

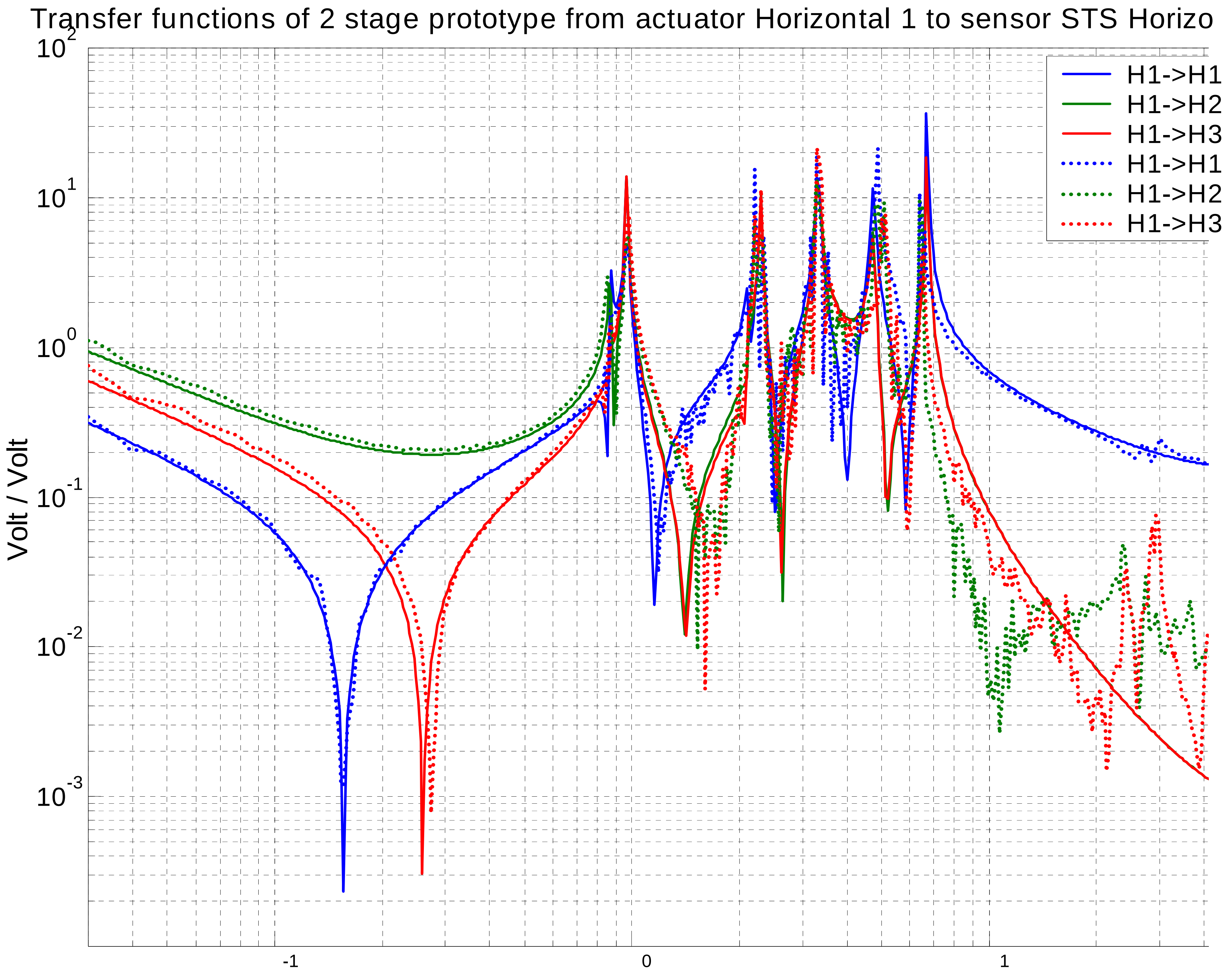
The controller can be cross-compiled onto dSPACE hardware and used on the real system.



Transfer functions from actuator Horizontal 1 to sensor STS Horizontal 1
H1->H
H1->H
Volt / Volt
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
-1
0
1



Transfer functions of 2 stage prototype from actuator Horizontal 1 to sensor STS Horizo
Volt / Volt
$10^{2}$
$10^{1}$
$10^{0}$
$10^{-1}$
$10^{-2}$
$10^{-3}$
-1
0
1
H1->H1
H1->H2
H1->H3
H1->H1
H1->H2
H1->H3



Transfer functions of 2 stage prototype from actuator Horizontal 1 to sensor STS Horizo
Volt / Volt
10^2
10^1
10^0
10^-1
10^-2
10^-3
-1
0
1
H1->H1
H1->H2
H1->H3
H1->H1
H1->H2
H1->H3

# Conclusion

- A linear dynamic model of advanced LIGO isolation system directly based on very simple physics principles.
- This model is used to analysis the dynamics of several prototype systems and to design control laws for them.
- This model is also used to design future LIGO isolation systems. Based on our experience, we feel confident of the predictions which the model made.