Corner reflectors and QND Measurements in gravitational wave antennae

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We propose Fabry-Perot cavity with corner reflectors instead of spherical mirrors. We demonstrate that thermo-elastic and thermo-refractive noise in corner reflector (CR) is smaller than in spherical mirrors. We consider the stability of main mode of cavity with CR to distortions of different kinds.

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Preambula

The existing to-day's multi-layer dielectric coating on optical mirrors have
the reflectivity $(1 - R) \simeq 10$ ppm.
The best obtained value is $(1 - R) \simeq 1$ ppm.
The prospect for near future $(1 - R) \simeq 10^{-9}$ (?).

In Advanced LIGO with the $(1 - R) \simeq 10$ ppm and $L \simeq 4$ km the ring down time of FP cavity is $\tau^*_{FP} \simeq 1$ sec.

This value permits to obtain squeezing factor

$$\xi = \sqrt{\frac{\tau_{av}}{\tau^*_{FP}}} = \sqrt{\frac{5 \times 10^{-3}\ \text{s}}{1\ \text{s}}} \simeq 7 \times 10^{-2}$$

The value ξ is important if QND will be used in LIGO.

Recent "anti-discovery":

Thermal expansion factor of amorphous Ta_2O_5:
$\alpha_{Ta_2O_5} \simeq 5 \times 10^{-6}\ K^{-1}$

Thermoelastic noise in $20-40$ layers coating will limit the LIGO sensitivity at the level

$$\sqrt{S_h^{TD\,coat}(\omega)} \simeq \Big(0.6 \div 1.4\Big) \times 10^{-24}\ \text{Hz}^{-1/2} \quad (1)$$

It is close to Standard Quantum Limit:

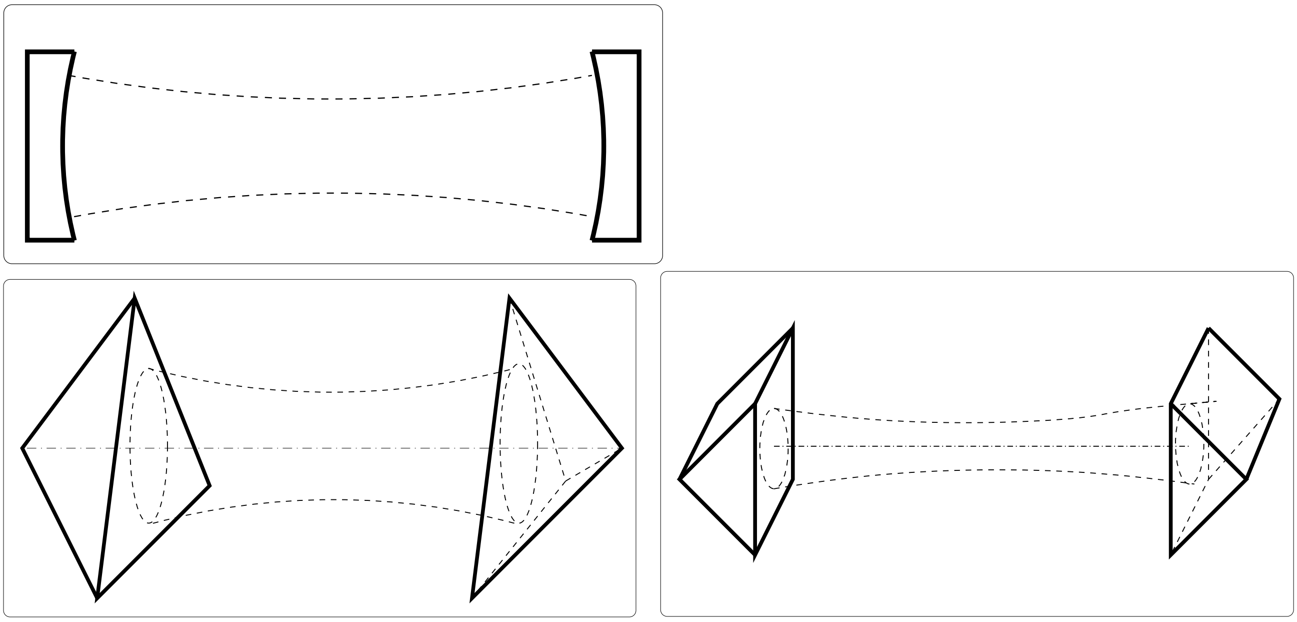
$$\sqrt{S_h^{SQL}(\omega)} = \sqrt{\frac{8\hbar}{m\omega^2 L^2}} \simeq 2 \times 10^{-24}\ \text{Hz}^{-1/2}, \quad (2)$$

if $m = 40$ kg, $L = 4$ km, $\omega = 2\pi \times 100\ \text{s}^{-1}$.

V. B. Braginsky, S. P. Vyatchanin, Physics Letters A312, (2003) 215;
V. B. Braginsky, A. A. Samoilenko, Physics Letters A315, (2003) 175;
G.Cagnoli et al, LIGO Document G03195-00 (2003).

Ambula

We propose Fabry-Perot cavity with corner reflectors (CR) instead of spherical mirrors. This substitution may allow to circumvent SQL.



These types of reflectors were well known among the jewelers at least from 16-th century (see e.g. autobiography by Benvenutto Cellini).

In the 70-s of the previous century corner reflectors (CR) installed on the Moon allowed to test the principle of equivalence for the gravitational defect of mass by laser ranging.

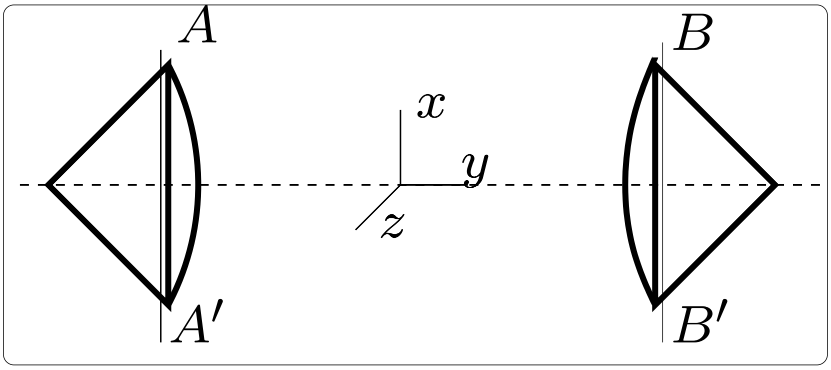
CR may be manufactured from SiO_2 —
$\alpha_{SiO_2} \simeq 5 \times 10^{-7}$ K^{-1}.
Snellius law permits because $n_{SiO_2} \simeq 1.45$

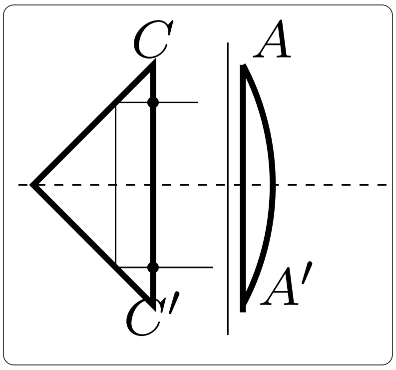
The Stability and the Distorsion

of optical mode in FP cavity with CR

CR may be realized by triheadral prism (three facets) of by two facets prism (“roof” type).

Optical modes in FP cavuty with CR are stable if the “foot” (the bottom) have *lense shape*.





The distorsion of optical mode may be a source of additional noise in FP cavity (non elaborated problem).

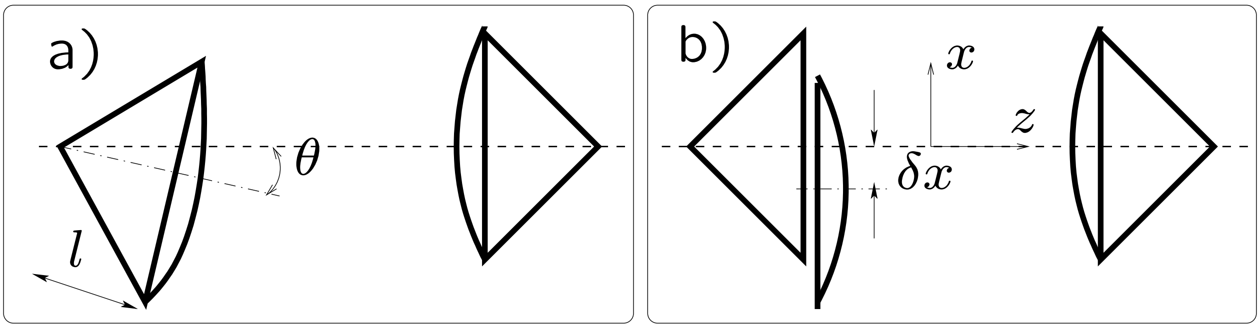
The distortion may be caused by

$\theta -$ tilt angle, $\epsilon -$ expose angle, $\delta x -$ displacement.

The distorsion may be described by *additional* terms in the distribution of light in beam spots:

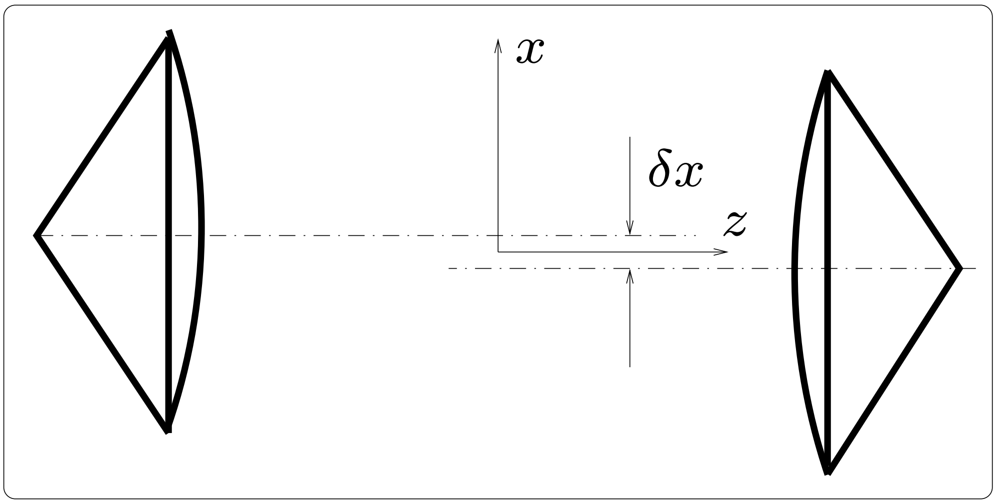
monopole $+\alpha_1$(dipole) $+\alpha_2$(quadrupole) $+\ldots$

Distortion due to the Tilt of CR

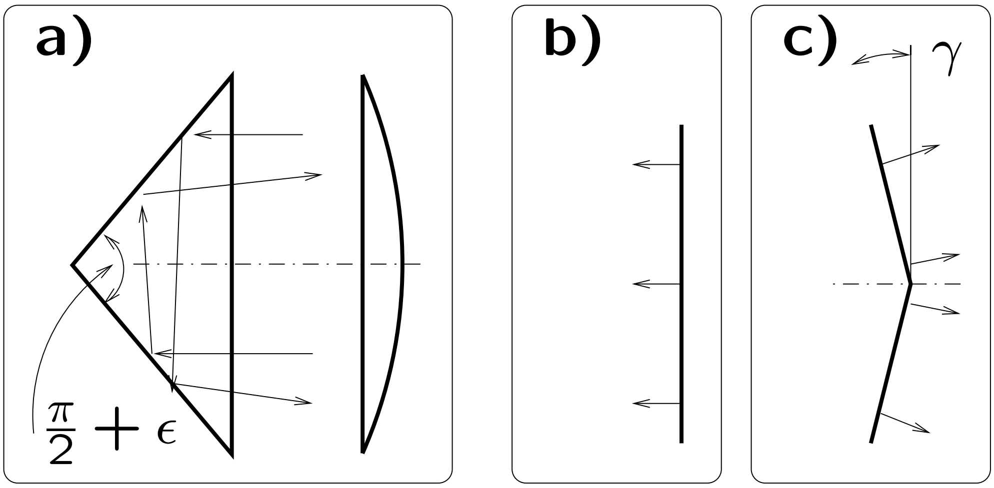


Small tilt of the left CR around its head is equivalent to untilted reflector and displaced lens.

The Distortion due to the Displacement of CR



The Distortion of Expose Angle



(a). Expose perturbation: $\epsilon \neq 0$. It produces the transformation of incident plane wave front (b) into a "broken" front of reflected wave (c) with angle $\gamma = 2\epsilon$ for 2-hedral prism or with angle $\gamma = 2\epsilon\sqrt{2/3}$ for 3-hedral prism (if only one plane is declined).

Comparison of CR and SM

We compare the distortion coefficients α^{CR} for FP cavity with CR and distortion coefficients α^{SM} for cavity with spherical mirrors.

Parameters:
radius of of laser beam $R_b \simeq 6$ cm
g-factor of cavity $g = 0.982$,
distance from foot to top of CR $l = 20$ cm

Net results of calculations

The tilt:

$$\begin{aligned} \alpha_1^{\text{tilt, SM}} &= 0.013\left(\frac{\theta}{10^{-8}}\right), \\ \alpha_1^{\text{tilt, CR}} &= 1.2\times 10^{-7}\left(\frac{\theta}{10^{-8}}\right), \end{aligned}$$

The displacement:

$$\begin{aligned} \alpha_1^{\text{disp, SM}} &= 0.0059\left(\frac{\delta x}{0.1\,\text{cm}}\right), \\ \alpha_1^{\text{displ, CR}} &= 0.06\left(\frac{\delta x}{0.1\,\text{cm}}\right) \end{aligned}$$

Expose angle for CR cavity:

$$\alpha_2^{\text{expose}} = 0.11\left(\frac{\gamma}{10^{-6}}\right).$$

Conclusion: CR cavity is more stable to tilt and less stable to displacement than SM cavity.

CR are approximately “equal” to SM.

Optical inhomogenity in CR

Today $\delta n \simeq 2\times 10^{-7}$, characteristic length $\simeq$ 10 cm, $\gamma = 2\delta n/n \simeq 2\times 10^{-7} \Rightarrow \alpha_2^{\text{inhomo}} \simeq 0.011$. Acceptable.

Optical losses

1. Assuming 0.1 ppm per cm we may expect losses in the bulk of CR which corresponds to $(1-R) \simeq 10$ ppm.

2. Diffraction on the edge:
$$(1-R)_d \simeq \frac{0.4\lambda}{R_b} \simeq 7 \text{ ppm} \tag{3}$$
λ is the optical wavelength, $R_b = 6$ cm – radius of laser beam.

3. Losses on nonperfect edge ($\Delta s \simeq 0.5\,\mu$m):
$$(1-R)_{\text{non-perfect}} \leq \frac{\Delta s}{R_b} \simeq 8 \text{ ppm}$$

4. Losses in anti-reflective coating. To keep the reflection from the lense shaped "foot" of CR it is necessary to cover it by 2-4 anti reflective layers of coating. This will give the value of $(1-R) \simeq 10$ ppm.

Thermo-refractive noise

Thermodynamic fluctuations of temperature produce fluctuations of phase of light traveling inside the CR through dependence of refractive index n on temperature T: $\beta = dn/dT \neq 0$.

V. B. Braginsky, M. L. Gorodetsky, and S. P. Vyatchanin, *Physics Letters* **A 271**, 303-307 (2000)

With $R_b = 6$ cm, $l_c = 10$ cm and gaussian distribution of the light density:

$$\sqrt{S_h(\omega)} \simeq 0.5 \times 10^{-24}\ \text{Hz}^{-1/2} = \frac{1}{4}\sqrt{S_h^{\text{SQL}}(\omega)}.$$

If $R_b = 10$ cm and mesa-shaped beam, then

$$\sqrt{S_h(\omega)} \simeq 0.1\sqrt{S_h^{\text{SQL}}(\omega)}.$$

Conclusion

Alternative solution: may be exist new material for coating, i.e. $\alpha_{\text{coating}} \simeq 5 \times 10^{-7}$ K^{-1} and $n \geq 2$?

Nonsolved problems:

1. The coupling of cavity with readout system. Intra-cavity meter?
2. The coupling of cavity with laser pumping. May be use of very thin dielectric grating on the surface of the facet?
3. The polarization characteristics of CR.
4. Mesa-shaped beam — what will be the fee?

Additional argument in favor of CR:
Univ. of Glasgow + Stanford Univ. + Iowa Univ. + Syracuse Univ. + LIGO Lab =
= multi-layer coating decreases the Q_{mech} of mirror's internal modes.