Low pumping energy mode of the “optical bars”/“optical lever” topologies of gravitational-wave antennae

1. The Energetic Quantum Limit
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F.Ya.Khalili, arXiv:gr-qc/0304060

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1 The Energetic Quantum Limit

Several versions of the traditional (extracavity) topology of laser gravitational-wave antennae which allow to overcome the SQL in wide band have been proposed.

Examples of these topologies are:

1. *Variation Quantum Measurement* (homodine detection with frequency-dependent local oscillator phase)

H.J.Kimble, Yu.Levin, A.B.Matsko, K.S.Thorne and S.P.Vyatchanin, Physical Review D **65**, 022002 (2002)

2. Different implementations of the *Quantum Speedmeter* scheme (measurement of the test masses velocity instead of the position)

V.B.Braginsky, M.L.Gorodetsky F.Ya.Khalili and K.S.Thorne, Physical Review D **61**, 4002 (2000)

P.Purdue, arXiv:gr-qc/011104

P.Purdue, Y.Chen, arXiv:gr-qc/0208049

Y.Chen, arXiv:gr-qc/0208051

F.Ya.Khalili, arXiv:gr-qc/0211088

All these methods suffer from the very high optical power circulating in the interferometer arms, which also depends sharply on the required sensitivity.

This dependence is described by the *Energetic Quantum Limit*:

$$\Delta\mathcal{E}_{\rm inter}\tau_{\rm inter} \geqslant \frac{\hbar}{2}\,. \qquad (1)$$

V.B.Braginsky, F.Ya.Khalili, *Quantum Measurement.* Cambridge University Press, 1992.

V.B.Braginsky, M.L.Gorodetsky, F.Ya.Khalili and K.S.Thorne, in *Gravitational waves. Third Edoardo Amaldi Conference*, pages 180–189, 2000.

In the case of the laser gravitational-wave antennae, this limit looks as follows: the optical energy in the interferometer can not to be smaller than

$$\mathcal{E} = \mathcal{E}_{\rm SQL} \times \frac{\zeta^2}{2\xi^2}\,, \tag{2}$$

where $\zeta \leqslant 1$ is the pumping field phase squeezing factor, and

$$\mathcal{E}_{\rm SQL} = \frac{ML^2\Omega^3}{2\omega_o}\,, \qquad\qquad \xi = \frac{h}{h_{\rm SQL}}\,.$$

Consider Advanced LIGO values of the parameters:

$$M = 40\,{\rm Kg}\,, \qquad\qquad L = 4\,{\rm Km}\,,$$

$$\Omega = 2\pi \times 100\,{\rm s}^{-1}\,, \qquad\qquad \omega_o = 2 \times 10^{15}\,{\rm s}^{-1}\,.$$

In this case

$$\mathcal{E}_{\rm SQL} \approx 40\,{\rm J} \quad \Leftrightarrow \quad W_{\rm SQL} = \frac{c}{4L}\,\mathcal{E}_{\rm SQL} \approx 0.75{\rm MW}\,. \tag{3}$$

2 The “optical bars”/“optical lever” intracavity schemes

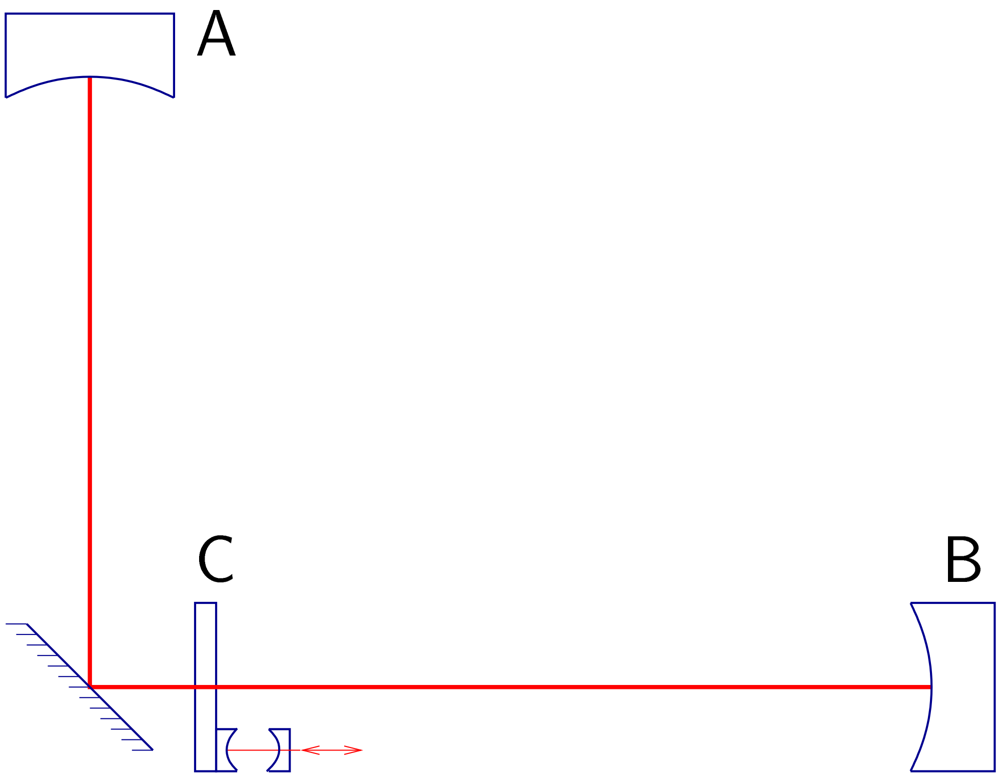
In order to increase sensitivity beyond the SQL keeping $\mathcal{E}$ relatively low, the optical pumping field in a squeezed quantum state ($\zeta < 1$) have to be used.

However, only modest values of the squeezing factor $\zeta \gtrsim 0.3$ have been obtained experimentally yet.

At the same time, squeezed state can be prepared *directly inside* the interferometer using QND measurement.

The idea of the intracavity detection: Displacement of the end mirrors produces redistribution of the optical pumping field inside the interferometer; this redistribution (*i.e.* variation of the optical energy density) can be detected in a QND way — for example, using the pondermotive meter.

Optical bars Optical field in the antenna arms act as a two "springs" with rigidity K which drag the local mirror C when gravitational wave shifts the end mirrors A,B.



$$K = \frac{2\omega_o \mathcal{E}}{L^2 \Omega_B}, \qquad (4)$$

$$\Omega_B = \frac{cT}{L}. \qquad (5)$$

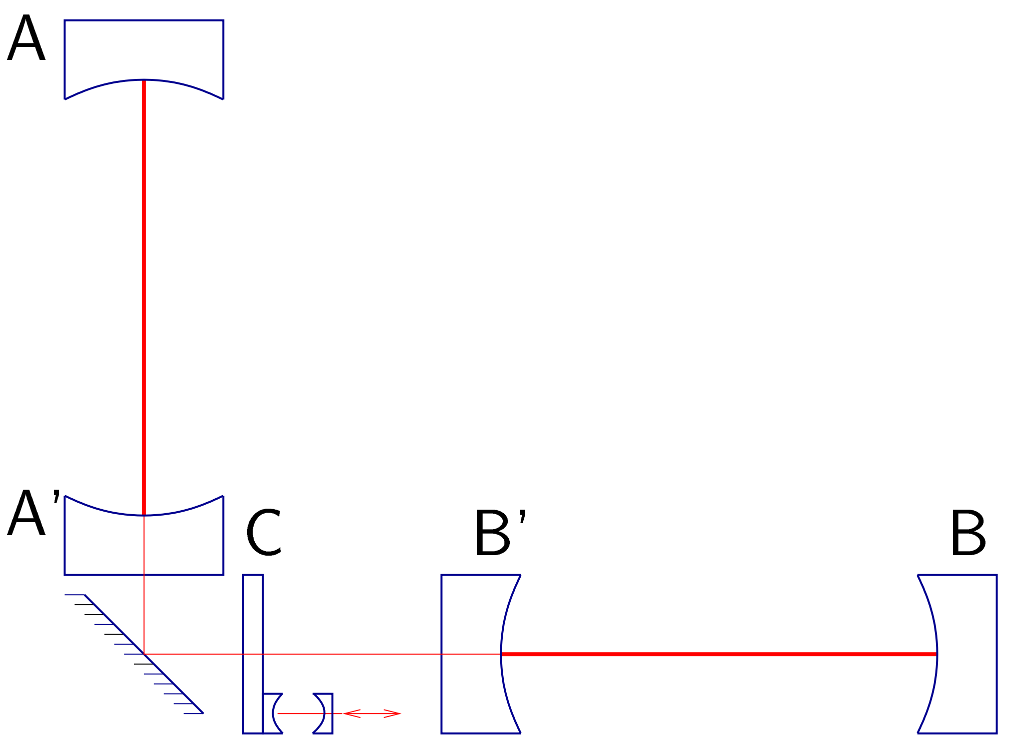
Disadvantage of this scheme:

$$K \geqslant M\Omega^2 \ \Rightarrow \qquad (6)$$

$$\mathcal{E} \geqslant \mathcal{E}_{\text{thesh}} \approx \mathcal{E}_{\text{SQL}}. \qquad (7)$$

V.B.Braginsky, M.L.Gorodetsky, F.Ya.Khalili, Physics Letters A **232**, 340 (1997).

Optical lever Modified version of the "optical bars" scheme, which allows to increase the local mirror displacement but still does not solve the energy problem.



The gain in the displacement

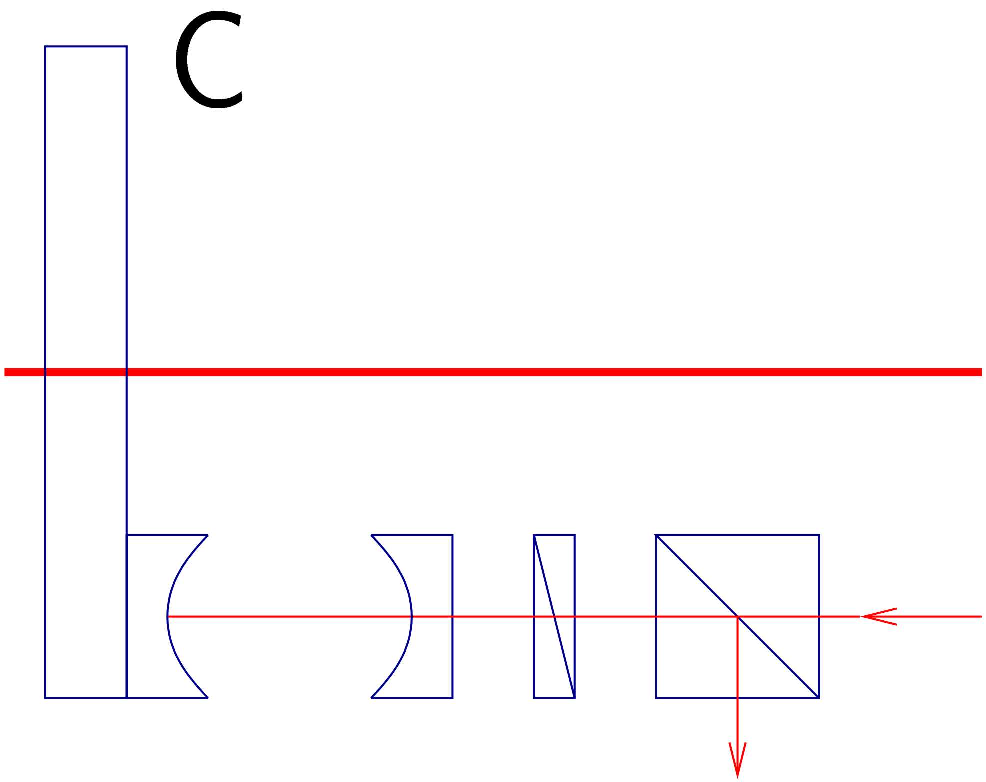
$$\digamma = \sqrt{\frac{2M}{m}} = \frac{2}{\pi}\mathcal{F}\,. \qquad (8)$$

$\mathcal{F}$ is the finesse of the cavities AA' and BB';
M is the mass of the end mirrors;
m is the mass of the local mirror C.

F.Ya.Khalili, Physics Letters A **298**, 308 (2002).

3 Local meter with cross-correlated noises

The key element of the intracavity topologies is the *local meter* which monitors the local mirror C position. For example, it can be implemented as an additional small-scale optical interferometer.



It can be described by its back-action noise $\hat{F}_{\rm meter}(t)$ and its measurement noise $\hat{y}_{\rm meter}(t)$ which spectral densities satisfy the uncertainty relation

$$S_y S_F - S_{yF}^2 \geqslant \frac{\hbar^2}{4} \,. \qquad (9)$$

The local meter output signal (in spectral form) is equal to

$$\tilde{y}(\Omega) = \frac{Lh(\Omega)}{2} + \frac{\hat{F}_{\rm meter}(\Omega)}{-2M\Omega^2} + \frac{\hat{F}_{\rm meter}(\Omega)}{K} + \hat{y}_{\rm meter}(\Omega)\,. \tag{10}$$

None-correlated noises Suppose first, that $S_{yF} = 0$ (no correlation). Suppose also that $K < M\Omega^2 \Leftrightarrow \mathcal{E} < \mathcal{E}_{\rm SQL}$. In this case spectral density of the meter total noise is equal to

$$S_{\rm total} \geqslant \frac{\hbar}{K} \sim S_{\rm SQL} \times \frac{M\Omega^2}{K} > S_{\rm SQL} \equiv \frac{\hbar}{2M\Omega^2},. \tag{11}$$

Therefore, the optical energy have to be large: (*wrong conclusion!*)

$$K \geqslant M\Omega^2 \;\Rightarrow$$
$$\mathcal{E} \geqslant \mathcal{E}_{\rm thesh} \approx \mathcal{E}_{\rm SQL}\,. \tag{12}$$

Correlated noises If the measurement noise contains the part proportional to the back-action force:

$$\hat{y}_{\rm meter} = \hat{y}^{(0)}_{\rm meter} - \frac{\hat{F}_{\rm meter}}{K}\,, \tag{13}$$

then the main back-action term $\hat{F}_{\rm meter}/K$ in equation (10) vanishes:

$$\tilde{y}(\Omega) = \frac{Lh(\Omega)}{2} + \frac{\hat{F}_{\rm meter}(\Omega)}{-2M\Omega^2} + \hat{y}^{(0)}_{\rm meter}(\Omega)\,, \tag{14}$$

There is no K in this formula, therefore, K and $\mathcal{E}$ can be small!

This time- and frequency-independent cross-correlation can be introduced rather easily by the proper setting of the local oscillator phase for the local meter.

4 The sensitivity limitations

The most fundamental sensitivity limitations are imposed by the optical losses and by the local meter noises.

Optical losses

$$\xi^2_{\rm loss} \equiv \frac{S_h^{\rm loss}}{S_h^{\rm SQL}} = \frac{M\Omega^2\gamma}{2\omega_o\mathcal{E}} = \frac{\mathcal{E}_{\rm SQL}}{\mathcal{E}}\,\frac{\gamma}{\Omega}\,, \tag{15}$$

where γ is the losses rate and

$$S_h^{\rm SQL}(\Omega) = \frac{4\hbar}{M\Omega^2L^2}\,. \tag{16}$$

$$1-R \sim 10^{-5} \Leftrightarrow \gamma \lesssim 1\,{\rm s}^{-1} \sim 10^{-3}\,\Omega\,, \qquad \mathcal{E} = 0.1\mathcal{E}_{\rm SQL} \;\;\Rightarrow \tag{17}$$

$$\xi_{\rm loss} \sim 0.1\,. \tag{18}$$

SQL-limited local meter: $h \geqslant h_{\mathrm{SQL}}$, but the optical energy can be smaller than $\mathcal{E}_{\mathrm{SQL}}$.

Optimization gives, that:

$$m^* \equiv \frac{2Mm}{2M+m} = 12M\frac{\Omega_{\max}^4}{\Omega_B^4}\,, \tag{19}$$

$$\mathcal{E} = \frac{3}{2}\frac{ML^2\Omega_{\max}^4}{\omega_o\Omega_B} = 3\mathcal{E}_{\mathrm{SQL}}\frac{\Omega_{\max}}{\Omega_B}\,, \tag{20}$$

where $\Omega_{\max}$ is the maximal signal frequency.

If $T \approx 0.1$ and $\Omega_{\max} = 2\pi\times 10^2\,\mathrm{s}^{-1}$, then

$$\Omega_B \approx 7.5\times 10^3\,\mathrm{s}^{-1}\,,$$

$$\mathcal{E} \approx 0.25\mathcal{E}_{\mathrm{SQL}}\,,$$

$$m^* \approx 25\,\mathrm{g}\,.$$

QND local meter The small-scale optical interferometer can be converted into a QND meter by using Stroboscopic-Variation Measurement (SVM) technique. This sophisticated but relatively simple to implement method permits to filter out the back-action noise by using periodic modulation of the local oscillator phase and/or the pumping power of the local meter. The modulation frequency has to be higher than the upper frequency of the gravitational-wave signal (a few kilohertz).

S.L.Danilishin, F.Ya.Khalili and S.P.Vyatchanin, Physics Letters A **278**, 123 (2000).

S.L.Danilishin, F.Ya.Khalili, Physics Letters A **300**, 547 (2002).

In this case the sensitivity is limited by the optical losses only and can exceed the SQL. Values of m^* and Ω_B can vary in wide range and should be chosen from technological reasons.

Suppose that $\mathcal{E} = 0.1\mathcal{E}_{\rm SQL}$. In this case typical numerical examples are the following (all intermediate values are possible too):

Heavy local mirror:

$$T = 0.01\,,$$
$$\Omega_B \approx 750\,{\rm s}^{-1}\,,$$
$$10\,{\rm Kg} \lesssim m^* \lesssim 16\,{\rm Kg}\,.$$

Small local mirror:

$$T = 0.1\,,$$
$$\Omega_B \approx 7500\,{\rm s}^{-1}\,,$$
$$10\,{\rm g} \lesssim m^* \lesssim 700\,{\rm g}\,.$$

5 Conclusion

Regime of the optical bars/optical lever intracavity topologies with cross-correlated local meter noises looks rather promising for implementing in the third generation of gravitational-wave antennae. It allows to obtain sensitivity better than the SQL, and it can do this using rather moderate value of the optical pumping energy: tens kilowatts of the circulating power only, instead of megawatts or tens of megawatts.

At the same time, the key element of all intracavity topologies — the local meter — have to be explored intensively, both experimentally and theoretically.

The goal: sensitivity ten times better than in the Advanced LIGO:

$$h_\omega \sim 10^{-25}\,\mathrm{Hz}^{-1/2} \quad \Leftrightarrow \quad \delta x_\omega \sim 2\cdot 10^{-20}\,\mathrm{cm}\cdot\mathrm{Hz}^{-1/2}\,. \tag{21}$$

The "optical lever" topology allows to increase the local mirror displacement δy by factor

$$\mathcal{F} \sim 100 \;\Rightarrow\; \delta y_\omega \sim 2\cdot 10^{-18}\,\mathrm{cm}\cdot\mathrm{Hz}^{-1/2}\,. \tag{22}$$

It have to be noted that sensitivity

$$\delta y_\omega \sim 5\cdot 10^{-18}\,\mathrm{cm}\cdot\mathrm{Hz}^{-1/2} \tag{23}$$

have been obtained already in bar antenna experiments.