Scaling law in signal-recycled interferometers

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Based on:
Buonanno and Chen, gr-qc/0208048

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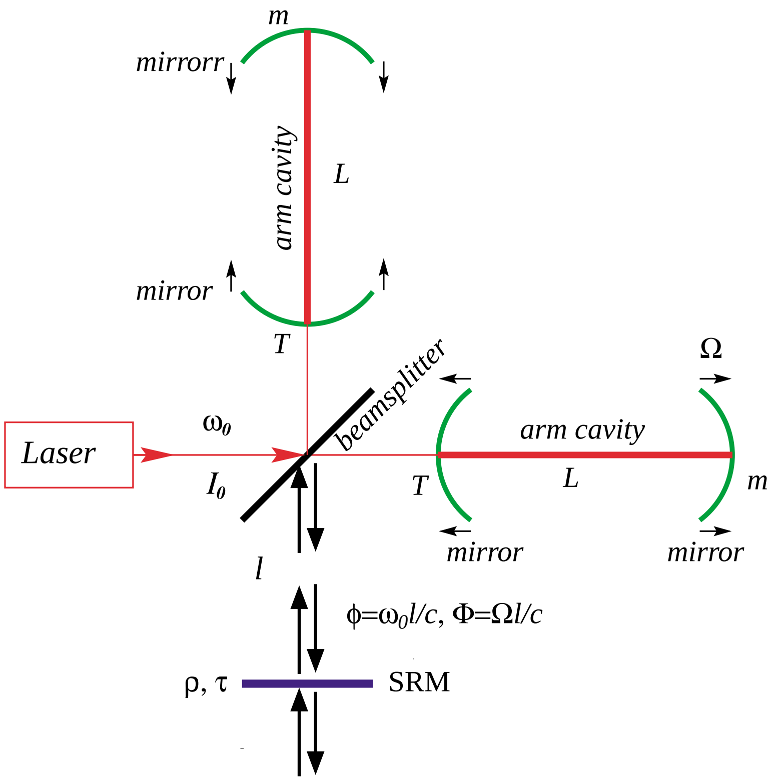
Signal-recycled interferometer of Advanced LIGO



The Presence of a Signal-Recycling Mirror (SRM)

[At all optical powers]

- Modifies optical resonant frequency and storage time. [Drever, 82; Meers, 88; Mizuno, 95.]

[At high enough optical power, e.g., in LIGO-II]

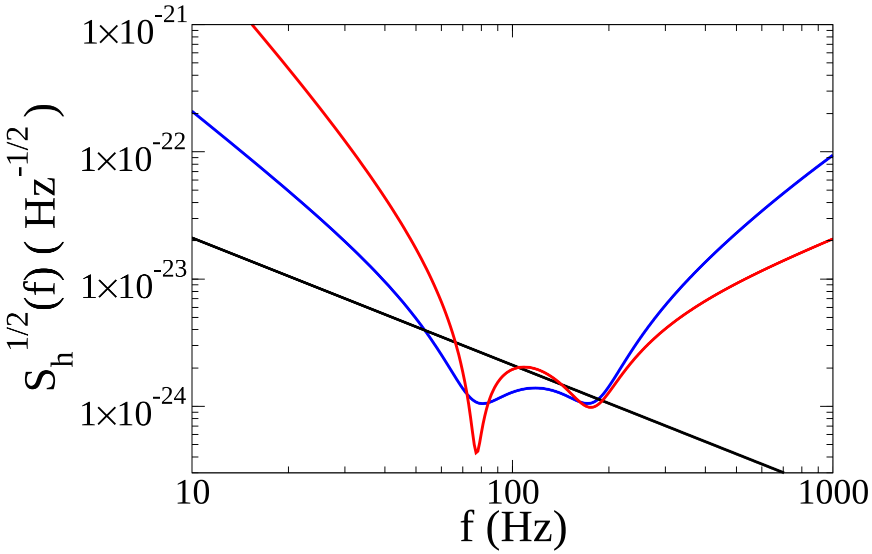
- Creates optomechanical coupling that changes test-mass dynamics. [Buonanno and Chen 01–02]
- Allows the interferometer to beat the SQL [Buonanno and Chen 01–02]

Conv (basic) ω_0, I_0, L, T, m

SR (extended) ρ, τ, l, $\phi = [\omega_0 l/c]_{\mathrm{mod}2\pi}$, $\Phi \equiv \Omega l/c \to 0$

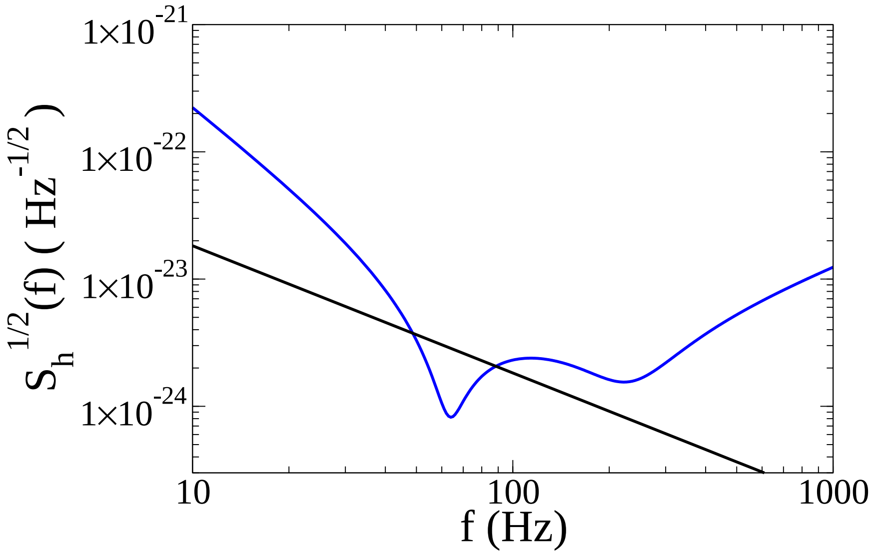
Exploring the parameter space: *scaling law?*

[Buonanno and Chen 01–02, Fritschel, Shoemaker, Strain . . .]



Example given by Buonanno and Chen:

$I_{\rm BS} = 10\,{\rm kW}$
$T = 0.033,\ \gamma_{\rm arm} = 2\pi \times 100\,{\rm Hz}$
$\rho = 0.9,\ \phi = \pi/2 - 0.47$

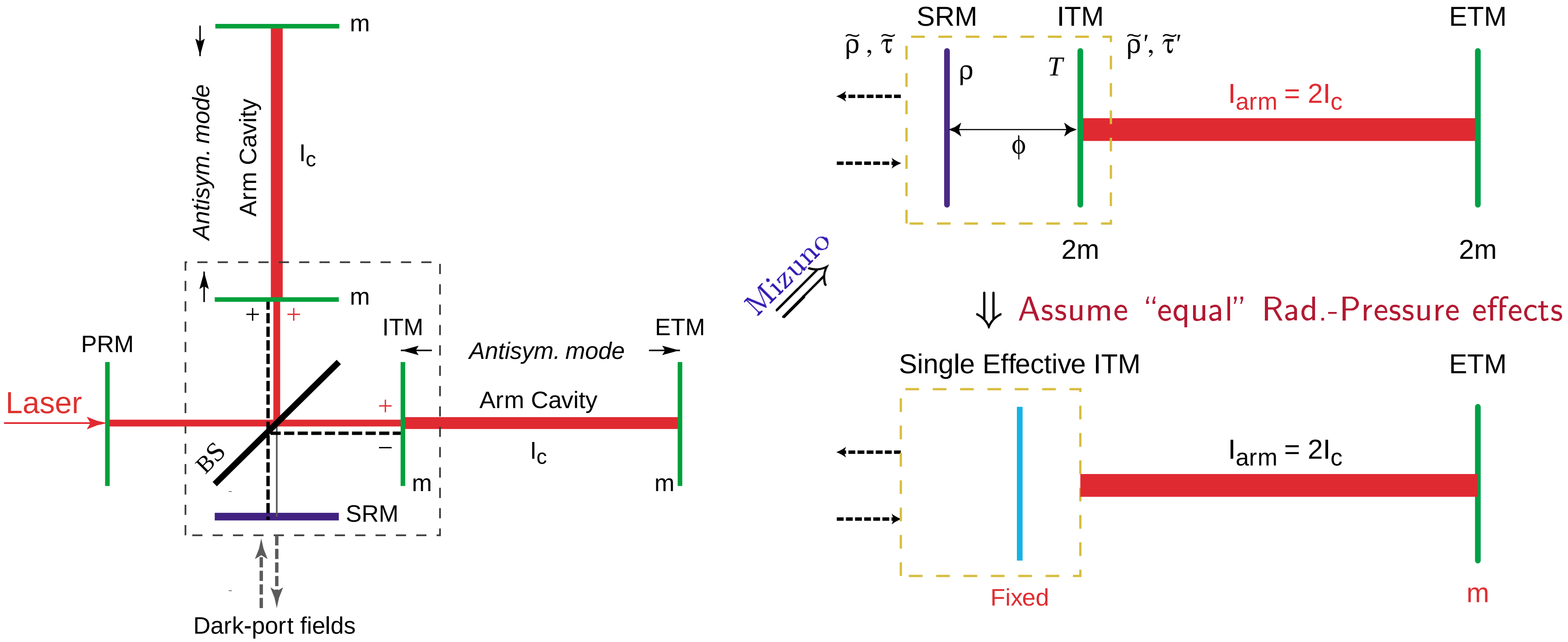


Reference design by LIGO experimentalists:

$I_{\rm BS} = 2.5\,{\rm kW}$
$T = 0.005,\ \gamma_{\rm arm} = 2\pi \times 15\,{\rm Hz}$
$\rho = 0.964,\ \phi = \pi/2 - 0.06$

Mapping SR interferometer to single-detuned cavity

[Mizuno, 95; Rakhmanov, 00; Khalili, 01; Buonanno and Chen, 02]



Optics: *well known,* $(T,\ \rho,\ \phi) \rightarrow \widetilde{\rho}' \rightarrow \omega_{\rm opt\ res} = \omega_0 - \lambda - i\epsilon$

Opto-Mechanics: *almost trivial,* [Approximation made, though]

$$I_c,\ m,\ L \rightarrow \iota_c \equiv \frac{8\omega_0 I_c}{mLc} \sim \Omega_{\rm GW}^3$$

The use of the characteristic description

[Buonanno and Chen, 02]

- Previous results of SR interferometers at high powers [Buonanno and Chen, 00–01] written in characteristic parameters: equations much simpler, more physical. For example, the *optical spring* constant:

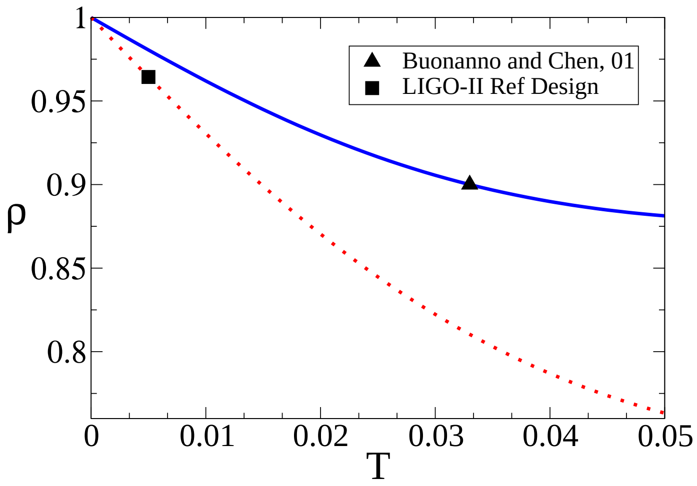
$$\frac{K_{\mathrm{opt}}(\Omega)}{m/4} = \frac{-\lambda\iota_c}{(\Omega-\lambda+i\epsilon)(\Omega+\lambda+i\epsilon)} \qquad \cdots = \Omega^2$$

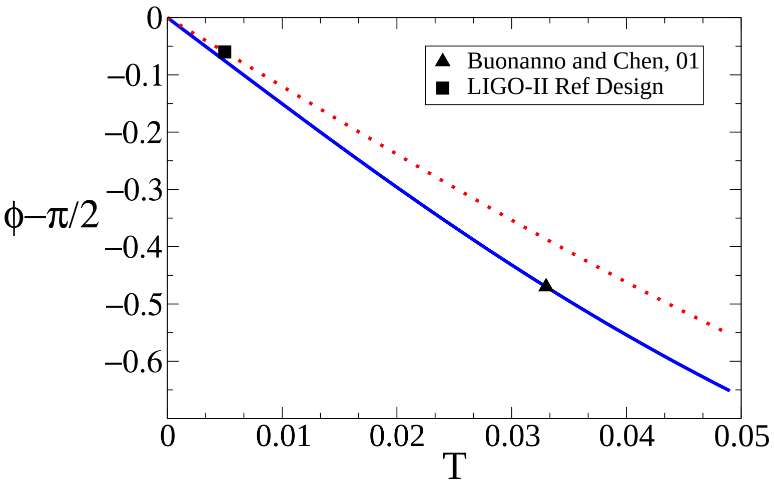
  See Buonanno and Chen, gr-qc/0208048 for more nice results.

- Scaling laws:
  - Optical, confirmed that, with I_c, m, L fixed, scaling laws in the low power regime still valid here: $(\lambda, \epsilon) = \mathrm{const}$. Can be used for LIGO-II optimization.
  - Opto-Mechanical, one more scaling relation: $\iota_c \equiv \frac{8\Omega_0 I_c}{mLc} = \mathrm{const}$. Relating LIGO-II to experiments in other regimes.

Using the Optical scaling law: simplifying LIGO-II optimization

For configurations with the same I_c, m and L, fix λ and ϵ, and:





A: $(\lambda, \epsilon) = 2\pi \times (195\,\mathrm{Hz}, 25\,\mathrm{Hz})$, contains $(T, \rho, \phi) = (0.033, 0.9, \pi/2 - 0.47)$. [Buonanno and Chen, 01–02]

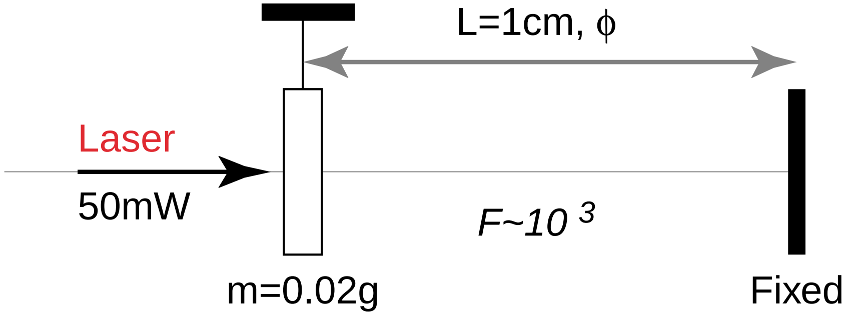
B: $(\lambda, \epsilon) = 2\pi \times (228\,\mathrm{Hz}, 69\,\mathrm{Hz})$, contains $(T, \rho, \phi) = (0.005, 0.964, \pi/2 - 0.06)$. [LIGO-II Reference Design.]

Using the full Optomechanical scaling law

Linking LIGO-II and table-top experiments

Table-top experiment proposed by Braginsky, Khalili and Volikov (2001), single detuned cavity with one movable mirror. [Directly analyzed by Khalili, (2001)]

. . . In a very different regime from LIGO



now fits into the same formalism as LIGO-II:

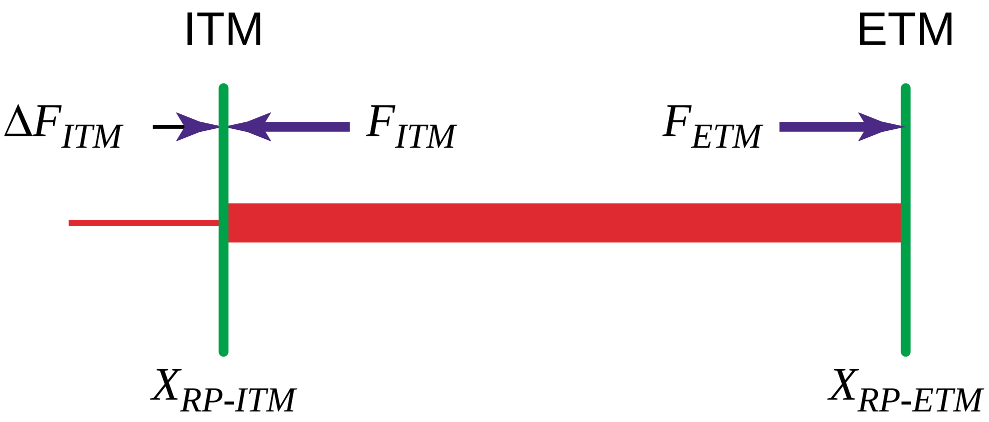
$\epsilon \sim \lambda/\sqrt{3}$	$\iota_c^{1/3}$	Ω
$5 \times 10^7\,\mathrm{s}^{-1}$	$10^5\,\mathrm{s}^{-1}$	$10^4\,\mathrm{s}^{-1}$

Approximations made in the quantum optical analysis of LIGO interferometers

So far: leading order in T and $\Omega L/c$.

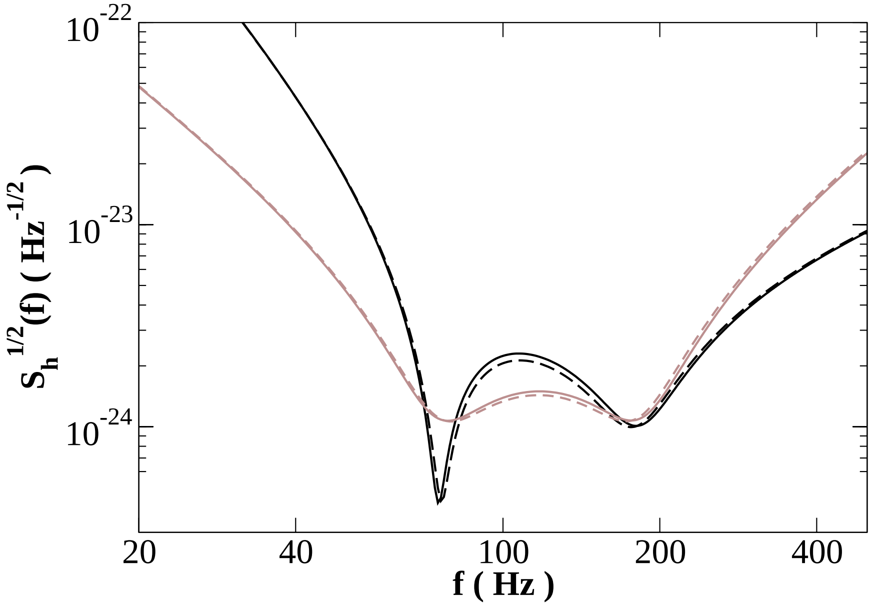
Errors associated:

- Difference in the ITM and ETM contributions to radiation-pressure effects ignored. *Still to be analyzed.*



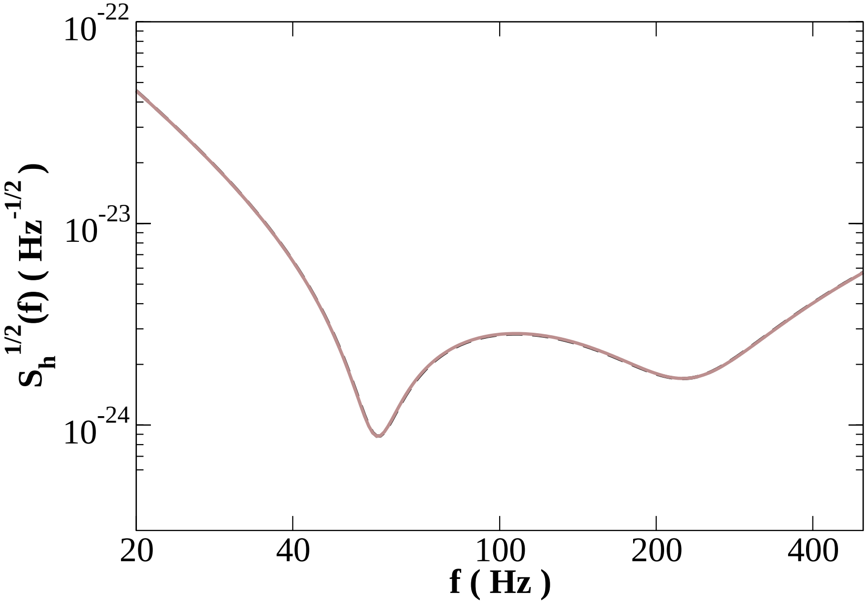
- Propagations of optical fields less accurate. *Resolved by this work.*

Leading-order in T vs all orders in T



Example given by Buonanno and Chen:

$T = 0.033$, ρ=0.9, $\phi = \pi/2 - 0.47$



LIGO-II Reference design:

$T = 0.005$, ρ=0.964, $\phi = \pi/2 - 0.06$

Summary

- SR interferometer *with high laser power* (with very short SR cavity) mapped into a single detuned cavity
- Scaling law among experimental parameters
- Scaling law helps:
  - presenting calculation results in a simpler way, more physical (!)
  - optimization for LIGO-II
  - relating LIGO-II to table-top experiments
- Remaining issue: difference in the contributions of ITM and ETM to raditaion-pressure effects.