LIGO: The Laser Interferometer Gravitational-wave Observatory

General Relativity...

Einstein...

A new window...

Astrophysics...

The R-modes in Neutron Stars with Magneto-viscous Boundary Layers

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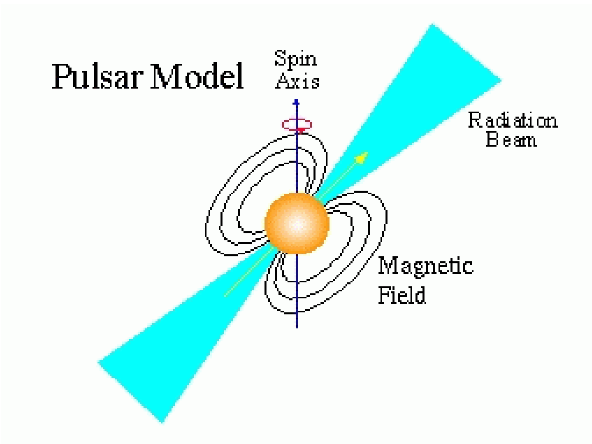
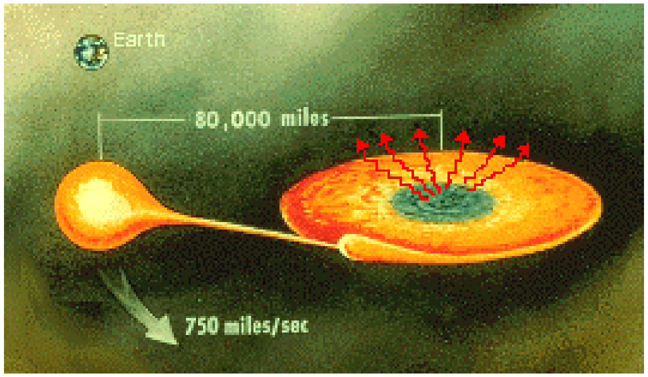
G020557-00-W

Possible Periodic Sources of Gravitational Waves



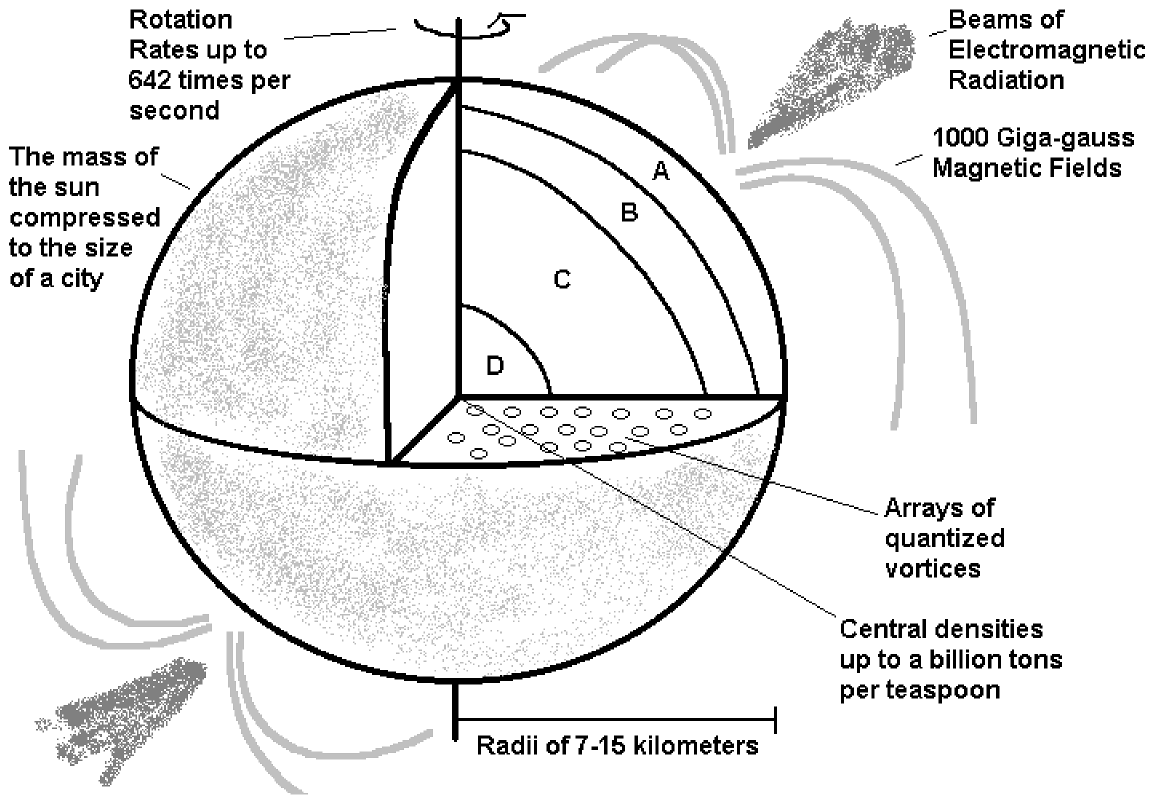


LMXBs

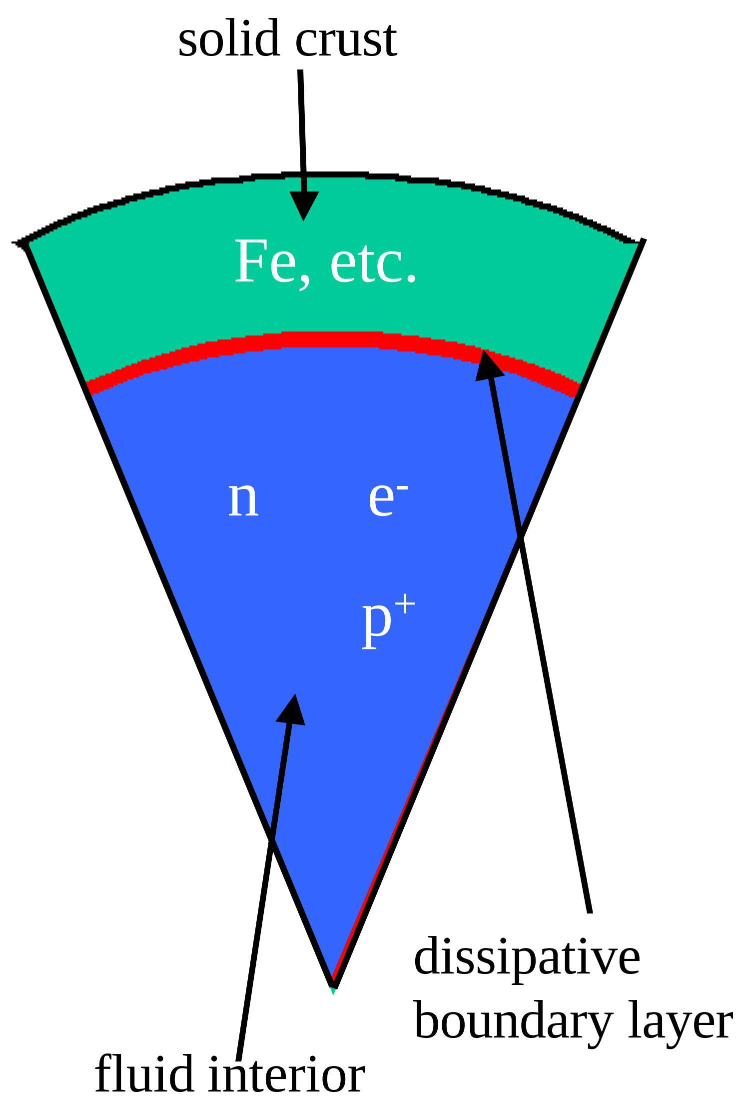
- **Pulsars** and **LMXBs** contain objects with the mass of the sun compressed to size of city; i.e., compact ($2GM/Rc^2 \sim .2$) and ultra dense ($10^{14\text{-}15}$ g/cm^3).
- **Neutron stars**: (superfluid) neutrons, (superconducting) protons, normal electrons, plus exotic particles (e.g., hyperons).
- **Strange stars**: up, down, & strange quarks.
- These objects spin rapidly and have intense magnetic fields.

Photos: http://antwrp.gsfc.nasa.gov; http://imagine.gsfc.nasa.gov

Neutron Star Basics



Simple Neutron Star Models



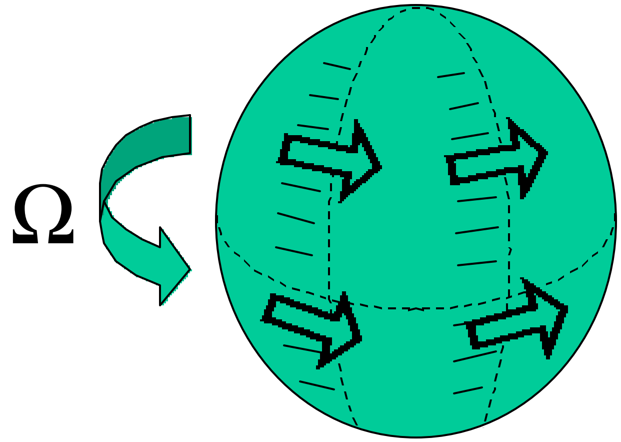
Courtesy Justin Kinney

Gravitational-radiation Driven Instability of Rotating Stars

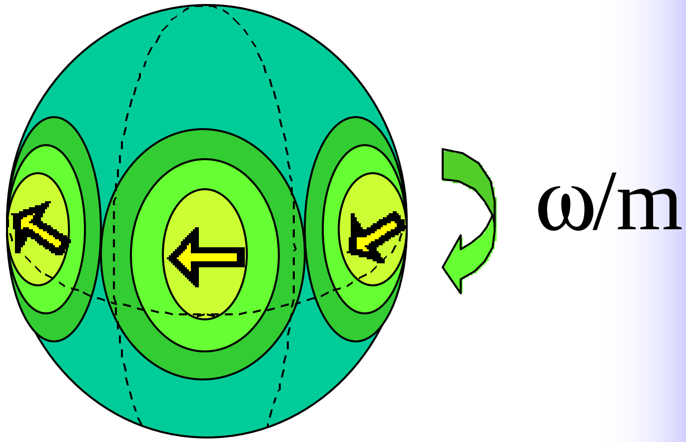
- GR tends to drive all rotating stars unstable!
- Internal dissipation suppresses the instability in all but very compact stars.

Perturbations in Rotating Neutron Stars

- Neutron star rotates with angular velocity Ω > 0.
- Some type of “wave” perturbation flows in the opposite direction with phase velocity ω/m, as seen in rotating frame of star.
- Perturbations create oscillating mass and current multipoles, which emit GR.



Rotating neutron star.

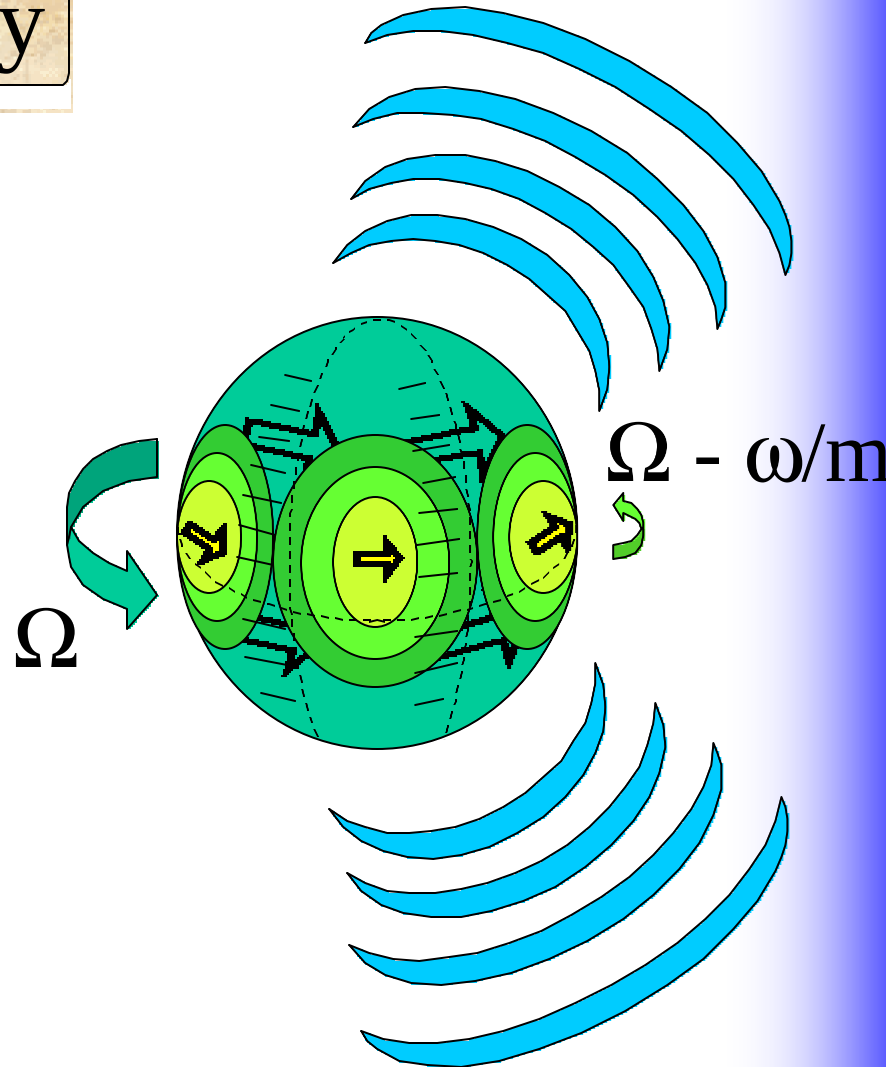


Perturbations in rotating frame.

Courtesy Justin Kinney

GR Causes Instability

- If Ω - ω/m > 0, star "drags" perturbations in opposite direction.
- GR carries away angular momentum.
- GR backreaction pushes on perturbations in direction they want to travel.
- This *increases* their amplitude!



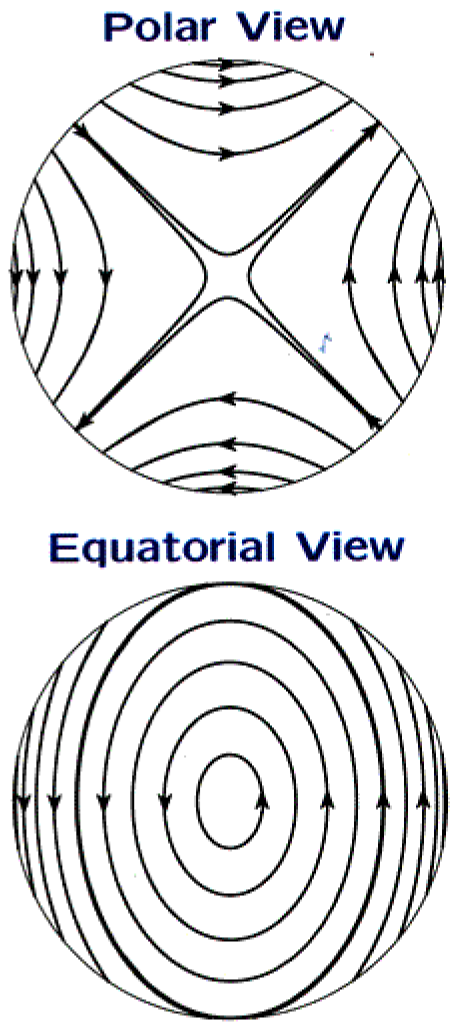
Star drags perturbations in opposite direction. GR drives mode unstable.

Courtesy Justin Kinney

The R-modes

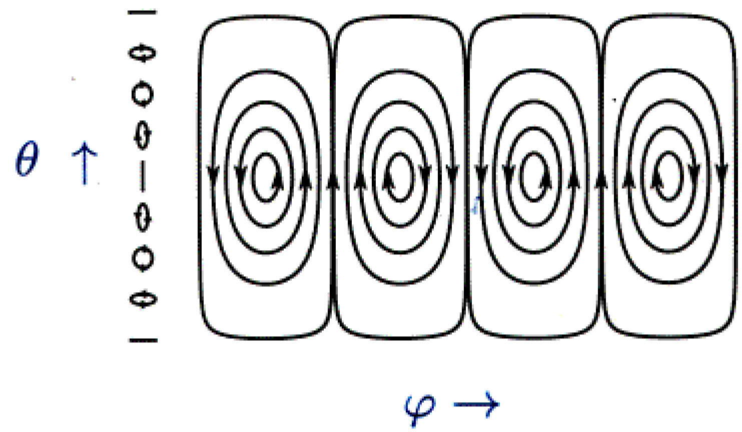
- The r-modes correspond to oscillating flows of material (currents) in the star that arise due to the Coriolis effect. The r-mode frequency is proportional to the angular velocity, W .
- The current pattern travels in the azimuthal direction around the star as exp$(iwt + im j)$.
- For the $m = 2$ r-mode:
  - Phase velocity in the corotating frame: $-1/3\,W$
  - Phase velocity in the inertial frame: $+2/3\,W$

Flow Pattern for the $m = 2$ r-mode



Courtesy Lee Lindblom

Fluid Motion in the $m = 2$ r-mode



- The flow pattern is shown along with the small elliptical paths (on the left) of individual fluid elements. The flow pattern moves (to the left) past the fluid particles as the mode evolves.

Courtesy Lee Lindblom

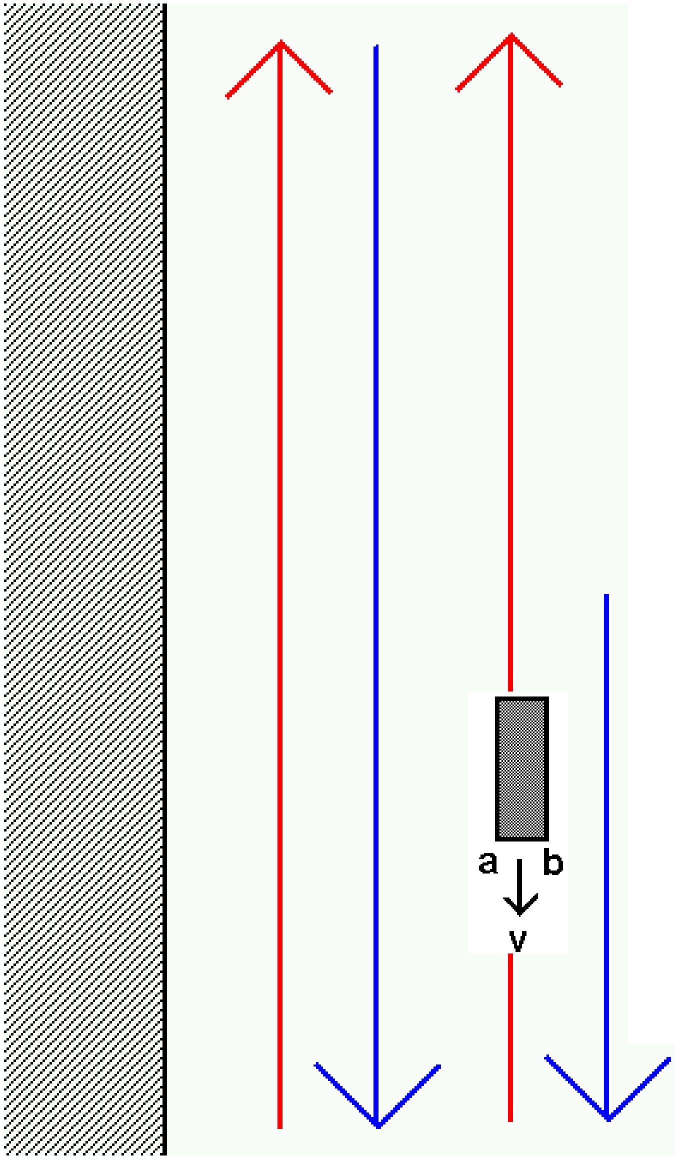
R-mode Instability Calculations

- Gravitational radiation tends to make the r-modes grow on a time scale τ_{GR}
- Internal friction (e.g., viscosity) in the star tends to damp the r-modes on a time scale τ_{v}
- The shorter time scale wins:
  - $\tau_{\mathbf{GR}} < \tau_{\mathbf{V}}$: Unstable!
  - $\tau_{\mathbf{GR}} > \tau_{\mathbf{V}}$: Stable!

Magnetic Effects on Viscous Boundary Layers

- Previously it has been shown that viscous boundary layer damping may be the most important suppression mechanism of the r-modes in neutron stars <u>with a solid crust</u> (Bildsten and Ushomirsky, ApJ **529**, L33 (2000)
- Magnetic effects on the viscous boundary layer were expected to be important at high temperatures.

LIGO



- The Shear Force:

$$F = \eta \frac{\left(\frac{\Delta v_y}{\Delta x}\right)_b - \left(\frac{\Delta v_y}{\Delta x}\right)_a}{\Delta x} A \Delta x,$$

where η is the viscosity.

- Newton's 2nd Law:

$$\frac{\partial v_y}{\partial t} = \frac{\eta}{\rho}\frac{\partial^2 v_y}{\partial x^2}.$$

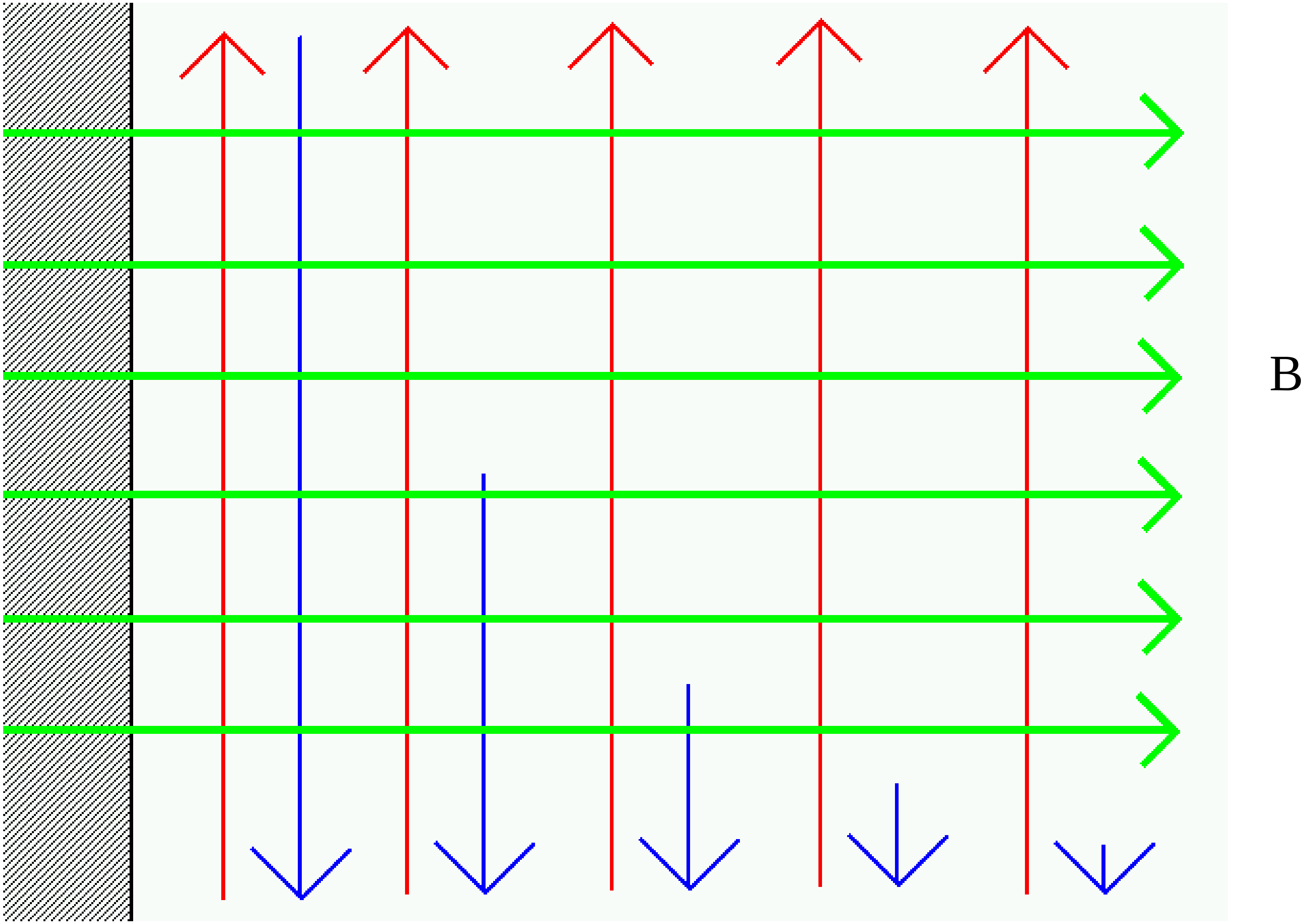
- For $v_y = \exp(i\omega t)\exp(ikx)$ the boundary layer thickness is given by:

$$d \equiv \frac{i}{\mathrm{Im}(k)} = \sqrt{\frac{2\eta}{\rho\omega}}.$$

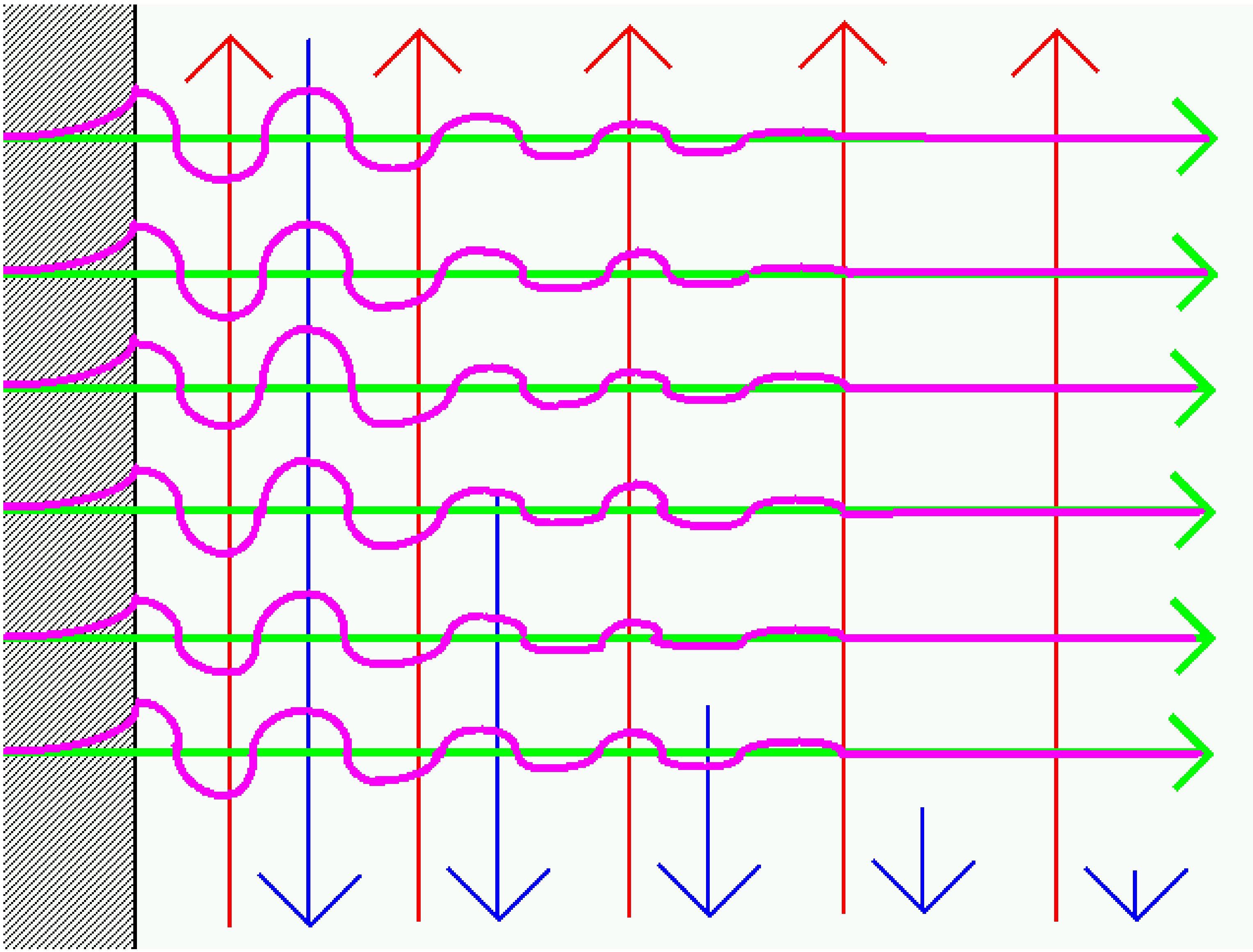
- VBL Damping Rate:

$$\frac{1}{\tau_v} \sim \eta |k|^2 d \sim \frac{\eta}{d}.$$

Add Magnetic Field…



Magneto-viscous Boundary Layer With Alfven Waves



LIGO

- Magnetohydrodynamic equations:

$$\partial_t \delta\vec{v} + 2\vec{\Omega}\times\delta\vec{v} = -\vec{\nabla}\delta U + \frac{1}{\rho}\left(\frac{\delta\vec{J}}{c}\times\vec{B}\right) + \frac{1}{\rho}\vec{\nabla}\cdot(2\eta\delta\overleftrightarrow{\sigma}),$$

$$\partial_t \delta\vec{B} = \vec{\nabla}\times(\delta\vec{v}\times\vec{B}),$$

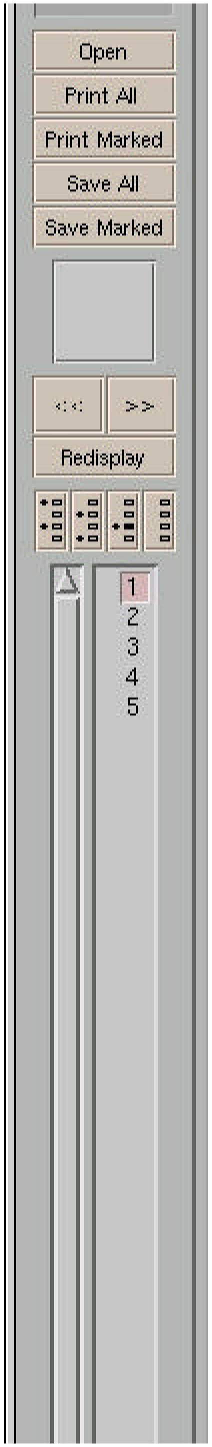
$$\vec{\nabla}\cdot\delta\vec{B} = 0.$$

- Approximate viscous boundary layer solution:

  * Keep only radial derivatives.

  * Let $\delta\tilde{v} \sim \exp[ik(r - R_c)]$.

$$k_\pm = K_\pm\sqrt{\frac{\Omega}{\frac{V_A^2}{\kappa\Omega} + \frac{i\eta}{\rho}}}$$



- Gravitation-radiation growth time-scale:

$$\frac{1}{\tau_{\rm GR}} = \frac{\omega(\omega + m\Omega)}{2E} \sum_{l \geq m} N_l \, \omega^{2l} \left(|\delta D_{lm}|^2 + |\delta J_{lm}|^2 \right),$$

where

$$\delta D_{lm} = \int r^l \delta\rho Y^*_{lm} \, d^3x.$$

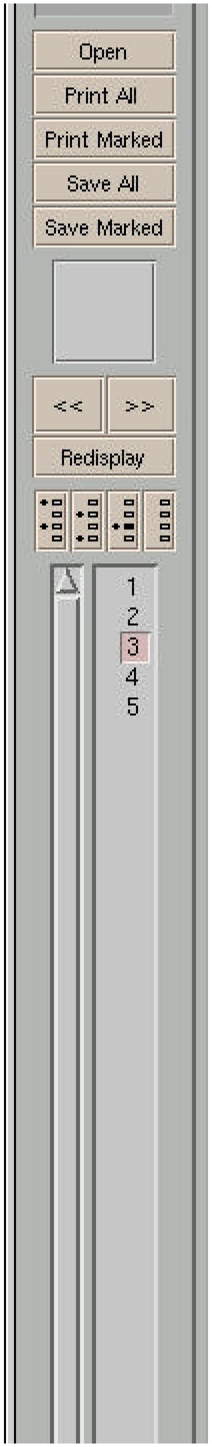
$$\delta J_{lm} = \int r^l (\rho \delta\vec{v} + \delta\rho \vec{v}) \cdot (\vec{Y}^B_{lm})^* \, d^3x.$$

- Shear viscosity damping time-scale:

$$\frac{1}{\tau_v} = \frac{1}{2E} \int 2\eta(\rho, T) \delta\sigma^{ab} \delta\sigma^*_{ab} \, d^3x.$$

where

$$\delta\sigma_{ab} = \frac{1}{2} \left[\nabla_a \delta v_b + \nabla_b \delta v_a - \frac{2}{3} g_{ab} \nabla_c \delta v^c \right].$$



- Viscous boundary layer equations:

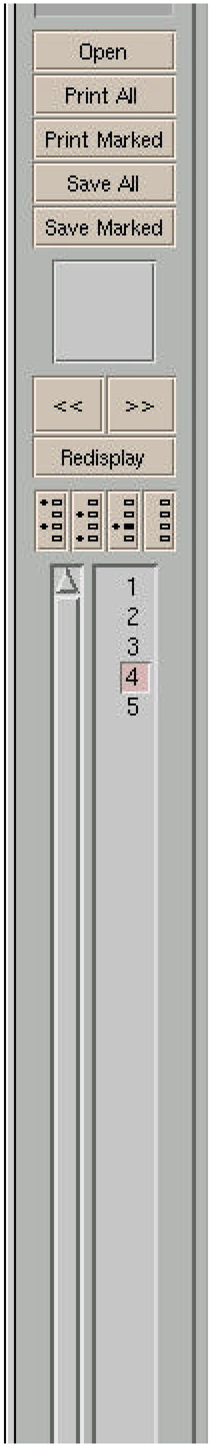
$$\delta\sigma^*_{ab}\delta\sigma^{ab} = \frac{1}{2}R_c^2(|\partial_r\delta\tilde{v}^\theta|^2 + |\partial_r(\sin\theta\delta\tilde{v}^\phi)|^2),$$

$$\tau_v = \frac{2\pi}{\eta\mathcal{I}}\frac{2^{m+3}(m+1)!}{m(2m+1)!!}\int_0^{R_c}\rho\left(\frac{r}{R_c}\right)^{2m+2}dr,$$

$$\mathcal{I} = \int_0^{2\pi}\int_0^{\pi}[|k_+|^2 d_+(1-\cos\theta)^2$$

$$+|k_-|^2 d_-(1+\cos\theta)^2]\sin^{2m-1}\theta d\theta d\phi,$$

$$k_\pm \equiv \frac{2\pi}{\lambda_\pm} + \frac{i}{d_\pm},$$



- Define VBL wavelength and VBL thickness:

$$k_\pm \equiv \frac{2\pi}{\lambda_\pm} + \frac{i}{d_\pm},$$

- Taylor expand for $\frac{V_{\rm A}^2}{\kappa\Omega} > \frac{i\eta}{\rho}$:

$$k_\pm = K_\pm \sqrt{\frac{\kappa\Omega^2}{V_{\rm A}^2}} \left[1 - \frac{i}{2}\frac{\eta}{\rho}\frac{\kappa\Omega}{V_{\rm A}^2}\right].$$

- Wavelength = distance Alfvén wave travels in one rotation; many of these wavelengths exist within a thick boundary layer:

$$\frac{\lambda}{2\pi} = \frac{3.7\,\mathrm{cm}}{|K_\pm|\kappa^{1/2}} B_{12}\Omega^{-1}_{2000\pi}\rho^{1/2}_{1.5e14},$$

$$d = \frac{3.4\times 10^5\,\mathrm{cm}}{|K_\pm|\kappa^{3/2}} B_{12}^3\Omega^{-2}_{2000\pi}\rho^{-11/4}_{1.5e14}T^2_{10}.$$

- Good for magnetic fields larger than the following lower bound:

$$B \geq (4.6\times 10^9\,\mathrm{G})\kappa^{1/2}\Omega^{1/2}_{2000\pi}\rho^{9/8}_{1.5e14}T^{-1}_{10}.$$

- Equation for the critical angular velocity:

$$\tau_{\mathrm{GR}} = \tau_v.$$

- Gravitational-radiation growth rate for the $m = 2$ r-modes:

$$\frac{1}{\tau_{\mathrm{GR}}} = 0.24\,\mathrm{s}^{-1} \left(\frac{\Omega}{\Omega_o}\right)^6,$$

where

$$\Omega_o = \sqrt{\pi G \bar{\rho}},$$

$$\Omega_{\mathrm{max}} \cong \frac{2}{3}\Omega_o,$$

- Magneto-viscous boundary layer damping rate for the $m = 2$ r-modes:

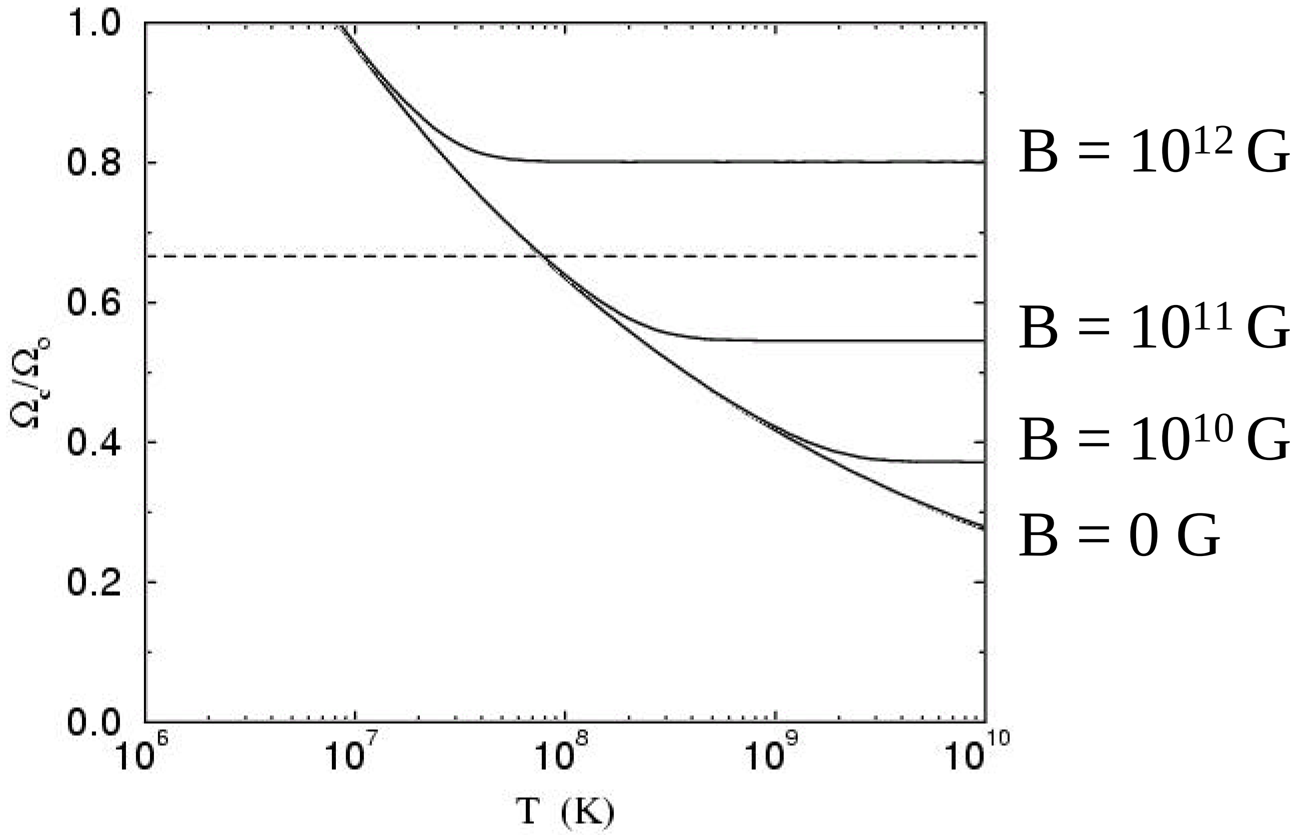
$$\frac{1}{\tau_v} \sim \eta |k|^2 d \sim \eta \frac{d}{\lambda^2} \sim 0.062\,\mathrm{s}^{-1}\, B_{12}.$$

- Critical angular velocity:

$$\frac{\Omega_c}{\Omega_o} = 0.8 B_{12}^{1/6}.$$

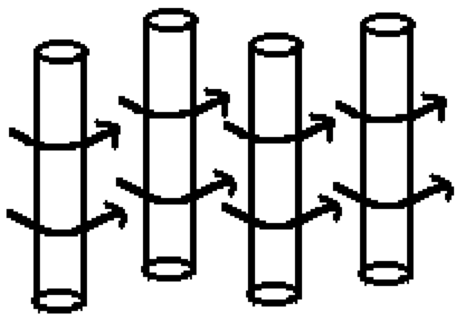
MVBL Critical Angular Velocity

Mendell, Phys. Rev. D 64, 044009 (2001)



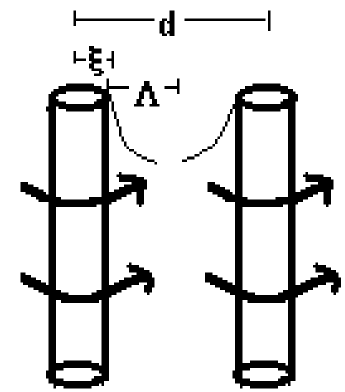
Quantized Vortices

- **Circulation and flux are quantized in superfluids and type II superconductors respectively.**



$$\oint \vec{v} \cdot d\vec{\ell} = 2pn \frac{\hbar}{2m_n}$$

$$\oint \vec{A} \cdot d\vec{\ell} = 2pn \frac{\hbar c}{2e}$$



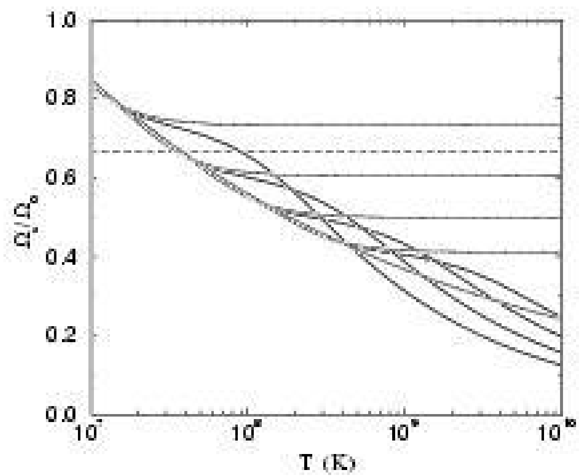
- **Arrays of quantized vortices form. Typical vortex densities are 10^6 neutron vortices per cm^2 and 10^{20} proton vortices per cm^2. For neutron vortices the velocity field is long range and varies inversely with distance from the vortex. For proton vortices the velocity and magnetic fields are short range and are characterized by a decay length known as the London depth, L ~ 10^{-11} cm. The vortex core radii = x ~ 10^{-11-12} cm. The vortex cores consist of normal fluid.**

Superfluid Effects

- No slip boundary condition replaced with vortex pinning.
- Alfven waves are replaced with cyclotron-vortex waves.
- Magnetic effects are similar to ordinary fluid case, but important for $B \geq 10^9$ G T_8^{-2}.
- Mutual friction is important too.

Critical Angular Velocity

Superfluid Case



B = 10^{12} G
B = 10^{11} G
B = 10^{10} G
B = 10^{9} G
B = 0 G

Justin Kinney and Gregory Mendell gr-qc/0206001; to appear in Phys Rev D.

Spin Cycles of LMXBs

Simple Evolution Equations

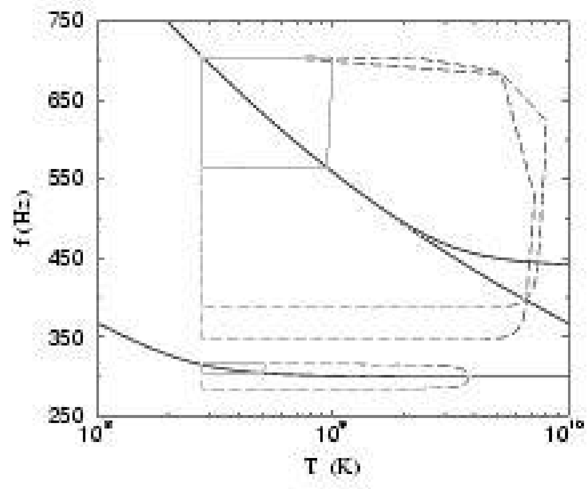
$$\frac{1}{\alpha}\frac{d\alpha}{dt} = F_g - F_v + F_g K_c \alpha^2 - \frac{1}{2}F_a, \tag{5.1}$$

$$\frac{1}{\Omega}\frac{d\Omega}{dt} = -2F_g K_c \alpha^2 + F_a, \tag{5.2}$$

$$C(T)\frac{dT}{dt} = W_{\rm diss} + K_n \dot{M} c^2 - L_\nu(T). \tag{5.3}$$

Spin Cycles of LMXBs

Ordinary Fluid Case



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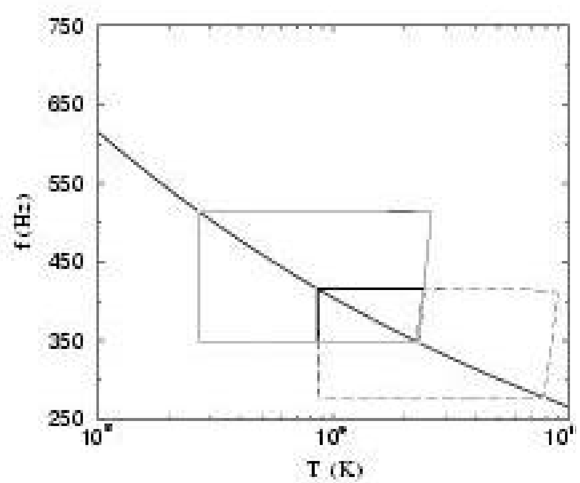
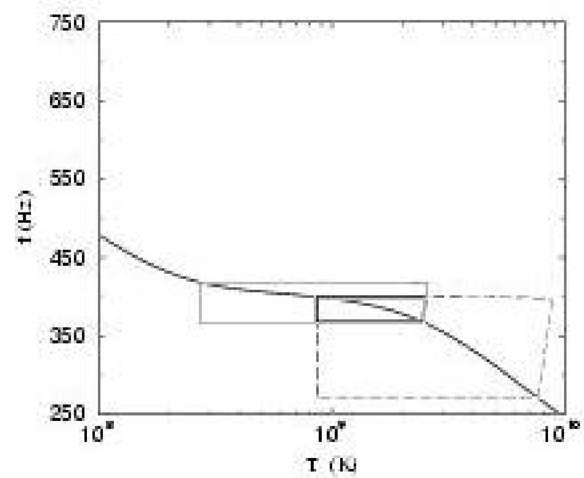
Spin Cycles of LMXBs

Superfluid Case

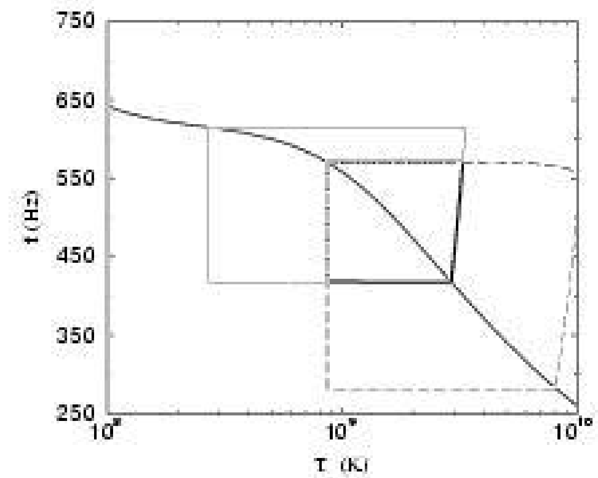
$B = 0$ G



$B = 10^9$ G



$B = 10^{10}$ G



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