University of Washington

Quantum System Engineering Group

Title: Positive P-Representations of the Thermal Operator with Applications in Noise and Backaction Simulations

Authors: Joseph L. Garbini, John A. Sidles, John Jacky

A positive P-representation for the spin-j thermal density matrix is given in closed form. The representation is constructed by regarding the wave function as the internal state of a closed-loop control system. A continuous interferometric measurement process is proved to einselect coherent states, and feedback control is proved to be equivalent to a thermal reservoir. Ito equations are derived, and the P-representation is obtained from a Fokker-Planck equation. Langevin equations are derived, and the force noise is shown to be the Hilbert transform of the measurement noise. The formalism is applied to magnetic resonance force microscopy (MRFM) and gravity wave (GW) interferometry. Some unsolved problems relating to drift and diffusion on Hilbert spaces are noted.

URLS: quant-ph/0401165 and quant-ph/0211108

At: LSC/LIGO Livingston Laboratory
Date: March 18, 2004
Presenter: John Sidles
URL: http://courses.washington.edu/goodall/MRFM

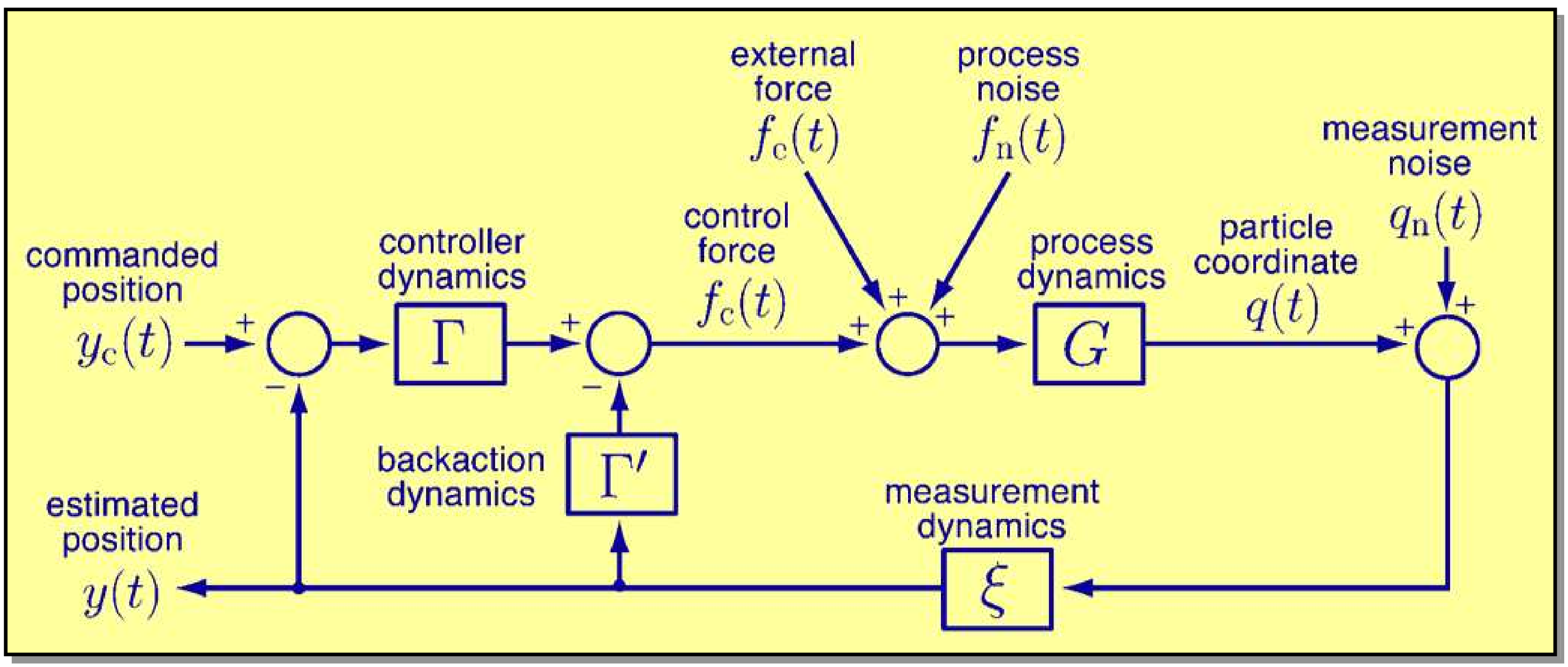
LIGO-G040162-00-Z

Background: from the UW LSC MOU:

2) Continue work on the efficient quantum simulation techniques described in quant-ph/0401165, "Positive P-Representations of the Thermal Operator from Quantum Control Theory", by J. A. Sidles.

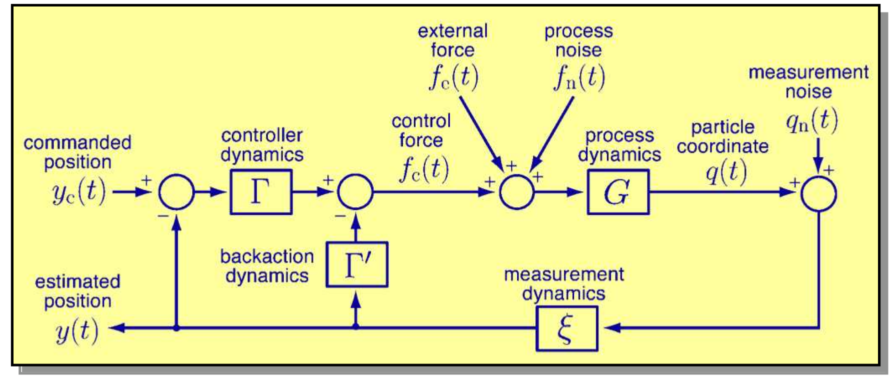
3) Work to establish the formal equivalence (or alternatively, the inequivalence) of the above formalisms to operator-based and field-theoretic quantum descriptions of test mass observation.

Deliverable: quantum control model in closed form



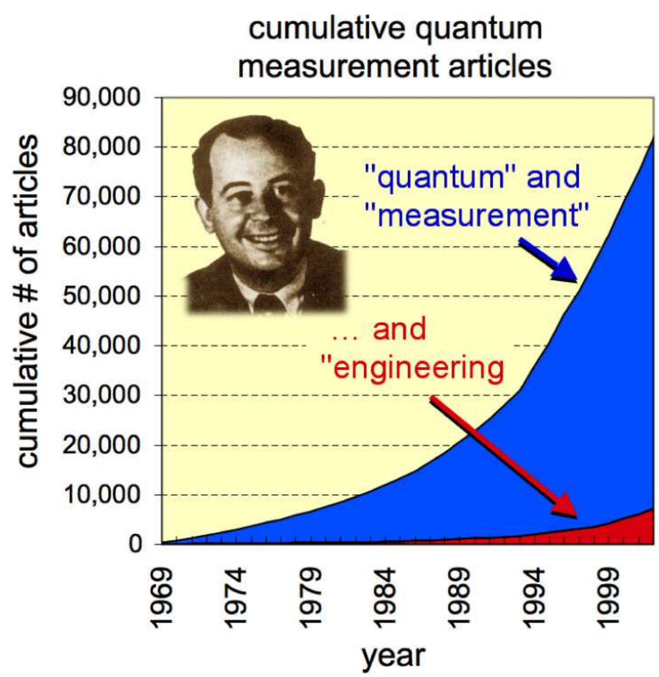
A "Reverse Engineering" Approach to Quantum Control

- Start with control theory
  - Specify a general block diagram
  - Equivalent to non-Markovian linear dynamical equations
- Construct the unique path integral that generates the control equations
  - This uniquely specifies the noise (up to a "Hilbert ambiguity").



quant-ph/0211108

- Identify the physics
  - LIGO I: Linear but non-Markovian physics
  - Single-spin: Markovian but non-linear
  - Next generation of devices: both kinds!
- Prove equivalence to the literature
  - New physics makes us uneasy … we embrace quantum "orthodoxy"
- Our non-intellectual working style
  - We test everything numerically, and prove theorems only to explain what we see.



Closed-form Quantum System Design Rules

- Heat baths are control loops
- Coherent states are *einselected* (Zurek)
- The SQL is rigorously enforced
- Process noise is the Hilbert Transform of measurement noise
- **IBM** / iOSCAR is semi-classical

Design e2e with confidence:

Control loops in high-power advLIGO/CEGO/VIRGO will be stable and quiet

Larmor-type single-proton quantum biomicroscopy is going to work.

control bath total noise

$$S^{\text{total}}_{f_{\text{N}} f_{\text{N}}} = \frac{2m\omega_0}{Q}\, \hbar\omega_0 \coth\left(\frac{\hbar\omega_0}{k_{\text{B}}T}\right)$$

einselection rate equation

$$\operatorname{tr} E[\dot{\boldsymbol{\sigma}}] = -\operatorname{tr} E[\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}^\star]$$

positive P-representation

$$P_j(\hat{\boldsymbol{t}}) = Q^{-1}_{j+1}(-\hat{\boldsymbol{t}})$$

Standard quantum limit (SQL)

$$S_{f^{\text{N}} f^{\text{N}}}(\omega) S_{q^{\text{N}} q^{\text{N}}}(\omega) = (\tfrac{1}{2}\hbar)^2$$

Hilbert Correlation (HC)

$$S_{q^{\text{N}} f^{\text{N}}}(\omega) = \tfrac{1}{2} i\hbar \operatorname{sgn}\omega$$

SNR for iOSCAR energy filter

$$\text{SNR} = \frac{32}{\pi^4}\left(\frac{f^2_{\text{spin}}}{S^{\text{total}}_{f_{\text{N}} f_{\text{N}}}}\right)^2 T_{\text{mod}}\, T_{\text{av}}$$

Positive P-Representations of the Thermal Operator from Quantum Control Theory

arXiv:quant-ph/0401165 v1 27 Jan 2004

UW Goals for 2004

- E2e of iOSCAR
  - Model IBM single-spin effort
  - No thermal baths; everything is a control loop.
- E2e of HIV/CD4 receptor scan
  - Based on 1992 Drobny/Garbini/Sidles device design
  - Rickover Strategy
    - Run e2e quantum simulations on the Pentek control boards themselves
    - Main challenge is dipole-dipole interactions in complex biomolecules
- E2e of high-power, high-finesse low-frequency FP cavities
  - Include torsional effects in path integral
  - Local all poles and zeros analytically
  - Show equivalence to existing formalisms
  - Provide assurance that proposed CEGO and advLIGO designs will be quiet and stable.