LIGO-G020376-00-Z

Stochastic UL Group E7 Report

LSC Stochastic Sources Upper Limit Group

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Outline

- Background information: Definition of stochastic GW background, ...
- Preliminary E7 investigations
- Results of hardware injections, production E7 analysis
- Future plans for S1 and beyond

$$\Omega_0 \leq 7.7 \times 10^4 \quad \text{for} \quad 40 \text{ Hz} < f < 215 \text{ Hz}$$

Stochastic GW Background

- Random GW signal produced by a large number of weak, independent, unresolved GW sources.
- Detect by cross-correlating output of two GW detectors.
- Strength specified by ratio of energy density in GWs to total energy density needed to close the universe:

$$\Omega_{\rm gw}(f) := \frac{1}{\rho_{\rm critical}} \frac{d\rho_{\rm gw}}{d \ln f} = \frac{10\pi^2}{3H_0^2} f^3 S_{\rm gw}(f) \quad (\Omega_0 = {\rm const})$$

- Current upper limits:
  - Low freq constraints from isotropy in CMBR and msec pulsar timing.
  - Broad band constraint from standard model of big-bang nucleo-synthesis: $\Omega_{\rm gw}(f) \leq 1 \times 10^{-7}$ in LIGO band
  - Garching-Glasgow IFOs (Compton et al, 1994): $\Omega_{\rm gw}(f) \leq 3 \times 10^5$
  - EXPLORER & NAUTILUS (Astone et al, 1999): $\Omega_{\rm gw}(907{\rm Hz}) \leq 60$

Cross-correlation statistic

- Look for a cross-correlated GW signal in output of two detectors (assumes noise uncorrelated with signal and noise in other detector):

$$Y_Q = \int_0^T dt_1 \int_0^T dt_2\, h_1(t_1)\, Q(t_1 - t_2)\, h_2(t_2) = T \int_0^\infty df\, \tilde{h}_1^*(f)\tilde{Q}(f)\, \tilde{h}_2(f)$$

- Mean due to cross-correlated SB signal:

$$\mu = T \int_0^\infty df\, \gamma(f) S_{\mathrm{gw}}(f)\, \tilde{Q}(f) \quad (= \Omega_0\, T)$$

- Variance dominated by noise in individual detectors:

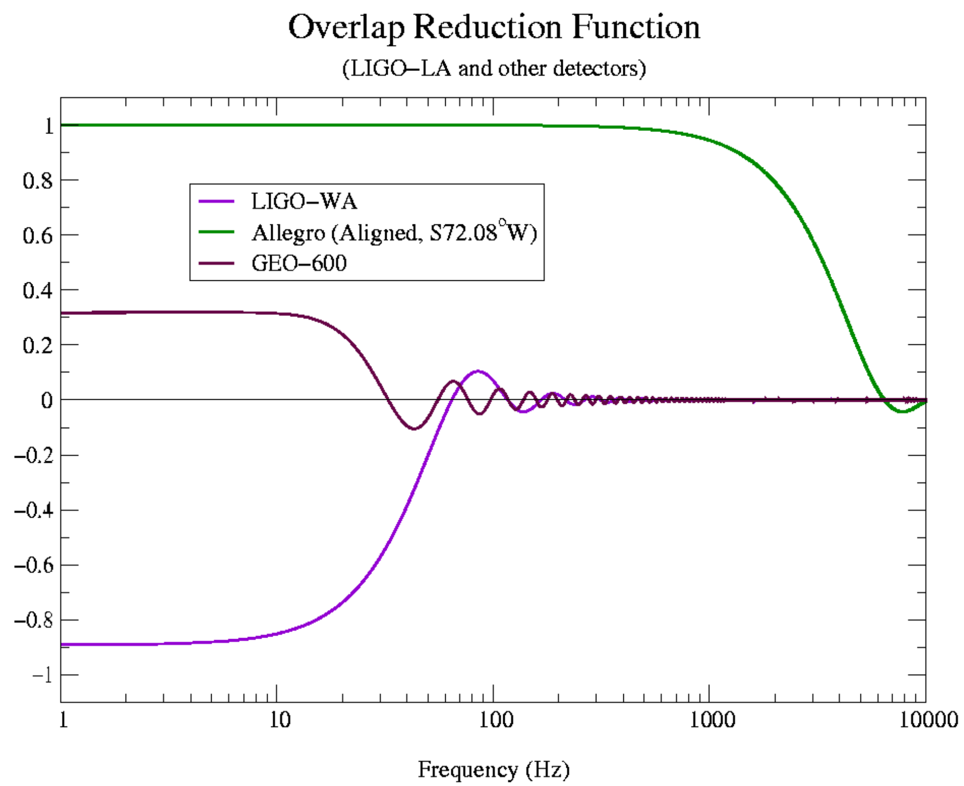
$$\sigma^2 \approx \frac{T}{2} \int_0^\infty df\, P_1(f) |\tilde{Q}(f)|^2\, P_2(f)$$

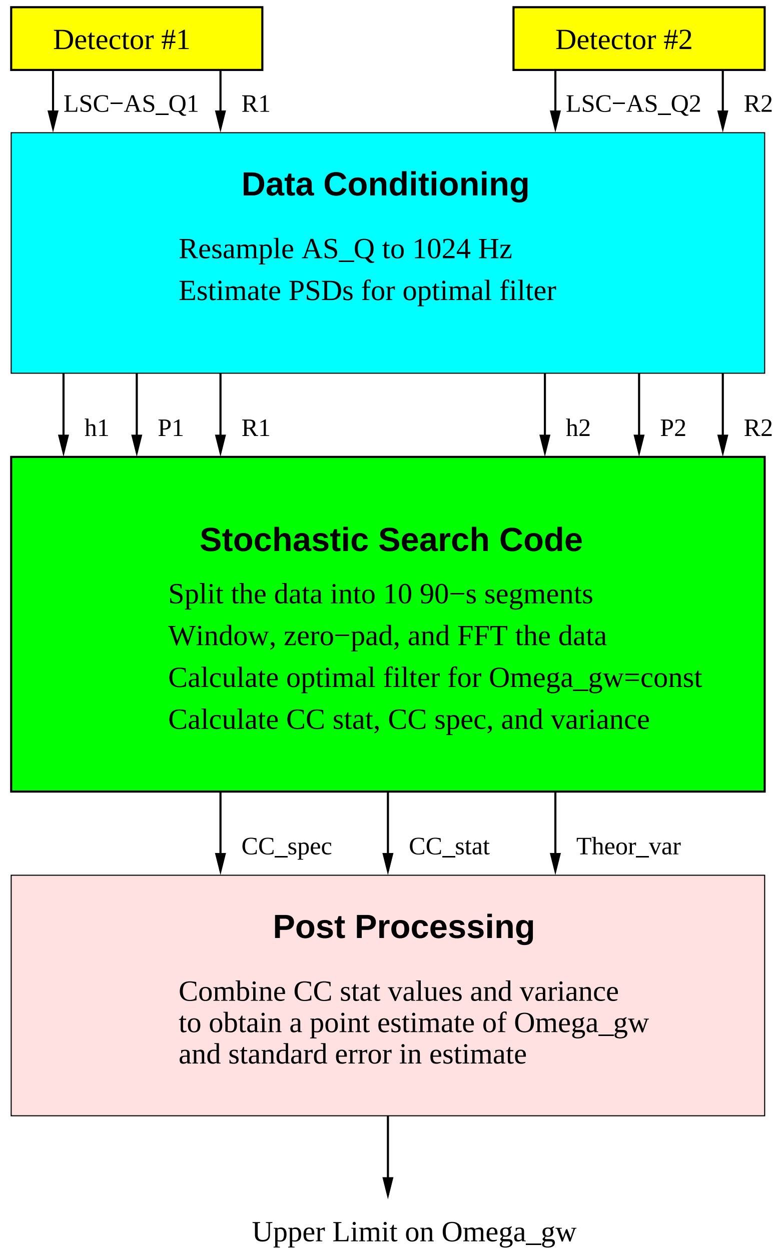
- Optimal filter maximizes SNR ($\propto \sqrt{T}$):

$$\tilde{Q}(f) \propto \frac{\gamma(f)\, S_{\mathrm{gw}}(f)}{P_1(f) P_2(f)} \propto \frac{\gamma(f)\, f^{-3}\, \Omega_{\mathrm{gw}}(f)}{P_1(f) P_2(f)}$$

Overlap reduction function: $\gamma(f)$

Reduction in sensitivity due to separation and orientation of the detectors:




Detector #1
Detector #2
LSC−AS_Q1
R1
LSC−AS_Q2
R2
Data Conditioning
Resample AS_Q to 1024 Hz
Estimate PSDs for optimal filter
h1
P1
R1
h2
P2
R2
Stochastic Search Code
Split the data into 10 90−s segments
Window, zero−pad, and FFT the data
Calculate optimal filter for Omega_gw=const
Calculate CC stat, CC spec, and variance
CC_spec
CC_stat
Theor_var
Post Processing
Combine CC stat values and variance
to obtain a point estimate of Omega_gw
and standard error in estimate
Upper Limit on Omega_gw

Setting an upper limit

Optimally-filtered CC statistic values:

$$Y_{Q1\,1}, Y_{Q1\,2}, \cdots Y_{Q1\,10},\ \ Y_{Q2\,1}, Y_{Q2\,2}, \cdots Y_{Q2\,10},\ \ \cdots\ \ Y_{QI\,1}, Y_{QI\,2}, \cdots Y_{QI\,10},\ \ \cdots$$

Sample mean and sample standard deviation:

$$\bar{Y}_{QI} := \frac{1}{10}\sum_{J=1}^{10} Y_{QIJ}\ ,\quad s_I := \left(\frac{1}{9}\sum_{J=1}^{10}(Y_{QIJ} - \bar{Y}_{QI})^2\right)^{1/2}$$

Weighted average and standard error:

$$\bar{Y}_Q := \frac{\sum_{I=1}^{M} \lambda_I \bar{Y}_{QI}}{\sum_{J=1}^{M} \lambda_J}\ ,\quad \hat{\sigma} := \frac{1}{\sqrt{10}}\frac{\left(\sum_{I=1}^{M} \lambda_I^2 s_I^2\right)^{1/2}}{\sum_{J=1}^{M} \lambda_J}$$

where $\lambda_I = \sigma_I^{-2}$.

90% CL upper limit:

$$\Omega_0 \le (\bar{Y}_Q + 1.28\,\hat{\sigma})/T$$

Preliminary E7 investigations

- Expected upper limit: $\Omega_0 \leq 1.4 \times 10^5$ for 40 Hz $< f <$ 215 Hz for 70 hrs of coincident H2-L1 E7 data.

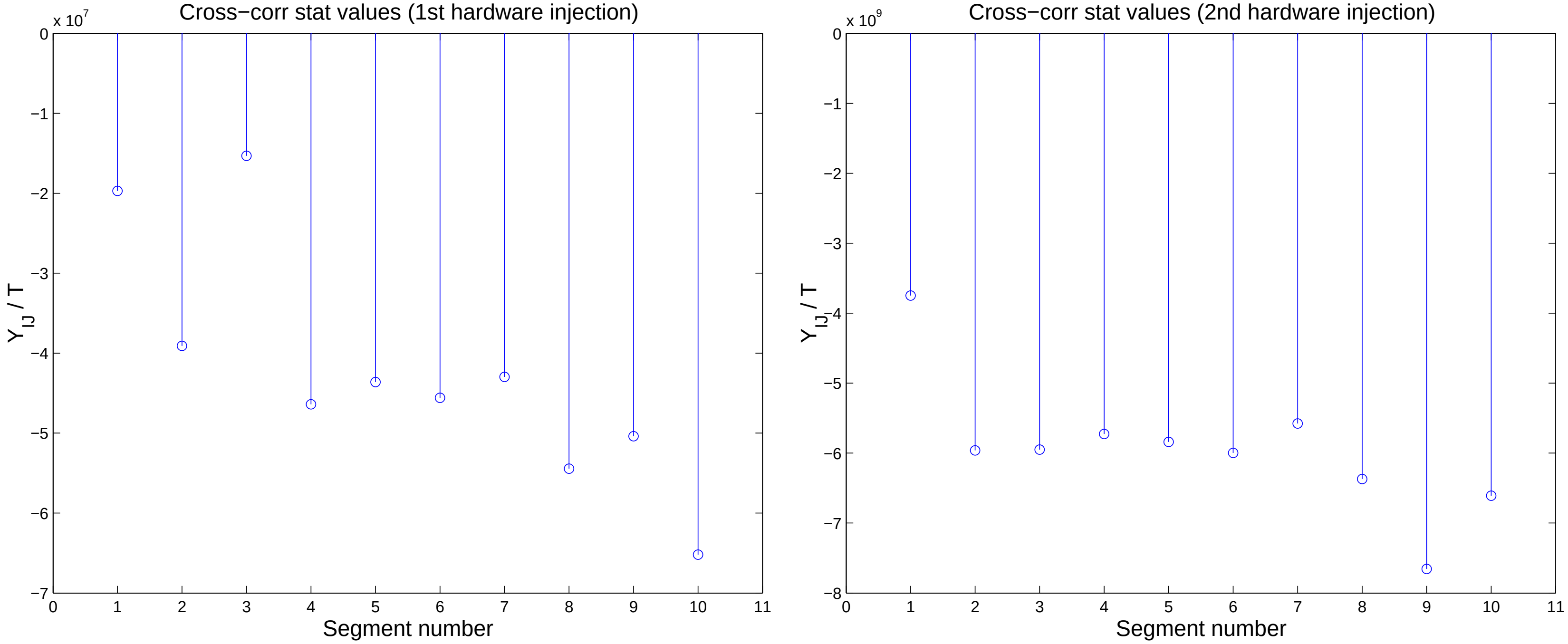
- Windowing: Required due to large dynamic range of LSC-AS_Q. Use pure Hann windows without overlap.

- High-pass filtering: Unnecessary if we restrict CC integral from 40 to 215 Hz. Fractional error in CC stat values ~ 1 part in 10^3.

- Line removal: Regression method implemented in datacondAPI. Excellent suppression for high SNR lines in PEMs, but effect on CC stat values not sufficiently characterized for E7 analysis.

- 60 Hz mains correlations: Not coherent over long time scales. No shift in mean, but still want to suppress lines to reduce variance.

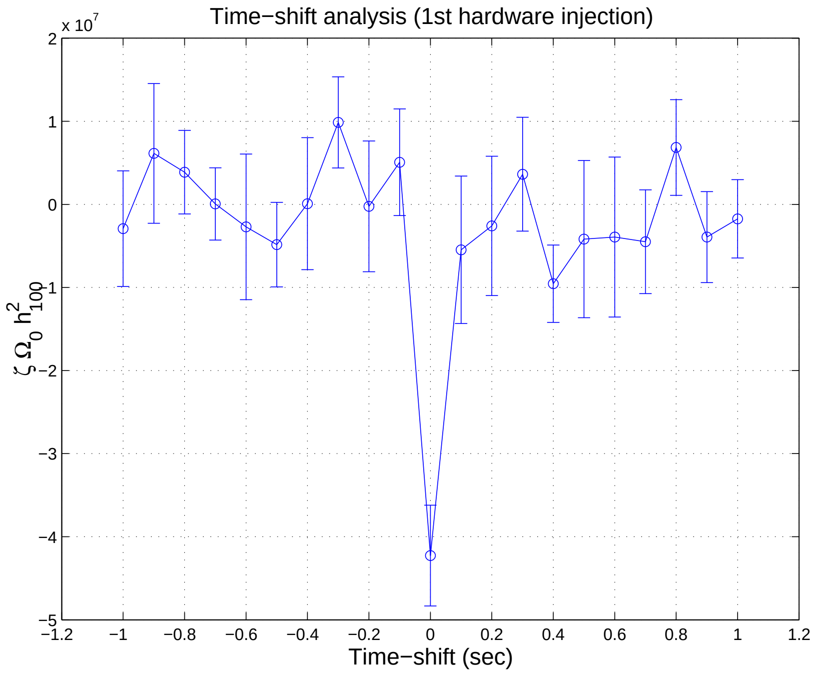
Hardware injection CC stat values

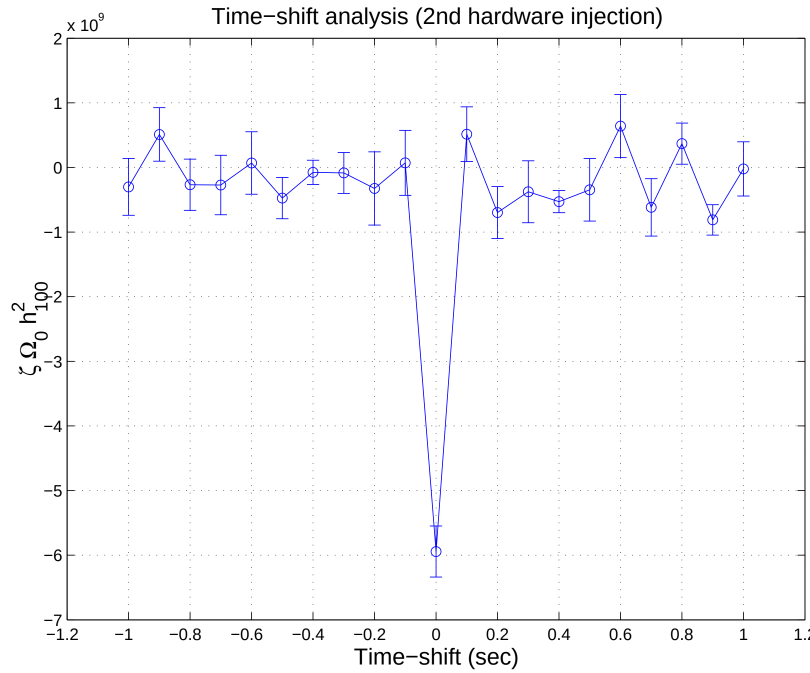


Point estimates: $\hat{\Omega}_0 = -4.2 \times 10^7$ and -5.9×10^9, respectively.

Negative due to relative sign ambiguity in calibration and/or injection between LLO and LHO.

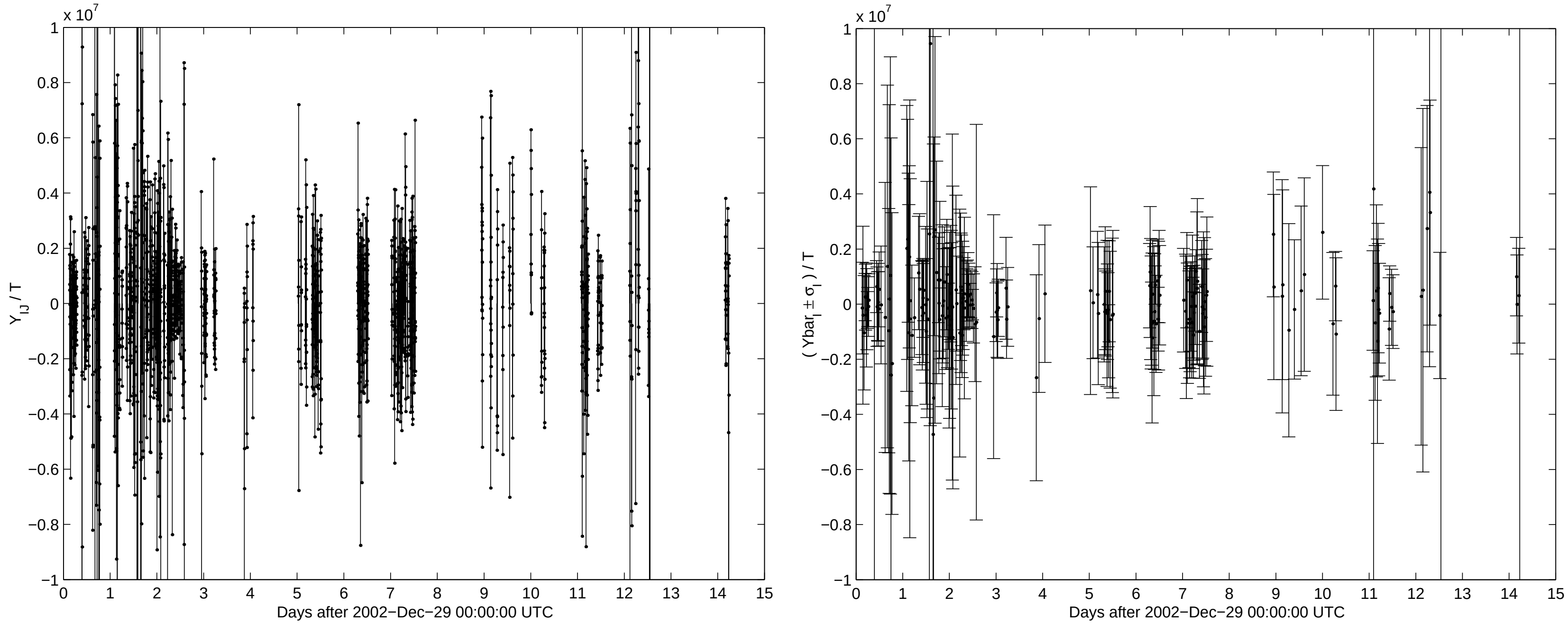
Hardware injection time-shift analysis





Point estimates $\hat{\Omega}_0$ with 90% CL error bars as a function of time-shift.

Production E7 analysis

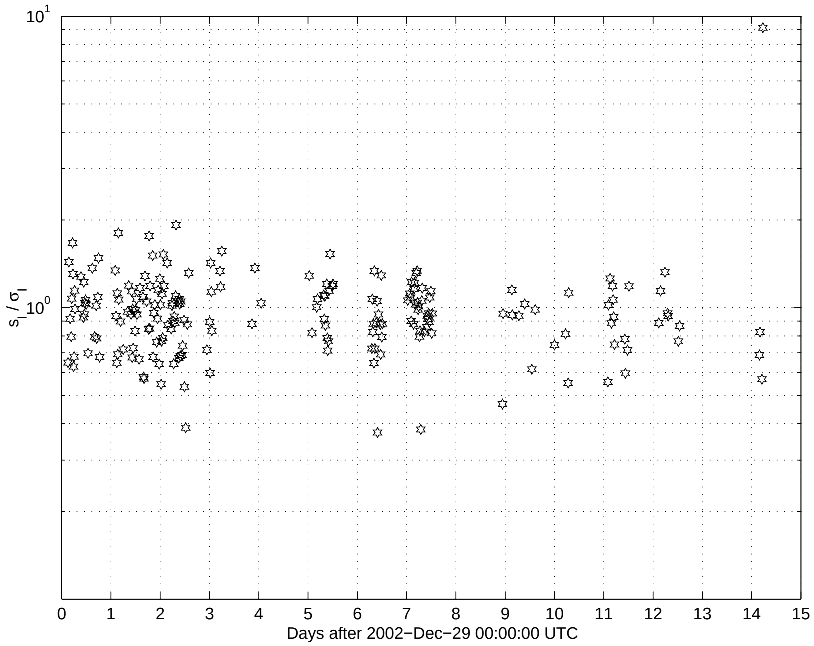


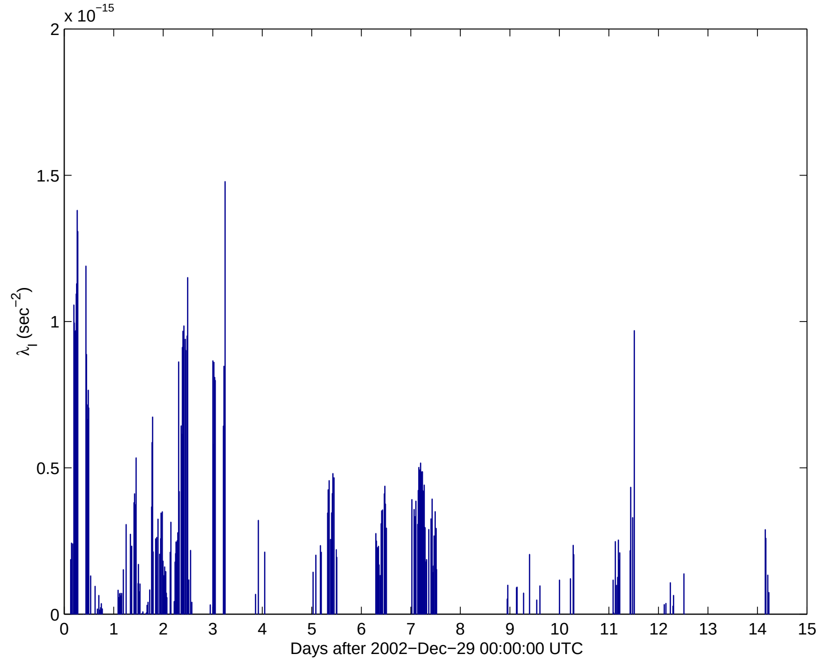
Y_{QIJ}/T (for all E7) and $(\bar{Y}_{QI} \pm s_I)/T$ (for each 15-minute chunk of data).

Point estimate, std error: $\widehat{\Omega}_0 = -1.9 \times 10^4$, $\hat{\sigma} = 4.5 \times 10^4\,T$

90% CL upper limit: $\Omega_0 \leq 7.7 \times 10^4$ for 40 Hz $< f <$ 215 Hz.

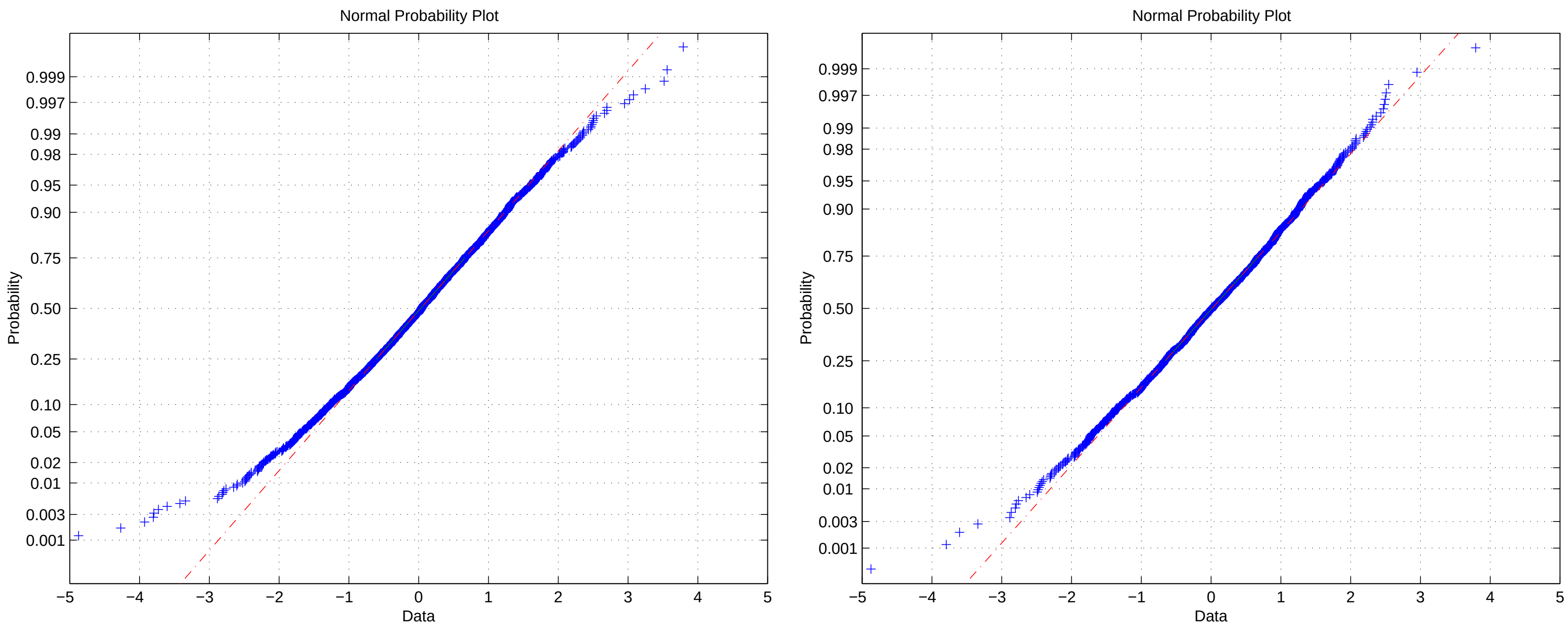
Production E7 analysis





Ratio s_I/σ_I and weighting factor $\lambda_I = \sigma^{-2}$ for each 15-min chunk of data.

Production E7 analysis

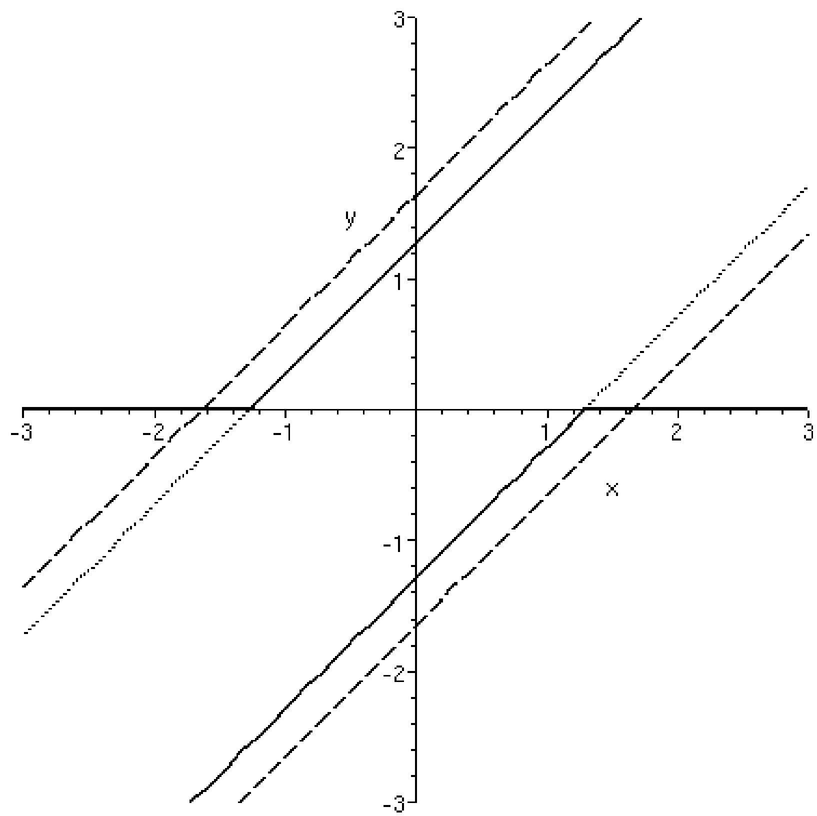


Gaussianity plots of $(Y_{QIJ} - \bar{Y}_Q)/\sigma_I$ for: (i) all E7 and (ii) the quietest 15-min chunks (which contribute 90% of weighted average).

Mean = -0.005 and -0.003, standard deviation = 1.22 and 1.02, respectively.

Future Plans (for S1 and beyond)

- Spliced Hann windows: Should improve upper limit by a factor of ~ 1.5.
- Monte Carlo simulations: Test within LDAS data analysis pipeline.
- Line removal: Need to characterize performance on CC stat values.
- Sign-correlation statistic: Use as alternative robust detection method for stochastic GW backgrounds and for broad-band CC noise studies.
- ALLEGRO-LLO correlation (E7 data): ALLEGRO rotated during E7; provides a method for estimating CC env noise component.
- GEO-LIGO correlation: Will not improve upper limit by much, but should increase our understanding of inter-continental CC env noise.


y
x
-3
-2
-1
1
2
3
3
2
1
-1
-2
-3