Mode mismatch in LIGO

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- **Plan:**
  - » introduction to mode-mismatch problem
  - » time domain modal model
  - » e2e's implementation
  - » example: 2-mir cavity, LIGO
  - » Plan for studying effects of thermal lensing
  - » observations from simulation runs

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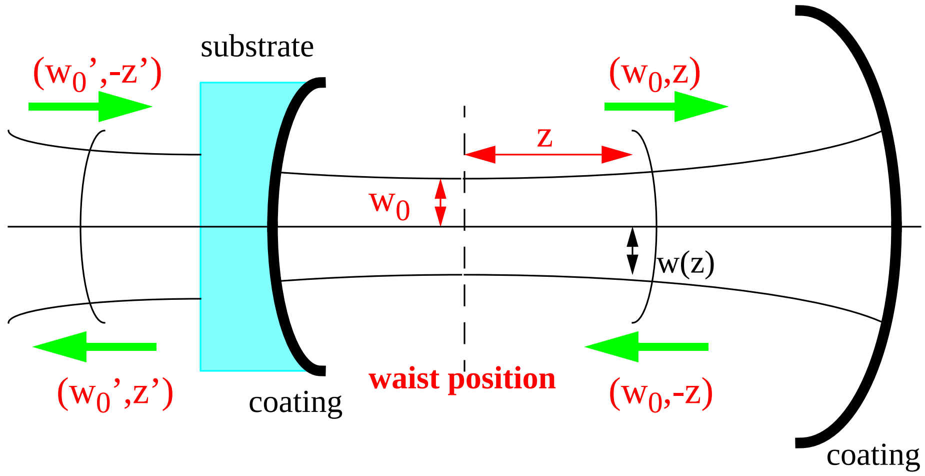
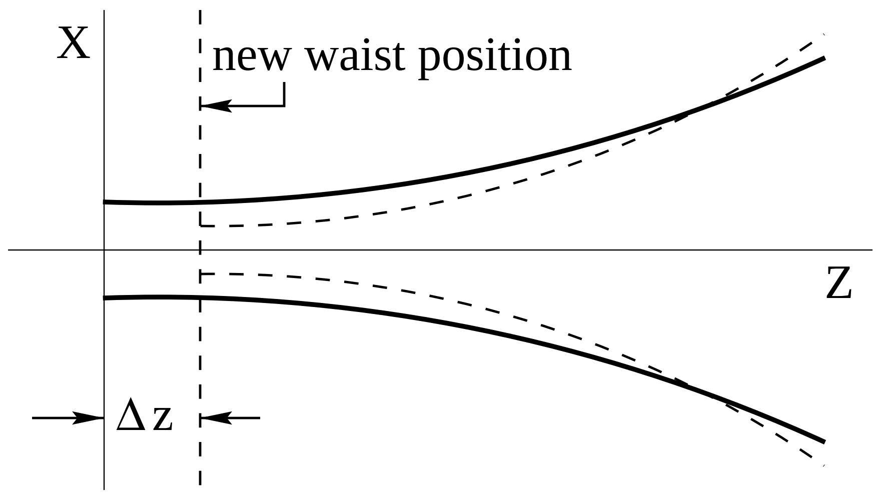
Mode Mismatch



- Field carries two modal info: waist-size, dist-to-waist
- Modal basis changes
  - **after passing thru lens/curved mirror**
  - **on reflection at an angle from a curved mirror**



- Mode mismatch:
  - **change in waist size**
  - **shift in beam position**

Modal model

- compute eigenfn Umn for the unperturbed system

$$E(x, y, z, t) = \exp(i\omega t) \cdot E(x, y, z)$$

$$E(x, y, z) = \sum a_{mn}(z, \bar{z}) \cdot U_{mn}(x, y, z)$$

$$a_{mn}(z, \bar{z}) = \bar{a}_{mn} \cdot \exp[-ik \cdot (z - \bar{z})] \exp[i(m + n + 1)(\eta_{00}(z) - \eta_{00}(\bar{z}))]$$

$$U_{mn}(x, y, z) = u_m(x, z) \cdot u_n(y, z)$$

$$u_m(x, z) = \left(\frac{2}{\pi}\right)^{\frac{1}{4}} \frac{1}{\sqrt{2^m m! w(z)}} \cdot H_m\left(\left(\frac{\sqrt{2}x}{w(z)}\right) \exp\left[-x^2\left(\frac{1}{w(z)^2} + \frac{ik}{2R(z)}\right)\right]\right)$$

- Simplification: separate longitudinal propagation from misalignment effects (perturbation)
- Perturbation: matrix operator acting on a complex vector space (transfers energy between transverse modes only)

- Laguerre-Gauss modes ?? - not needed

Operation

The operation on a field, like reflection from a tilted mirror or change of Hermite-Gaussian base like displacing the waist position, can be represented by the mode decomposition matrix, $M^{Op}_{mm'}$:

$$
\begin{aligned}
Op[E(x, y, \bar{z})] &= \sum_{mn} Op[\bar{a}_{mn} \cdot U_{mn}(x, y, \bar{z})] \\
&= \sum_{mn} \bar{a}_{mn} \cdot Op[u_m(x, \bar{z})] Op[u_n(y, \bar{z})] \\
&= \sum_{mn} \bar{a}'_{mn} \cdot U_{mn}(x, y, \bar{z})
\end{aligned}
$$

$$
\begin{aligned}
\bar{a}'_{mn} &= \int dx dy U_{mn}{}^*(x, y, \bar{z}) Op[E(x, y, \bar{z})] \\
&= \sum_{m'n'} \bar{a}_{m'n'} \int dx u_m{}^*(x, \bar{z}) Op[u_{m'}(x, \bar{z})] \int dy u_n{}^*(y, \bar{z}) Op[u_{n'}(y, \bar{z})] \\
&= \sum_{m'n'} \bar{a}_{m'n'} \cdot M^{Op}_{mm'} \cdot M^{Op}_{nn'}
\end{aligned}
\tag{1}
$$

perturbation effects

- Initial beam : k - mode no. ; w - waist

$$AU_0$$

- Rotation (r) :

$$A\left[U_0 + jr\frac{kw}{\sqrt{2\pi}}U_1\right]$$

- lateral displacement (d) :

$$A\left[U_0 + \left(\frac{2}{\pi}\right)^{1/2}\frac{d}{w}U_1\right]$$

- Waist-position mismatch (b) :

$$A\left[U_0 + j\frac{b}{2kw^2}\{U_0 + U_2\}\right]$$

- Waist-size mismatch (s) :

$$A\left[U_0 + \frac{s}{2w}U_2\right]$$

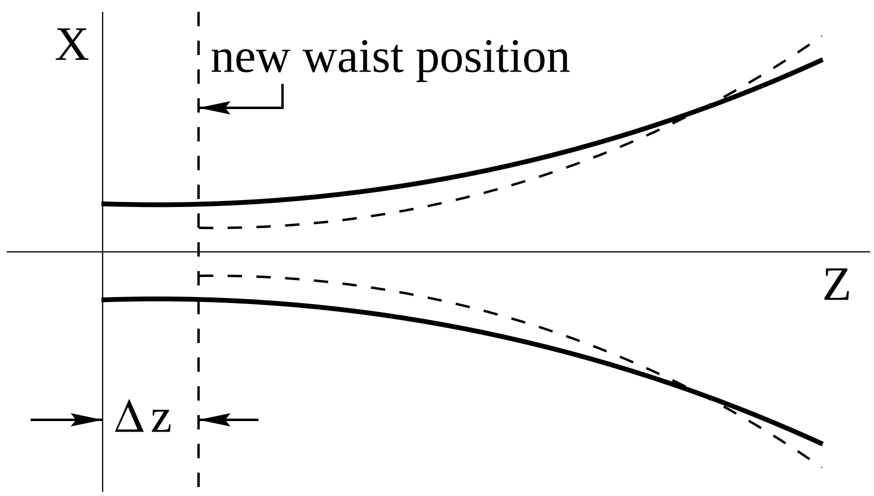
2-mir cavity

When the waist position of a field is shifted by Dz, the new field can be expressed by using the original field by shifting the z coordinate. Then up to the first order:

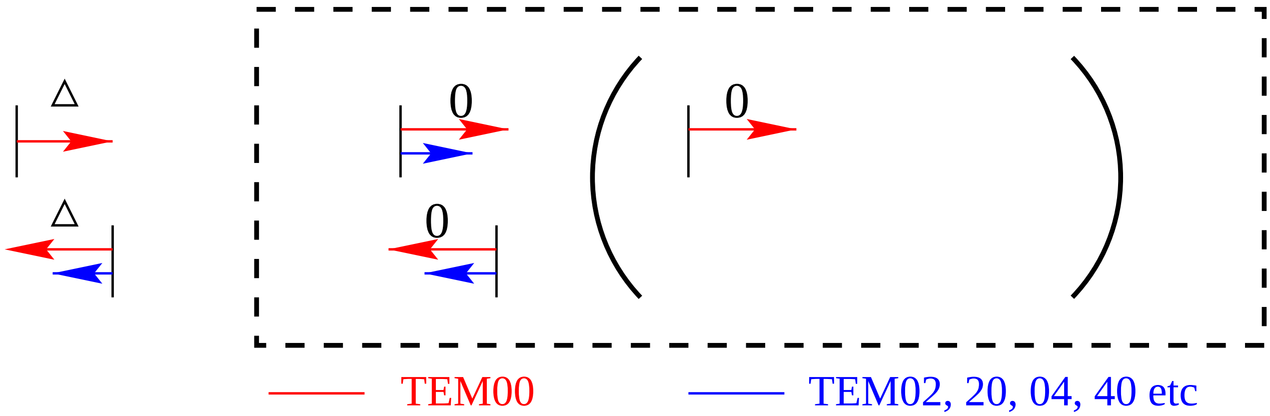
$$u_0(x, \bar{z} + \Delta z) = [a_0 \cdot u_0(x, \bar{z}) + a_2 \cdot u_2(x, \bar{z})]$$

$$a_0 = \frac{1}{\sqrt{1 - i\frac{1}{2}\frac{\Delta z}{z_0}}} \qquad a_2 = -i\frac{1}{2\sqrt{2}}\exp[i2\eta(\bar{z})]\frac{\Delta z}{z_0} \tag{2}$$

From these 2 coefficients, field at other z can be calculated



2-mir cavity



For the reflected TEM20 component from a 2-mir cavity:

$$[r_{00} + r_{20}\exp(i4\eta(\bar{z}))] \cdot \frac{1}{2\sqrt{2}}\frac{\Delta z}{z_o}$$

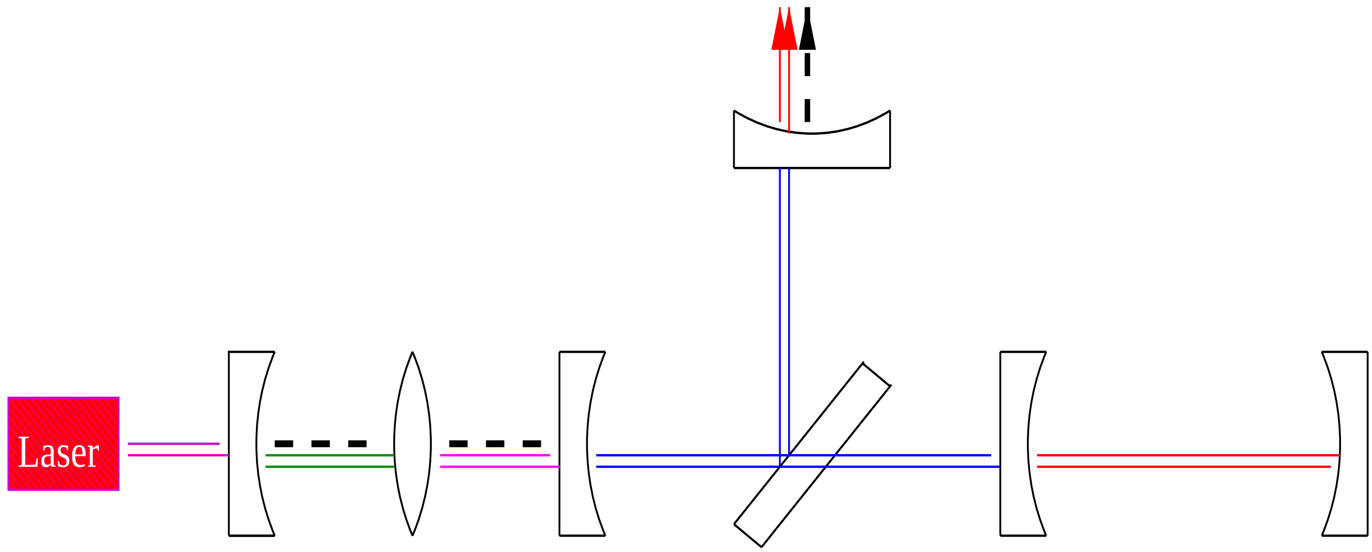
The reduction in the coupled TEM00 power

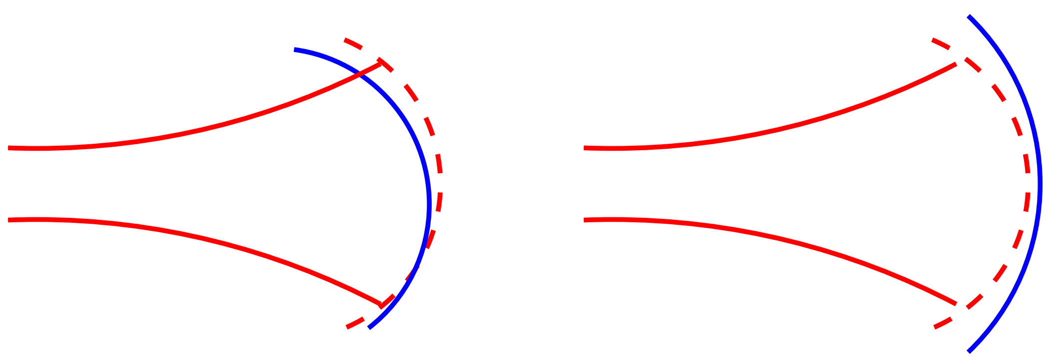
$$\left\langle\frac{\Delta P}{P}\right\rangle = \left(\frac{w_o'}{w_o} - 1\right)^2 + 2 \times \left(\frac{1}{2\sqrt{2}}\frac{\Delta z}{z_o}\right)^2$$

Time domain modal model

perturbation at surfaces

- propagate distance & perturbation at surface

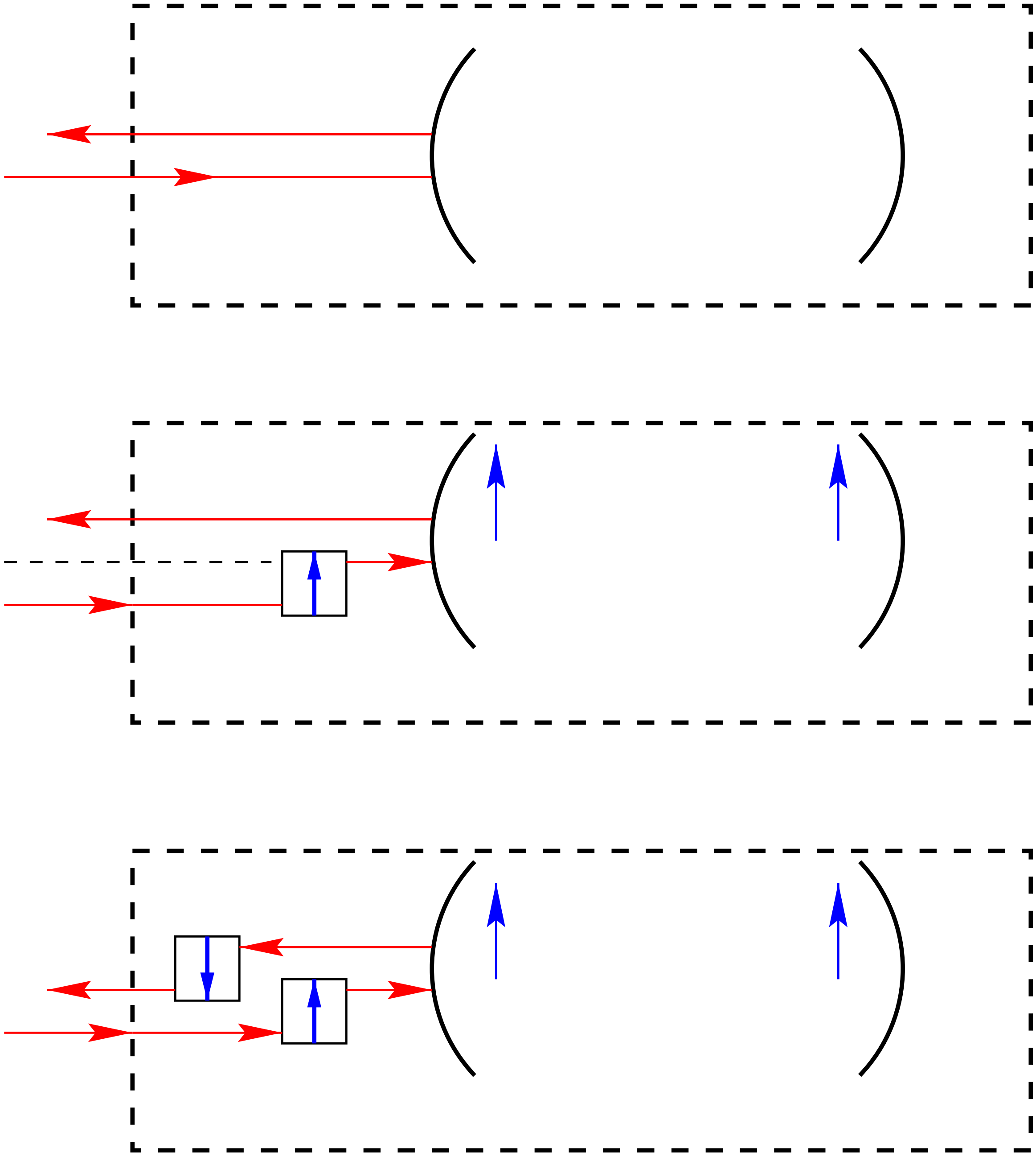




- Tilt, shift, curvature mismatch are treated using mode decomposition matrix

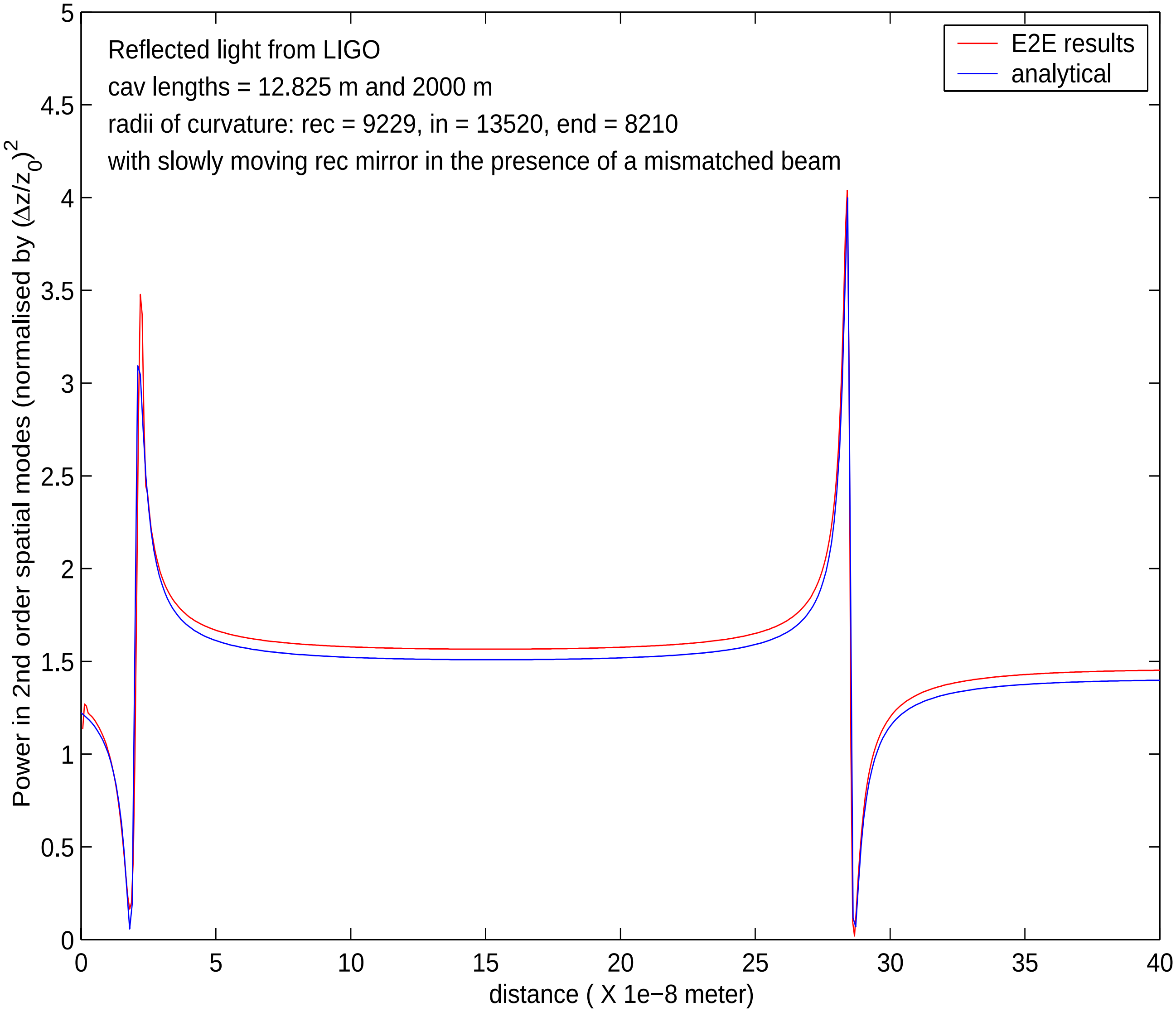
....just to remind the importance of proper basis while comparing analytical results with experiments

- A null experiment: lateral shift

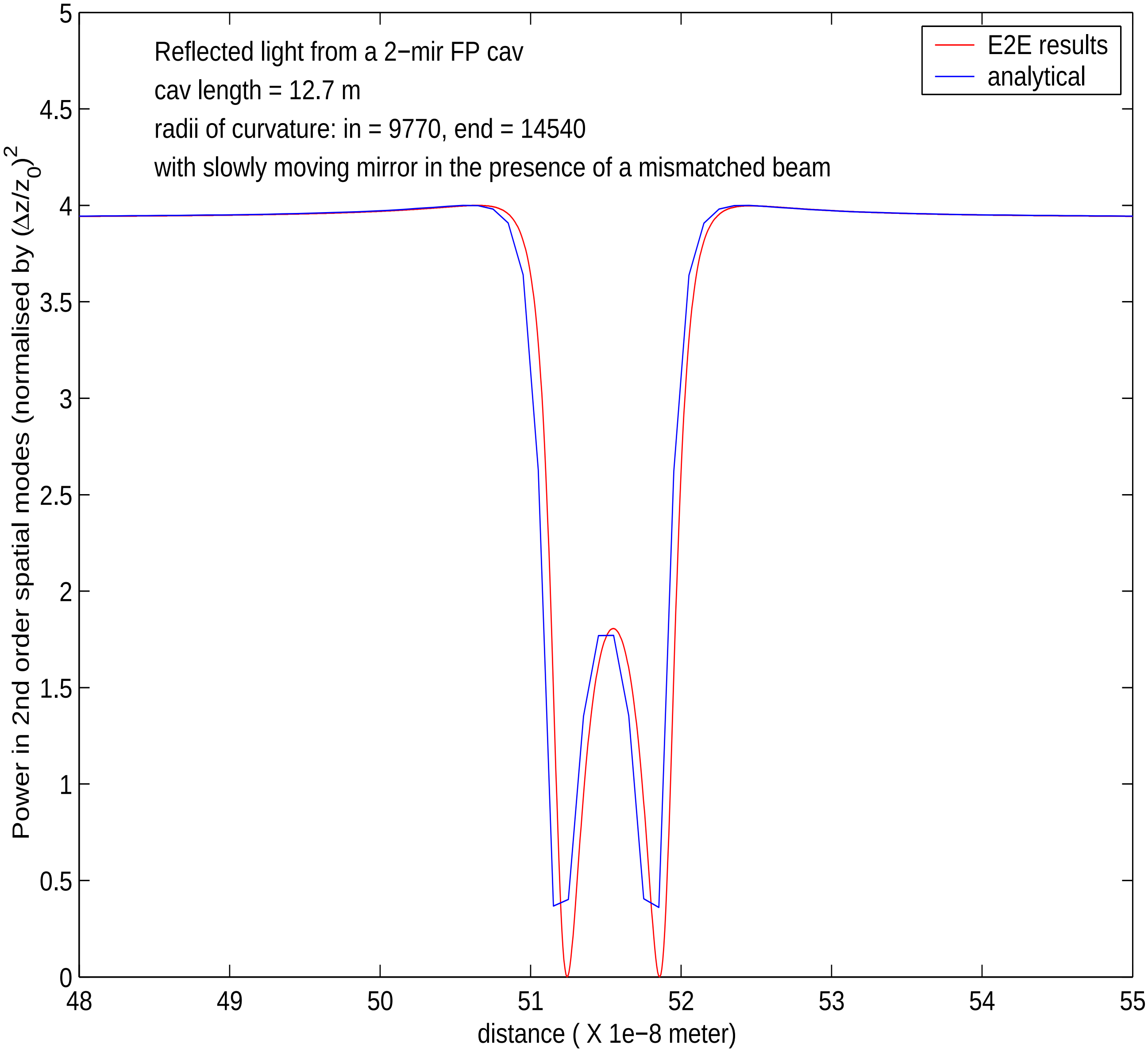


  - 3rd box is equivalent to the 1st box. The 2nd box is not

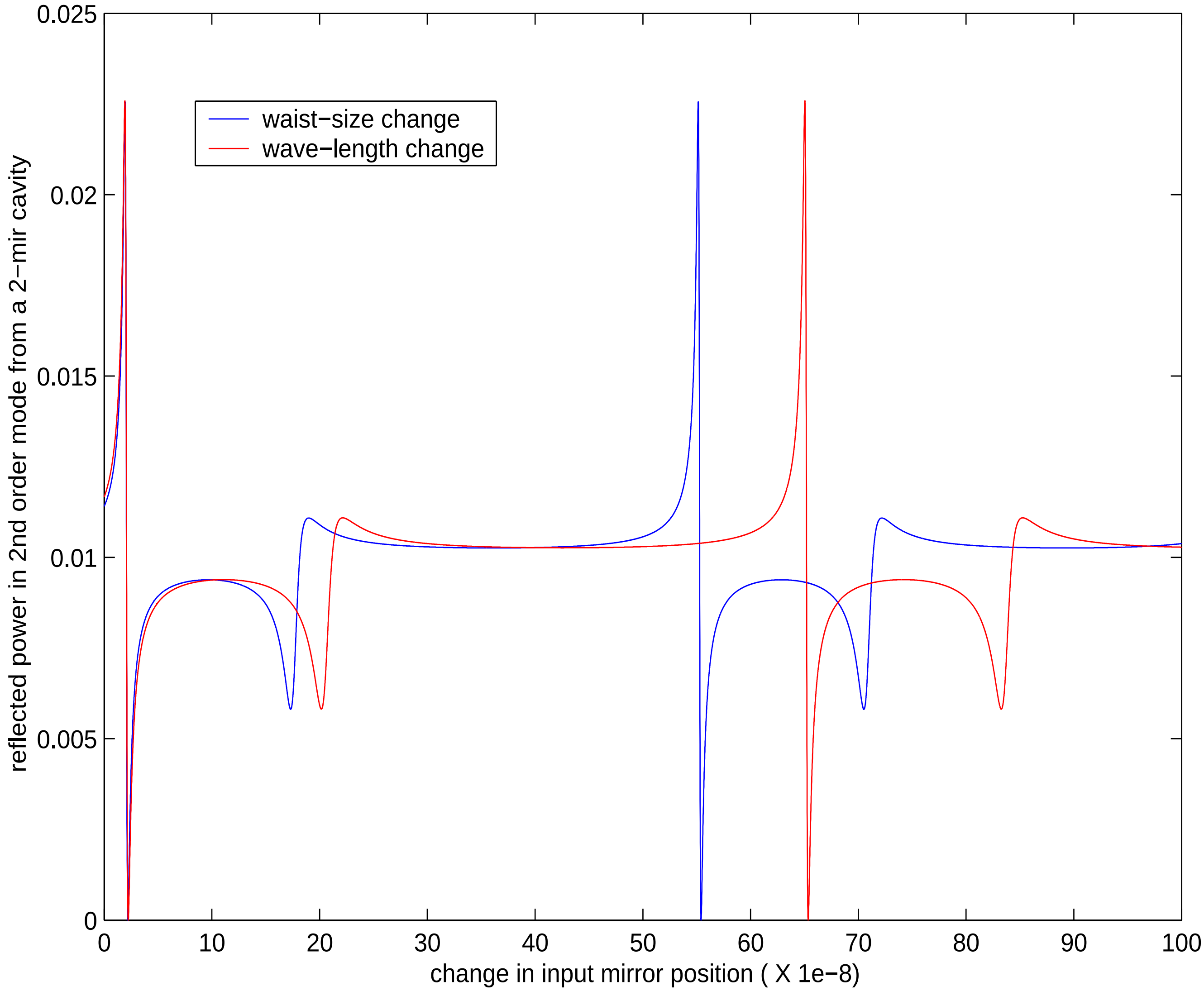
LIGO or HiFinesse 2-mir cavity



recycling cavity only



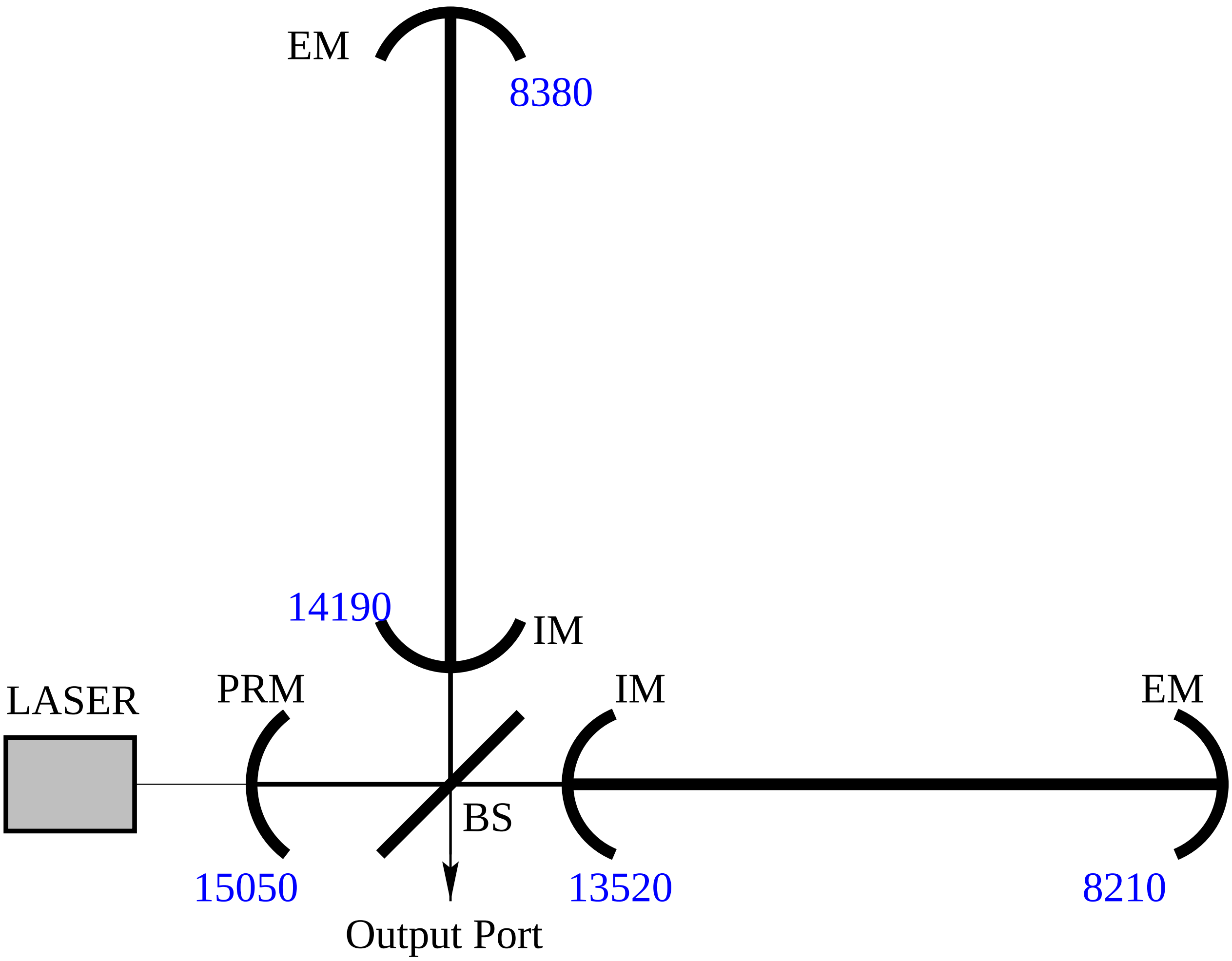
Effect of waist-size mismatch



$$\frac{\pi(w_o + \Delta w_o)^2}{\lambda} = \frac{\pi w_o^2}{(\lambda - \Delta\lambda)}$$

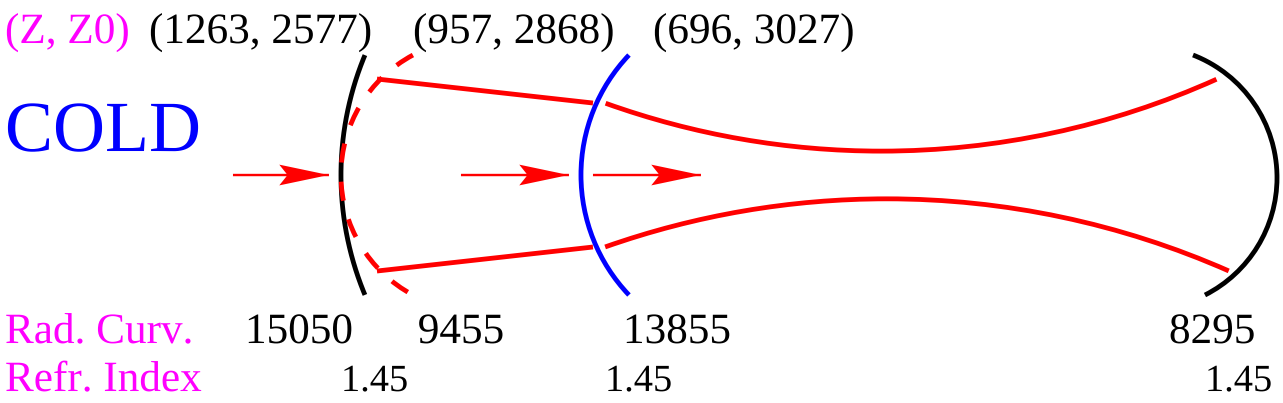
» mismatch in waistsize equiv. to change in frequency (keeping 'changed' Rayleigh range same in both cases) .... equivalence as far as modal content is concerned.

LIGO Interferometer

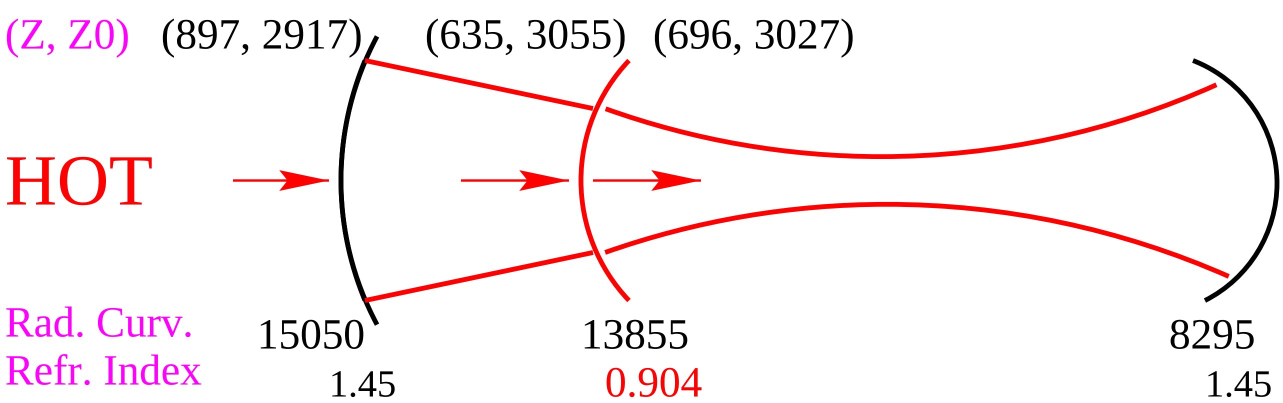


- Carrier mode-matching dominated by arm-cavity parameters; Sideband mode-matching determined by recycling cavity parameters
- Radius of curvature of the recycling mirror is chosen to minimize modal mismatch in hot state taking into account thermal lensing in input mirror.
- cold-state approximate mode matching corresponds to recycling mirror radius of curvature of ~9456 m

Han2k: Cold & Hot states



Z -> Dist. to Waist,   Z0 -> Rayleigh Range



Few Observations & Guess
from simulation runs

- Difference in curvatures of two arms does not add any extra problem of mode matching
- Runs suggest that gain parameters of lock acquisition need to be changed several times during the heating-up process
- Coupling of TEM00 from IOO to COC is not good at cold state. But it’s important to adjust it closer to the calculated matched modal state of hot IFO when lock acquistion starts

Simulation Plan

- Locking different IFO states in between cold and hot
  - how mirror positions, gain parameters change
- Studying mode profiles especially of sidebands
- Noise at dark port :
  - how much in comparison to residual tilt
- Comparison with actual data (preferably from a well-aligned cavity/IFO)