CRYOGENIC INTERFEROMETERS:

Some Good Reasons for Pursuing Them

Amaldi Conference
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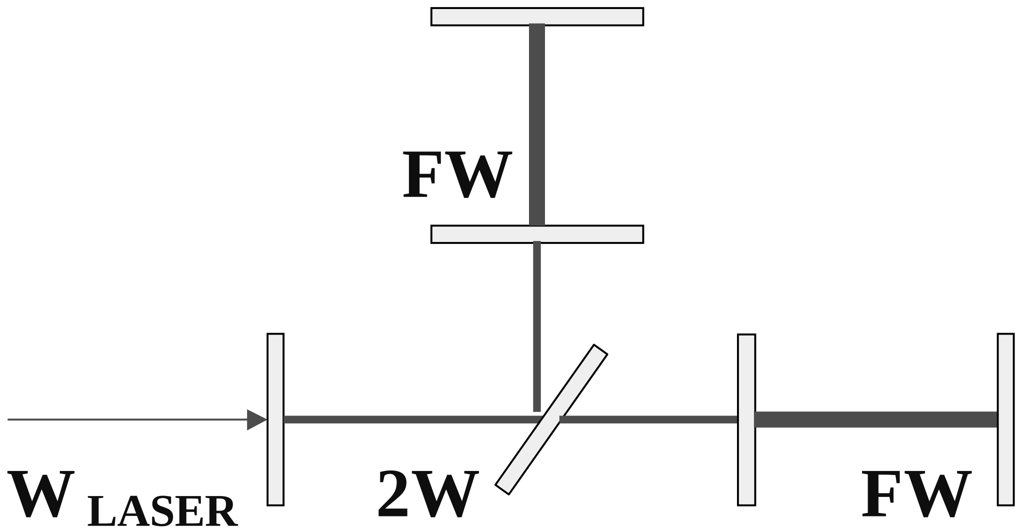
The Fabry-Perot Michelson optical scheme:

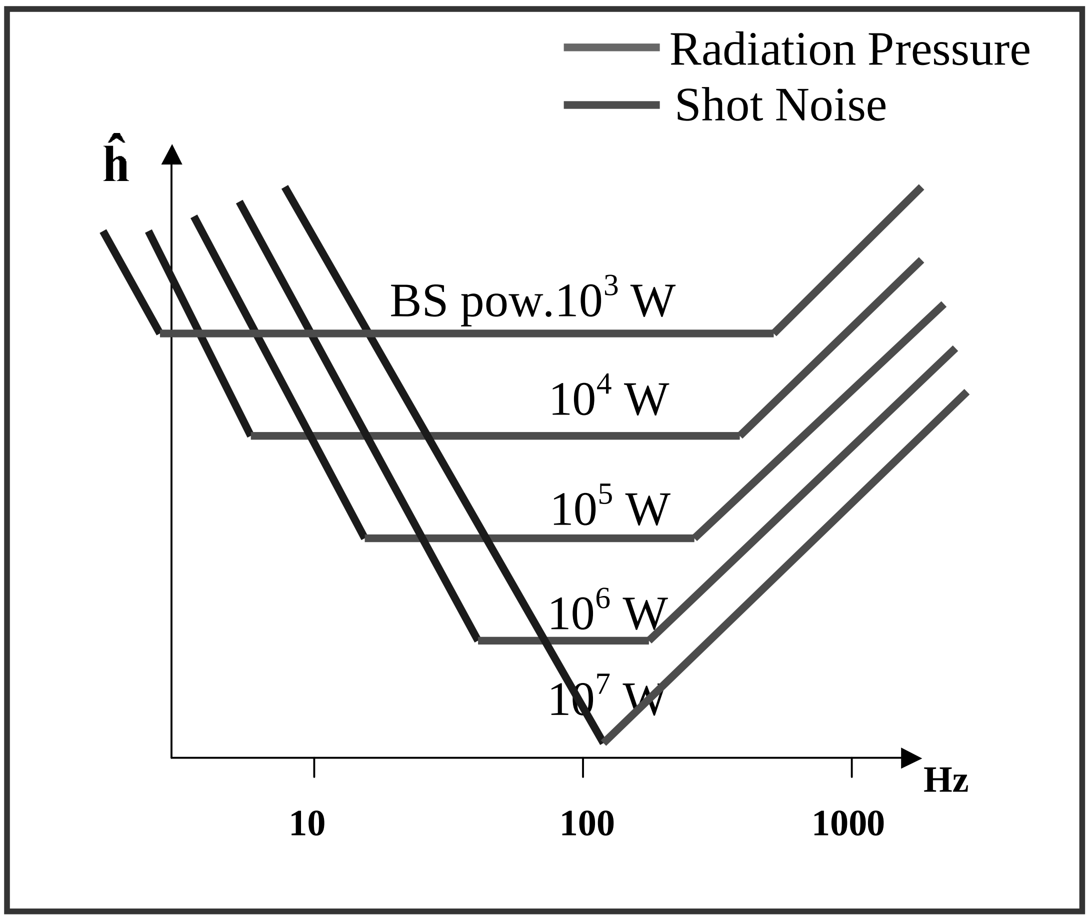


Where F is the finesse and W the power on the beam-splitter Radiation Pressure (RP) and Shot Noise (SN) are Quantum noises and undergoe Heisenberg Uncertainty Principle:

$$\tilde{h}^2 \geq \underbrace{\frac{F^2}{1+(F\omega L/c)^2}\frac{Wh\nu}{L^2c^2M^2\omega^4}}_{\text{RADIATION PRESSURE}} + \underbrace{\frac{1+(F\omega L/c)^2}{F^2}\frac{\lambda^2}{16\pi^2}\frac{h\nu}{L^2W}}_{\text{SHOT NOISE}}$$

The terms W and $$\frac{F^2}{+\left(F\omega L/c\right)^2}$$

have dramatic effects on the detector sensitivity and bandwidth:

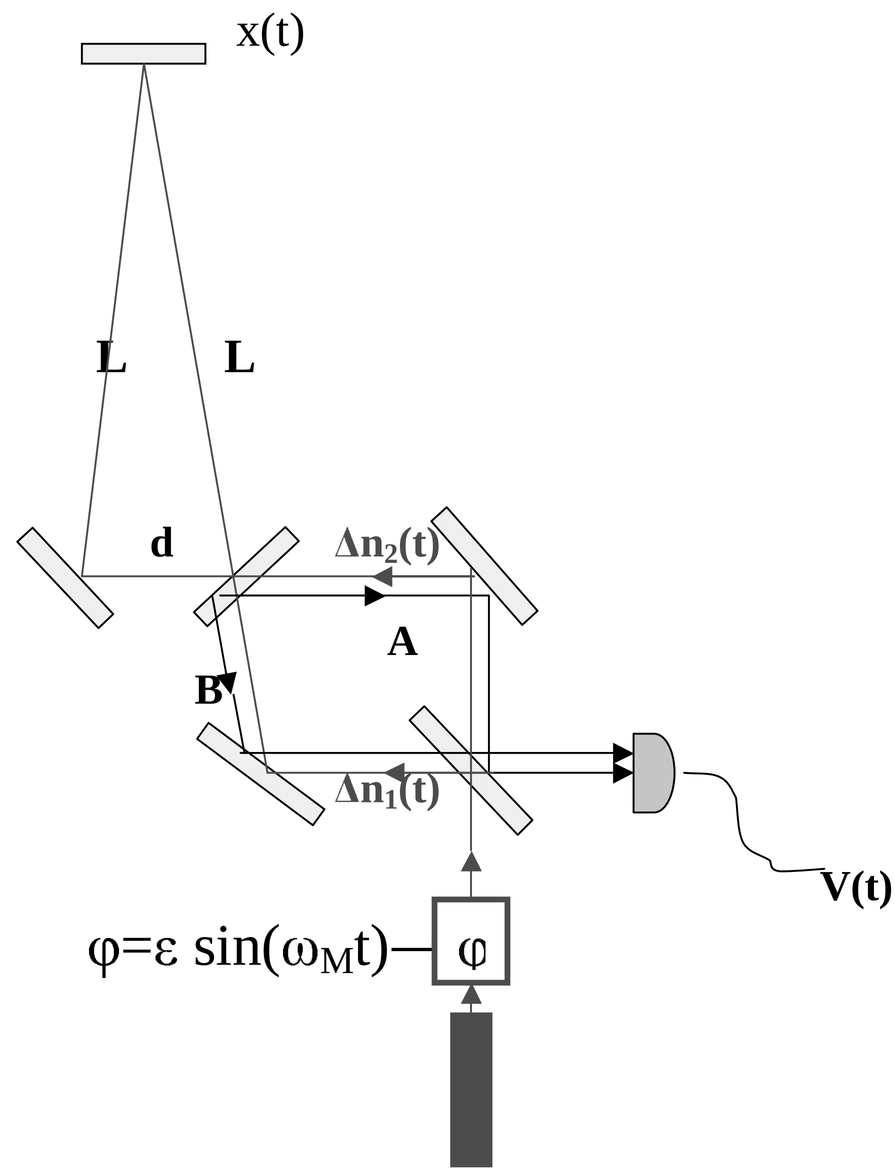


Recent development on Quantum-Non-Demolition (QND) techniques are extremely interesting. Sagnac ITF measures mirror speed dx/dt⇒iωx and RP fluctuations cancels out up to $(\omega d/c)^2$ terms.

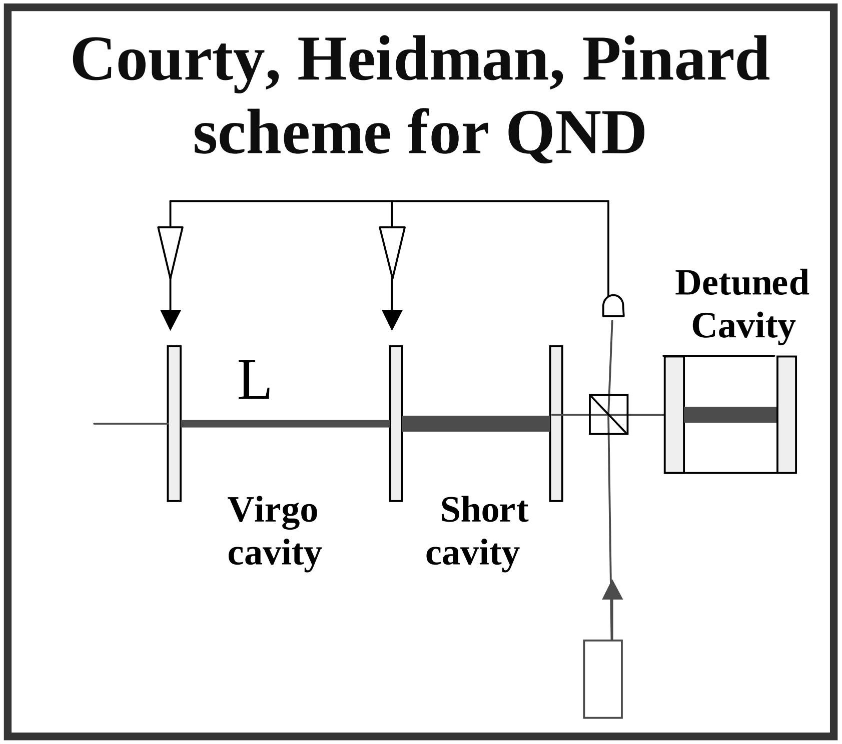
Unfortunately in Sagnac ITF Shot Noise is overwhelming (with respect to Michelson ITF) due to the $\omega d/c \sim O(10^{-5})$ term in the signal:

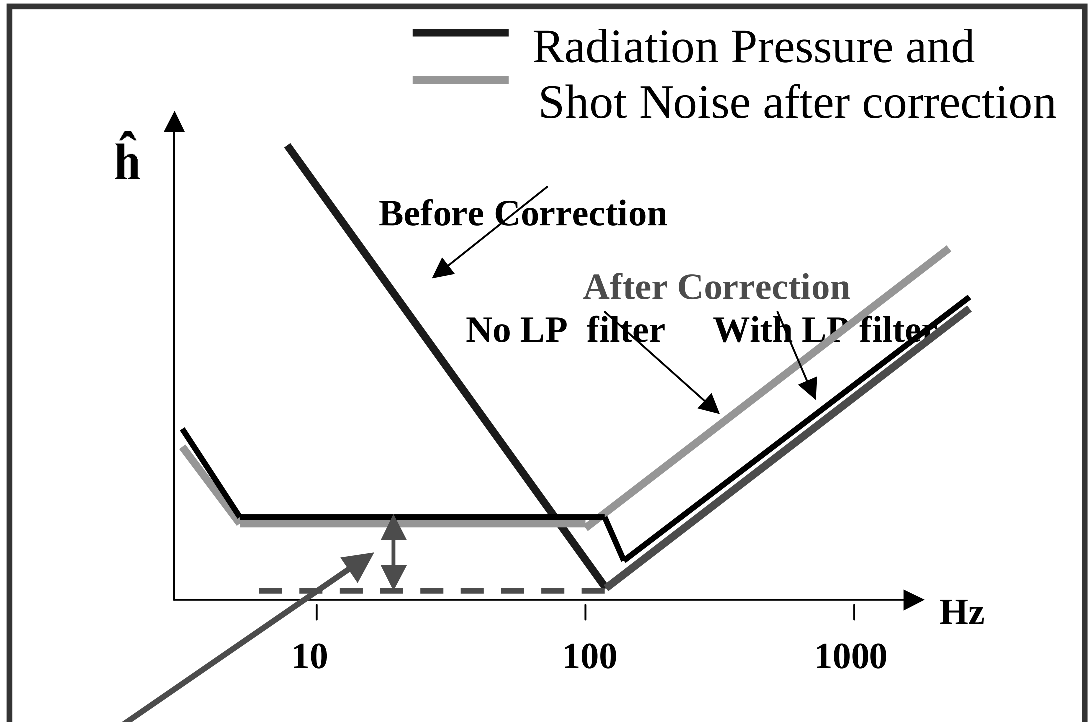
$$V(\omega) \sim \omega \frac{d}{c} F L h + \frac{\lambda}{4\pi} \sqrt{\frac{h\upsilon}{W}}$$

Sagnac are "natural" QND ITF



More suitable optical schemes have been investigated succesfully; as an example the Courty, Heidman, Pinard scheme:





This method should be used on each FP cavity and allows in priciple to reduce very strongly RP effects; the price to pay is anyway an increase of Shot Noise which can be reduced by increasing the power on Beam Splitter (BS).

After these considerations an optimistic statement could be the following:
Radiation Pressure effects can be transformed in a Shot noise increase and, in priciple, <u>this noise can be reduced by increasing the power on the BS</u>.

In an optimistic view, since <u>it seems</u> we can reduce strongly RP and SN problems by using QND methods and by increasing the power on BS, we are left to worry about two enormous problems:

1) **Thermal Noise**
2) **Lensing effects in the mirrors due to the power increase.**

Going to low temperature could be an interesting option giving several advantages.

6 good reasons for going to low temperature:

1)For reducing Phothermal noise

2)For reducing the thermodynamical noise

3)For reducing the thermoelastic noise

4)For reducing Electromagnetic coupling Mirror-Environment by making the superconductive the cryostat surrounding the mirror. (See Mitrofanov)

5)For Reducing TN

6)For reducing Mirror Optical Lensing

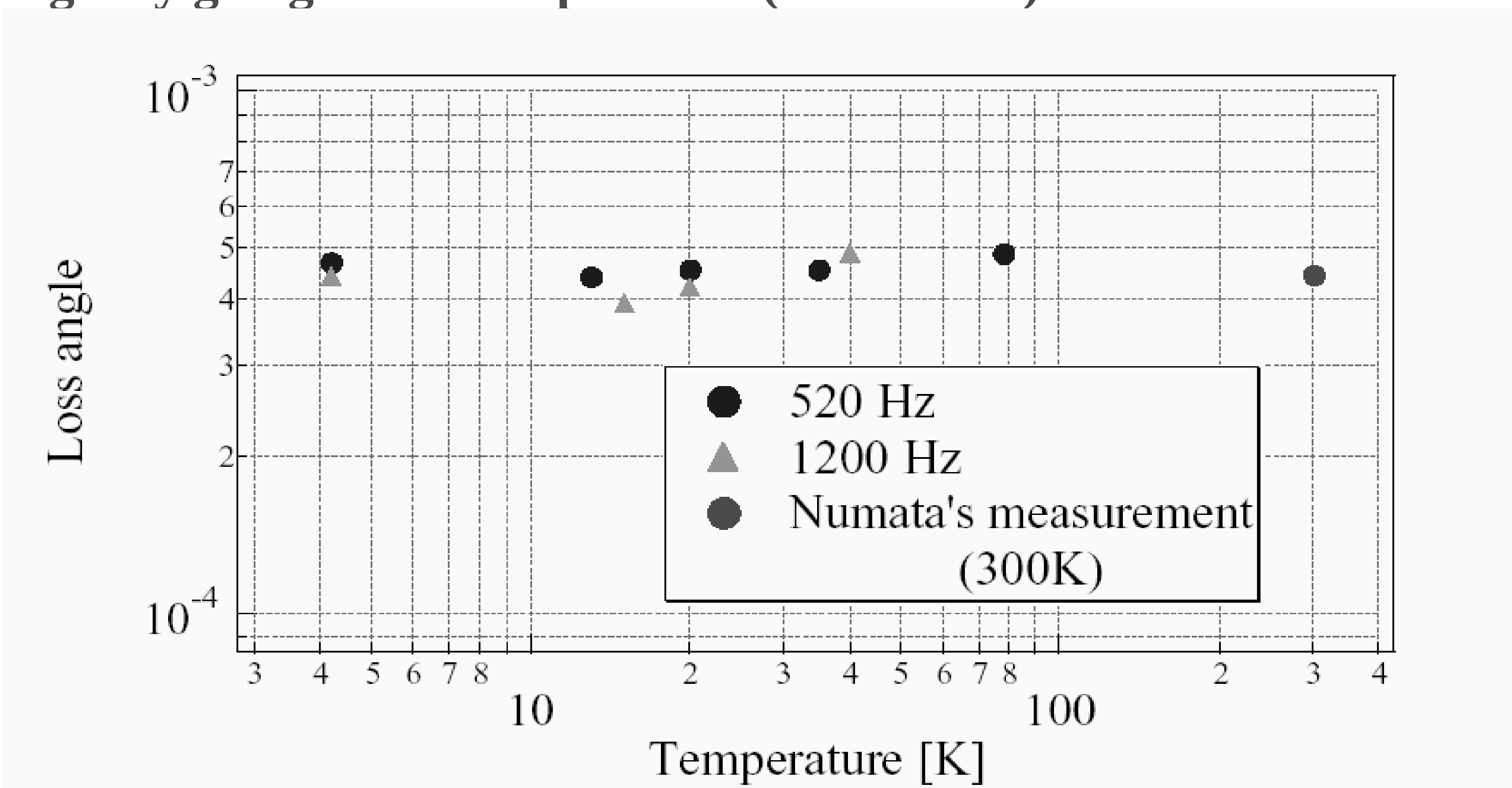
1) Pendulum and Bulk TN

The TN displacement noise due to pendulum and mirror bulk are expected to contains the term $(T\phi)^{1/2}$ and ϕ decreases as T decreases; we can expect at 4K a factor 1/100 TN noise reduction with respect 300 K.

2) Mirror Coating internal losses:

more complex seems the problem of coating internal friction and coating-mirror surface friction.

Very preliminary experimental data show no improvement on the coating loss angle by going to low temperature (Yamamoto):



These are very preliminary results and tests need to be repeated again with different bulk and coating materials (at present it is Ta_2O_5) and in particular to check what is the scaling law.
At present the following scaling law is considered valid

$$\phi_{Total} = \phi_{Bulk} + \frac{E_C}{E_B}\phi_{Coating}$$

Where E_C and E_B are the Coating and bulk stored energy respectively. By assuming the same Young's modulus for both coating and bulk, the displacement of the mirror is:

$$\tilde{x}_{Total}(\omega) = \frac{\sqrt{16 k_B T (\phi_{Bulk} + \frac{E_C}{E_B}\phi_{Coating})}}{\omega^{1/2} E^{1/2} w_0^{1/2}} \quad \text{m/Hz}^{1/2}$$

In any case even if $\phi_{Coating}$ does not decrease at low temperature but remain constant, as shown by Yamamoto, and limits the maximum Q reacheable,

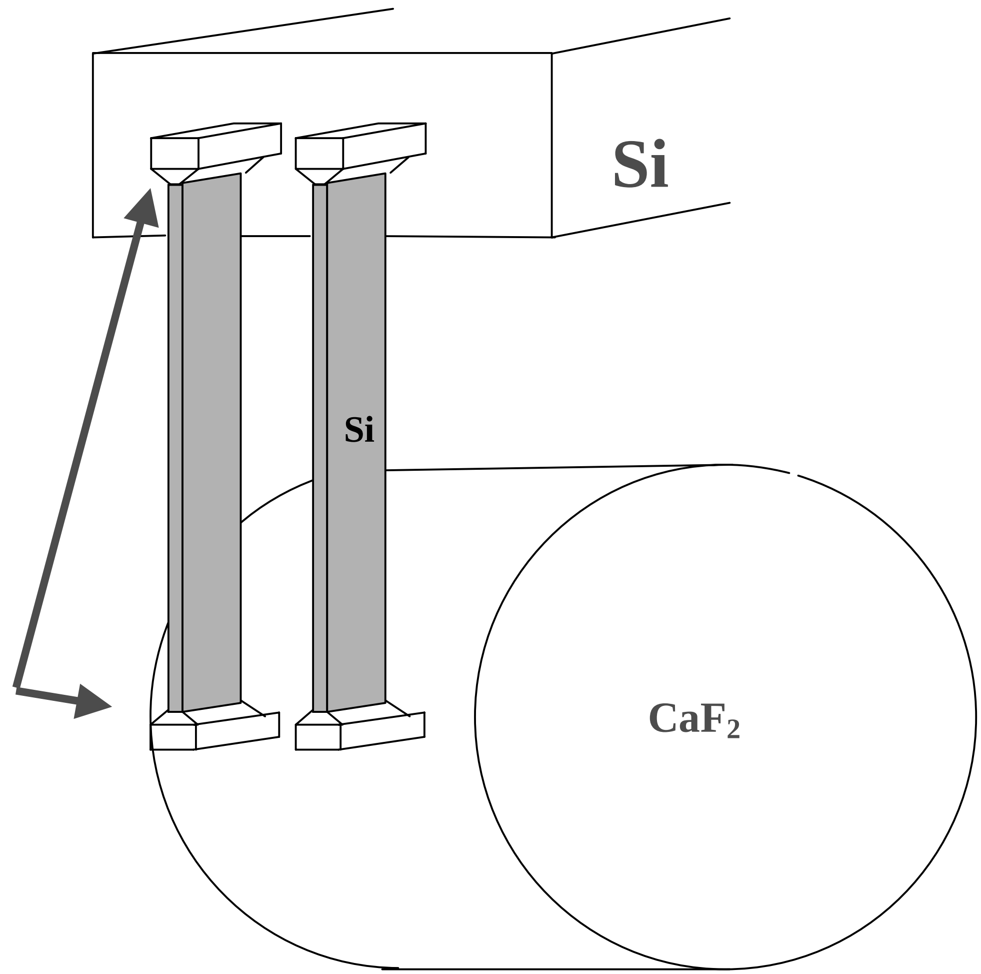
This is a key point: we may still try to reduce $\tilde{X}_{Total}$ by a factor ~ 10 by going to 4K

Some ideas on how to reach 4-5K

As an example: if we have 8-10 MW in the FP cavities and coating losses are ~5 10^{-8} we need to extract ~0.5 W @ 4 K

Let us suppose a mirror support like this:

How good is the Q of Material connections???



2) The equation for T is the following:

$$C_V \frac{dT}{dt} = \Delta W - \left[\beta(T)\frac{\sigma}{L} + 5{,}6.10^{-8}.4T^3 S_M \right](T - T_0)$$

Where $\beta(T)$ is the mirror supporting rods/wires thermal conductivity coefficient, σ the area where the heat goes trough, L the length of heat extraction device and C_v the specific heat , T_0 the thermostat temperature and S_M the total mirror surface.

In the stationary case dT/dt=0, we obtain:

$$- T_0 = \frac{\Delta W}{\beta(T)\frac{\sigma}{L} + 5{,}6.10^{-8}.4T^3 S_M}$$

As an example we consider two kinds of materials:

1)Mirrors in Fluorite (CaF_2) because is not Birefringent.

2)Mirror supporting hinges in Silicon (Si) because it has high thermal conductivity at low temperature and low Young modulus.

In the following we consider only lensing due to the thermal expansion coefficient and assume $dn/dT \sim 10^{-12} T^3$, negligeable.

1)Room Temperature Case

CaF_2 Ther. exp. coef. $\alpha_{CaF2}(300)=1,9\ 10^{-5}\ K^{-1}$

CaF_2 Ther. cond. coef. $\beta_{CaF2}(300)=10W/(m.K)$

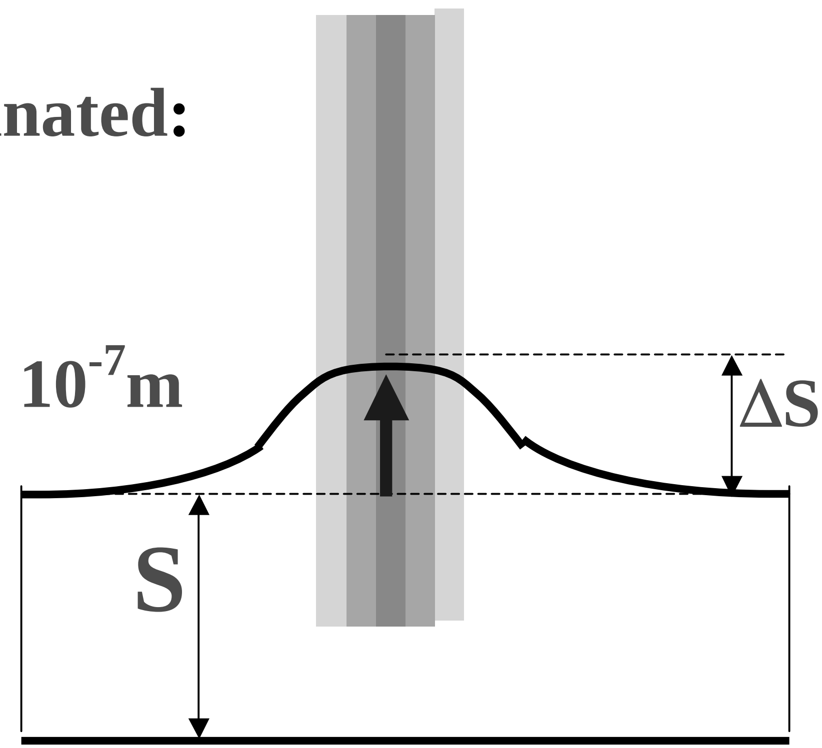
S_i Ther. cond. coef. $\beta_{Si}(300)=100W/(m.K)$

Mirror radiating area $S=0,3\ m^2$

ΔT@300K is Stefan-Boltzmann dominated:

$$\Delta T \sim \Delta W/1.8 \sim 0.1\ K$$

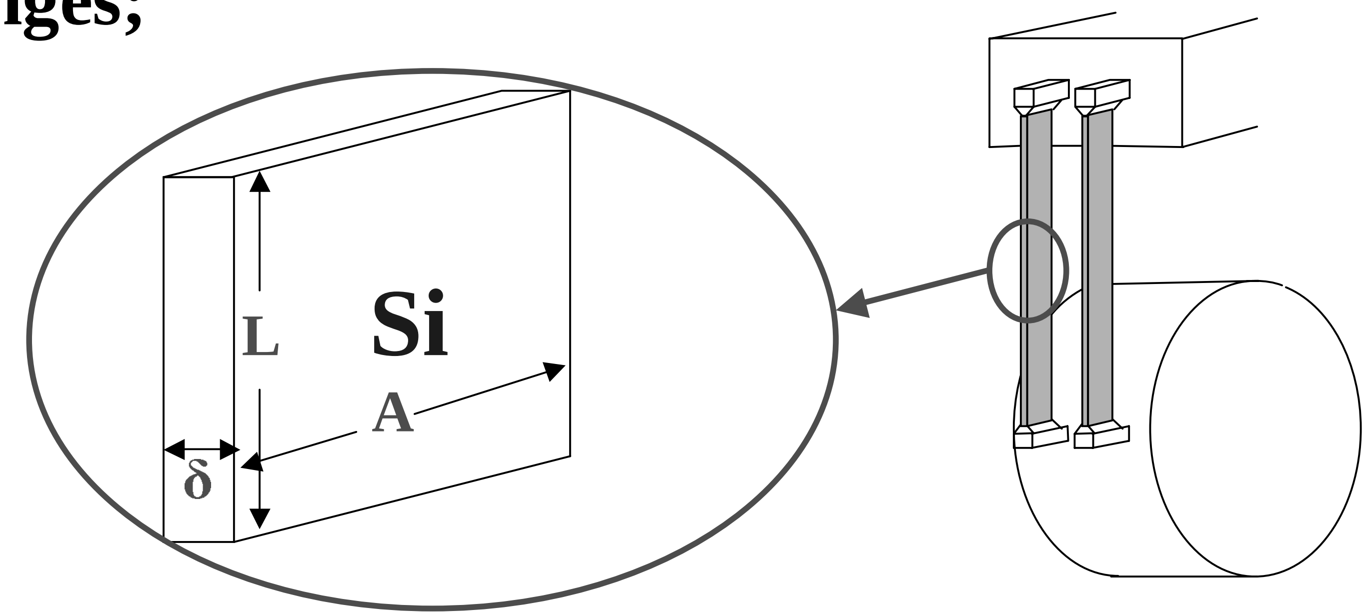
With $S=.1m \Rightarrow \Delta S = S\alpha_{CaF2}(300)\ \Delta T \sim 2\ 10^{-7} m$



2) Temperature 5K

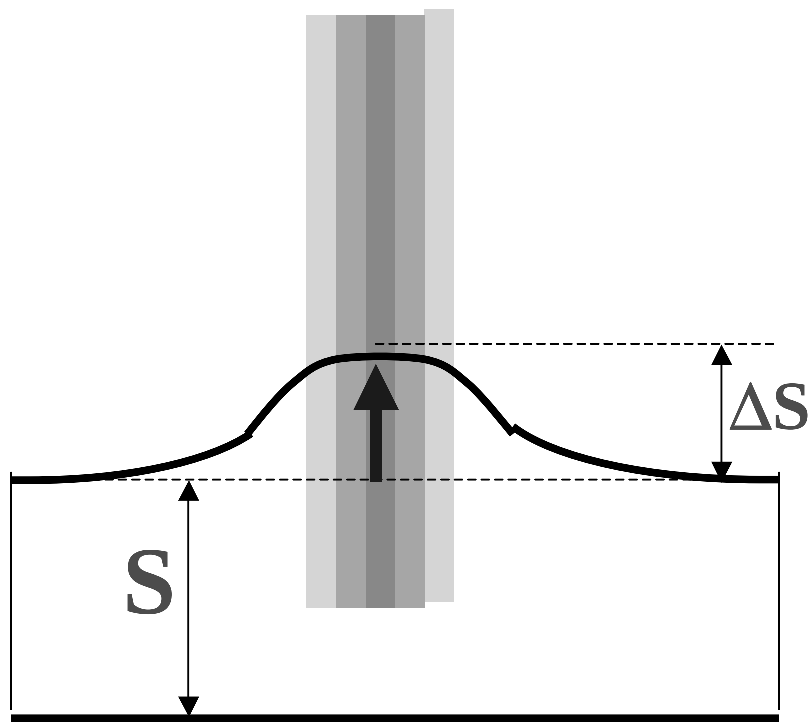
CaF_2 Ther. exp. coef. $\alpha_{CaF2}(5) = 10^{-9}\ K^{-1}$
CaF_2 Ther. cond. coef. $\beta_{CaF2}(5) = 600 W/(m.K)$
S_i Ther. cond. coef. $\beta_{Si}(5) = 600 W/(mK)$

As an example, for transporting 0,5 W away from mirror we may use 4 S_i hinges;



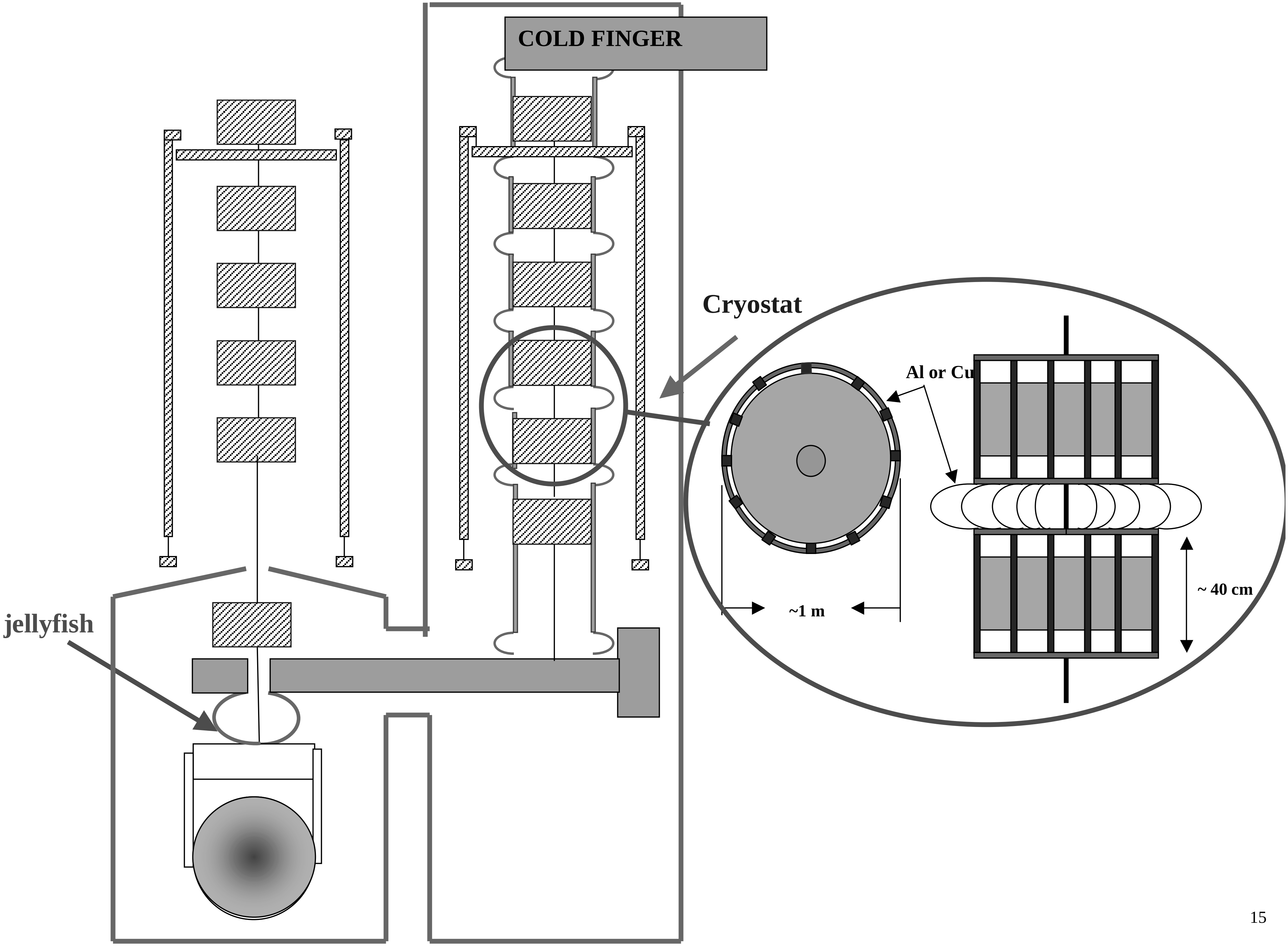
If we have $A = 3\ 10^{-2}$m, $L = 0,3$ m it follows $\delta = 1,6\ 10^{-3}$m. and $\Delta T = 1K$.

This corresponds to a negligeable lensing effect:

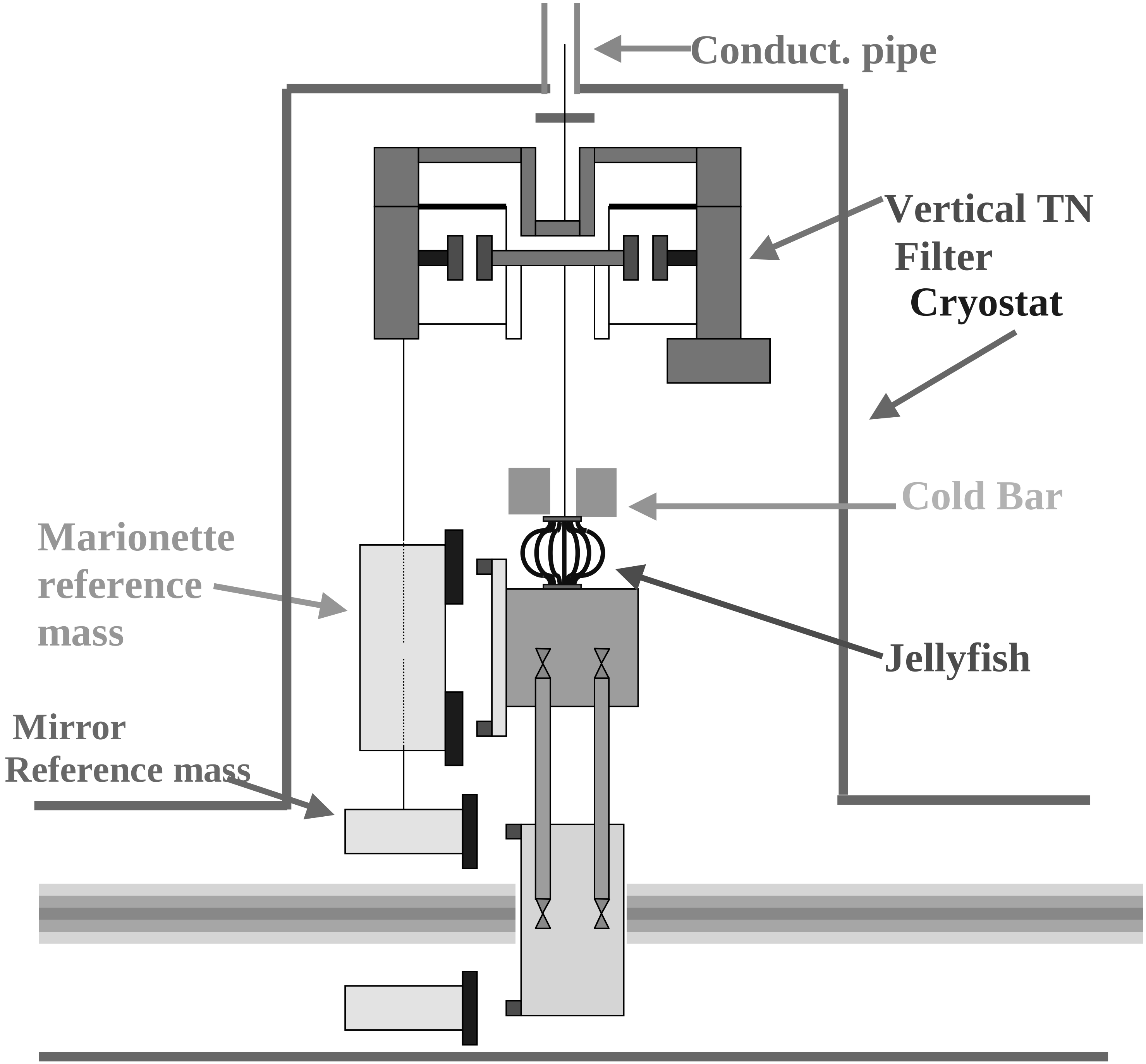


$$\Delta S = S\, \alpha_{CaF2}(5)\, \Delta T \sim 10^{-10} m$$

A problem is then how to extract ~0,5 W from mirrors preserving the Superattenuator softness.


COLD FINGER
Cryostat
Al or Cu
~1 m
~ 40 cm
jellyfish

Some Ideas for a cooled Last Stage



Yamamoto experiment needs to be repeated to check down to mK if there are phase transition.

The mechanism for coatings high loss angle is today not understoodt.

Test 4K cryo chamber by some Virgo groups-Leiden-ACIGA-GEO