Pulsar Detection and Parameter Estimation with MCMC - Six Parameters

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Goal of our Work

SN1987a Location known, but all other parameters unknown. Use Middleditch parameters (f=467.46Hz, df/dt=-2 to -3E-10 Hz/s) and look at f and 2f.
Spindown irregularities: df/dt ~ -5E-9

Heterodyne 1/60 Hz bandwidth: 5 Hz search with 300 processes. About one-day per processor search.

Developing Monte Carlo to set upper limits on h.

Also useful for radio observed pulsar: work in concert with time domain search. Will look at known pulsars where there is some uncertainty in frequency and df/dt values.

Code reproduces time domain results from S3.

Starting Point

The Starting point for the MCMC is the same likelihood and priors used in the LSC time domain (Bayesian) search

Remember Rejean's talk

Markov Chain Monte Carlo

- Computational Bayesian Technique
- Metropolis-Hastings Routine
- Estimate Parameters and Generate Summary Statistics (PDFs, cross correlation, etc)
- 6 Unknown Parameters (so far): h_0, ι, ψ, ϕ, δf, df/dt
- Initial Application SN1987a: Location known but other parameters not

Time domain method (Rejean)

- The phase evolution can be removed by heterodyning to dc.
  - Heterodyne (multiply by $e^{-i\,\Phi(t)}$) calibrated time domain data from detectors.
  - This process reduces a potential GW signal h(t) to a slow varying complex signal y(t) which reflects the beam pattern of the interferometer.
  - By means of averaging and filtering, we calculate an estimate of this signal y(t) every 1 minute (or 40 minutes) which we call B_k.

The B_k's are our data which we compare with the model

$$y(t) = \frac{1}{4} F_{+}(t) h_0 \left(1 + \cos^2 i\right) e^{i \quad 0} - \frac{i}{2} F_{X}(t) h_0 \left(\cos i\right) e^{i \quad 0}$$

Bayesian analysis (Rejean)

A Bayesian approach is used to determine the joint posterior distribution of the probability of the unknown parameters,

a=(h_0,ι,φ,ψ,δf,df/dt), via the likelihood:

Model

$$p\left(\{B_k\} \mid \vec{a}\right) \propto \exp\left[-\sum_k \frac{\left|B_k - y\left(t_k; \vec{a}\right)\right|^2}{2\ {}_k^2}\right]$$

¿

B_k's are processed data

Noise estimate

$$p\left(\vec{a} \mid \{B_k\}\right) \propto \mathrm{p}\left(\vec{a}\right) p\left(\{B_k\} \mid \vec{a}\right)$$

Posterior

Prior

Likelihood

Metropolis Hasting Routine

Generate a chain of parameter values

PDFs generated from the distribution of chain values

Quasi-Random (somewhat intelligent) walk through parameter space

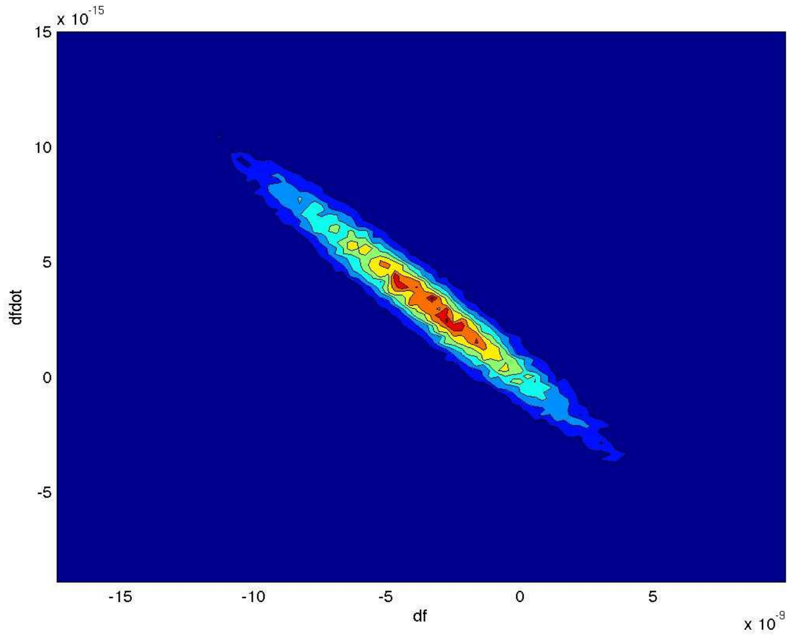
4 or 5 parameter search is straightforward

h_0, ι, ψ, ϕ, δf

6th parameter df/dt causes problems

Distributions for δf and df/dt are VERY narrow

Hard to find

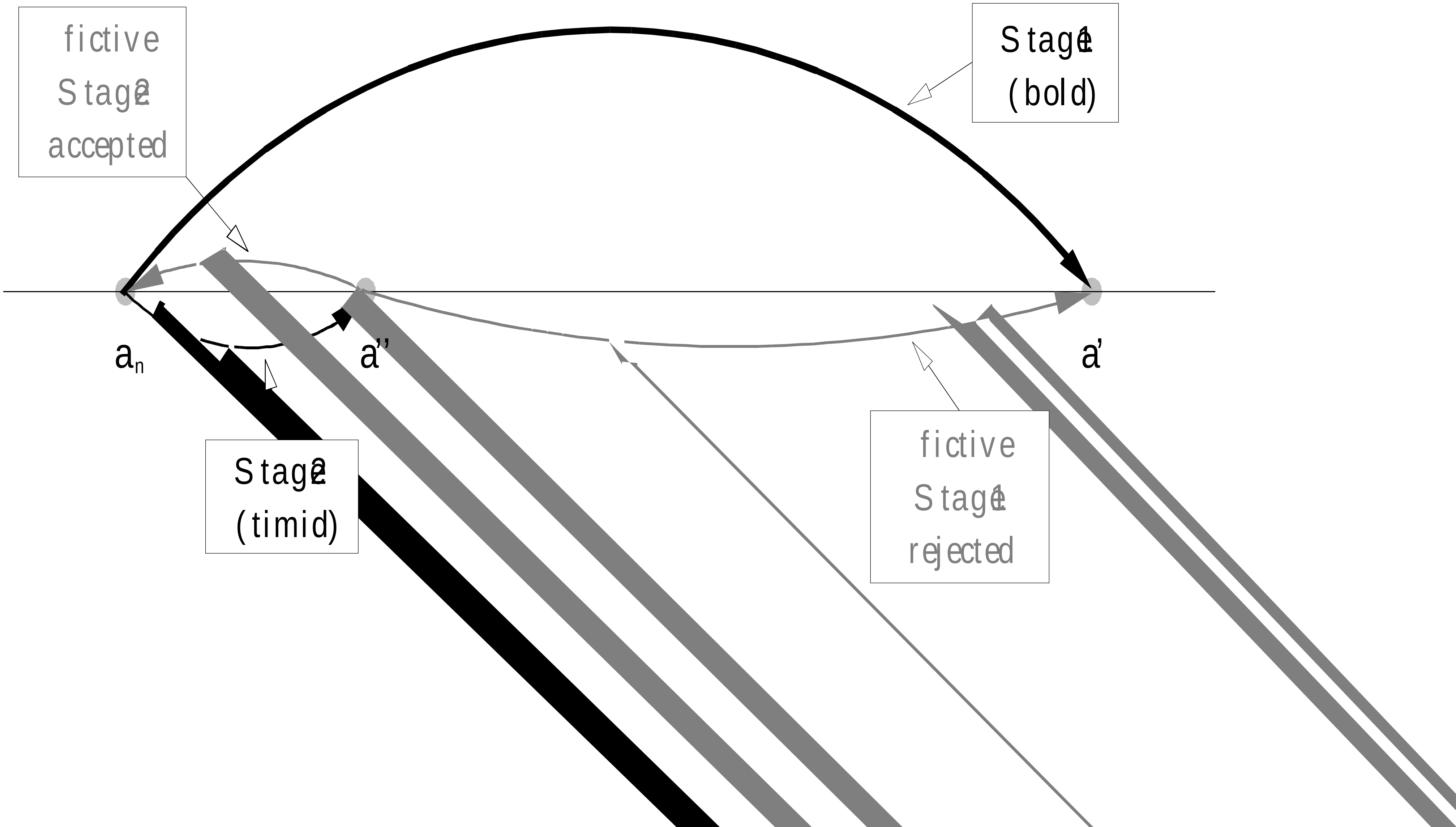


Reparametrization
New Variables

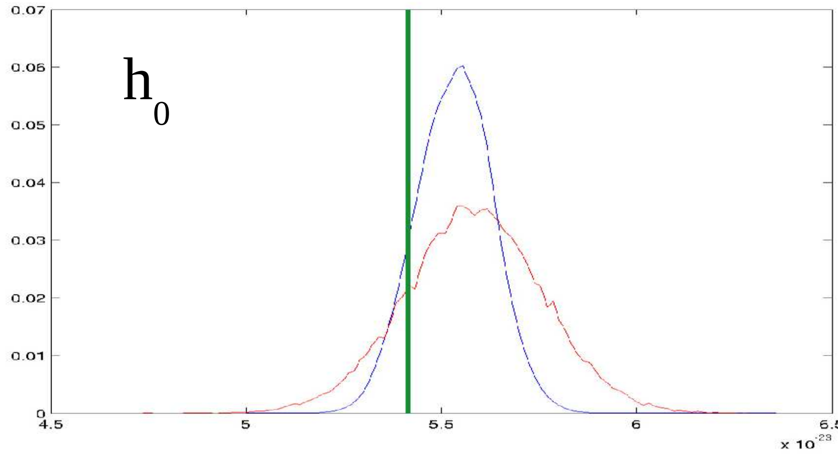
$$f_{start} = \delta f + (1/2)(df/dt)T_{start}$$

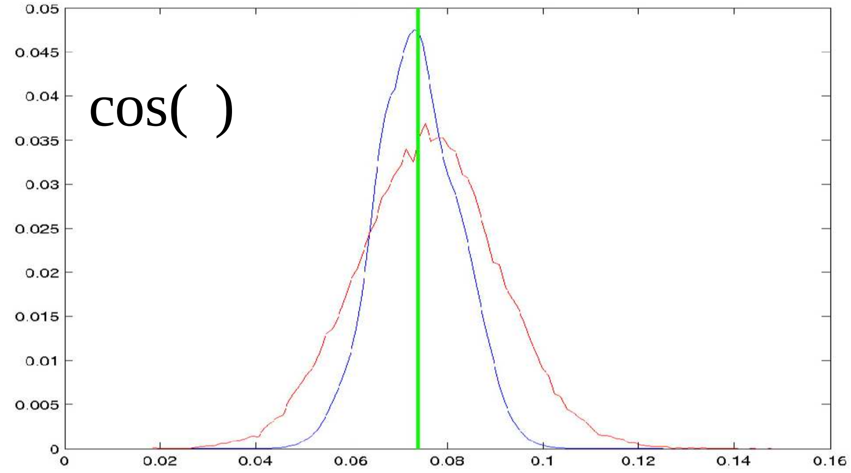
$$f_{end} = \delta f + (1/2)(df/dt)T_{end}$$

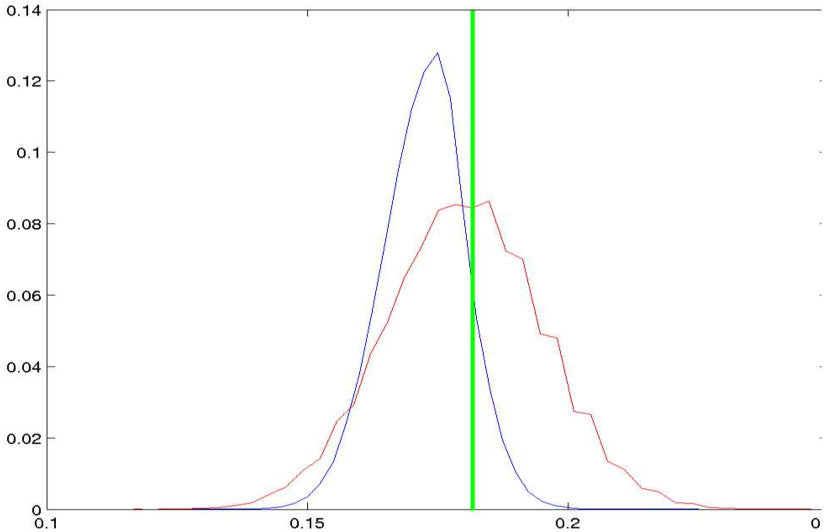
Delayed Rejection

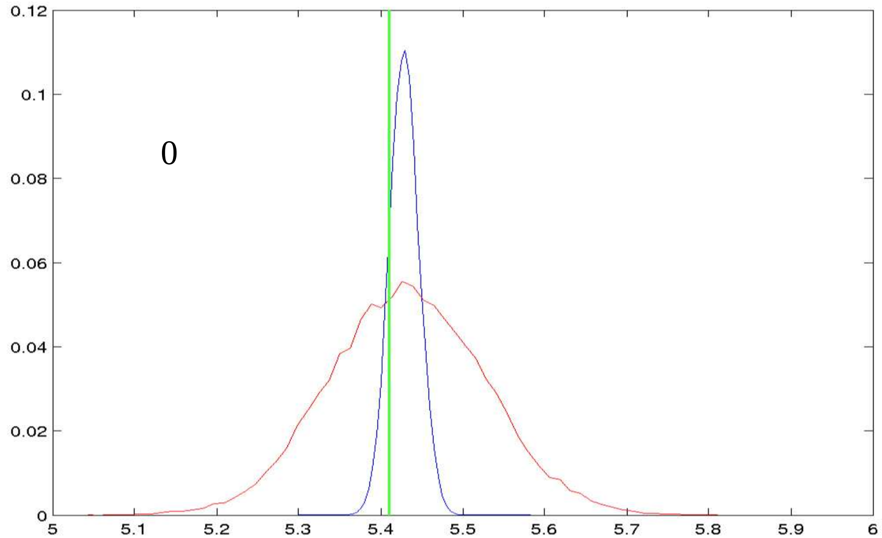


S3 Injection comparisons (PSR8)

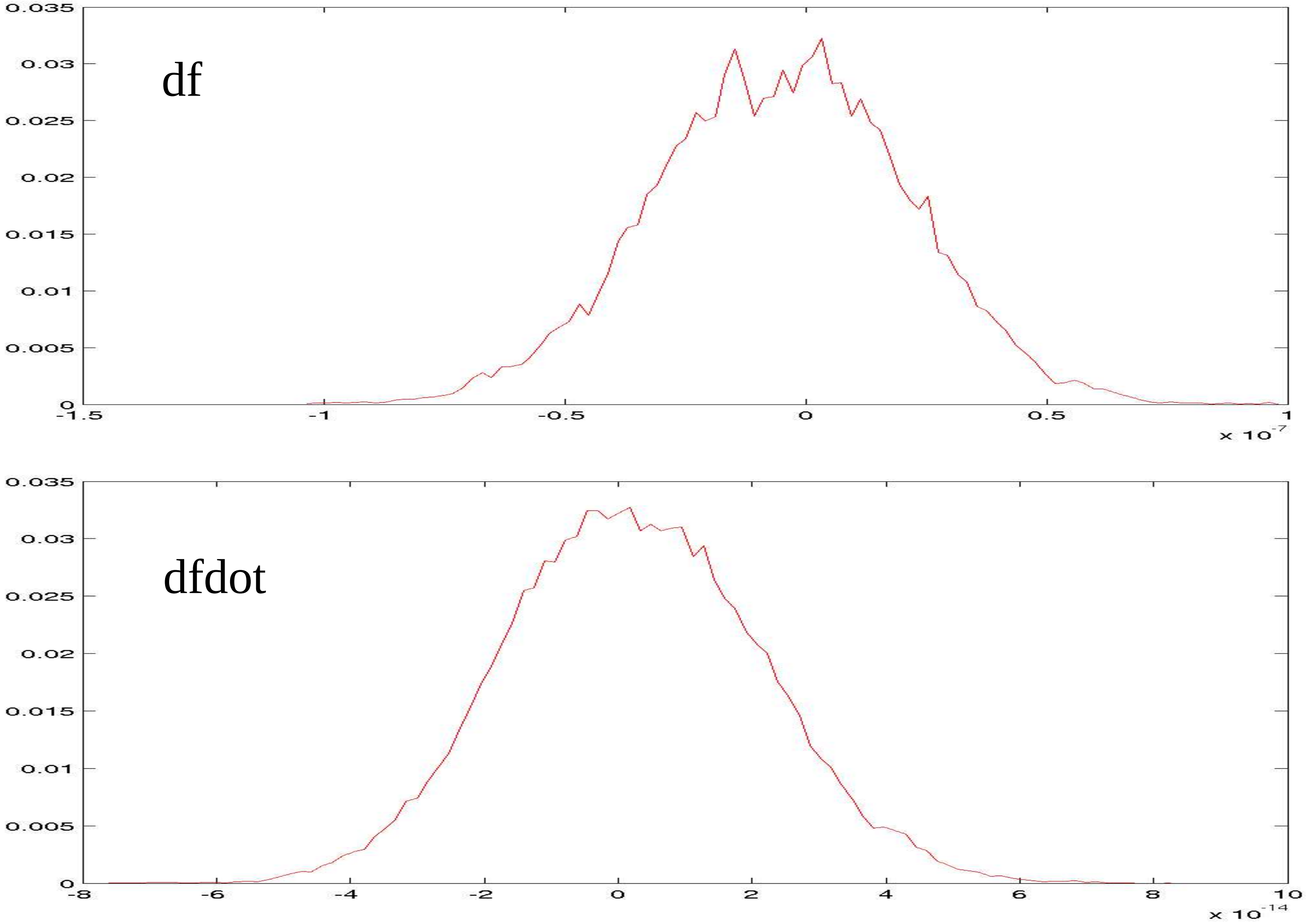




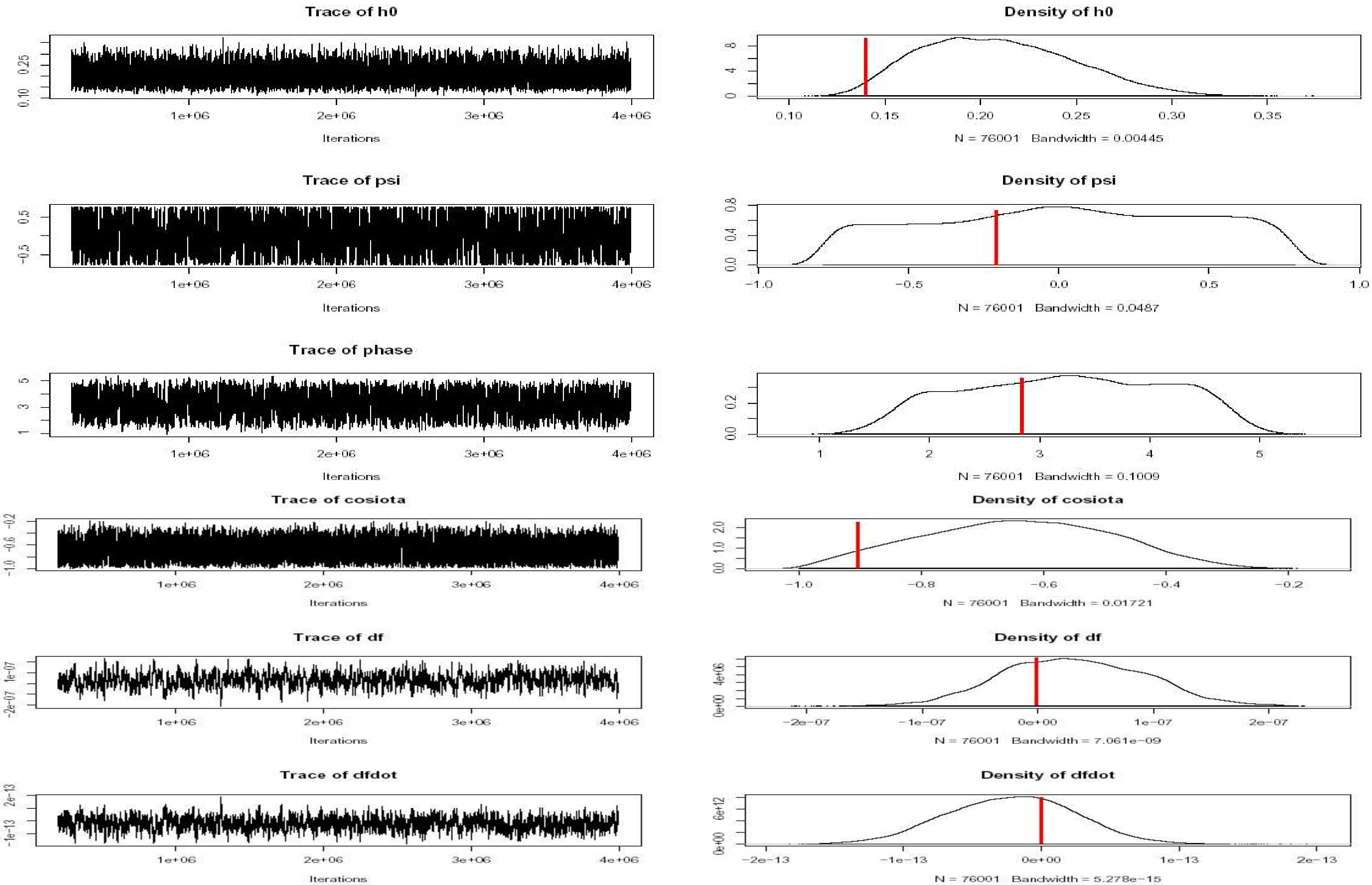




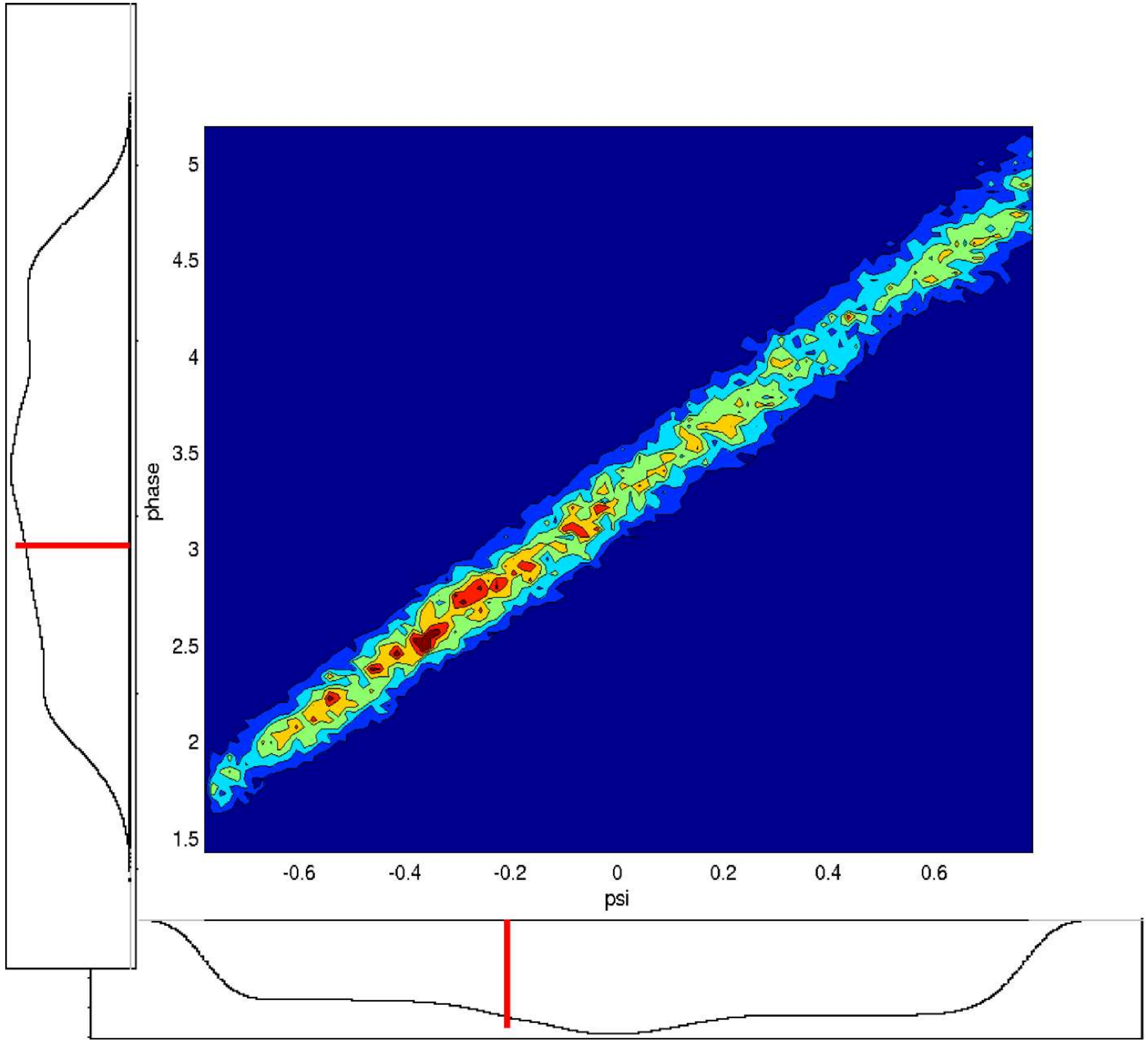
S3 Injections (PSR8)



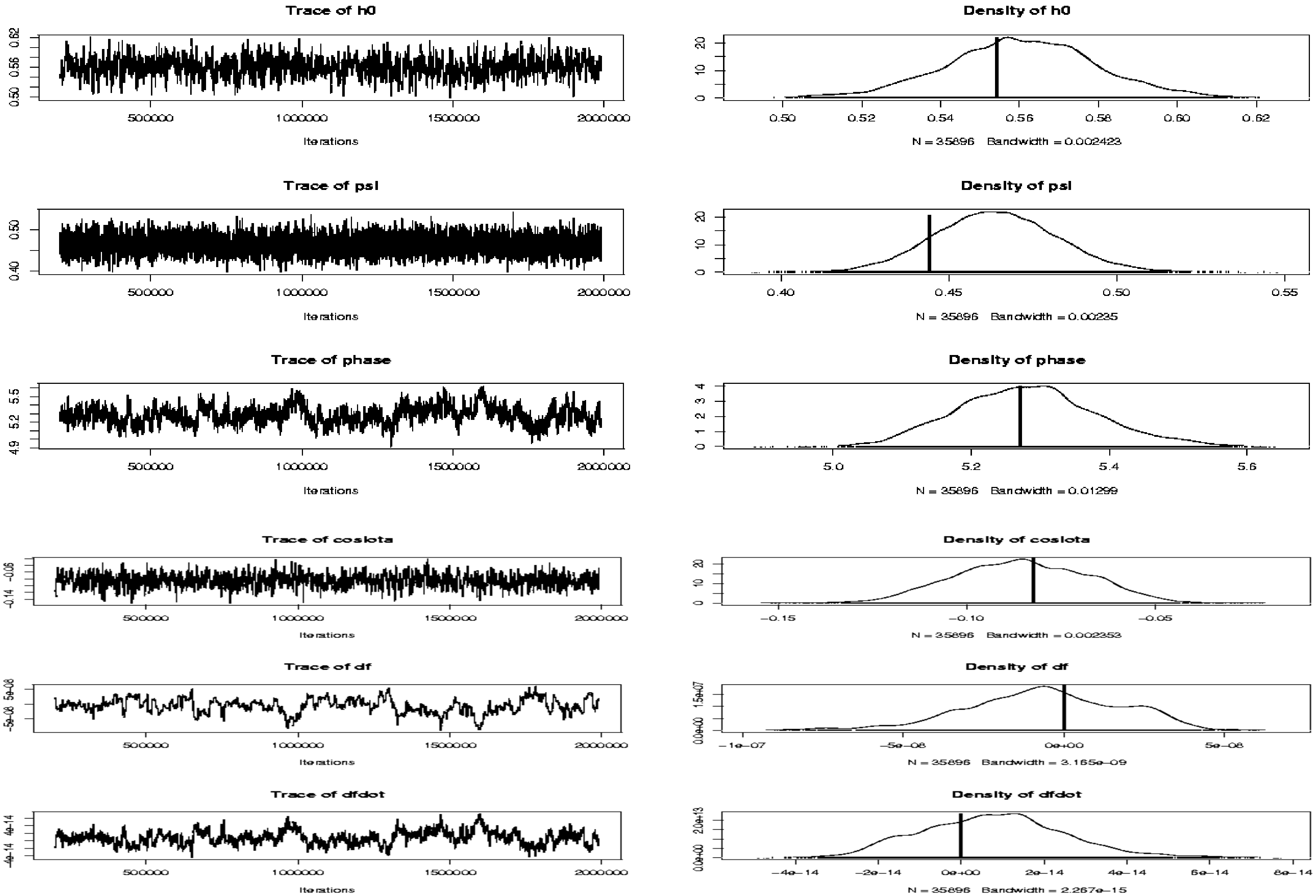
S3 Injections (PSR2)



Correlation



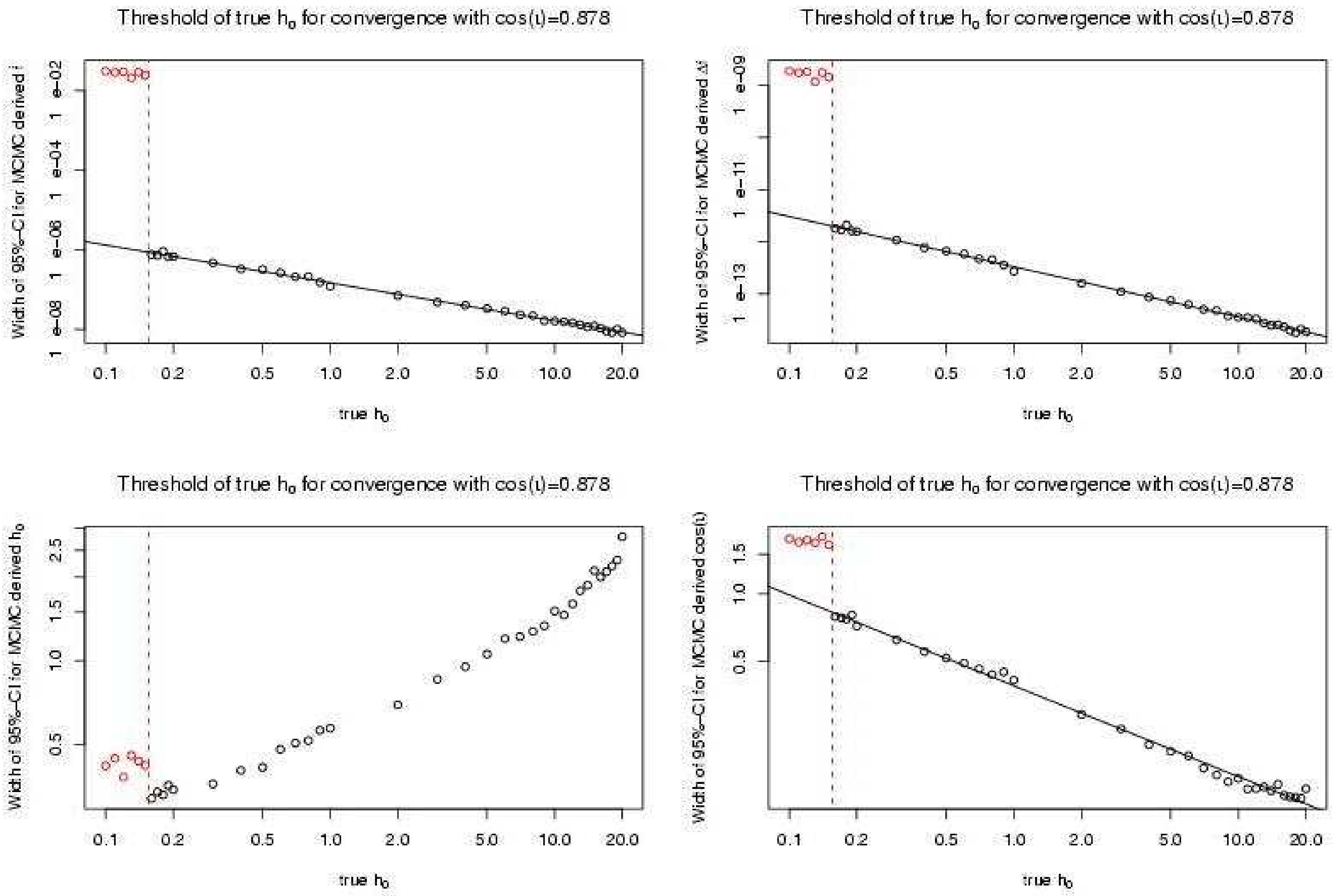
S3 Injections (PSR3)



Future Ideas

- Modify code to account for precession
  - Emission at f & 2f
  - Emission from binary pulsars
    - Additional parameters to be determined
  - Fuzzy searches over limited regions in sky.

Convergence Characteristics



Tested Code on Synthesized Data

δf	0.007 Hz
df/dt	-2.5e-10
d^2f/dt^2	0.0
h0 “varied”	
ψ	0.4
ϕ	1.0
ι	0.5

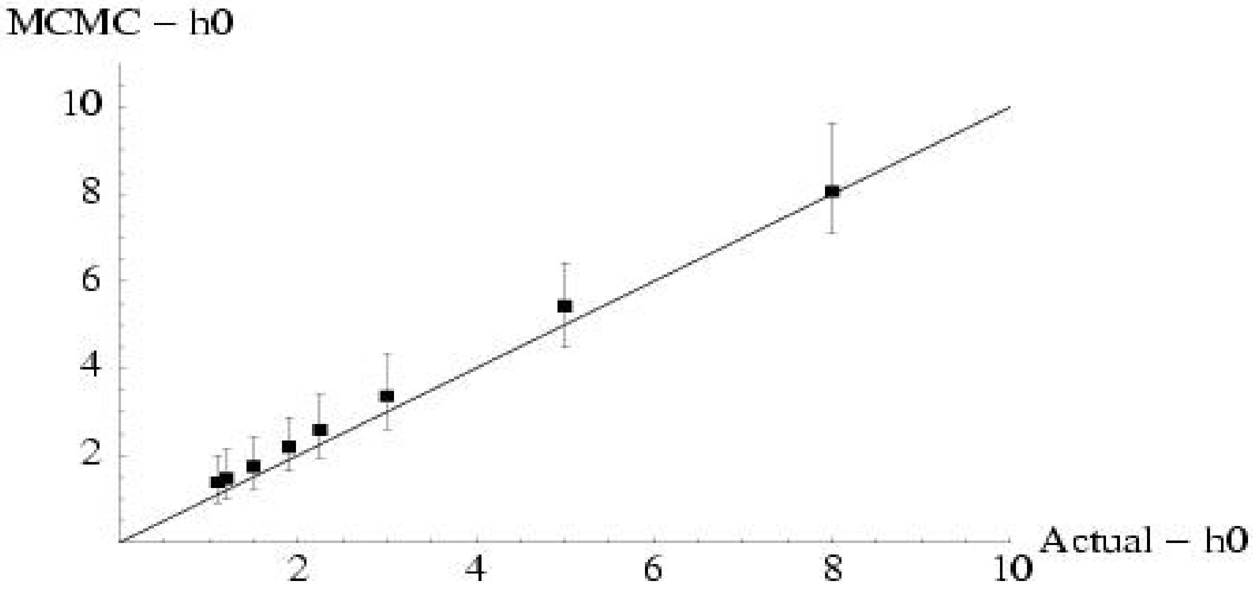
RA 1.23

DEC 0.321

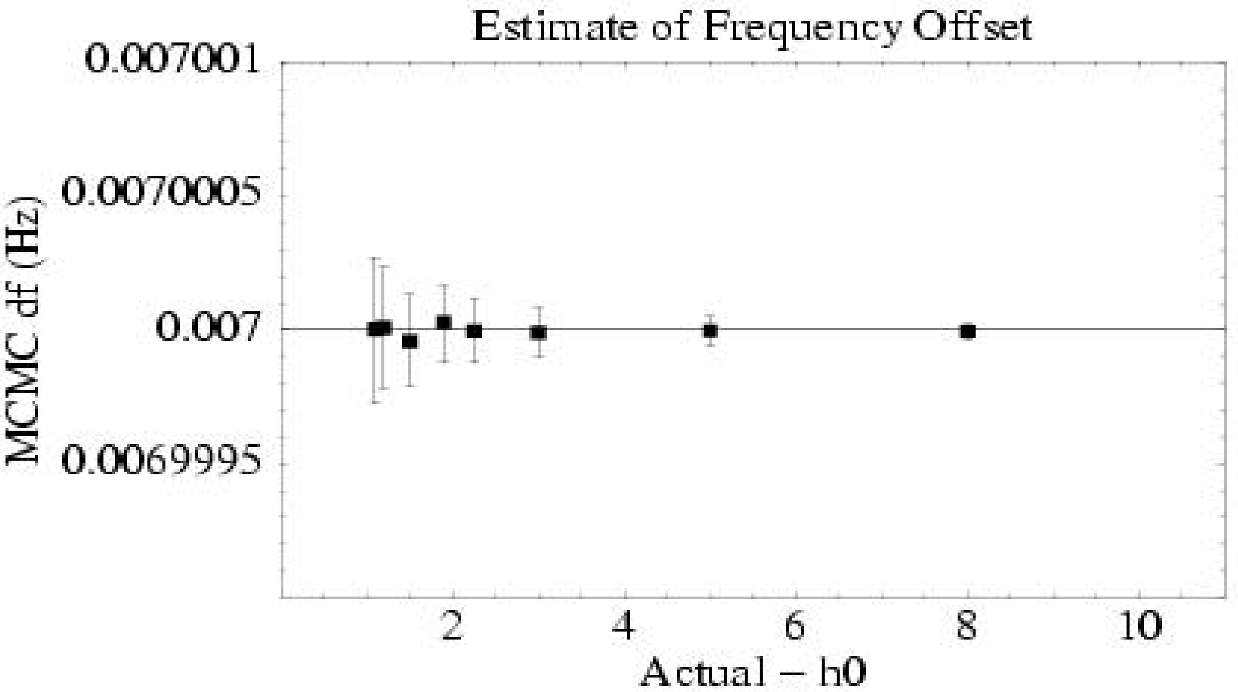
10 days worth of data

Noise $\sigma_k = 1.0$

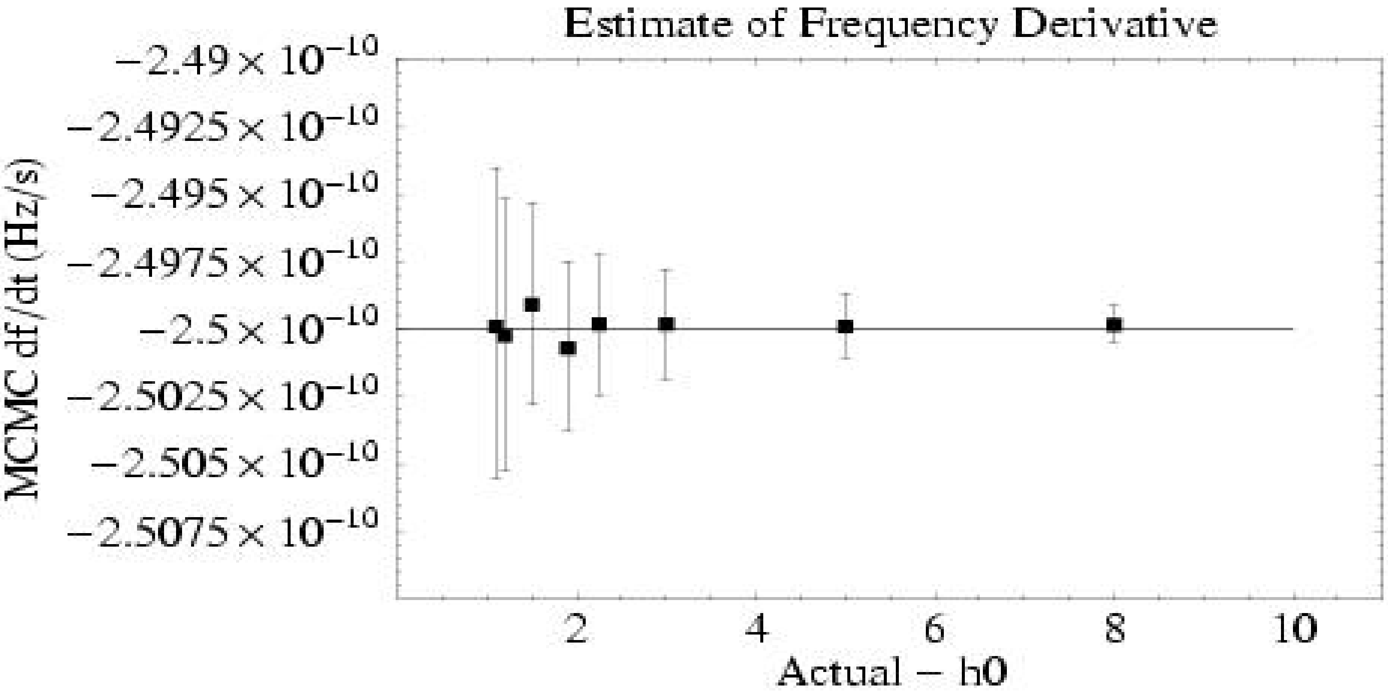
Estimate of h_0



Estimate of df



Estimate of df/dt



ABSTRACT

Presented is a Markov chain Monte Carlo technique for finding a laser interferometer detected pulsar signal and estimating 6 unknown parameters, including the pulsar frequency and frequency derivative. The technique used is called Delayed Rejection in Reversible Jump Metropolis-Hastings. This method will be explained, as well as noting how a simple extension to multiple computer processors will allow for a wider frequency band search. The goal of this research is to search for a possible (but radio quiet) source at a know location; for example, SN1987A. The code has been successfully demonstrated with synthesized data, and with LIGO S2 injected signals. The results of the code on these artificial signals will be presented.

Initial state $\boldsymbol{a}_0$.

At time n a new candidate $\boldsymbol{a}'$ is generated from the candidate generating pdf, $q(\boldsymbol{a}|\boldsymbol{a}_n)$; depends on the current state $\boldsymbol{a}_n$ of the Markov chain.

New candidate $\boldsymbol{a}'$ is accepted with a certain *acceptance probability* $\alpha(\boldsymbol{a}'|\boldsymbol{a}_n)$, also depending on the current state $\boldsymbol{a}_n$, given by

$$\alpha(\boldsymbol{a}'|\boldsymbol{a}_n) = \min\left\{\frac{p(\boldsymbol{a}')p(B_k|\boldsymbol{a}')q(\boldsymbol{a}_n|\boldsymbol{a}')}{p(\boldsymbol{a}_n)p(B_k|\boldsymbol{a}_n)q(\boldsymbol{a}'|\boldsymbol{a}_n)}, 1\right\} \tag{1}$$

For good efficiency a multivariate normal distribution is used for $q(\boldsymbol{a}'|\boldsymbol{a}_n)$.

The steps of the MH algorithm are therefore:

Step 0: Start with an arbitrary value $\boldsymbol{a}_0$

Step $n+1$: Generate $\boldsymbol{a}'$ from $q(\boldsymbol{a}|\boldsymbol{a}_n)$ and u from $U(0, 1)$
If $u \leq \alpha(\boldsymbol{a}'|\boldsymbol{a}_n)$ set $\boldsymbol{a}_{n+1} = \boldsymbol{a}'$ (acceptance)
If $u > \alpha(\boldsymbol{a}'|\boldsymbol{a}_n)$ set $\boldsymbol{a}_{n+1} = \boldsymbol{a}_n$ (rejection)