TITLE

Laser Beam Pointing in LIGO II

and its consequences for the Input Optics

Guido Müller

Department of Physics
University of Florida

Livingston, LA
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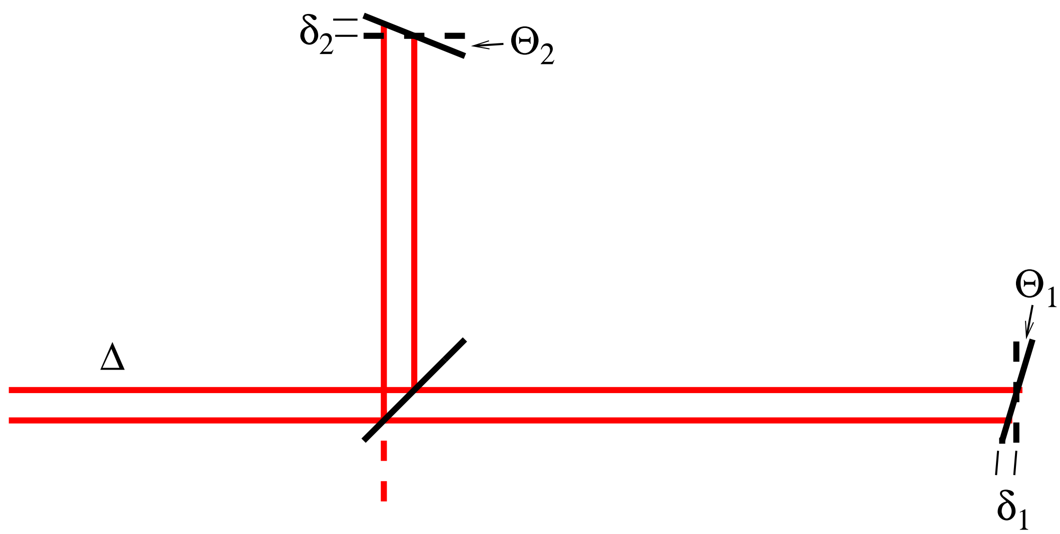
THE PROBLEM



Distances between tilted mirrors depend on beam position:

$$\delta_1 - \delta_2 = \Delta\left(\Theta_1 - \Theta_2\right)$$

Beam pointing in a misaligned IFO generates noise.

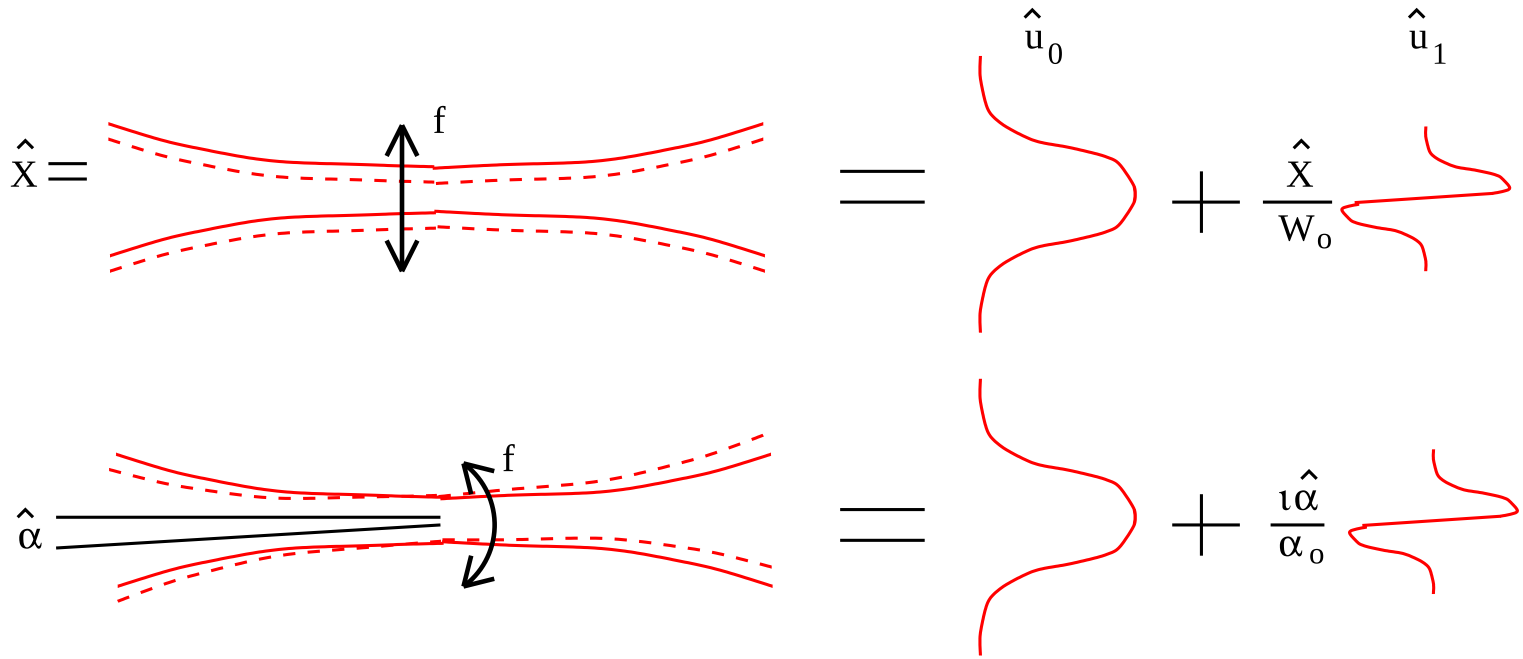
Definitions: **Differential tilt:** $\delta_1 = -\delta_2$

Common Tilt: $\delta_1 = \delta_2$

Involved Frequencies:

- **(mis-) alignement of mirrors is static**
- **Pointing of laser happens at GW-frequencies**

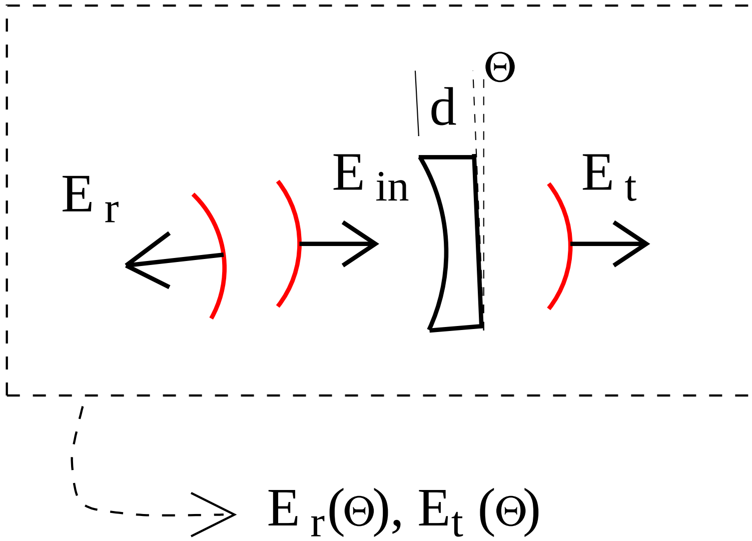
Pointing in the Modal Model



$$E_{in}(z_0) = E_{in} e^{i\omega_o t} \left(\frac{1}{\frac{a_1(f)}{2}\left(e^{i2\pi f t} + e^{-i2\pi f t}\right)} \right)$$

$$a_1(f) = \left[\frac{\hat{x}(f)}{w(z_0)} \left(1 + i\frac{z_0}{z_R}\right) + i\hat{\alpha}(f)\frac{\pi w(z_0)}{\lambda} \right]$$

TILTED MIRROR IN THE MODAL MODEL



It can be described with a 2x2 matrix acting on $\hat{u}_0$ and $\hat{u}_1$:

$$E_r = \hat{M}_r * E_{in}$$

$$\hat{M}_r = \begin{pmatrix} \sqrt{1-4\theta^2} & -2i\theta \\ -2i\theta & \sqrt{1-4\theta^2} \end{pmatrix} \quad \textbf{with } \theta = \frac{\pi w(z)}{\lambda}\Theta$$

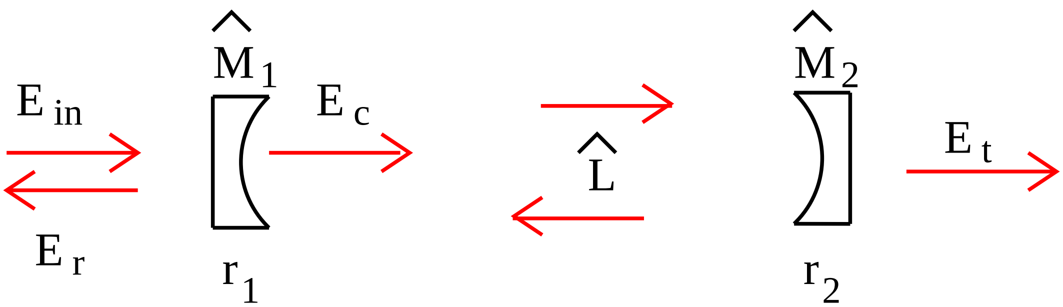
$$E_t = \hat{M}_t * E_{in}$$

$$\hat{M}_t = \begin{pmatrix} \sqrt{1-x^2} & x \\ -x & \sqrt{1-x^2} \end{pmatrix} \quad \textbf{with } x = \frac{d}{w}\frac{n-1}{n}\Theta$$

In interferometric GW-detectors ($\Theta = O(10^{-8})$):

$$\frac{\pi w(z)}{\lambda} \approx 10^5 \qquad \gg \qquad \frac{d}{w}\frac{n-1}{n} \approx 1$$

TILTED MIRRORS IN A CAVITY



Roundtrip Propagator:

$$\hat{P}_{RT} = r_1 r_2 \hat{M}_1 \hat{L} \hat{M}_2 \hat{L} \qquad \hat{L} = \begin{pmatrix} e^{i\omega\frac{L}{c}} & 0 \\ 0 & e^{i\omega\frac{L}{c}+i\phi_g} \end{pmatrix}$$

matrices do NOT commute !

Be careful with the sequence.

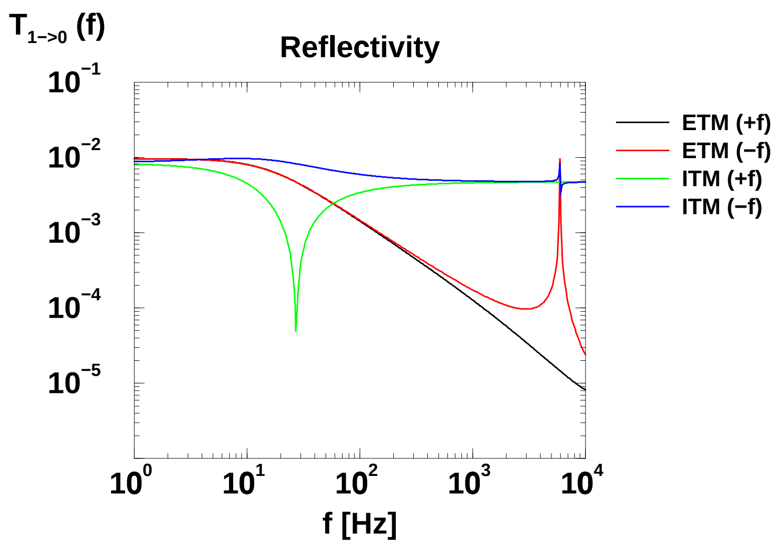
$$E_c = it_1 \left(\hat{U} - \hat{P}_{RT}\right)^{-1} E_{in}$$

$$E_t = -t_1 t_2 \hat{L} \left(\hat{U} - \hat{P}_{RT}\right)^{-1} E_{in}$$

$$E_r = \left(r_1 \hat{M}_1^{-1} - t_1^2 r_2 \hat{L} \hat{M}_2 \hat{L} \left(\hat{U} - \hat{P}_{RT}\right)^{-1}\right) E_{in}$$

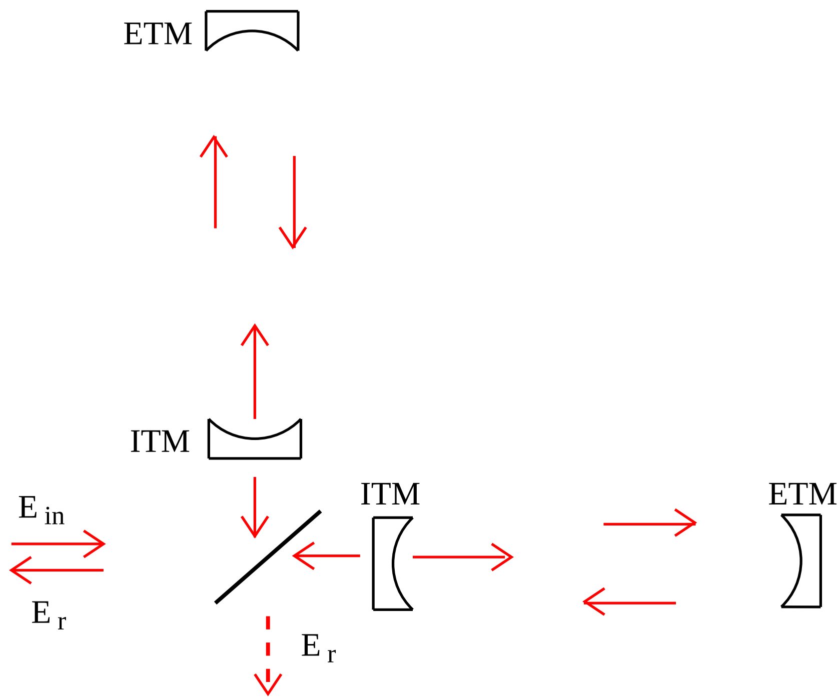
TILTED MIRRORS IN A CAVITY

We need to know:
How much of the 1-mode will show up
as a 0-mode at the detector (dark port)?

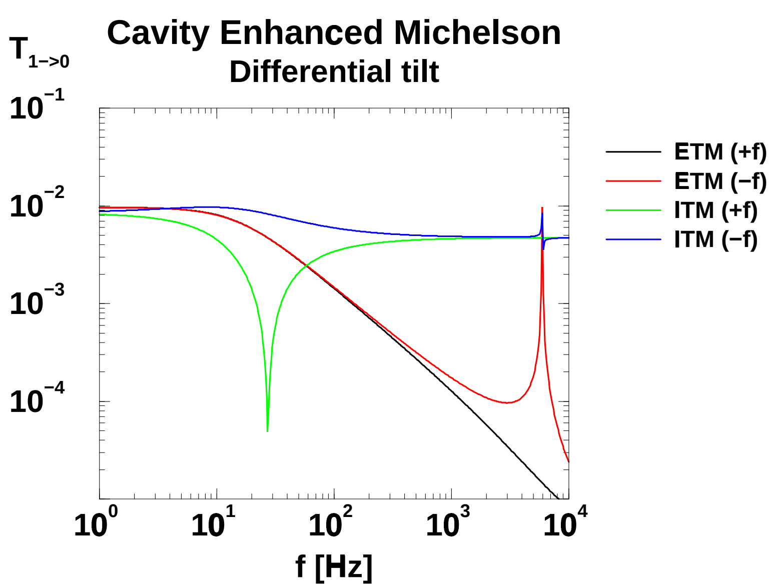


- **ETM tilt rolls off with cavity pol**
- **peak when 1-mode on resonance (Gouy phase)**
- **ITM tilt:** $E = rE_r + tE_c \rightarrow$ **Interference**
  - **overcoupled cavity: f** $<$ **Pol:** $tE_c > rE_r$
  - **f** $>$ **Pol:** $rE_r > tE_c$
  - **f** $\approx$ **Pol: constructive or destructive Interference**

Cavity enhanced Michelson (D)



Differential Tilt:



similar to cavity in reflection

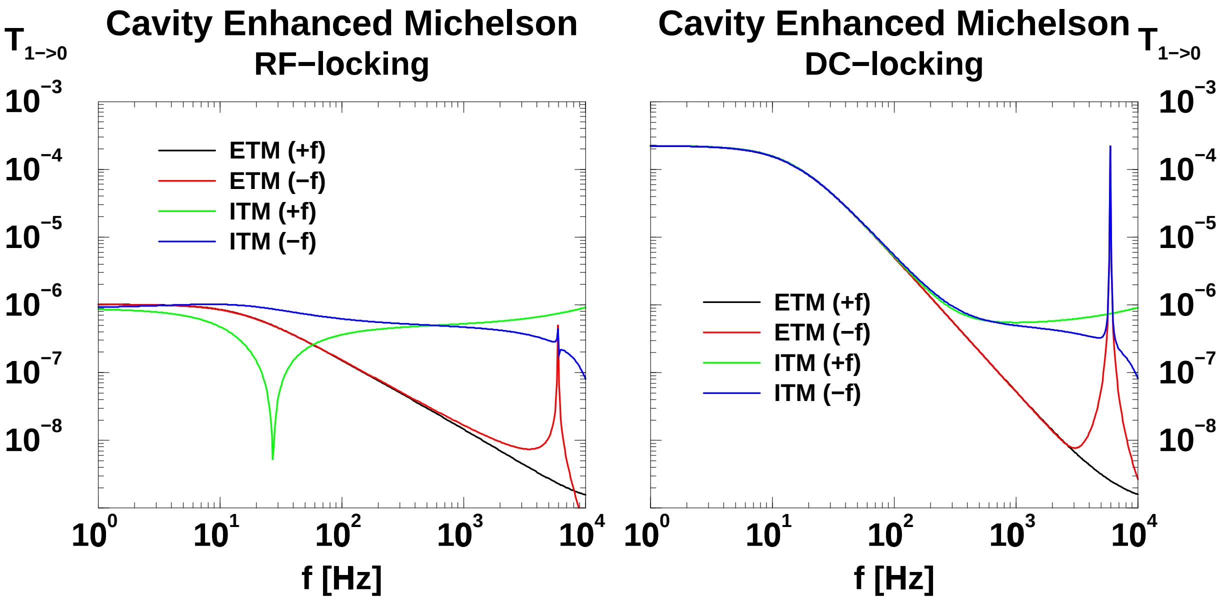
Cavity enhanced Michelson (C)

Common Tilt:

Difference between RF-locking and DC-locking:

1. **RF-locking**
   - **LO at dark port is RF-sideband**
   - **carrier** *completely* **dark**
2. **DC-locking**
   - **small differential offset in arm cavities ($\pm 10^{-11} m$)**
   - **carrier is its own LO**
   - **no RF-sideband necessary**

RF-locking:



$T_{Comm} = T_{Diff} \cdot$ **Asymm.**
here: Schnupp

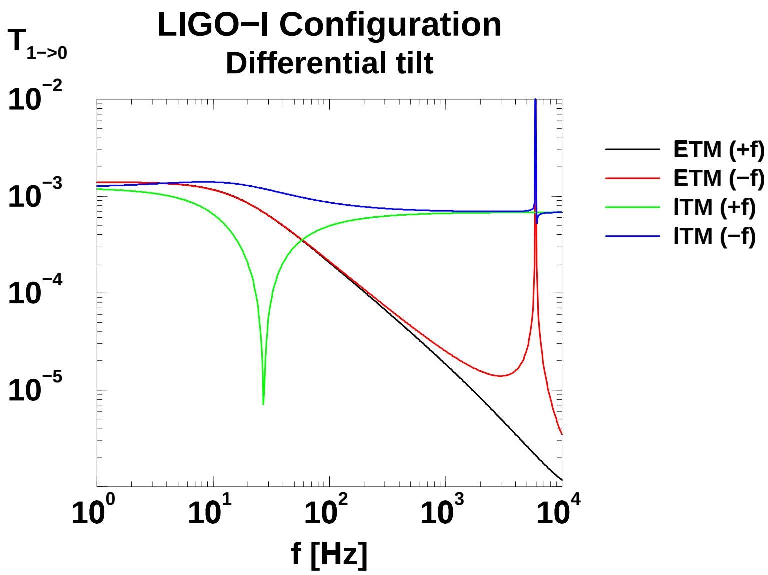
$f < f_c \qquad T_{DC} \approx 200 \cdot T_{RF}$

$f > f_c \qquad T_{DC} \approx T_{RF}$

LIGO-I CONFIGURATION (D)

Cavity enhanced Michelson interferometer with power recycling.

Differential Tilt:



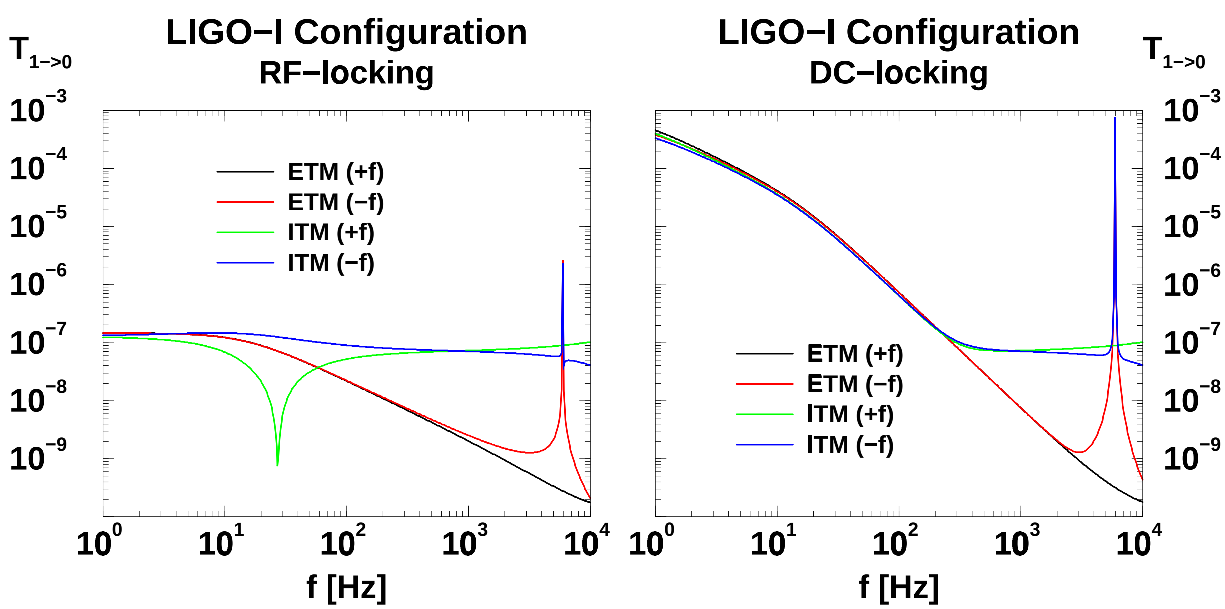
LIGO-I with LIGO-II parameter:
ITM: $T = 0.005$
PR: $T_{PR} = 0.08$

LIGO-I $\approx$ **Cavity enhanced MI x** $2\sqrt{T_{PR}}$

PR-filters the pointing

LIGO-I Configuration (C)

Common Tilt:



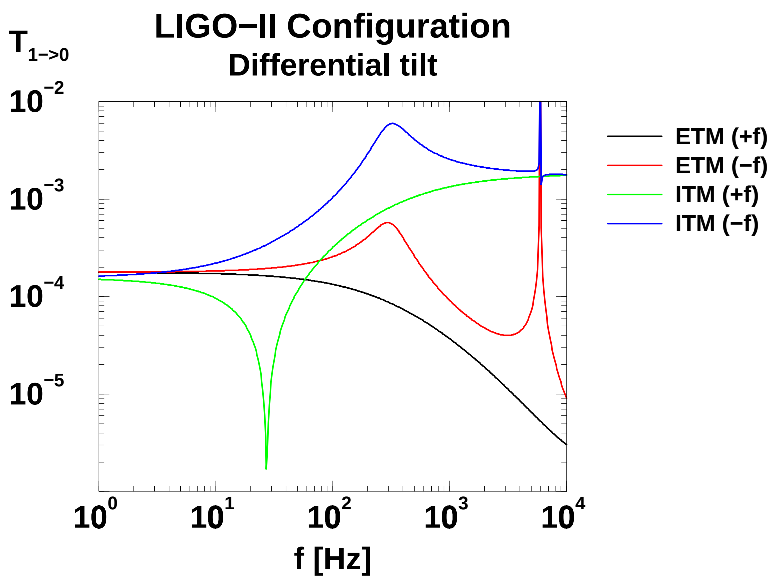
$T_{Comm} = T_{Diff} \cdot$ **Asymm.** $\qquad f < f_c \qquad T_{DC} \approx 7000 \cdot T_{RF}$

here: Schnupp $\qquad f > f_c \qquad T_{DC} \approx T_{RF}$

LIGO-II CONFIGURATION (D)

Dual Recycled Cavity enhanced Michelson interferometer

Differential Tilt:



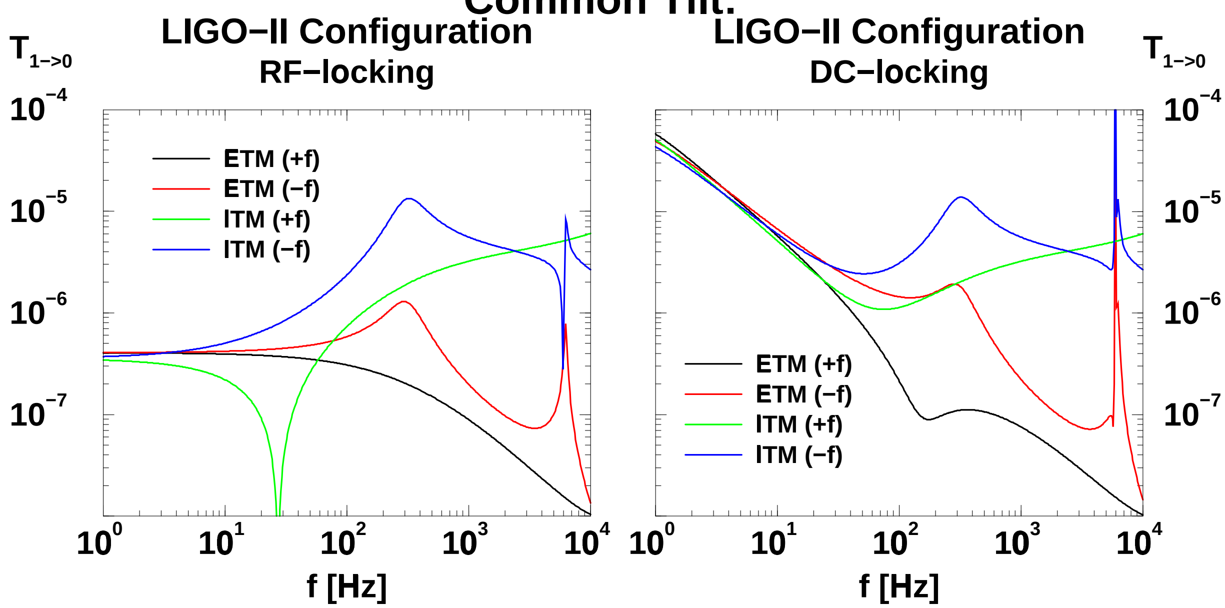
- **low frequencies** $f < 100Hz$**: SR-mirror filters noise**
- **above 100Hz: SR-cavity amplifies noise**

In General:

Like simple Cavity in reflection filtered by PR-mirror times Signal Recycling Transferfunction

Dual Recycled Cavity enhanced Michelson interferometer

Common Tilt:



$T_{Comm} = T_{Diff} \cdot$ **Asymm.**
here: Schnupp

$f < f_{cP} \qquad T_{DC} \approx 500 \cdot T_{RF}$
$f > f_c \qquad T_{DC} \approx T_{RF}$

Common Tilt in DC-locking:

- **low frequencies: gain of PR-cavity boosts pointing signal**

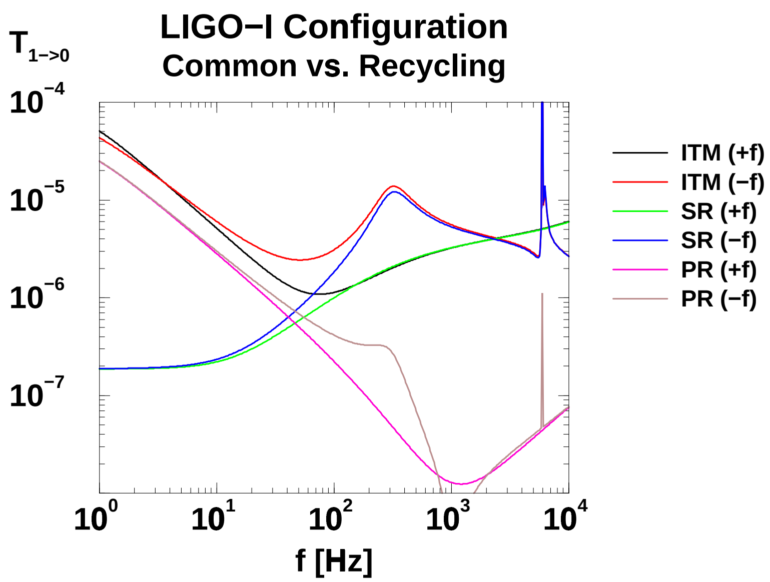
Summary:

- **Disadvantages for DC-locking at low frequencies**
- **Still less problematic than differential tilt at all frequencies**

Remark:
Additional asymmetries will boost Common Signals in RF-locking scheme.

RECYCLING MIRRORS

What happens if the recycling mirrors are tilted ?



PR-tilt:

- **at low frequencies: factor 1- 2 smaller than Common ITM tilt**
- **strong roll off to higher frequencies**

SR-tilt:

- **at low frequencies: far below Common ITM**
- **reaches Common ITM transferfunction at peak frequency**

Comments on this (not backed up by any analysis):

- **PR-mirror not a probem**
- **SR-mirror could be, will be hard to detect SR-tilt**

EQUIVALENT SHOT NOISE LIMIT

Detected Pointing Signal:

$$S_p(f) = E_{LO}^* \left(E_p(+f)e^{i2\pi ft} + E_p(-f)e^{-i2\pi ft}\right) + c.c.$$
$$= \Re\left[E_{LO}^* \left(T_{1->0}(+f)e^{i2\pi ft} + T_{1->0}(-f)e^{-i2\pi ft}\right) E_1(f)\right]$$

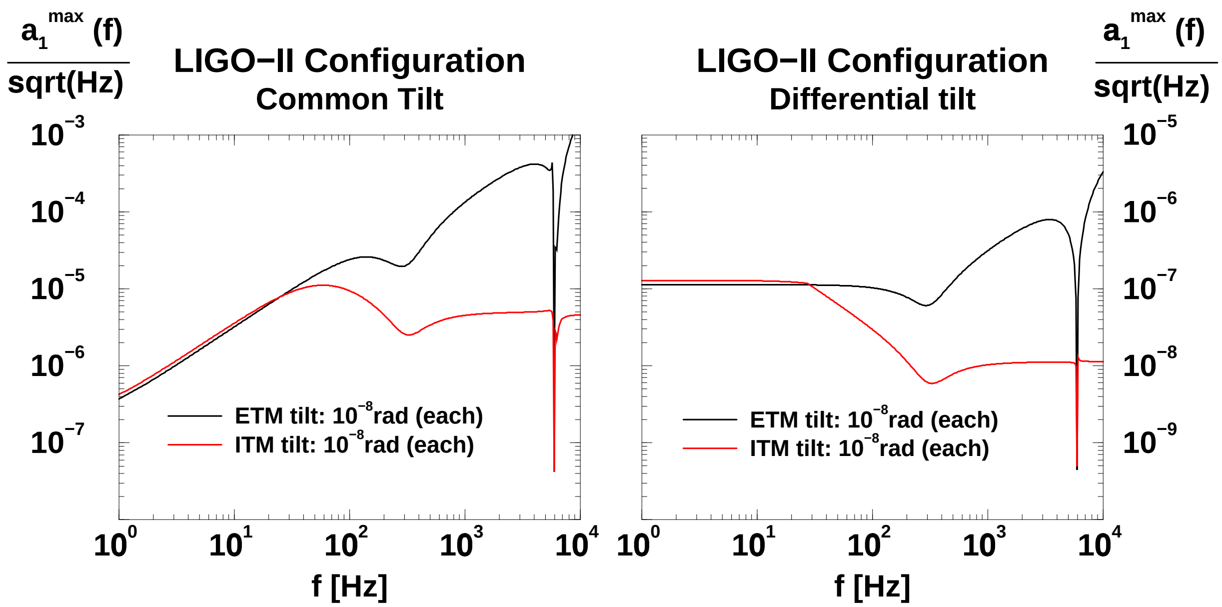
$$E_1(f) = a_1(f)\sqrt{n_{in}}, \qquad a_1(f) = x(f) + i\alpha(f)$$

Don't know the phase (?). Assume worst case:

$$S_p(f) \leq |E_{LO}| \left(|T_{1->0}(+f)| + |T_{1->0}(-f)|\right) |a_1(f)||E_{in}|$$

equivalent Shot Noise Limit (P=125W, $n_{in} = 6.3 \cdot 10^{20}/s$)

$$|a_1(f)| \leq \frac{1}{\left(|T_{1->0}(f)| + |T_{1->0}(-f)|\right)\sqrt{n_{in}}}$$



MODE CLEANING

The limit for differential ITM (the most crucial) tilt:

$$|a_1(30Hz)| < \frac{1.0 \cdot 10^{-7}}{\sqrt{Hz}} \frac{[10^{-8}]}{\Theta_D^{ITM}} \sqrt{\frac{[125W]}{P_{in}}}$$

$$|a_1(100Hz)| < \frac{3.4 \cdot 10^{-8}}{\sqrt{Hz}} \frac{[10^{-8}]}{\Theta_D^{ITM}} \sqrt{\frac{[125W]}{P_{in}}}$$

$$|a_1(300Hz)| < \frac{5.7 \cdot 10^{-9}}{\sqrt{Hz}} \frac{[10^{-8}]}{\Theta_D^{ITM}} \sqrt{\frac{[125W]}{P_{in}}}$$

Both dimensions and PSL specs:

f	$a^{max}_{10/01}(f)[Hz^{-1/2}]$	$a^{PSL}_{10/01}(f)[Hz^{-1/2}]$	a_1^{max}/a_1^{PSL}
30Hz	$5.0 \cdot 10^{-8}$	$4 \cdot 10^{-6}$	$3.8 \cdot 10^{-3}$
100Hz	$1.7 \cdot 10^{-8}$	$4 \cdot 10^{-6}$	$4.3 \cdot 10^{-3}$
300Hz	$2.8 \cdot 10^{-9}$	$4 \cdot 10^{-6}$	$7 \cdot 10^{-4}$

Mode cleaner as a passive filter:
Finesse: 2026, g-factor 0.407
Amplitude Transmission $TEM_{10} < 10^{-3}$

- **Leaves no safety factor (compared to 10 required for LIGO I).**
- **Requires that pointing in the GW-band is dominated by PSL.**

Not acceptable.
Three solutions:

- **active steering system**
- **lower offsets in mirror tilts**
- **additional (low finesse) mode cleaner**

LIGO-PARAMETER/LITERATURE

Parameter:

Arm Cavities: 4 km
$T_{ITM} = 0.005$
Rayleigh Range: 2000 m
inline short MI: 3.21 m
PR-arm: 5.34 m
$T_{PR} = 0.08$

DC-detuning: $\pm 10^{-11} m$
T_{ETM}**=100ppm**
Gouy phase: 0.5 rad
outline short MI: 2.79 m
SR-arm: 6.134 m
$T_{SR} = 0.06$

SR-detuning: 87.4deg from SR

beamsize: 4 cm
only scales the mixing angles of the tilted mirrors.

Literature:

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3. **Yaron Hefetz, N. Mavalvala, D. Sigg** *Principles of Calculating Alignment Signals in Complex Resonant Optical Interferometers* **LIGO-P960024-A-D, November 1996**
4. **P. Fritschel, N. Mavalvala, D. Shoemakher, D. Sigg, M. Zucker, G. Gonzales** *Alignment of an interferometric gravitational wave detectoor*, **Applied Optics, 37(28), pg. 6734-6747 (1998)**
5. **D. Sigg** *Modal model update 1..7(8)*, **LIGO-T***-D (1996-1997)**