Looking for Periodic Sources with LIGO

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The Physics

- The best candidates for periodic sources of gravitational waves are galactic neutron stars with a non-axisymmetric deformation
- The GW are emitted at twice the star rotation frequency
- With enough observation time, LIGO should be able to see stars with a deformation of order 10^{-6} at 1 kpc
- Such a deformation is not impossible, but it's near the upper end of the range of theoretical predictions from neutron stars crust models

The Physics

- On the other hand, accreting stars are very likely to be deformed enough to be copious emitters of GW; however, the data analysis is very hard!
- Isolated neutron stars are not the most likely to emit GW with enough strength to be visible on Earth, but at this time they are the only stars for which an all-sky blind search is possible.

The Signal

- In the source rest frame, the signal is only approximately monochromatic: the star spins down because angular momentum is lost to gravitational waves
- In the detector rest frame, the signal is modulated by Doppler shifts from the Earth's rotation and orbit around the Sun
  - Earth's rotation: $\Delta f \sim 10^{-6} f$
  - Earth's orbit: $\Delta f \sim 10^{-4} f$
- For f = 100 Hz, the signal is spread over 10^5 frequency bins for 4 months of observation.

The Signal

- Mathematically, the phase of the signal can be modeled as

$$\varphi(t) = 2\pi f(t)[t + M_1(t) + M_2(t)]$$

- The modulation functions are periodic in time, i.e.

$$M_1(t) = M_1(t + d)$$

$$M_2(t) = M_2(t + yr)$$

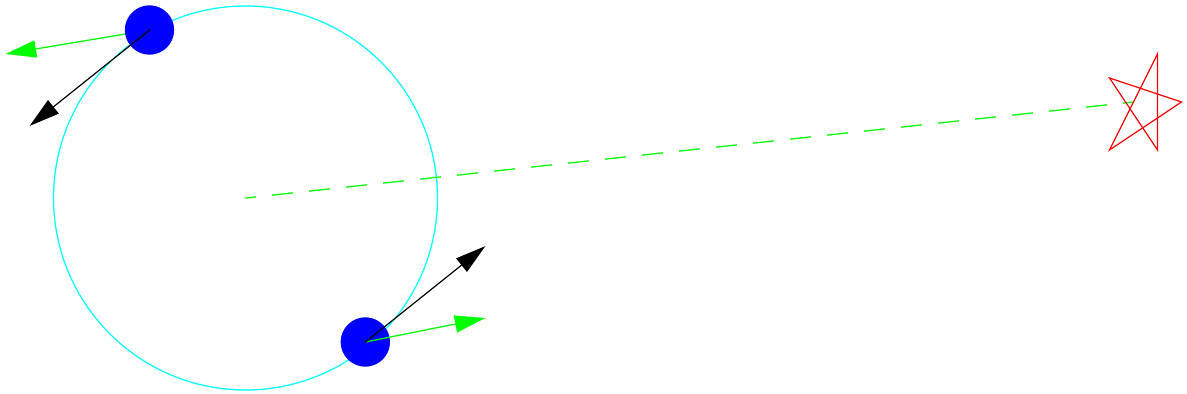
- They also obey (approximately for M_2)

$$M_1(t) = -M_1\left(t + \frac{d}{2}\right)$$

$$M_2(t) \cong -M_2\left(t + \frac{yr}{2}\right)$$

The Signal

- Geometrically, what this is saying is that for any position on the sky, the Doppler phase shift for the Earth's rotation (orbit) is equal and opposite at any two moments separated by 12 hours (6 months)



The Trick

- If we could add the phase of time shifted signals, we could remove the Doppler modulations independently of the sky position!

$$\psi(t) \equiv \varphi(t) + \varphi\left(t + \frac{d}{2}\right) = 2\pi f\left[2t + \frac{d}{2} + M_2(t) + M_2\left(t + \frac{d}{2}\right)\right]$$

$$\Lambda(t) \equiv \psi(t) + \psi\left(t + \frac{yr}{2}\right) = 2\pi f[4t + d + yr]$$

- In practice, the frequency is not constant, but the method still works well:

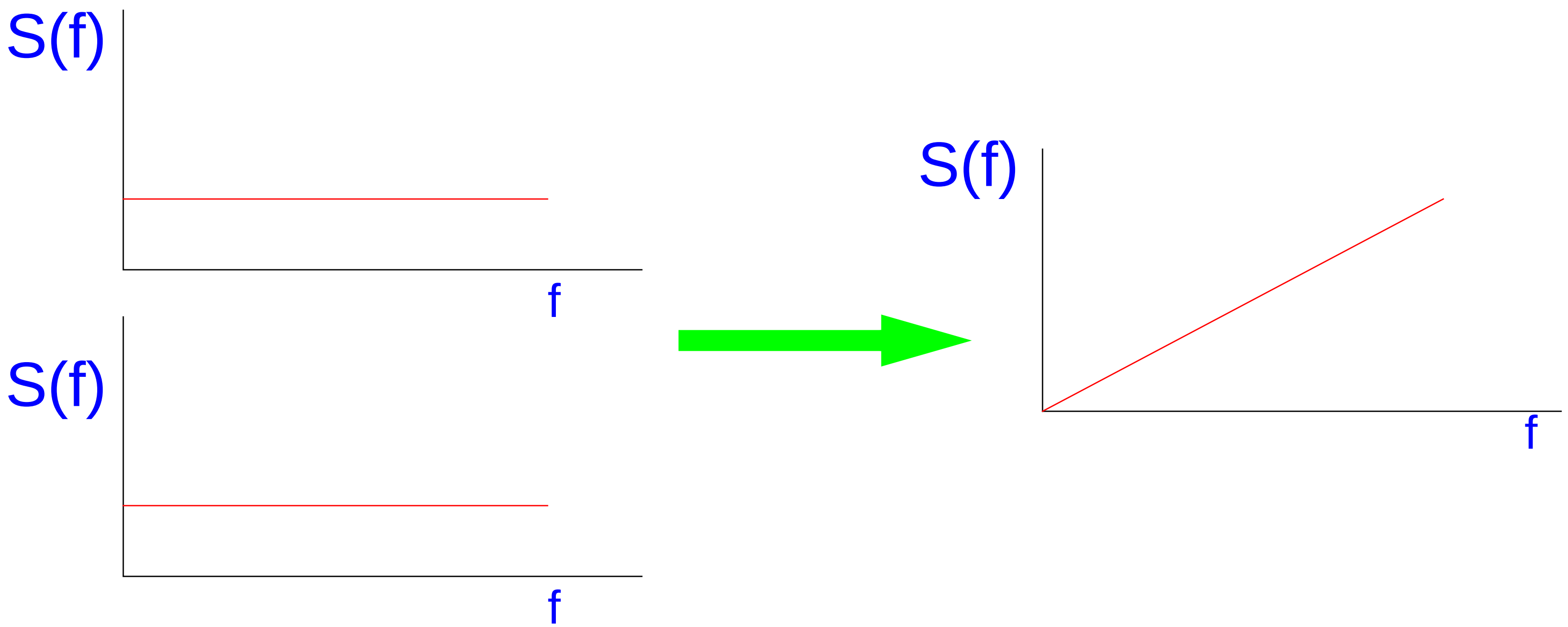
$$\frac{\Lambda(t)}{2\pi} = f(t)t + f(t_1)t_1 + f(t_2)t_2 + f(t_3)t_3 + M_1(t)[f(t) - f(t_1)] + \ldots$$

The Trick

- Multiplication is a natural way to add phases
- For $x(t) = h\,cos(\varphi(t))$, the quadrature $h\,sin(\varphi(t))$ is formed using the Hilbert transform:
  1. Fourier transform x
  2. Replace negative frequency region by negative of positive frequency region
  3. Inverse Fourier transform back to time domain
- The signal and its quadrature can be combined into a complex exponential
- Addition of phases becomes trivial

The Catch

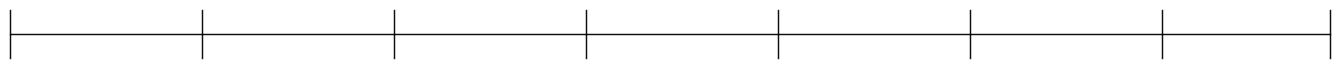
- Multiplying broadband random processes is rarely a very good idea...



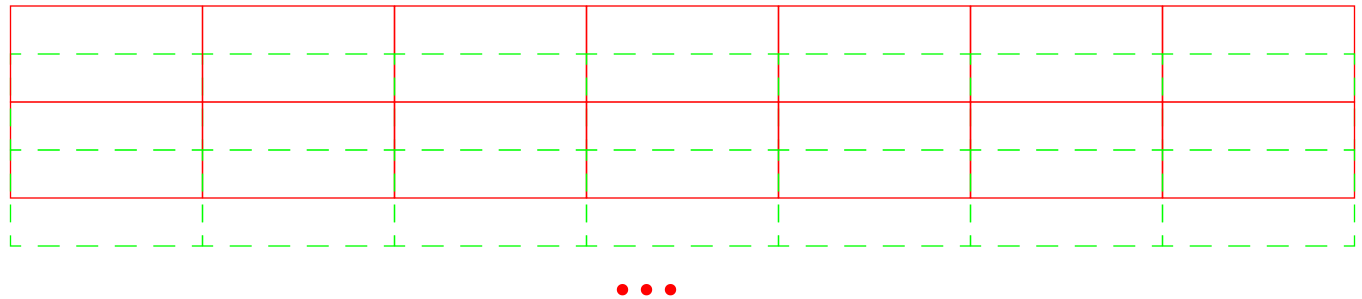
- Need to apply a bandpass filter. This breaks the symmetry by requiring some knowledge of the source position.

The Algorithm

1. Split the data into a set of segments of length T

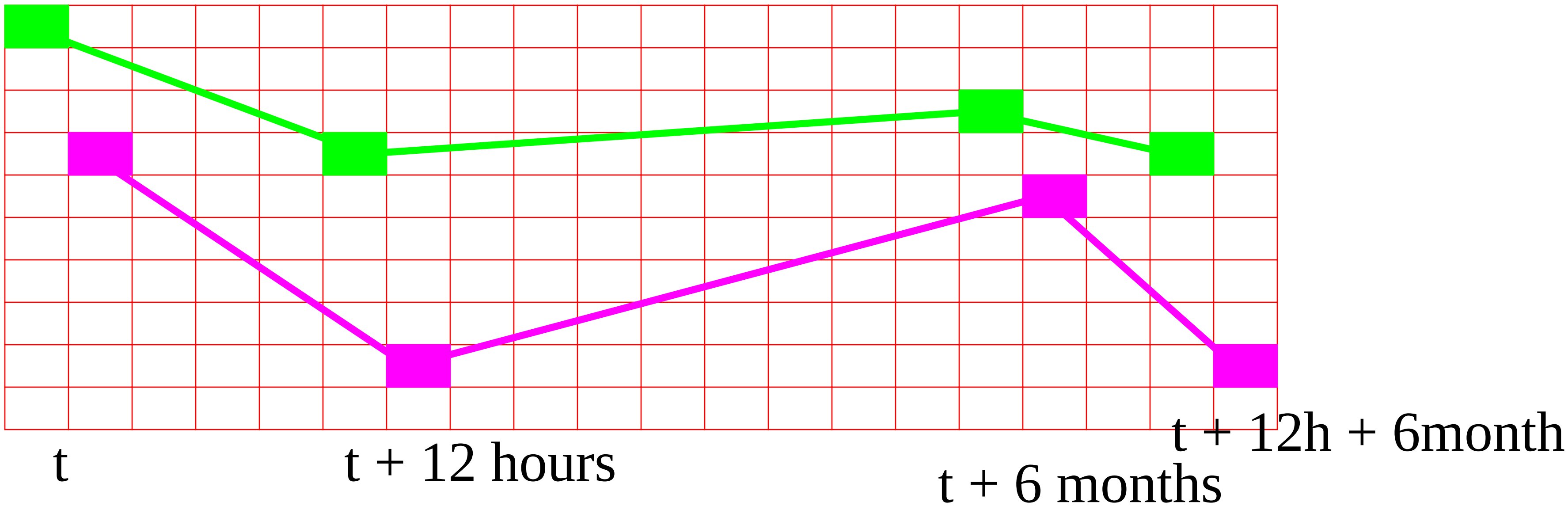


2. Filter every segment with a bandwidth B, for frequency intervals overlapping by B/2



The Algorithm

3. Pick a position on the sky
4. For that position, select the bandpass filtered segments corresponding approximately to the expected frequency and bandwidth of the signal
5. Multiply segments in groups of 4 (times t, t + 12 hours, t + 6 months, t + 12 hours + 6 months)



The Algorithm

6. Pick a spin-down parametrization
7. Resample Doppler-demodulated signal
8. Add spectra incoherently
9. Threshold on the power
10. Goto 6; Goto 3

How many trials?

- To determine the number of star positions to cover the whole sky, it is requested that the position be known with enough precision to select the right filtered segment from the bank
- For the spin-down portion, the number of trials is set by limiting the possible loss in signal-to-noise ratio, using the usual, geometrical approach

The Performances

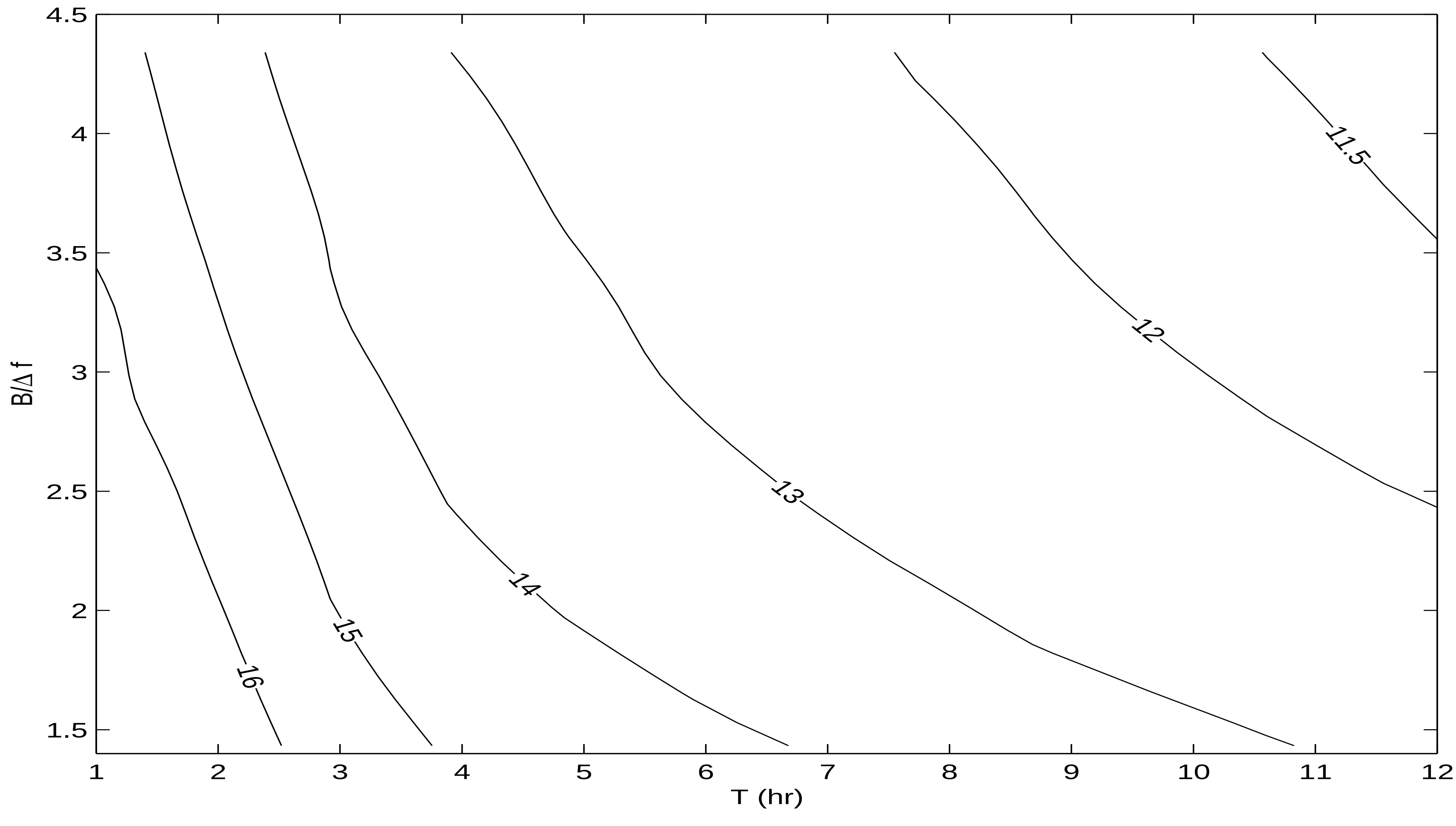
- Lowering the threshold on the power increases the false alarm rate, but decreases the false dismissal rate
- I define the ‘strain sensitivity of the search’ as the minimal detectable strain with false dismissal probability = 1/2
- The ‘optimal amplitude’ , for the case where the source position and spin-down characteristics are known exactly, is h = 4.2 at 99% confidence level (in units of ‘detector strain noise’)
- What I will quote for the sensitivity is the ratio

$$\Theta \equiv \frac{4.2}{\text{strain sensitivity of the search}}$$

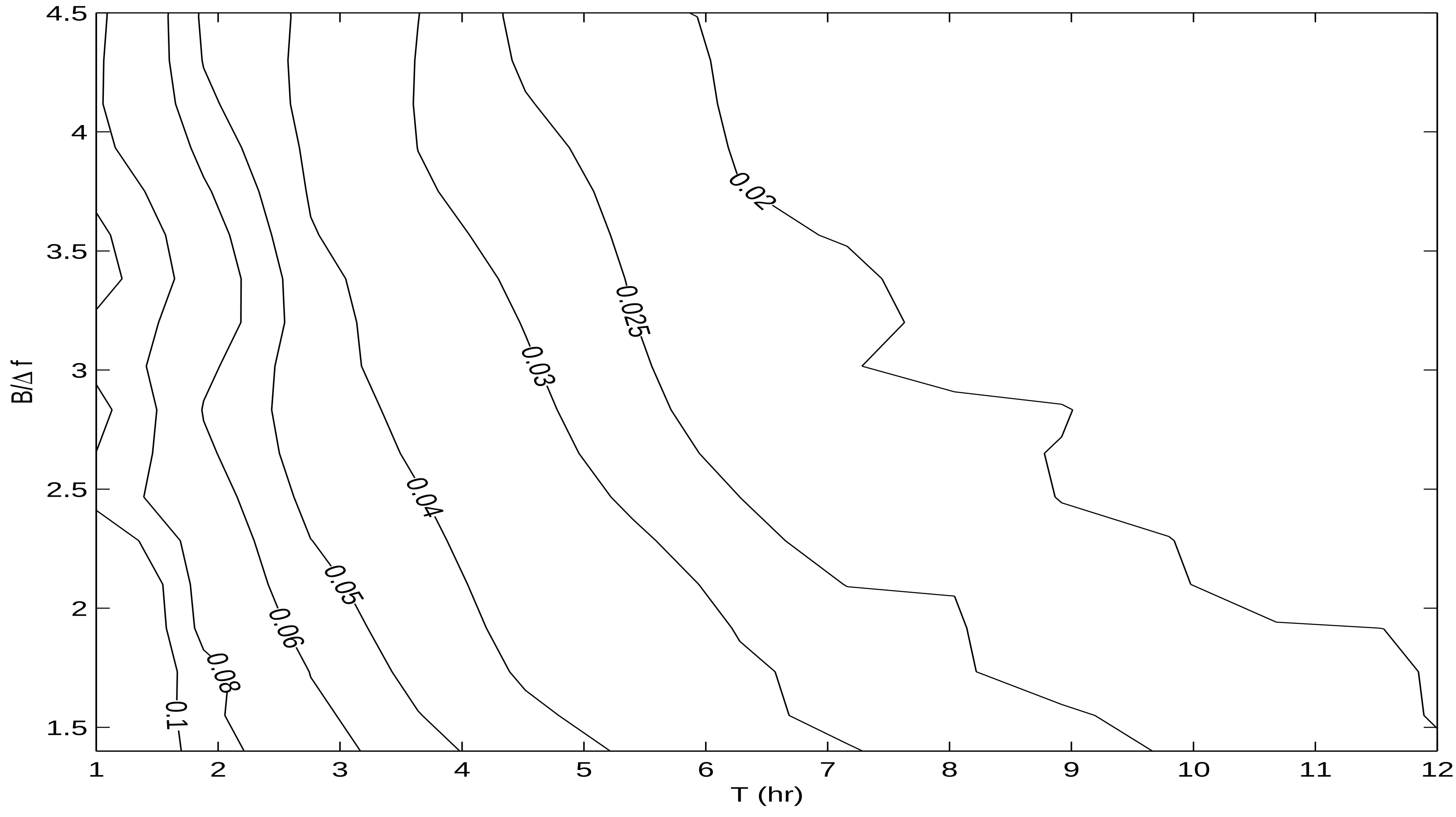
The Simulations

- The following parameters were used for a broadband all-sky search:
  - minimum frequency: 40 Hz
  - maximum frequency: 1 kHz
  - Spin-down time of the star: 5000 yr
  - Observation time: 1 yr
- And for a narrow-band search (e.g. narrow-band LIGO II), I used:
  - minimum frequency: 450 Hz
  - maximum frequency: 500 Hz
  - Spin-down & observation time as above

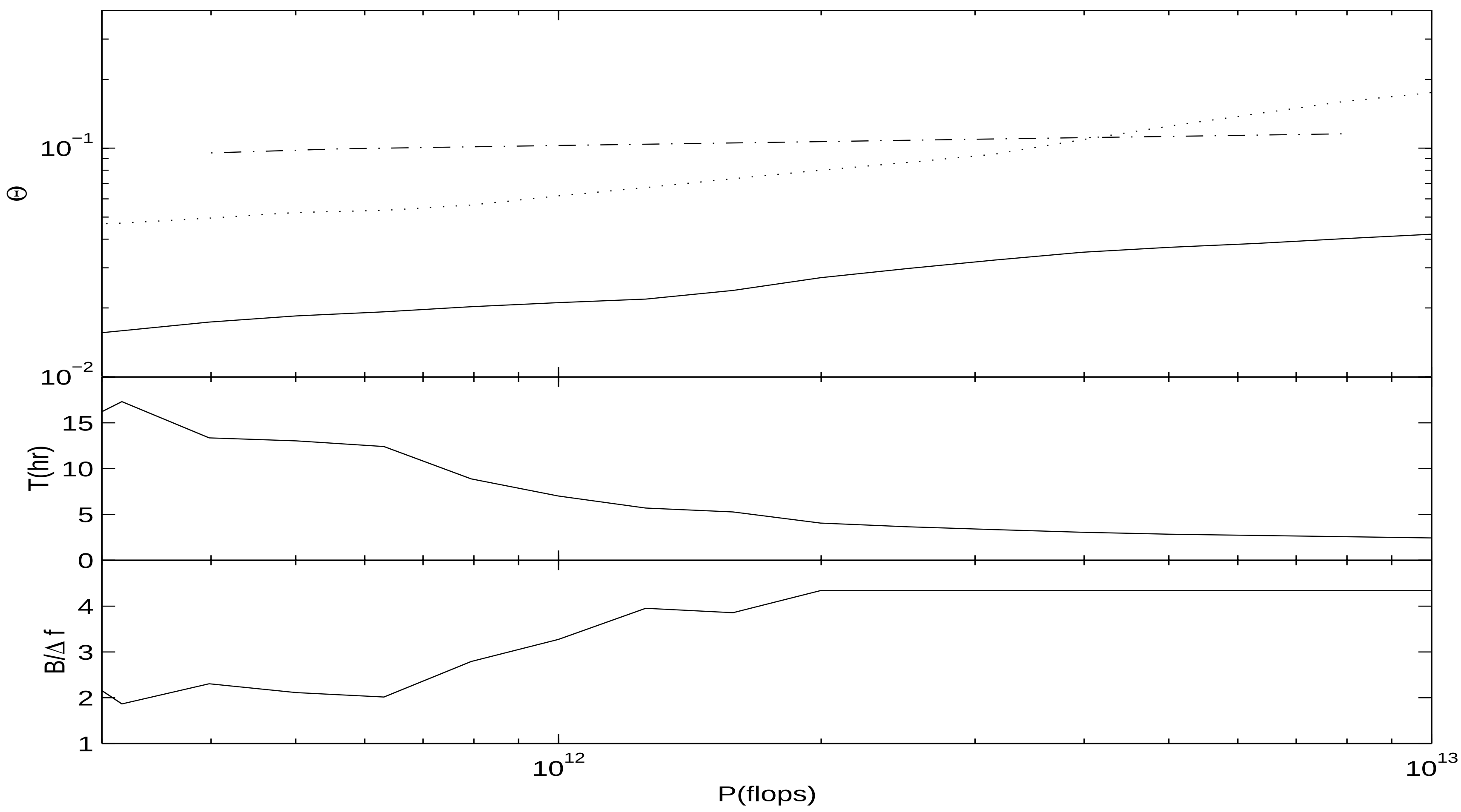
The Simulations



The Simulations



The Simulations



Conclusion

- The algorithm I have presented doesn't work better than the stack-slide algorithm for an all-sky, broad-band search. The bandpass requirements turn out to be too strict.
- It is not entirely clear how other algorithms (e.g. stack-slide) performances scale with the bandwidth of the search. My algorithm applies naturally to bandlimited searches, because it involves a search over frequency. It could be competitive for narrow-band searches.

- For more details, see Phys. Rev. D, **63**, 082004