Laser Interferometer Space Antenna (LISA)

Optimal Time-Delay Interferometry for LISA

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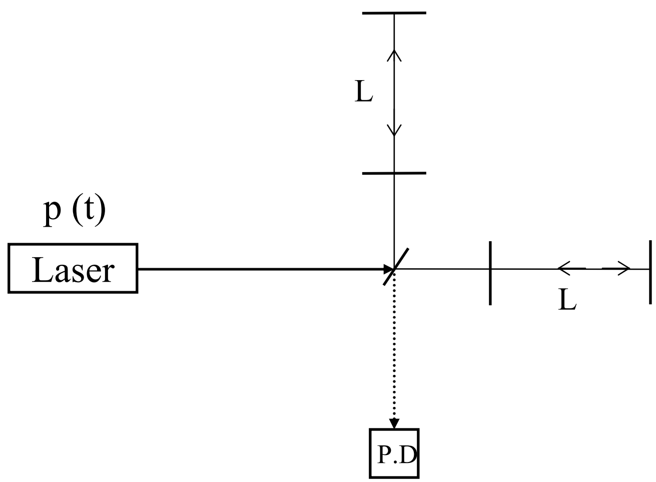
Earth vs. Space-Based Interferometers

- Earth-based interferometers have arm lengths essentially equal. This is in order to directly remove laser frequency fluctuations at the photodetector, where the two beams interfere.
- They operate in the long-wavelength limit ($L \ll \lambda$).
- By contrast, LISA will have arm lengths significantly different ($\Delta L/L \sim 10^{-2}$), with $L = 5 \times 10^6$ km.
- Over much of its sensitivity frequency-band, it will **not** operate in the long-wavelength regime.
- Time-of-flight delays in the response to the wave, and travel times along the beams in the detector must be allowed for, in order to derive a correct theory of the detector response.

Statement of The Problem

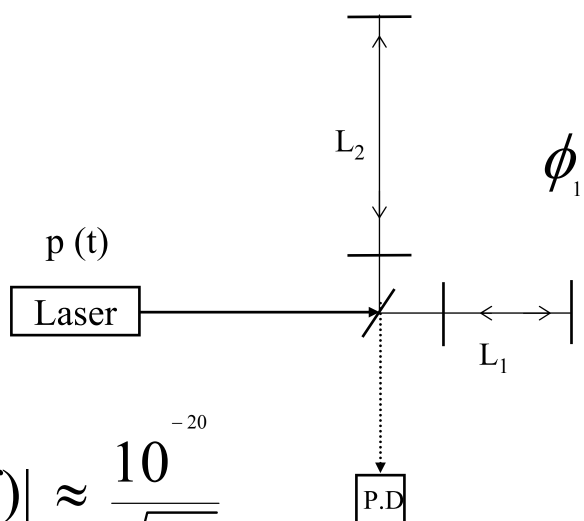
p (t) = Laser phase fluctuations

$$\frac{1}{2\pi\nu_0}\frac{dp(t)}{dt} = \left[\frac{\Delta\nu(t)}{\nu_0}\right]_{Laser} \equiv C(t)$$



$$\phi_1(t) = h_1(t) + p(t - 2L_1) + n_1(t)$$

$$\phi_2(t) = h_2(t) + p(t - 2L_2) + n_2(t)$$

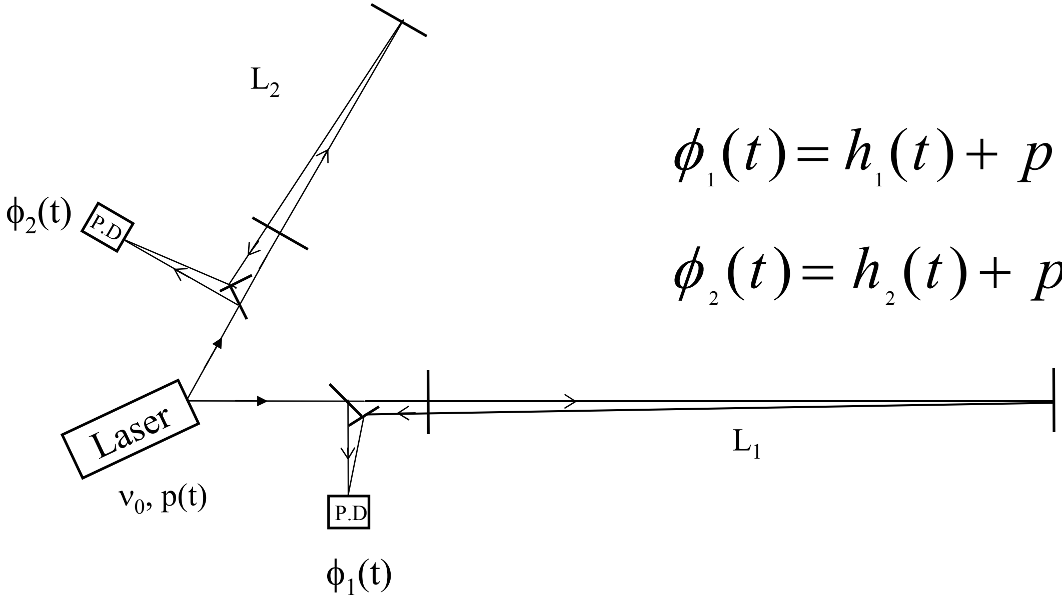


$$\phi_1(t) - \phi_2(t) \Rightarrow p(t - 2L_1) - p(t - 2L_2) \cong 2\frac{dp}{dt}\varepsilon L_1$$

$$|\tilde{h}(f)| \approx \frac{10^{-20}}{\sqrt{Hz}}$$

$$|\tilde{C}(f)| \approx \frac{10^{-13}}{\sqrt{Hz}}, \ \varepsilon \cong 3\times10^{-2} \Rightarrow \frac{5\times10^{-16}}{\sqrt{Hz}}$$

Unequal-arm Interferometers



$$\phi_1(t) = h_1(t) + p(t-2L_1) - p(t) + n_1(t)$$

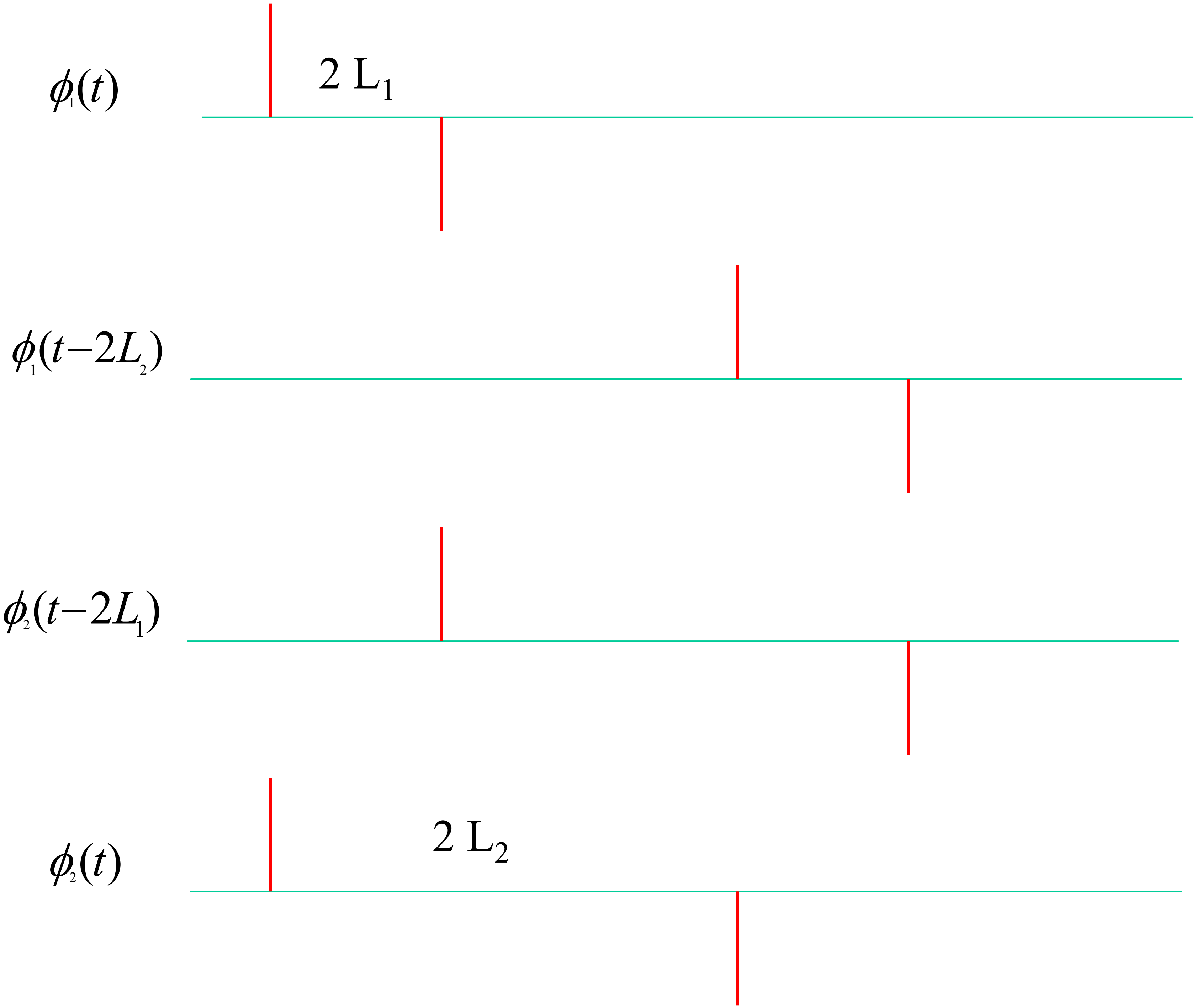
$$\phi_2(t) = h_2(t) + p(t-2L_2) - p(t) + n_2(t)$$

$$\phi_1(t) - \phi_2(t) = h_1(t) - h_2(t) + p(t-2L_1) - p(t-2L_2) + n_1(t) - n_2(t)$$

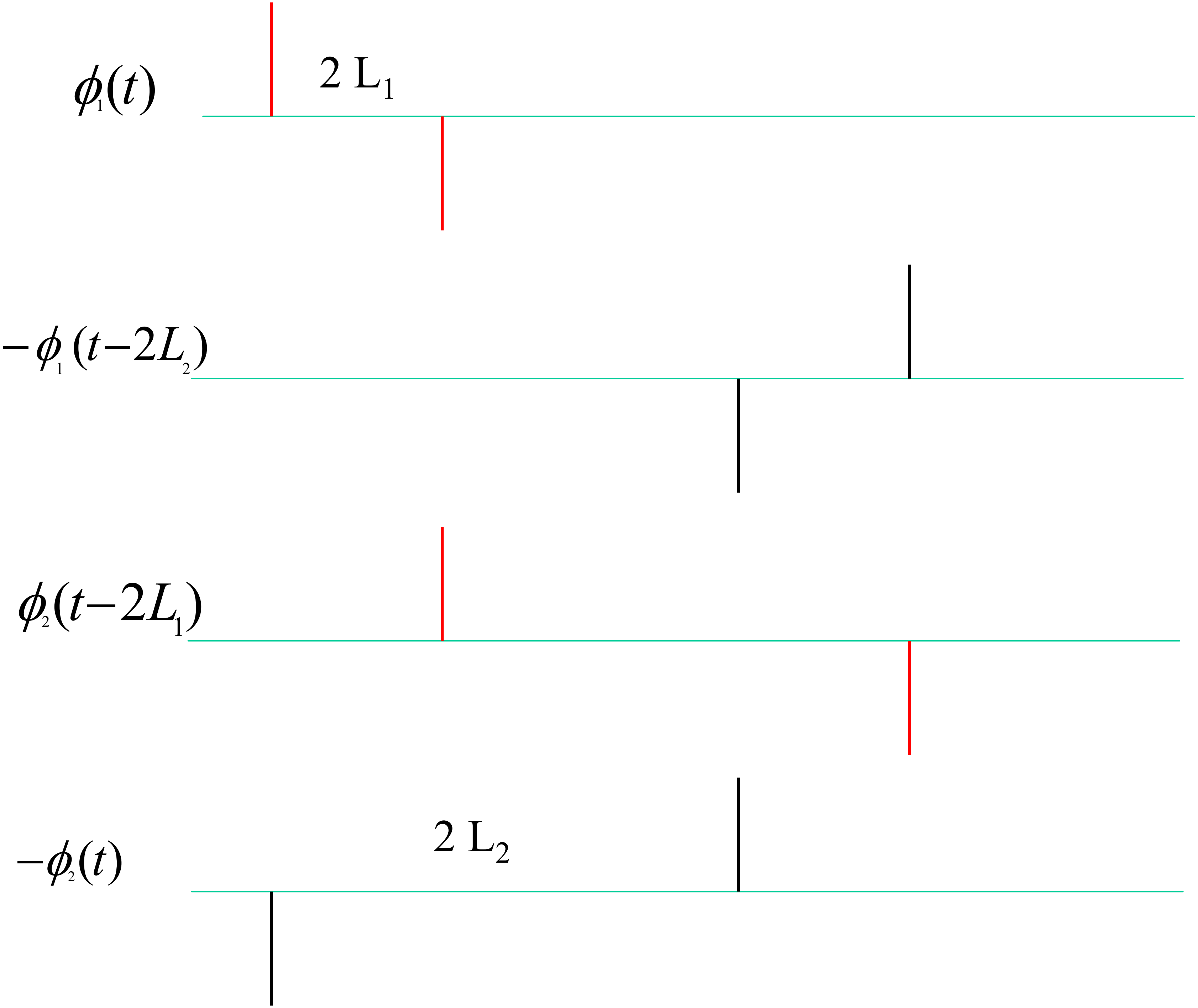
$$\phi_1(t-2L_2) - \phi_2(t-2L_1) = h_1(t-2L_2) - h_2(t-2L_1) + p(t-2L_1) - p(t-2L_2) + n_1(t-2L_2) - n_2(t-2L_1)$$

$$X(t) \equiv [\phi_1(t) - \phi_2(t)] - [\phi_1(t-2L_2) - \phi_2(t-2L_1)]$$

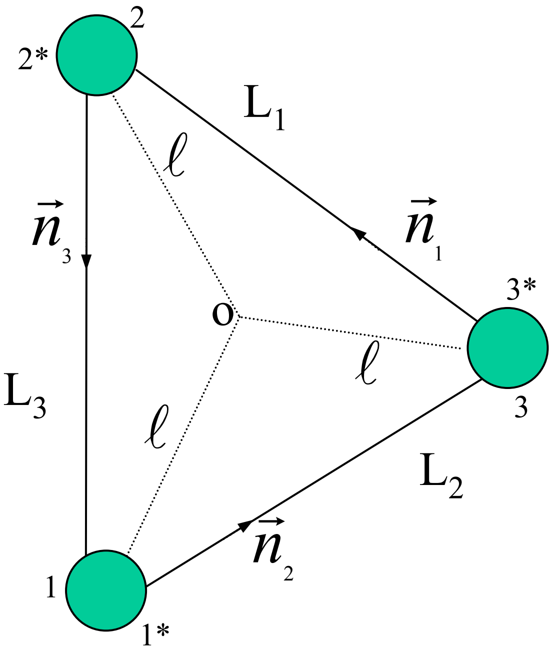
M. Tinto, & J.W. Armstrong, *Phys. Rev. D*, **59**, 102003 (1999).



$\phi_1(t)$
2 L_1
$\phi_1(t-2L_2)$
$\phi_2(t-2L_1)$
$\phi_2(t)$
2 L_2


$\phi_1(t)$
2 L_1
$-\phi_1(t-2L_2)$
$\phi_2(t-2L_1)$
$-\phi_2(t)$
2 L_2

Time-Delay Interferometry (T.D.I.)



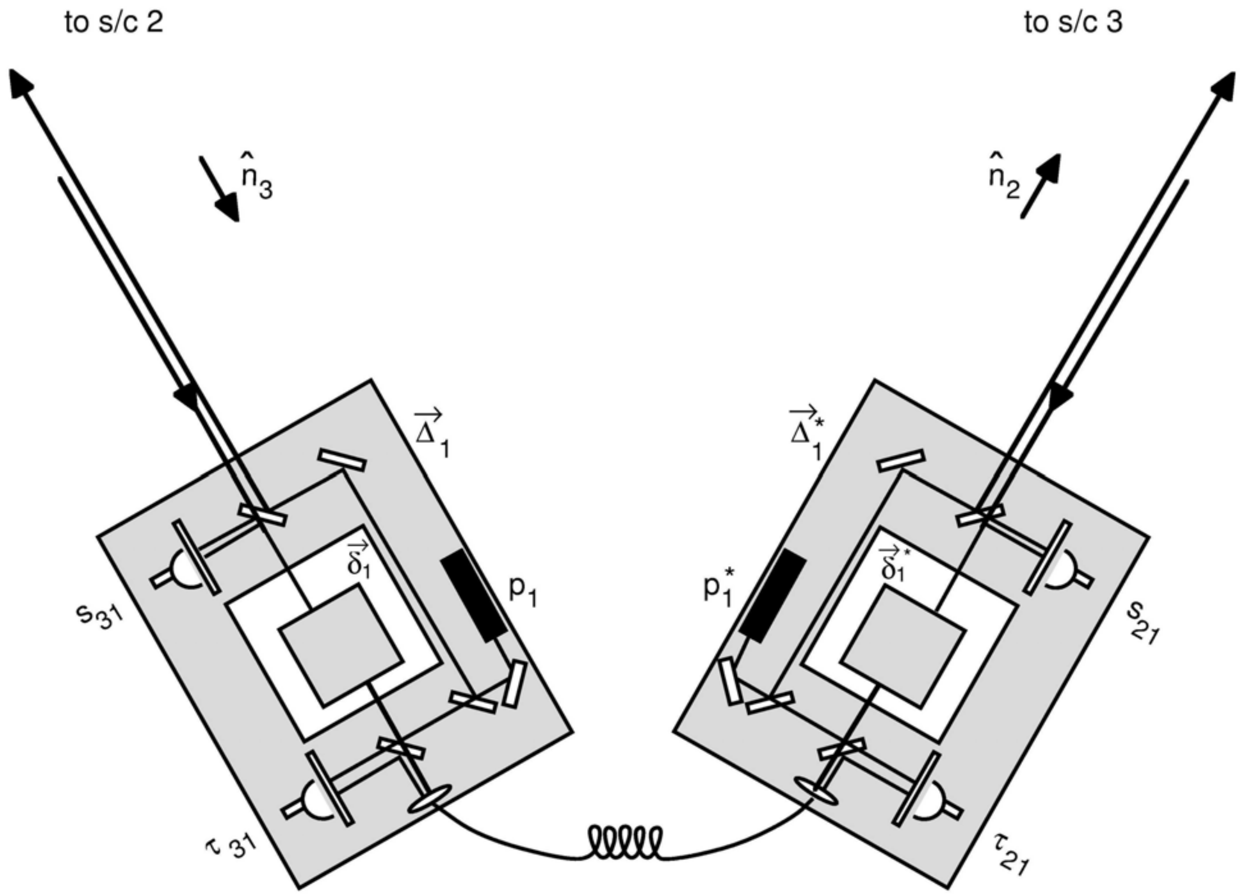
- It is best to think of LISA as a closed array of six one-way delay lines between the test masses.

- This approach allows us to reconstruct the unequal-arm Michelson interferometer, as well as new interferometric combinations, which offer advantages in hardware design, in robustness to failures of single links, and in redundancy of data.

M. Tinto: *Phys. Rev. D,* **53**, 5354 (1996); *Phys. Rev. D,* **58**, 102001 (1998)

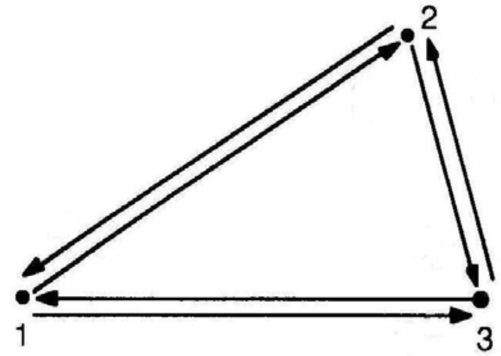
J.W. Armstrong, F.B. Estabrook, and M. Tinto: *Ap. J.,* **527**, 814 (1999).

Time-delay Interferometry & The Drag-free Configuration



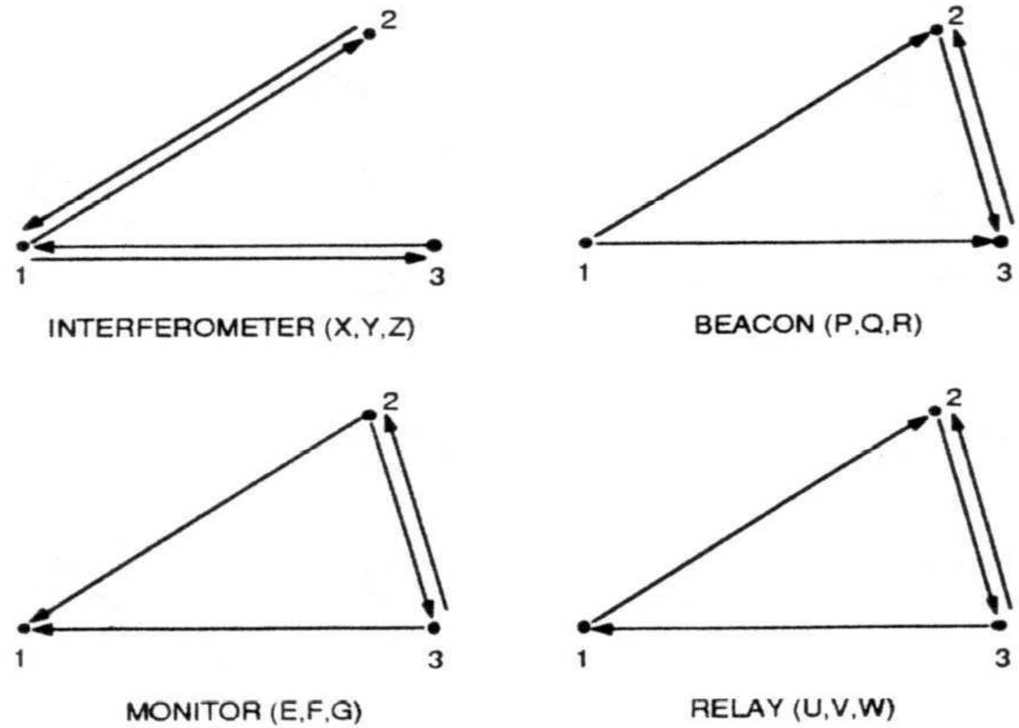
F.B. Estabrook, M. Tinto, and J.W. Armstrong: *Phys. Rev. D,* **62**, 042002 (2000)
M. Tinto, F.B. Estabrook, and J.W. Armstrong: *Phys. Rev. D*, **65**, 082003, (2002)

Six-Pulse Data Combinations



α, β, γ, ζ

Eight-Pulse Data Combinations



Data Combinations (Cont.)

- There are 6 optical benches, 6 lasers, 3 Ultra Stable Oscillators (USO), and a total of 18 Doppler time series observed.
- The 6 beams exchanged between distant spacecraft contain the information about the GW signal; the other 12 signals are for comparison of the lasers, relative optical bench motions within the spacecraft, and calibration of the USO phase noises affecting the interferometric observables.
- The four combinations $(\alpha, \beta, \gamma, \varsigma)$ can be regarded as the generators of the functional space of all possible interferometric combinations.

$$\varsigma - \varsigma_{,123} = \alpha_{,1} - \alpha_{,23} + \beta_{,2} - \beta_{,31} + \gamma_{,3} - \gamma_{,12}$$

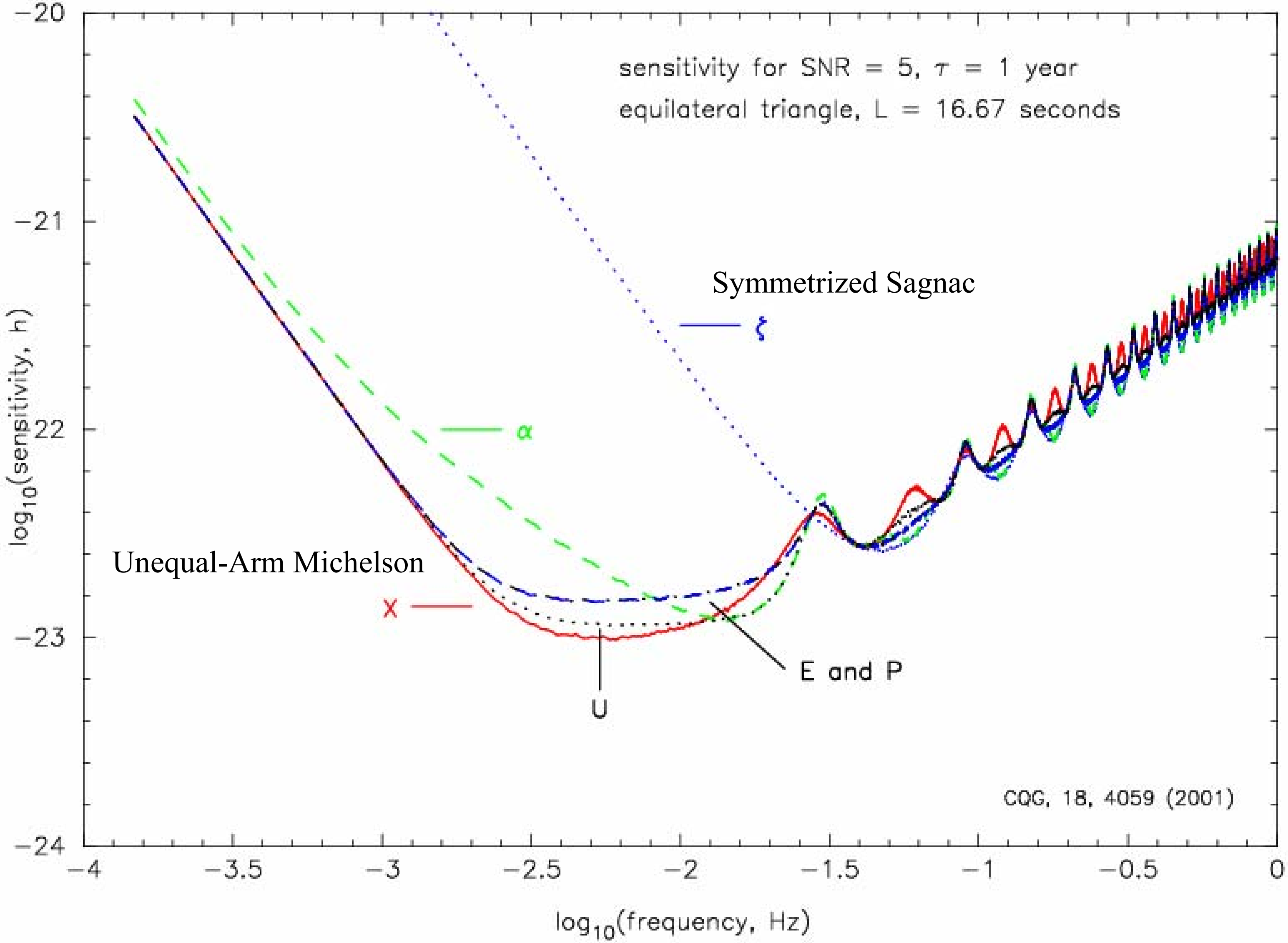
$$X_{,1} = \alpha_{,32} - \beta_{,2} - \gamma_{,3} + \varsigma$$

$$P = \varsigma - \alpha_{,1}$$

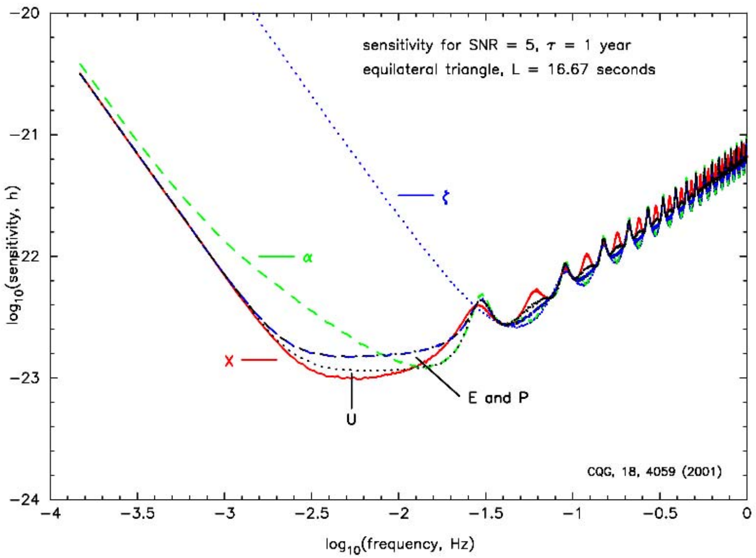
$$E = \alpha_{,1} - \varsigma_{,1}$$

$$U = \gamma_{,1} - \beta$$

J.W. Armstrong, F.B. Estabrook, and M. Tinto: *Ap. J.,* **527**, 814 (1999)
S.V. Dhurandhar, K.R. Nayak, and J-Y. Vinet, *Phys. Rev. D*, **65**, 102002 (2002)


sensitivity for SNR = 5, τ = 1 year
equilateral triangle, L = 16.67 seconds
Symmetrized Sagnac
ζ
α
Unequal-Arm Michelson
X
U
E and P
CQG, 18, 4059 (2001)
log10(sensitivity, h)
log10(frequency, Hz)
−20
−21
−22
−23
−24
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0

Optimal Interferometric Combinations



$$\eta = a_1(f)\tilde{\alpha} + a_2(f)\tilde{\beta} + a_3(f)\tilde{\gamma} + a_4(f)\tilde{\zeta}$$

$$[1 - e^{2\pi i f (L_1 + L_2 + L_3)}]\tilde{\zeta} = [e^{2\pi i f L_1} - e^{2\pi i f (L_2 + L_3)}]\,\tilde{\alpha} + [e^{2\pi i f L_2} - e^{2\pi i f (L_1 + L_3)}]\,\tilde{\beta} + [e^{2\pi i f L_3} - e^{2\pi i f (L_1 + L_2)}]\,\tilde{\gamma}$$

$$SNR_\eta^{\;2} = \int \frac{|\, a_1(f)\tilde{\alpha}_s + a_2(f)\tilde{\beta}_s + a_3(f)\tilde{\gamma}_s \,|^2}{<|\, a_1(f)\tilde{\alpha}_n + a_2(f)\tilde{\beta}_n + a_3(f)\tilde{\gamma}_n \,|^2>}\, df$$

- One should regard the SNR as a functional of the functions a_i (f), and extremize it with respect of them.

T. Prince, M. Tinto, S. Larson, J.W. Armstrong, *Phys. Rev. D*, **66**, 122002 (2002)

Optimal Interferometric…(Cont.)

$$SNR_\eta^{\;2} = \int \frac{a_i \, A_{ij} \, a_j^*}{a_r \, C_{rt} \, a_t^*} df$$

$$A_{ij} = X_i^{(s)} X_j^{(s)*} \quad ; \quad C_{ij} = \langle X_i^{(n)} X_j^{(n)*} \rangle$$

$$(X_1, X_2, X_3) \equiv (\alpha, \beta, \gamma)$$

- The stationary values of the SNR_η correspond to the stationary points of the integrand.
- Since the quadratic form **C** is non-singular, we can identify the stationary points of the integrand by using the **Rayleigh's Principle for Quadratic Forms**: *The stationary values of the integrand are attained at the eigenvalues of the matrix (**$C^{-1}A$**)*

Optimal Interferometric…(Cont.)

- Since **A** has rank 1, it follows that also $\mathbf{C}^{-1}\mathbf{A}$ has rank 1.
- The only non-zero eigenvalue of the matrix $\mathbf{C}^{-1}\mathbf{A}$ is therefore equal to Tr ($\mathbf{C}^{-1}\mathbf{A}$). This implies the following expression for the optimal signal-to-noise ratio:

$$SNR^2{}_{\eta_{opt.}} = \int X_i^{(s)^*} \, C^{-1}{}_{ij} \, X_j^{(s)} \, df$$

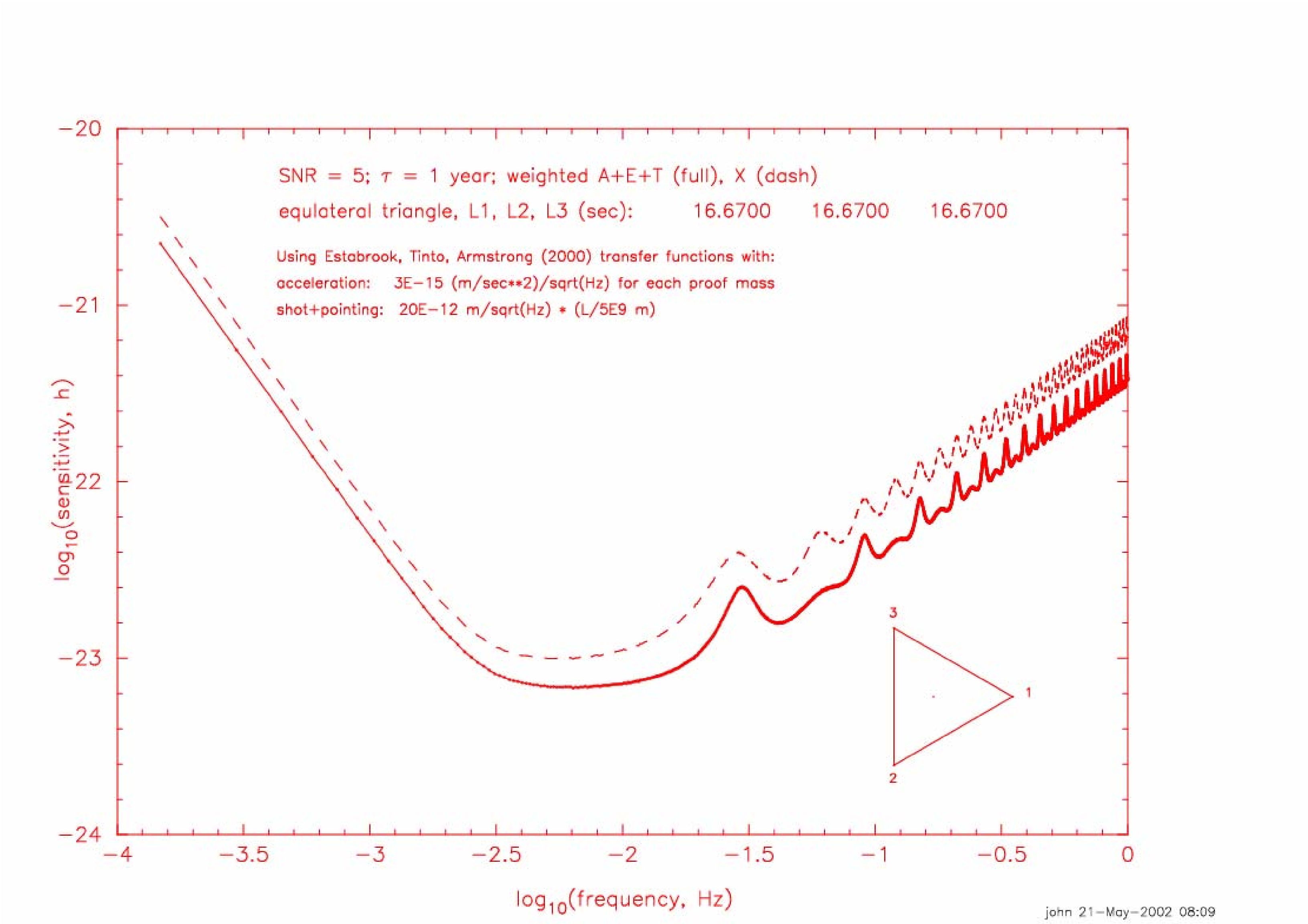
- We can change basis, and work with a triad of interferometric observables that have diagonal covariance matrix.

Optimal Interferometric….(Cont.)

$$\begin{bmatrix} \mathrm{A} \\ \mathrm{E} \\ \mathrm{T} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}}(\tilde{\gamma} - \tilde{\alpha}) \\ \frac{1}{\sqrt{6}}(2\tilde{\beta} - \tilde{\alpha} - \tilde{\gamma}) \\ \frac{1}{\sqrt{3}}(\tilde{\alpha} + \tilde{\beta} + \tilde{\gamma}) \end{bmatrix}$$

$$SNR_{\eta}^{\ 2} = \int \left[\frac{\mathrm{A}_s \mathrm{A}_s{}^*}{<\mathrm{A}_n \mathrm{A}_n{}^*>} + \frac{\mathrm{E}_s \mathrm{E}_s{}^*}{<\mathrm{E}_n \mathrm{E}_n{}^*>} + \frac{\mathrm{T}_s \mathrm{T}_s{}^*}{<\mathrm{T}_n \mathrm{T}_n{}^*>} \right] df$$

Optimal Interferometric….(Cont.)



Conclusions

- T.D.I. provides an exact method for canceling the leading noise source – laser phase fluctuations – in an interferometer with unequal, time-variable arms.
- It allows analysis of signals, noises, achievable sensitivities, and architectural design (including system-level tradeoffs between, e.g. Laser stability, arm length accuracy, stability of optical bench, Doppler shifts due to chosen orbits, USO stability).
- It provides robustness of the mission with respect to failures of subsystems.
- It shows existence of alternate LISA configurations offering potential design, implementation, or cost advantages.
- It gives a data combination (ζ) for assessing the LISA on-orbit instrumental noise performance.
- The LISA optimal sensitivity can be obtained by regarding LISA as a network of three Interferometers.