Numerical Modelling of Solid State Lasers

Talk at Laser working group session,15 Aug 2001

4:30h p.m.

LIGO-G010362-00-Z

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Outline

- Introduction
  - 200 W from a rod system
  - overview on modelling
- Thermal Modelling
  - temperature distribution
  - mechanical stress
  - thermal lensing
  - stress induced birefringence
- Propagation of the electromagnetic field through medium
  - split-step method
  - finite differencing
  - FFT method
  - Fox/Li approach
- Outlook/upcoming tasks

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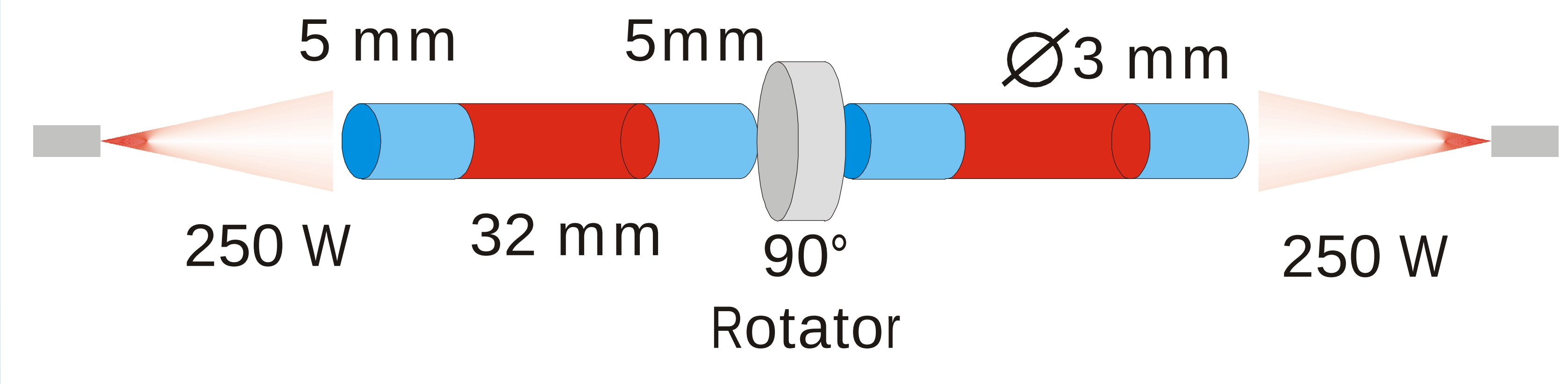
100/200 W from end-pumped rods

- LASER head consists of two rods (diam 3 mm, length 42 mm)



- diffusion bonding weakens thermal lens
- HR(808) coating on non-pumped ends for double pass

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Introduction/Overview

Numerical simulation of pump light distribution

- ray trace analysis
- analytic approximation to pump light distribution

Heat

Finite element modelling of

- Temperature distribution
- mechanical stress

Cooling

gain

Propagation of the laser field through medium

- finite difference method
- split step approach

k-vector

Calculation of thermo-optical properties

- thermal lensing
- stress induced birefringence
- depolarisation

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Pump light distribution

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Pump light distribution

1. Numerical results from ray trace analysis (e. g. ZEMAX-EE)

2. Approximation by analytic functions

   - take wavelength dependence of absorption coefficient and broad pump source spectrum into account

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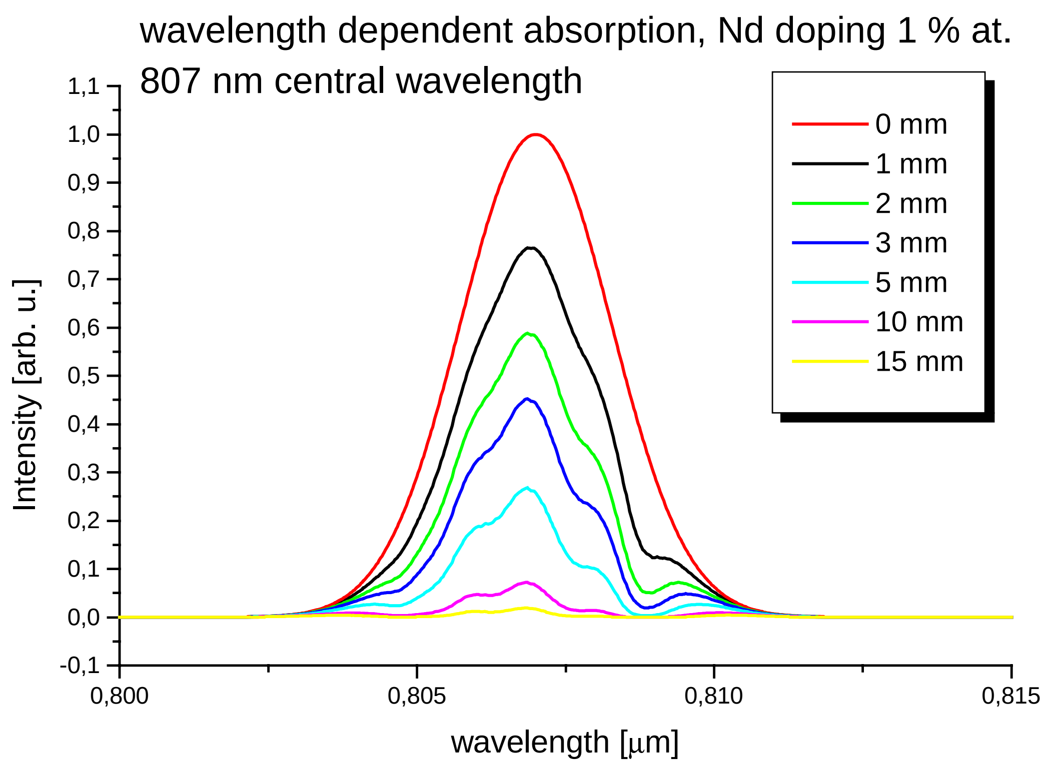
Pump light distribution



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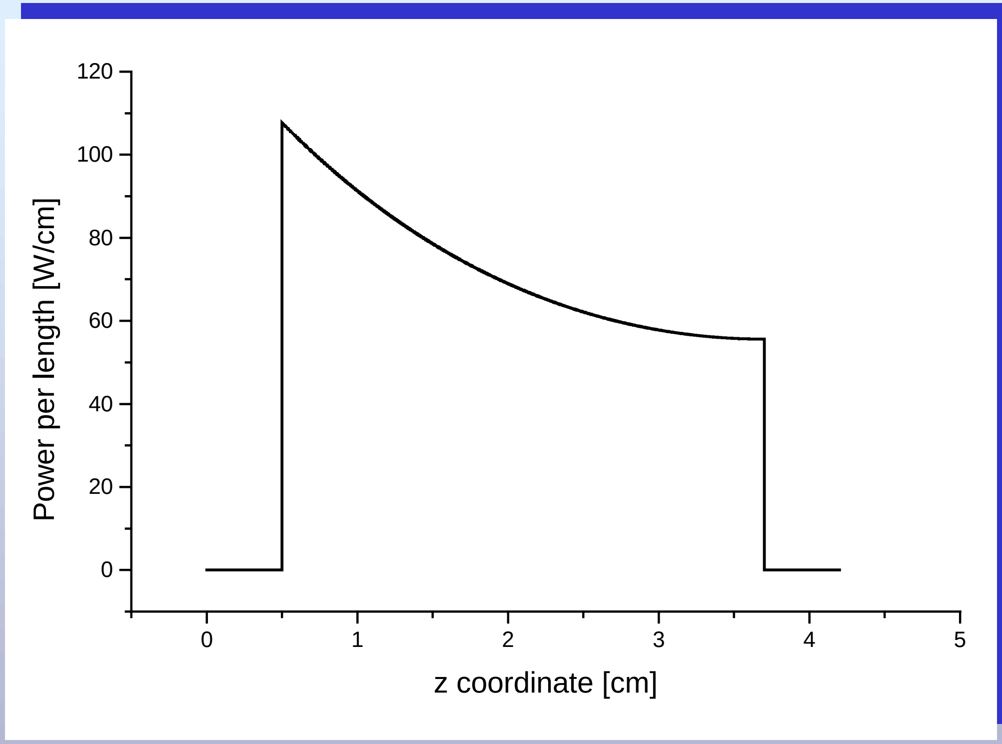
Pump light distribution

- Assume homogenous intensity over cross section



Thermal modelling

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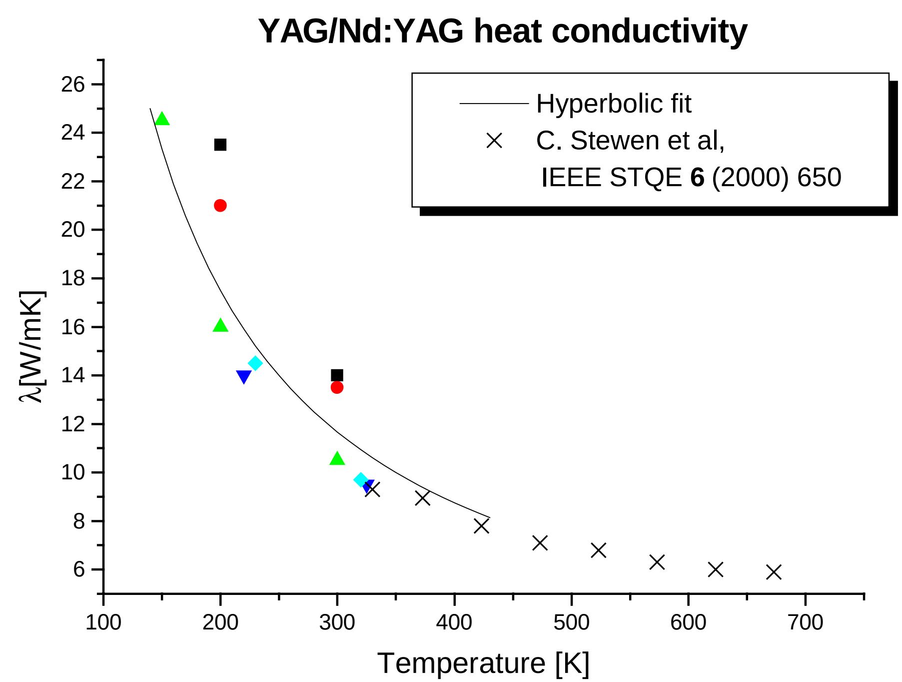
Thermal modelling/Temperature distribution

- Steady-state heat conduction equation

$$\nabla(\lambda(T)\cdot\nabla T(x,y,z))+Q(x,y,z)=0$$

$$Q=\frac{\text{abs. pump power}}{\text{unity volume}}\eta_h \qquad \eta_h=0.4$$



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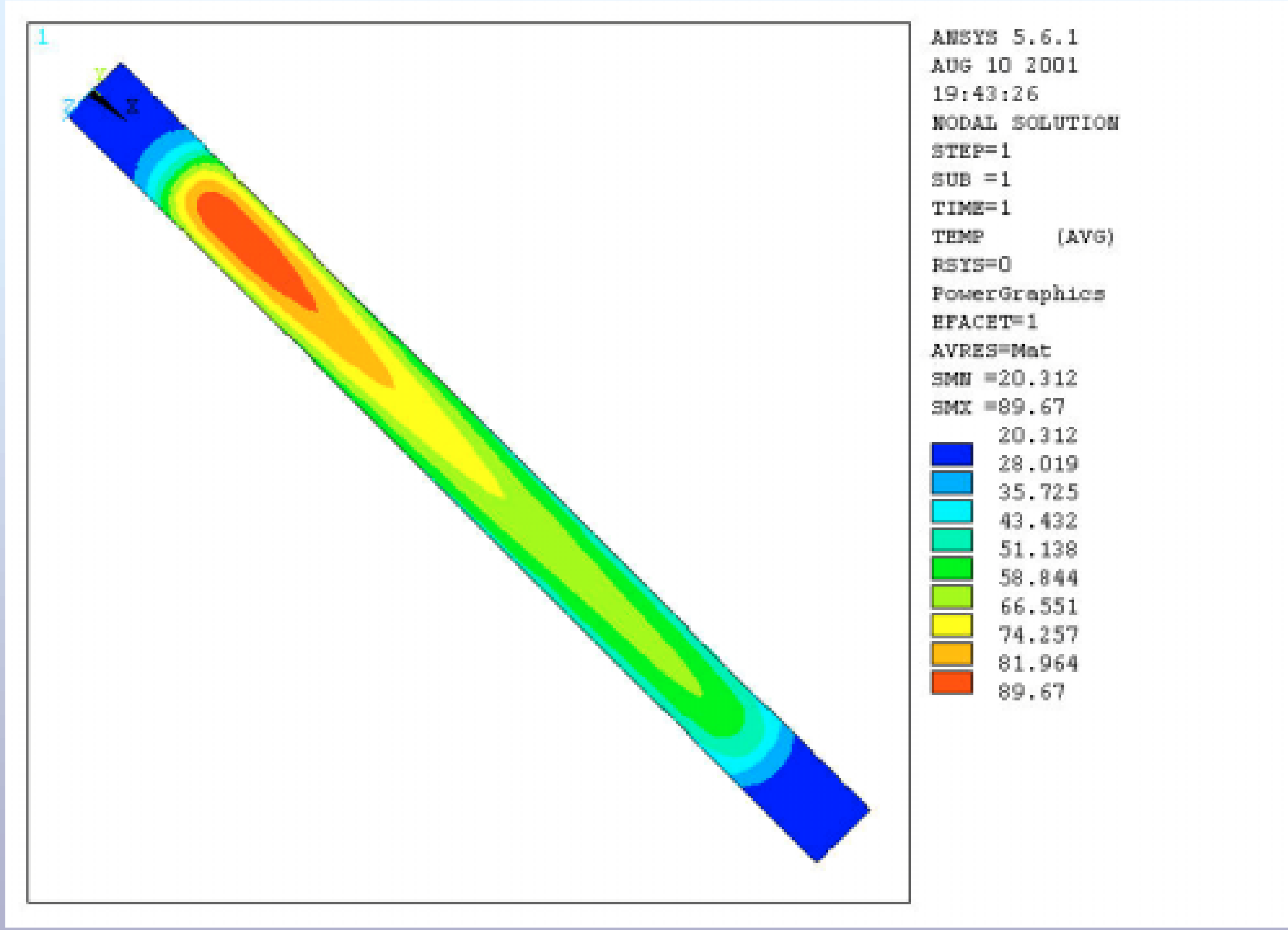
Thermal modelling/Temperatur distribution

- Solution by finite element method (ANSYS)



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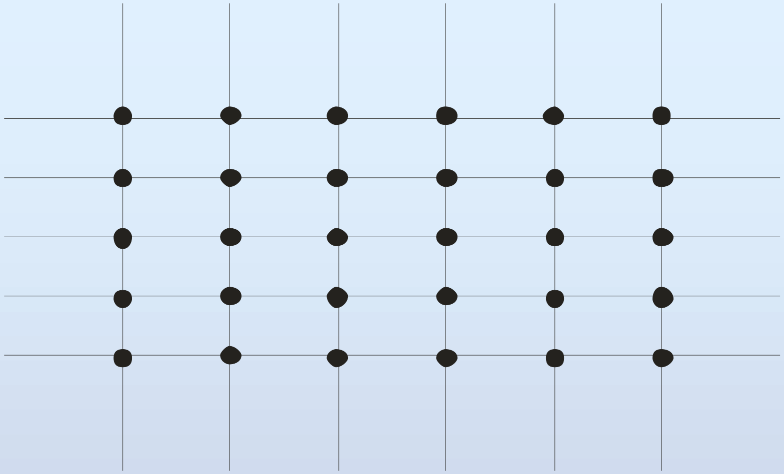
Thermal modelling/Mechanical stresses

•Distortion

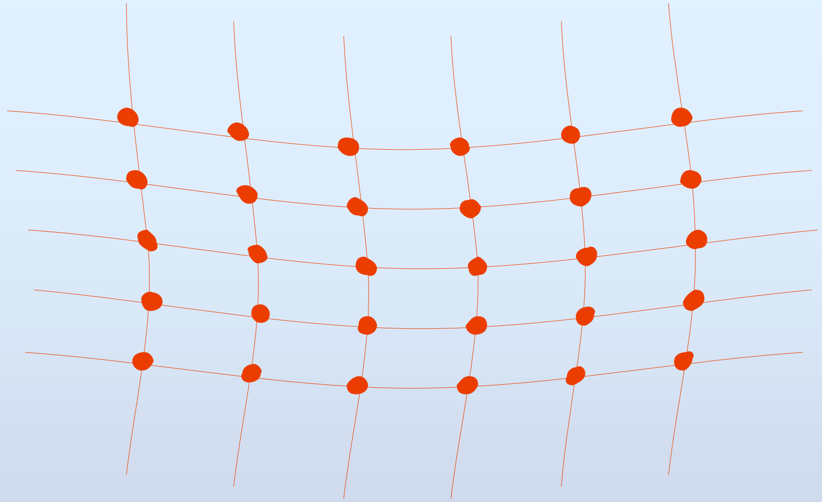
$\vec{r}$



Thermal modelling/Mechanical stresses

- Distortion

$$\vec{r} \rightarrow \vec{r} + \vec{u}(\vec{r})$$



- Displacement tensor

$$\varepsilon_{ij}(\vec{r}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

- Stress tensor

$$\sigma_{ij} = \frac{E}{1+\nu}\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\left(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}\right)\delta_{ij} - \frac{1+\nu}{1-2\nu}\alpha_T(T)T\delta_{ij}\right)$$

α_T temperature dependent

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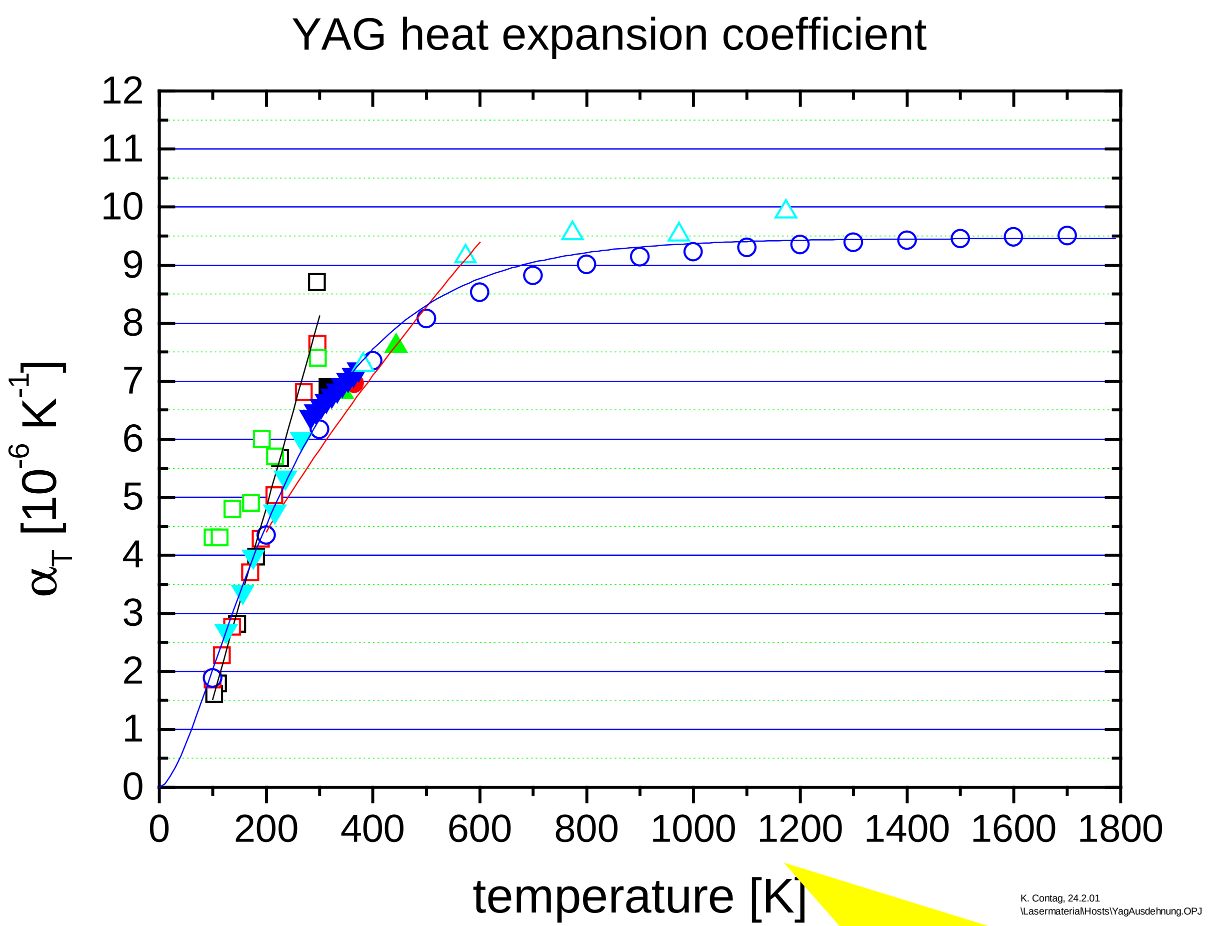
Thermal modelling/mechanical stress



•Data compiled by K. Contag, Stuttgart

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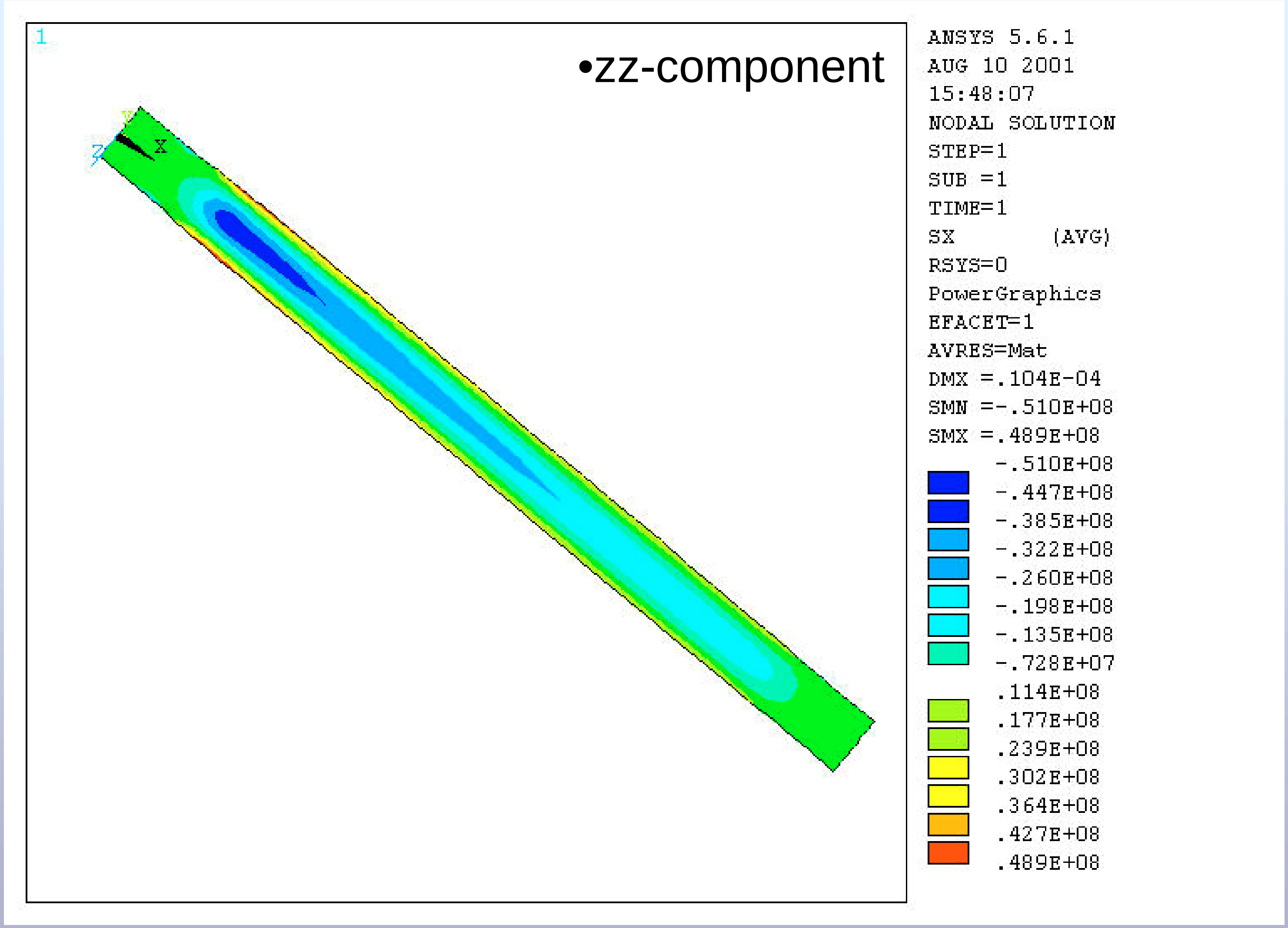
Mechanical stresses



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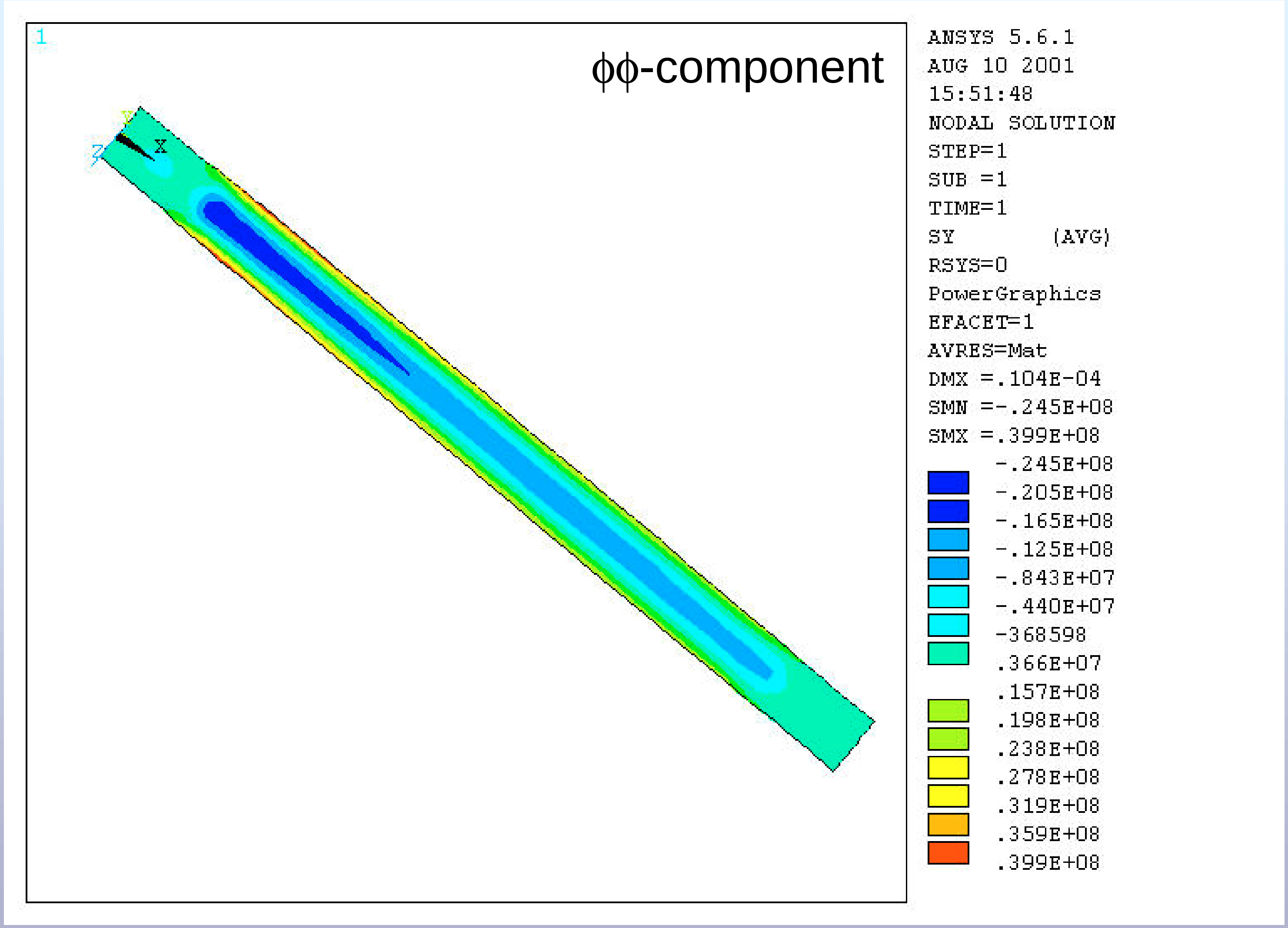
Mechanical stresses



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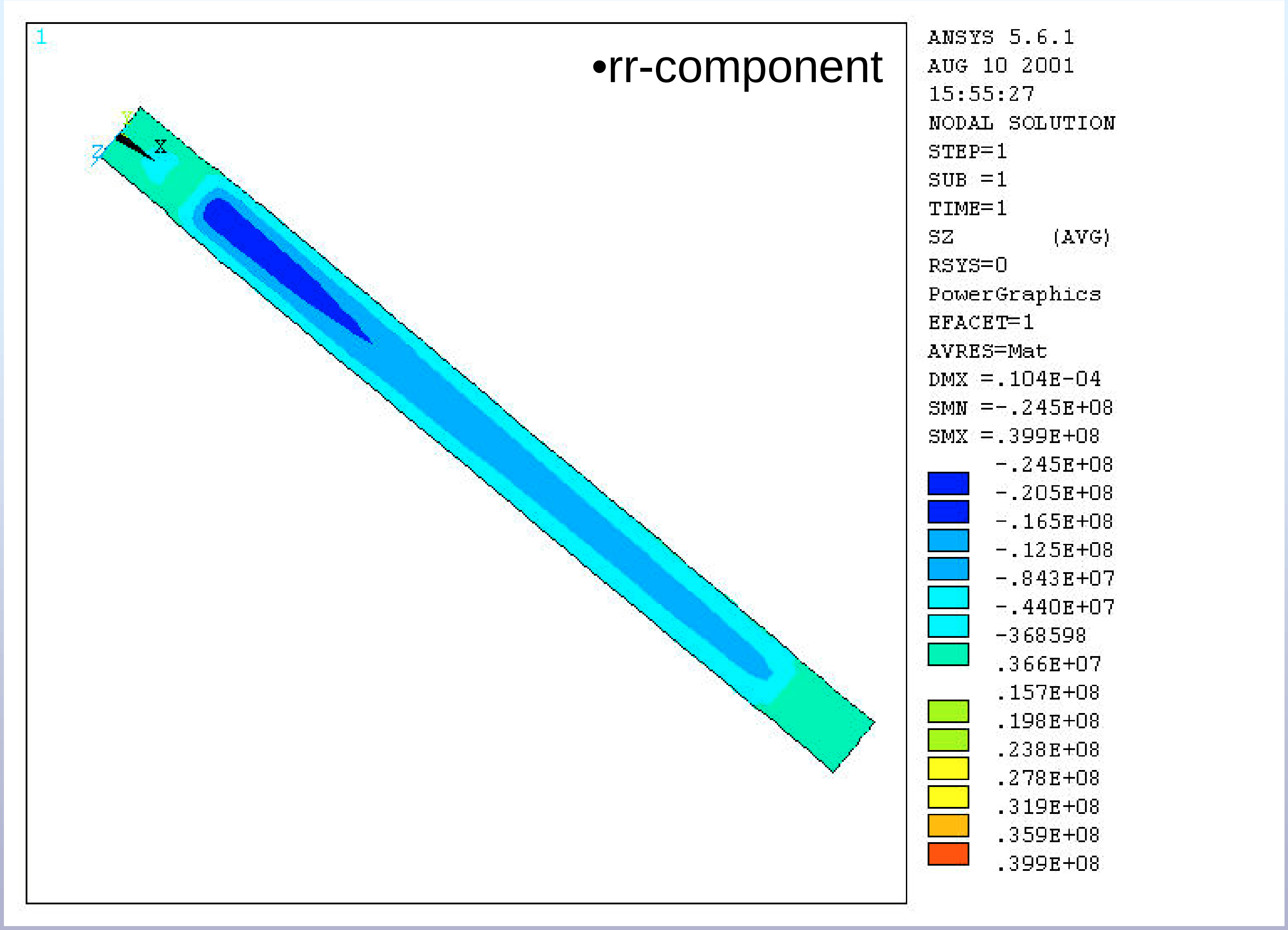
Mechanical stresses



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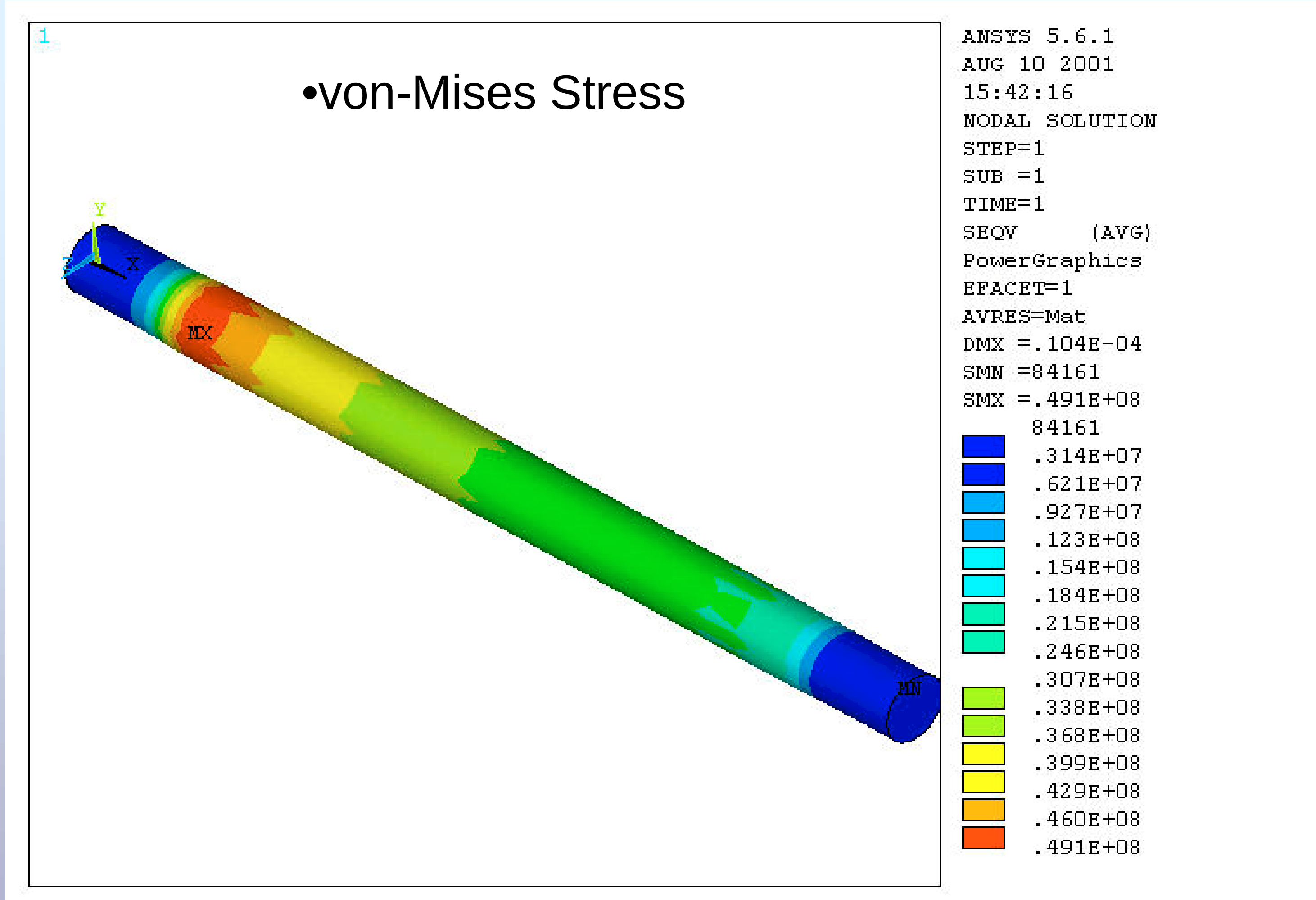
Mechanical stresses



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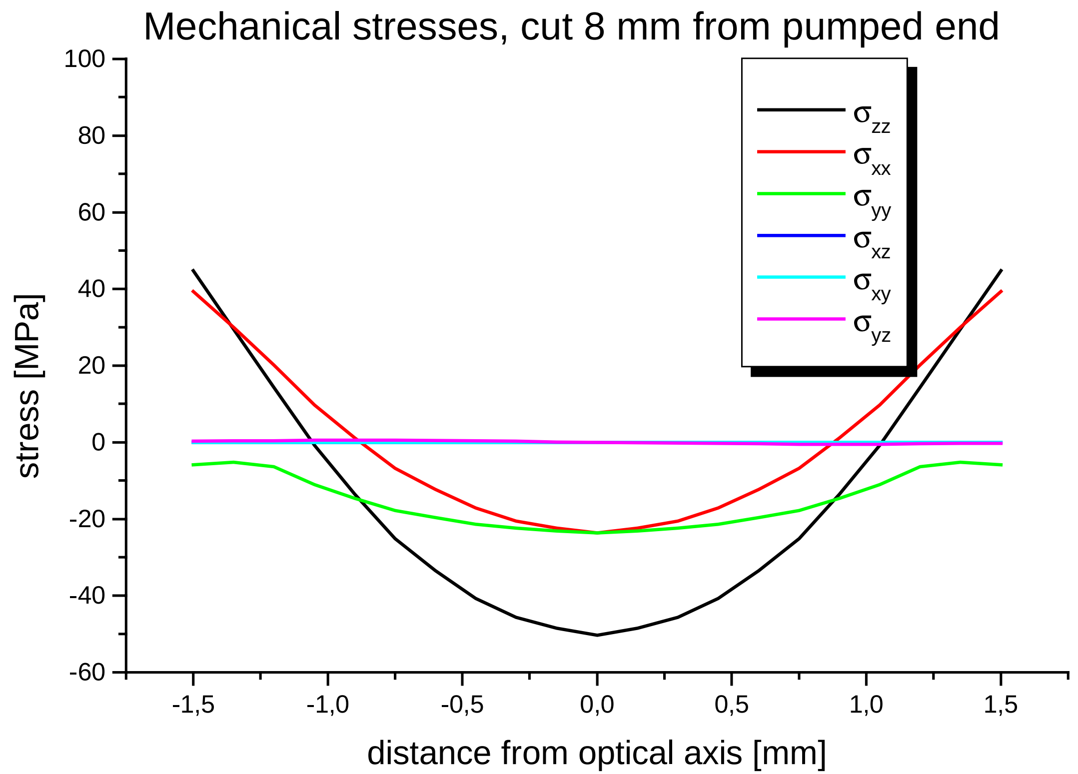
Mechanical stress



Well below fracture limit (130 thru 260 Mpa)

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Optical properties

Numerical simulation of pump light distribution

- ray trace analysis
- analytic approximation to pump light distribution

Heat

Finite element modelling of

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Cooling

gain

Calculation of thermo-optical properties

- thermal lensing
- stress induced birefringence
- depolarisation

k-vector

Propagation of the laser field through medium

- finite difference method
- split step approach

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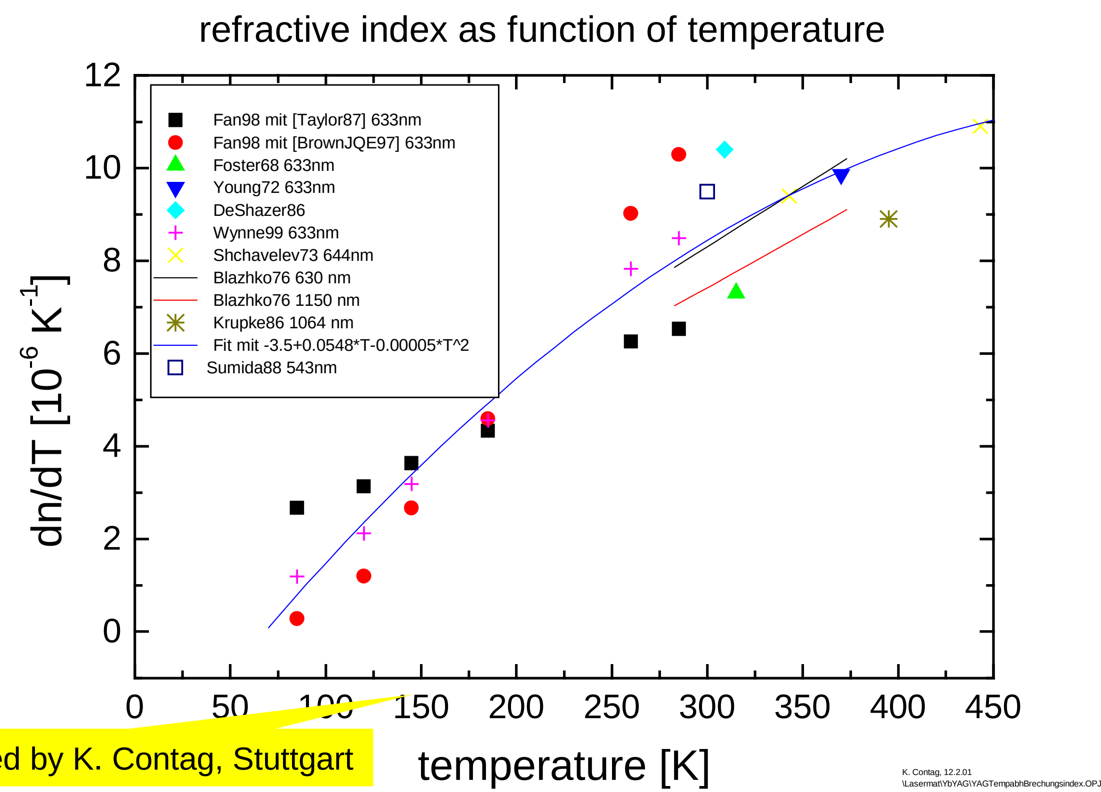
Thermal modelling/optical properties

- Thermal lensing

refractive index $n = n(T) = T \cdot \frac{dn}{dT}(T)$



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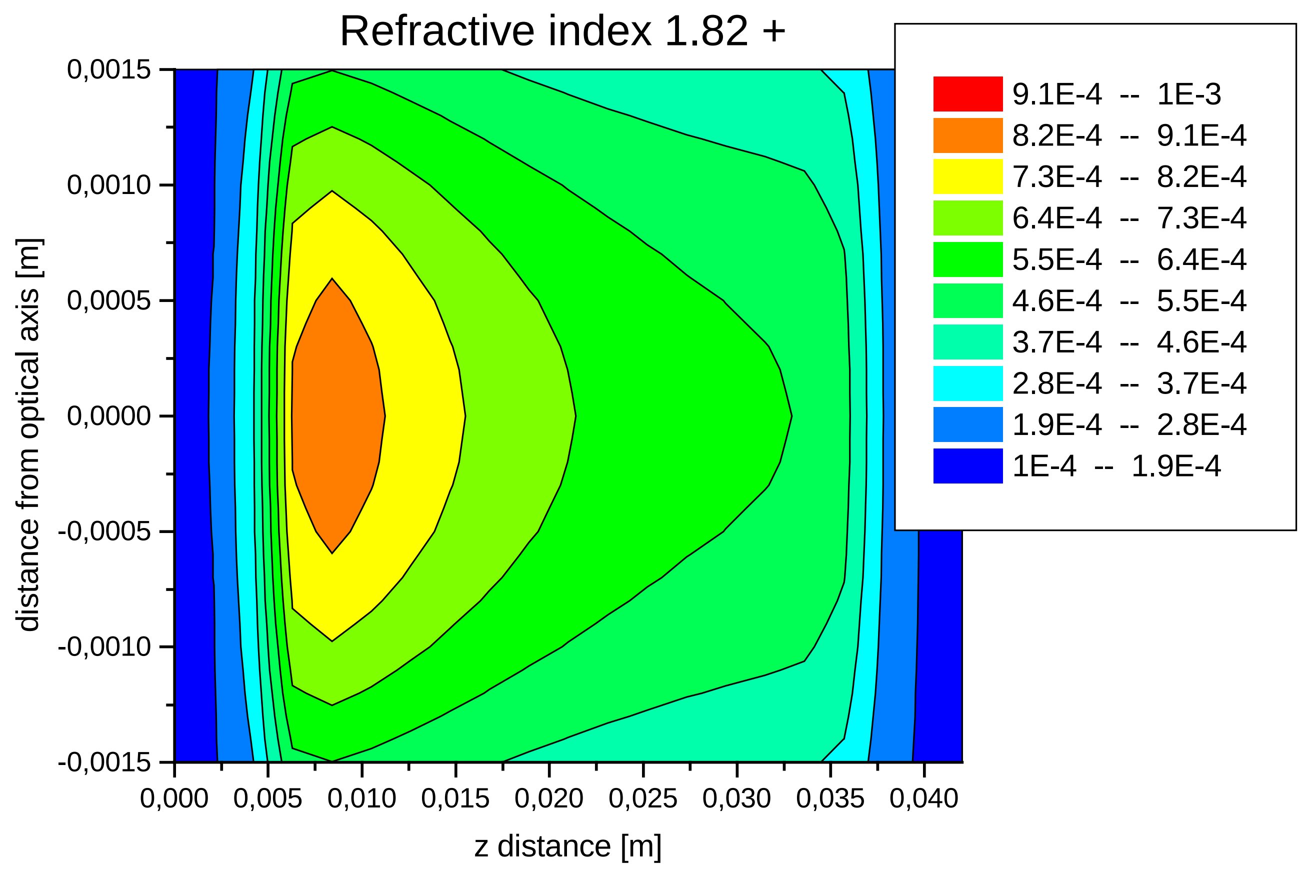
Thermal lensing



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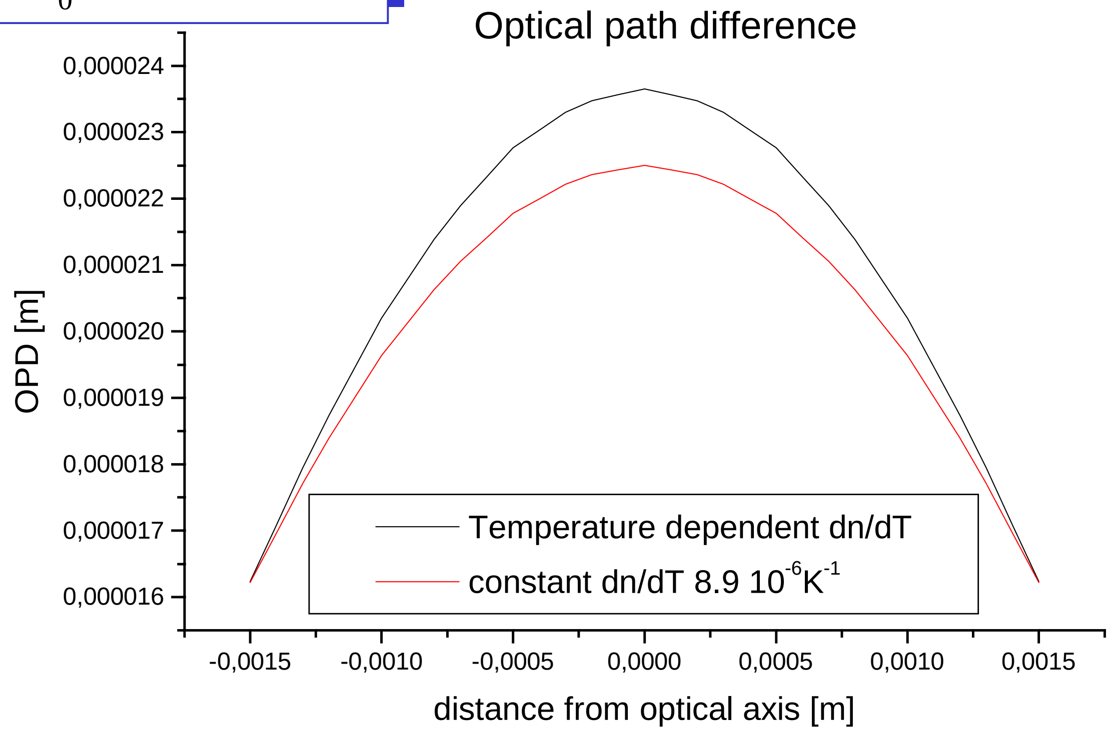
Thermal lensing

$$Opt.Path = \int_0^L dz(n(x,y,z;T))$$



$$D(r) = -\frac{d^2}{dr^2} OPD(r,\phi)$$

•Thermal lens 6.62 dioptries

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Thermal modelling/optical properties

- Stress induced birefringence

Refractive index ellipsoid

$$\sum_{i,j} B_{ij} x_i x_j = 1 \quad \text{principal axis} \quad \sum_i B_i x_i^2 = 1$$

$$B_{ij} = B_{0,ij}(T) + \sum_{k,l} \pi_{ijkl} \sigma_{kl}$$

•YAG: piezo-opt. Tensor has 8 independent coefficients

write as 6 x 6 matrix

$$B_i = B_{0,i}(T) + \sum_{j=1}^{6} \pi_j \sigma_{j,i}$$

•Eigenvalues: refractive indices in principal coordinates

•Eigenvectors: polarisation directions that are not mixed by index ellipsoid

•diagonalize transversal part

$$B_{ortho} = \begin{pmatrix} B_{xx} & B_{xy} \\ B_{xy} & B_{yy} \end{pmatrix}$$

$$B_{1/2} = 1/2\left(\left(B_{xx} + B_{yy}\right) \pm \sqrt{\left(B_{xx} - B_{yy}\right)^2 + 4B_{xy}^2} \right)$$

$$n_{1,2} = \frac{1}{\sqrt{B_{1,2}}}$$

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Thermal modelling/optical properties

- Stress induced b

Refractive index ellipsoid

$$\sum_{i,j} B_{ij} x_i x_j = 1$$ princip

$$B_{ij} = B_{0,ij}(T) + \sum_{k,l} \pi_{ijkl} \sigma_{kl}$$

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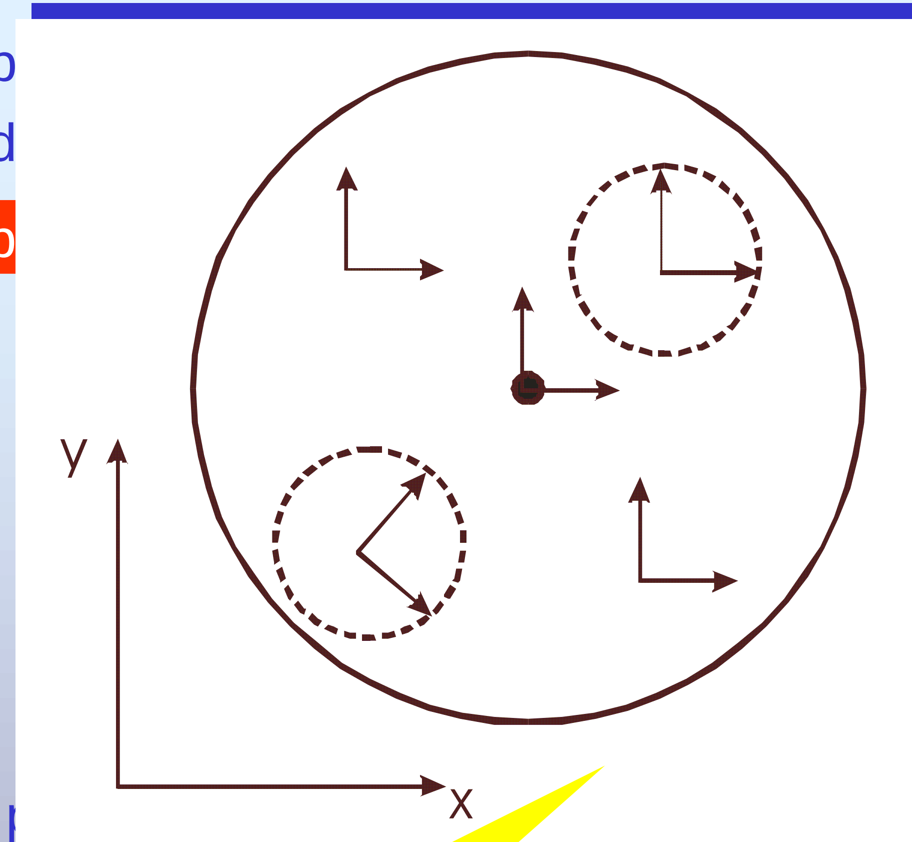
$$n_{1,2} \quad \sqrt{B_{1,2}}$$



•Cubic crystal/no stress

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Thermal modelling/optical properties

• Stress induced b

Refractive index ellipsoid

$$\sum_{i,j} B_{ij} x_i x_j = 1$$ princip

$$B_{ij} = B_{0,ij}(T) + \sum_{k,l} \pi_{ijkl}\sigma_{kl}$$

write as 6 x 6 matrix

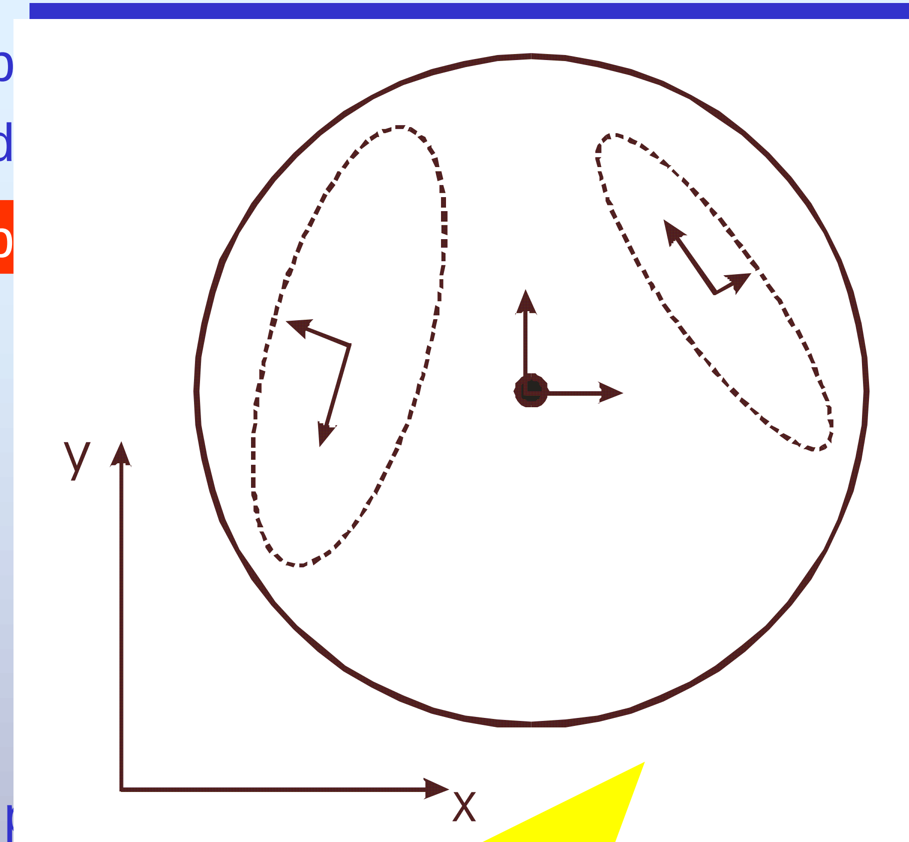
$$B_i = B_{0,i}(T) + \sum_{j=1}^{6} \pi_j \sigma_{j,i}$$

•diagonalize transversal

$$B_{ortho} = \begin{pmatrix} B_{xx} & B_{xy} \\ \dots \end{pmatrix}$$

$$B_{1/2} = 1/2\left(\left(B_{xx} + B_{yy}\right) \pm \sqrt{\dots + 4B_{xy}^2}\right)$$

$\dots_{1,2} \dots \sqrt{B_{1,2}}$



•Cubic crystal with stress induced birefringence

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Thermal modelling/optical properties

- Jones-matrix formalism
- Polarisation eigenstates are not mixed

$$\begin{pmatrix} E_+(x,y,z) \\ E_-(x,y,z) \end{pmatrix} = \begin{pmatrix} e^{ik_0 n_+ z} & 0 \\ 0 & e^{ik_0 n_- z} \end{pmatrix} \begin{pmatrix} E_+(x,y,0) \\ E_-(x,y,0) \end{pmatrix}$$

- x and y Polarised states are no eigenstates

$$\begin{pmatrix} E_x(x,y,z) \\ E_y(x,y,z) \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} e^{ik_0 n_+ z} & 0 \\ 0 & e^{ik_0 n_- z} \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} E_x(x,y,0) \\ E_y(x,y,0) \end{pmatrix}$$

- Turn onto principal axes
- Multiply
- Turn back

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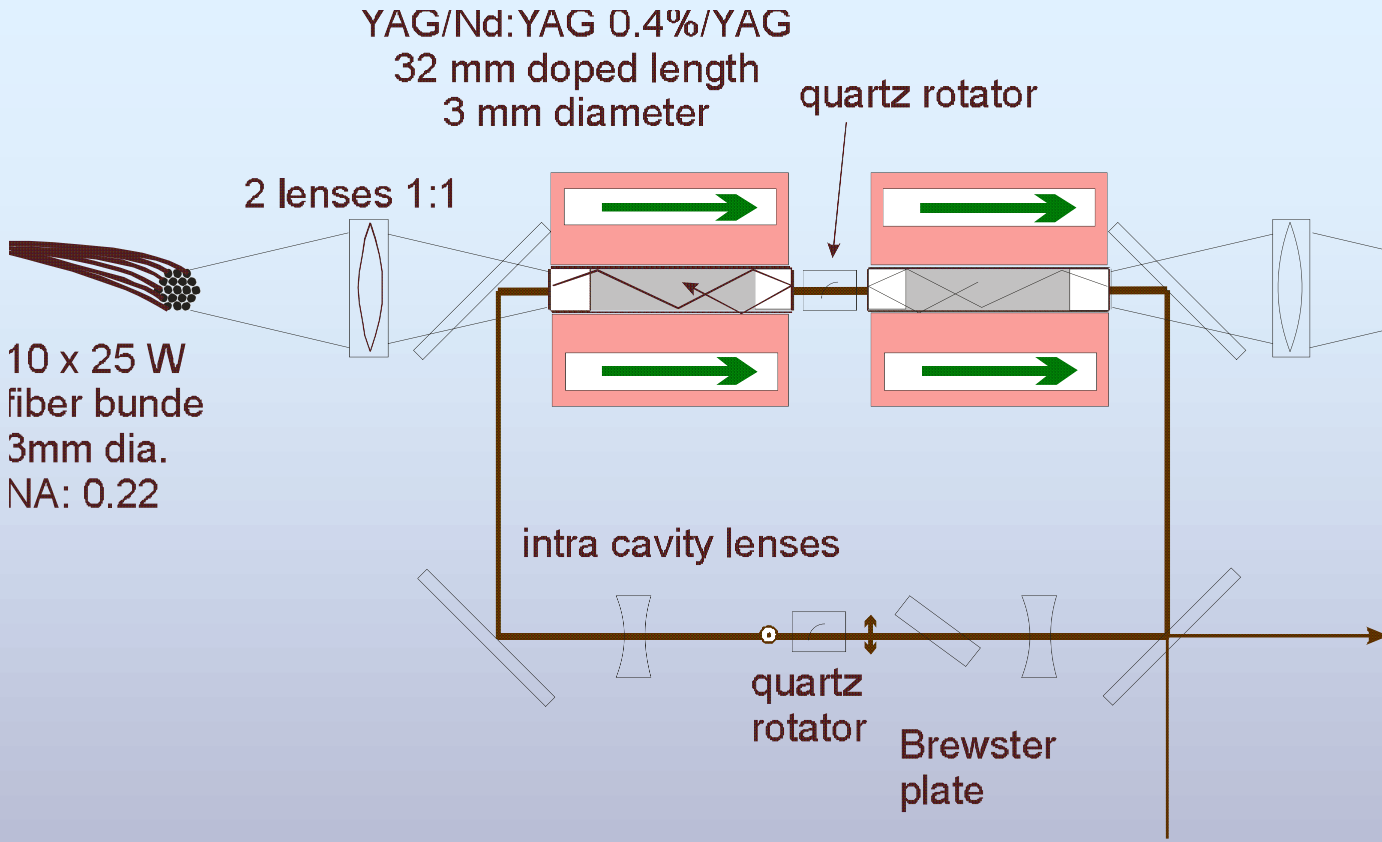
Concept of 100 W Laser



200 W by symmetric duplication

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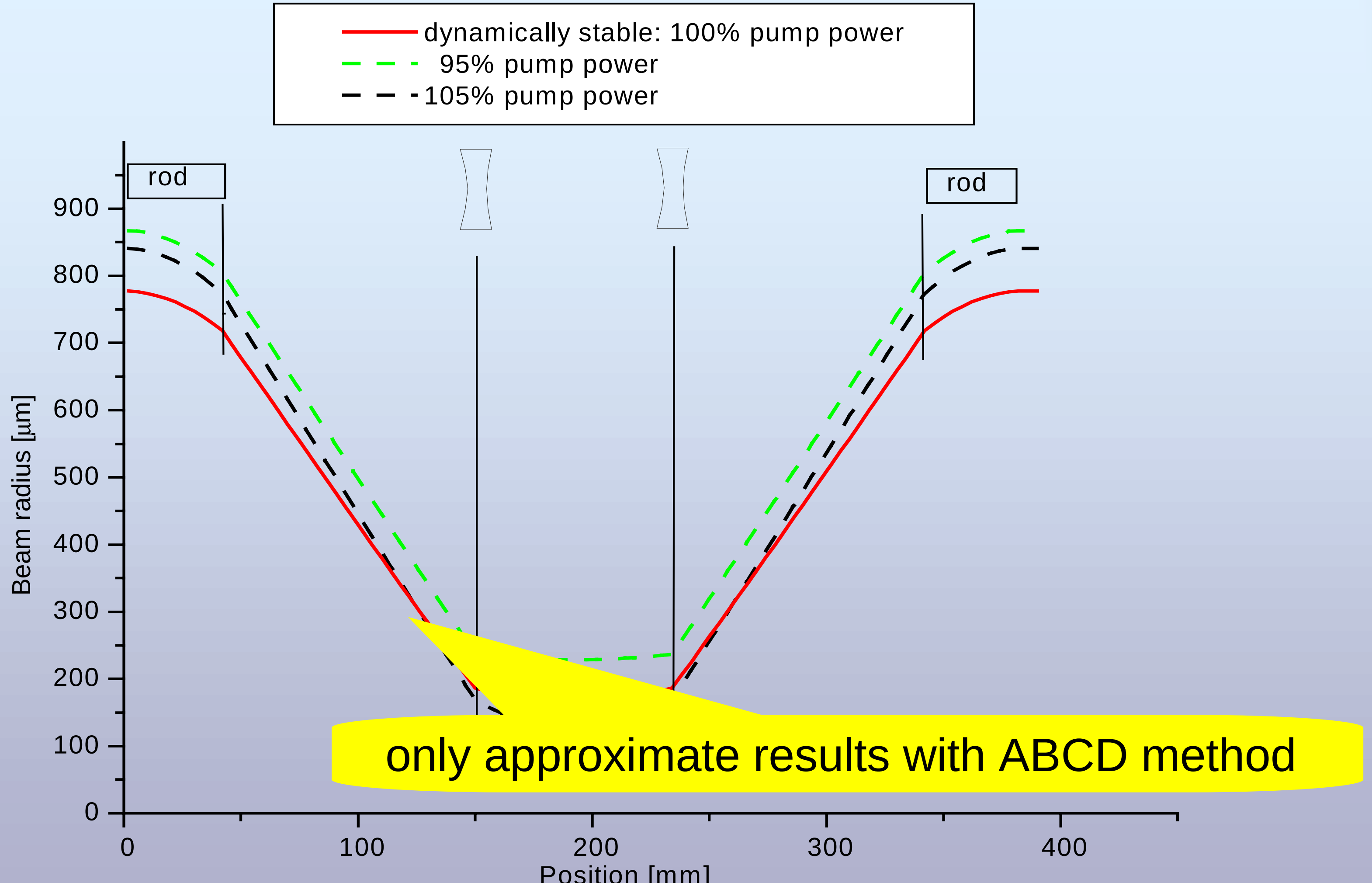
Mode Size in Resonator: (ABCD Matrices)



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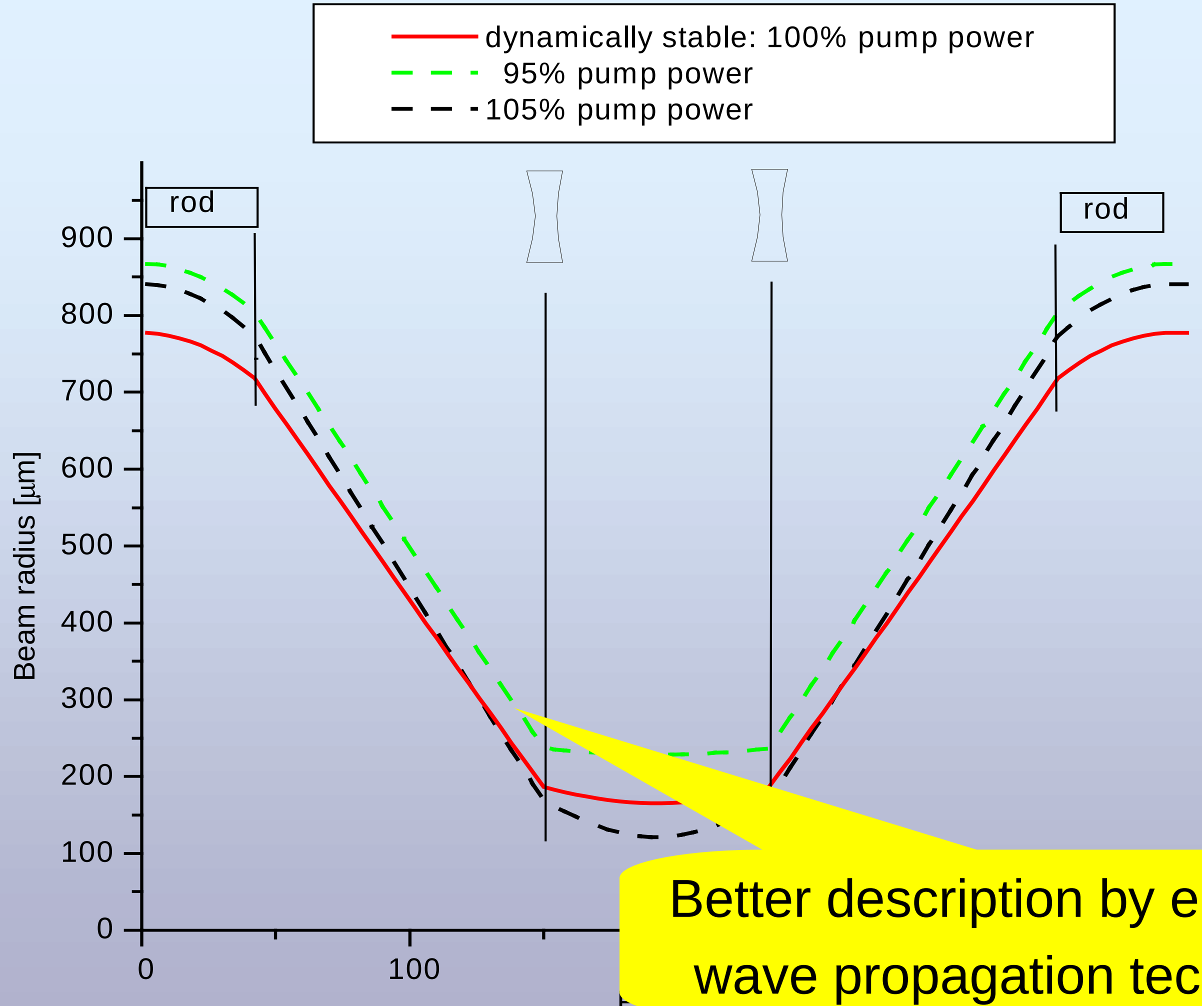
Mode Size in Resonator: (ABCD Matrices)



Better description by employing wave propagation techniques

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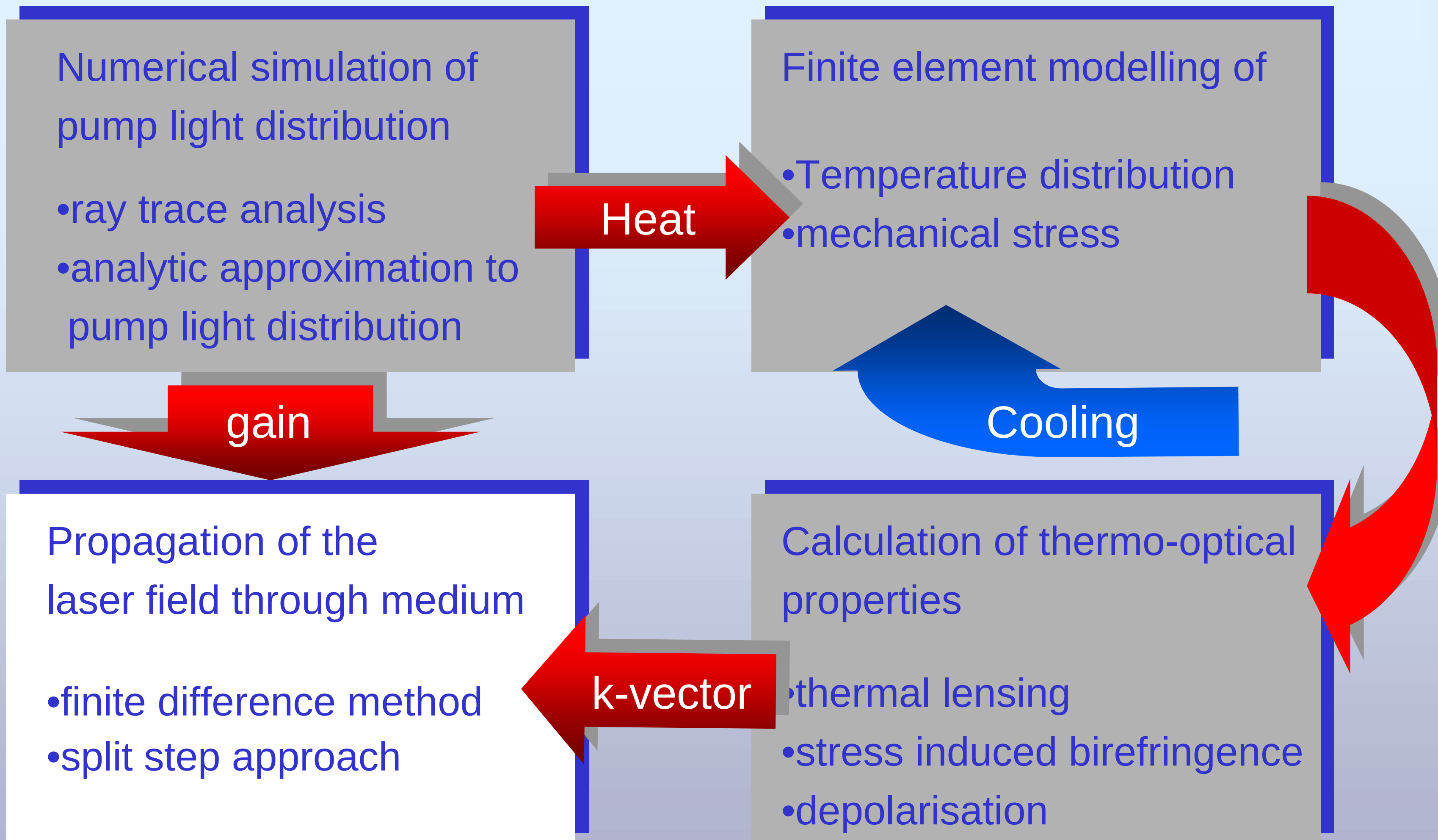
Wave field propagation



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Wave field propagation/split-step method

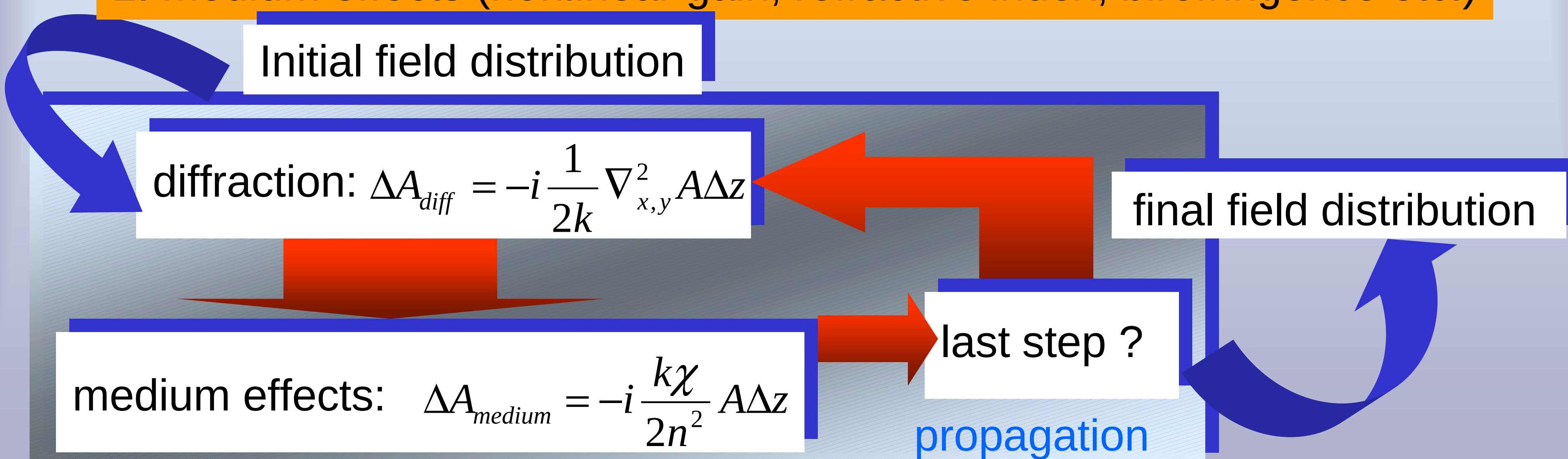
- Basic idea: write paraxial wave equation as:

$$\frac{\partial \vec{A}}{\partial z} = -i\frac{1}{2k}\nabla^2_{x,y}\vec{A} - i\frac{\mu\omega^2}{2k}\vec{P} = -i\frac{1}{2k}\nabla^2_{x,y}\vec{A} - i\frac{k\chi}{2n^2}\vec{A}$$

with $I = \left|\vec{A}\right|^2 = \frac{c\varepsilon}{2n}\left|\vec{E}\right|^2$

- Separation into two parts

1. pure diffraction
2. medium effects (nonlinear gain, refractive index, birefringence etc.)



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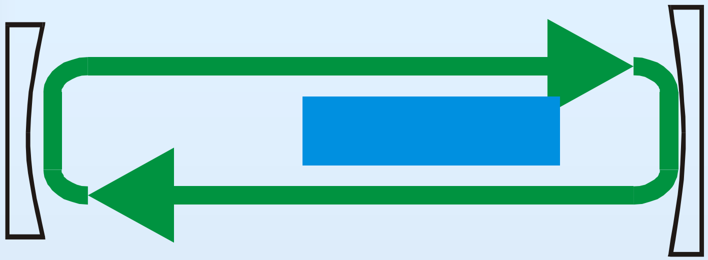
Wave field propagation/Fox-Li approach



- Eigenvalue equation

$$\hat{H}(E,x,y,z)E(x,y,z) = \gamma E(x,y,z)$$

not necessarily linear

- Iterative solution:

initial $E(x,y,z_0)$ (random)

pure diffraction

Medium effects:
- nonlinear gain
- ref. index $n(x,y,z)$
- spat. var. birefringence

medium

pure diffraction step to mirror 2

Output coupler → field

pure diffraction step to mirror 1

Pure diffraction to medium

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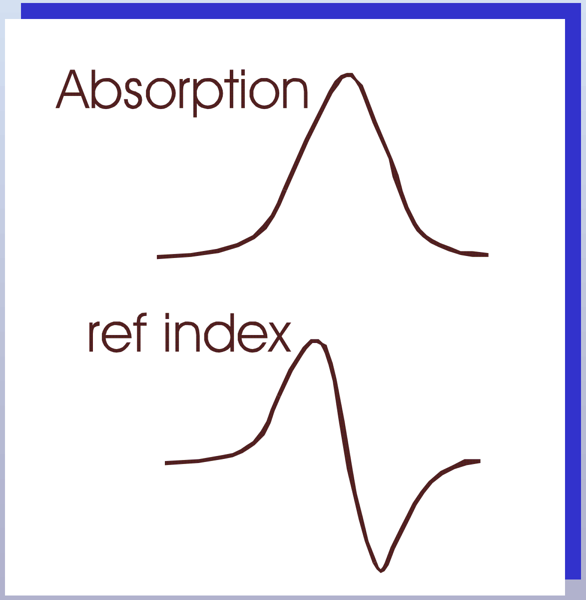
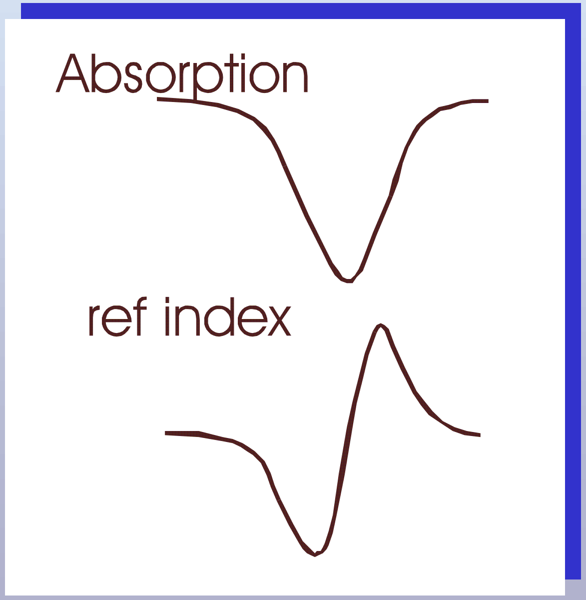
Outlook/upcoming tasks

Test and calibrate modelling with existing data

Further effects to be included

1. Gain related index guiding





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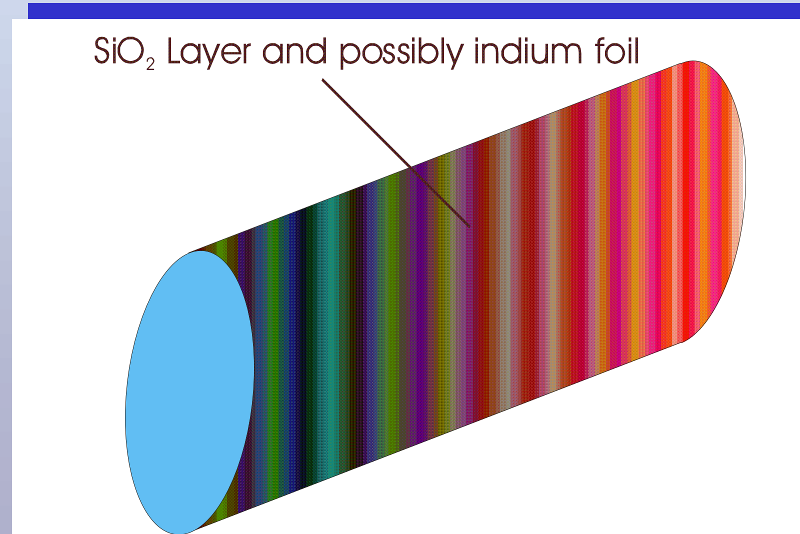
Outlook/upcoming tasks

Test and calibrate modelling with existing data

Further effects to be included

1. Gain related index guiding
2. Heating of coatings and indium foil by fluorescence



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