LIGO

Thoughts on Thermal Compensation and Stable Recycling Cavity Design for Advanced LIGO

Phil Willems

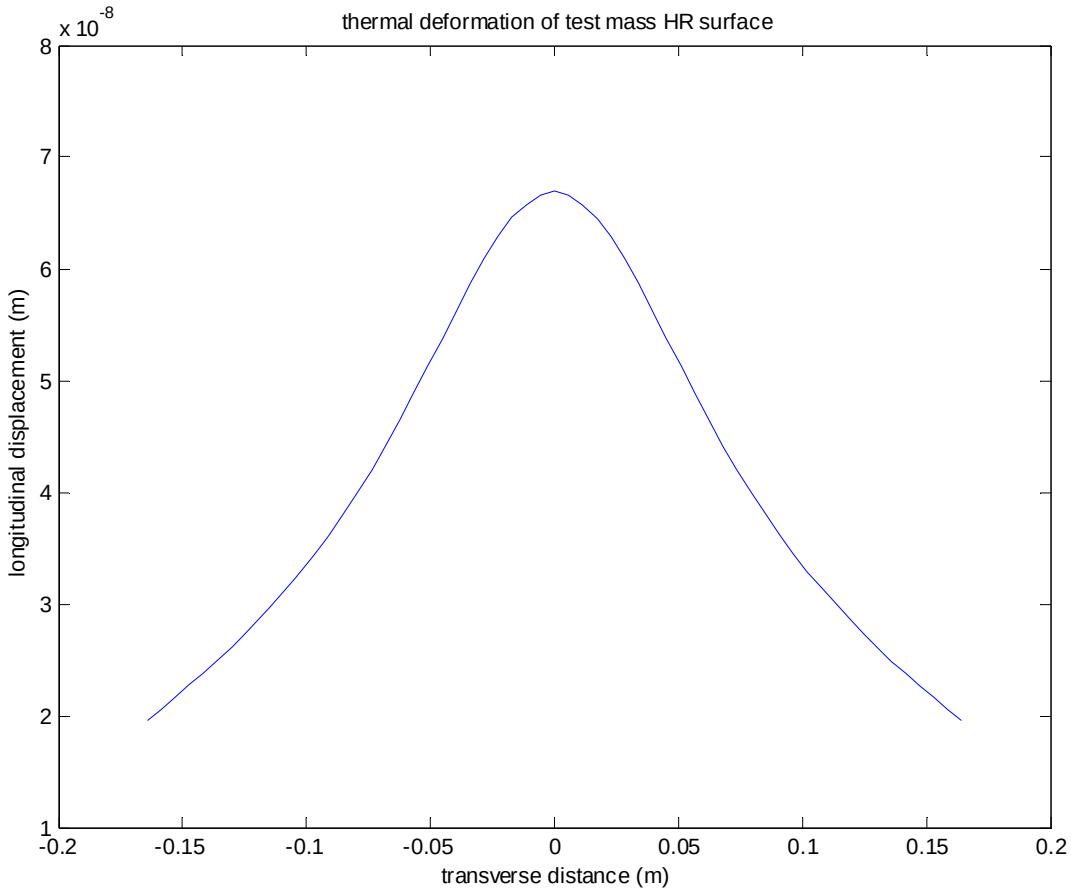
LIGO

Main Points

- Modes in thermally distorted arm cavities
  - » looks pretty good
- “Thermally invariant” stable recycling cavity design
  - » has its drawbacks but the GW sidebands sail through to output unharmed

Arm Cavity Modes

- Previously we have assumed a thermal radius of curvature when calculating arm cavity modes.
- As the thermal model shows, the surface change is not a pure curvature.
- Does this matter?

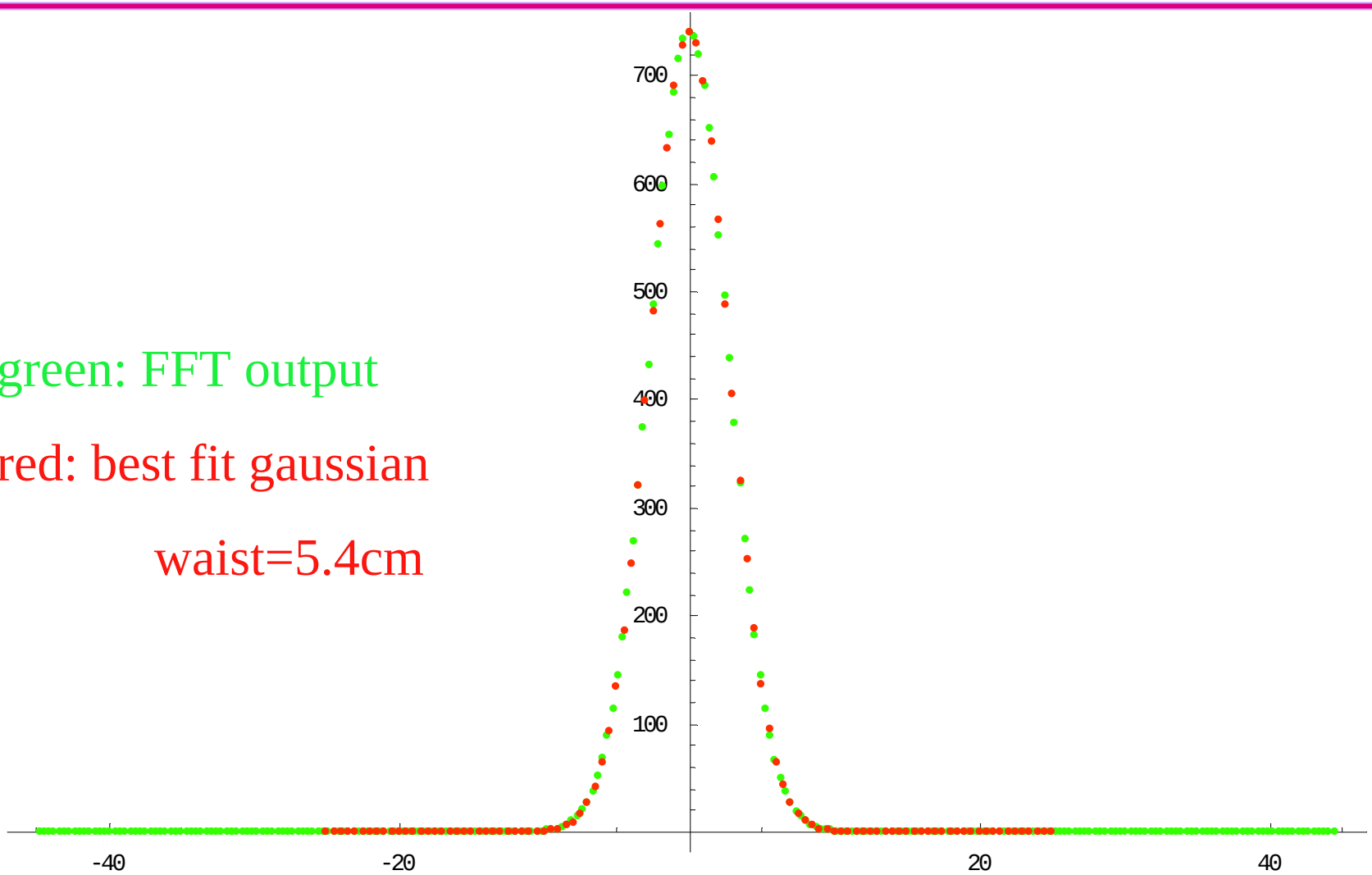


Thermally Distorted Arm Mode

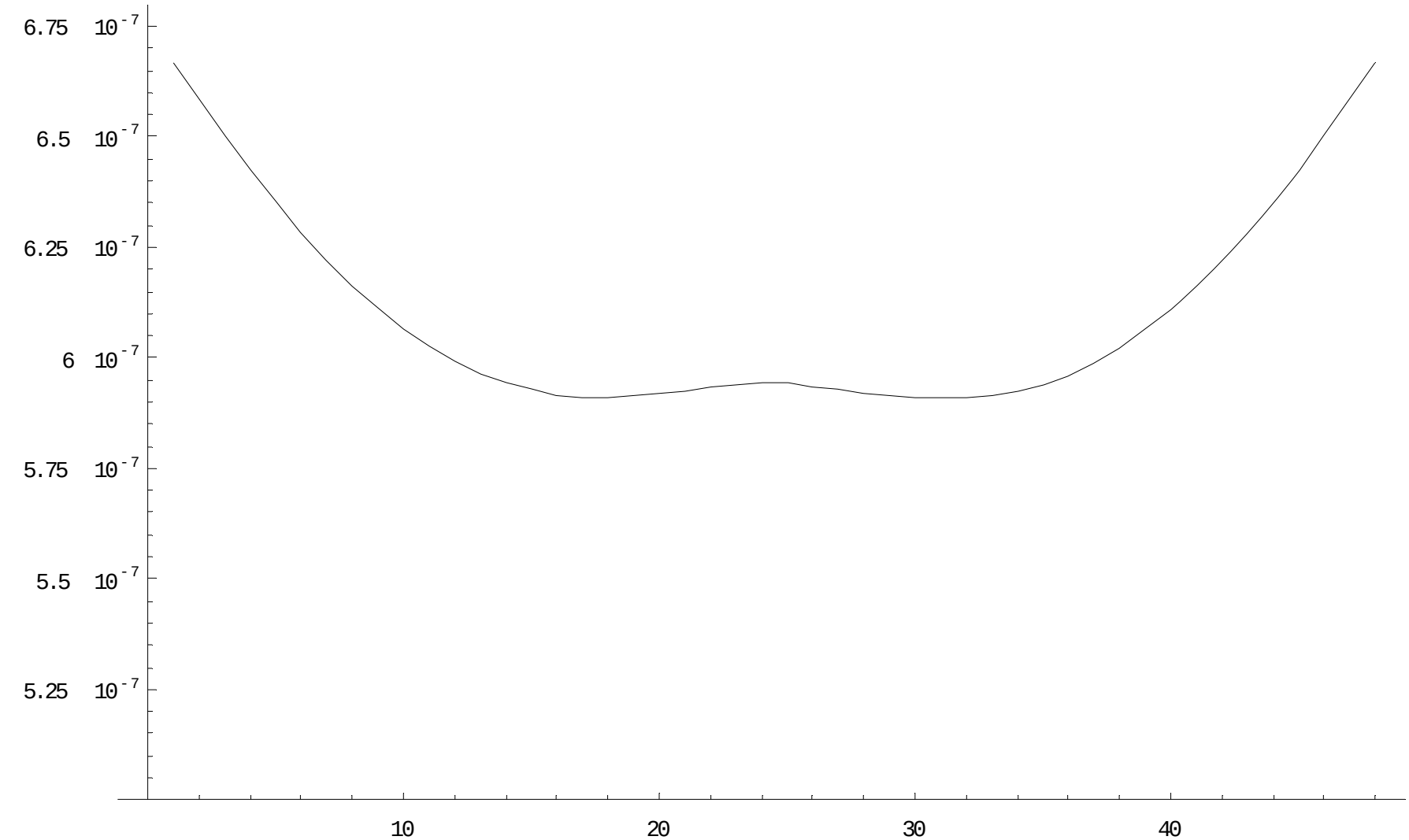
green: FFT output

red: best fit gaussian

waist=5.4cm

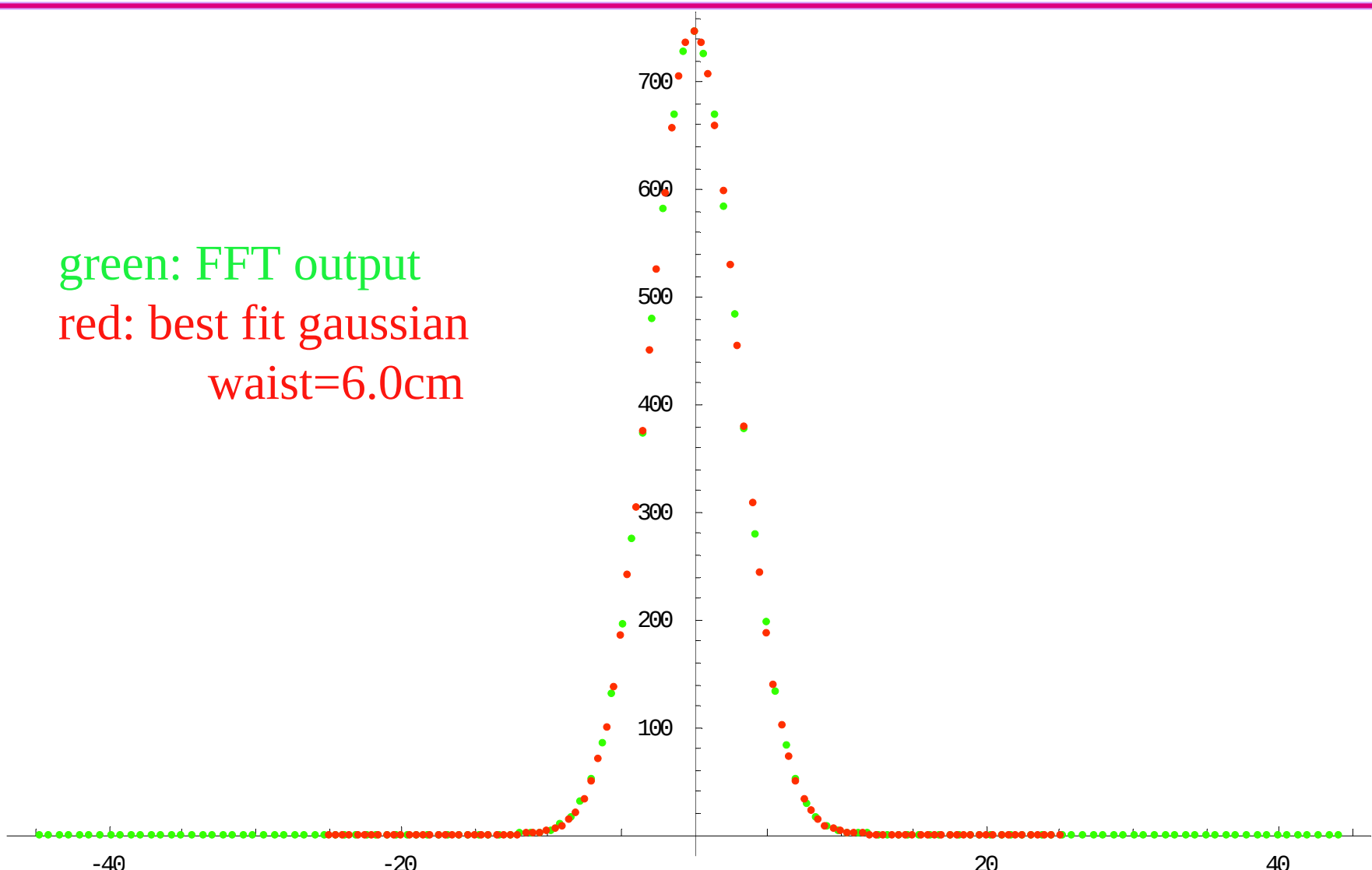


Deformation of Ring-Compensated Mirror



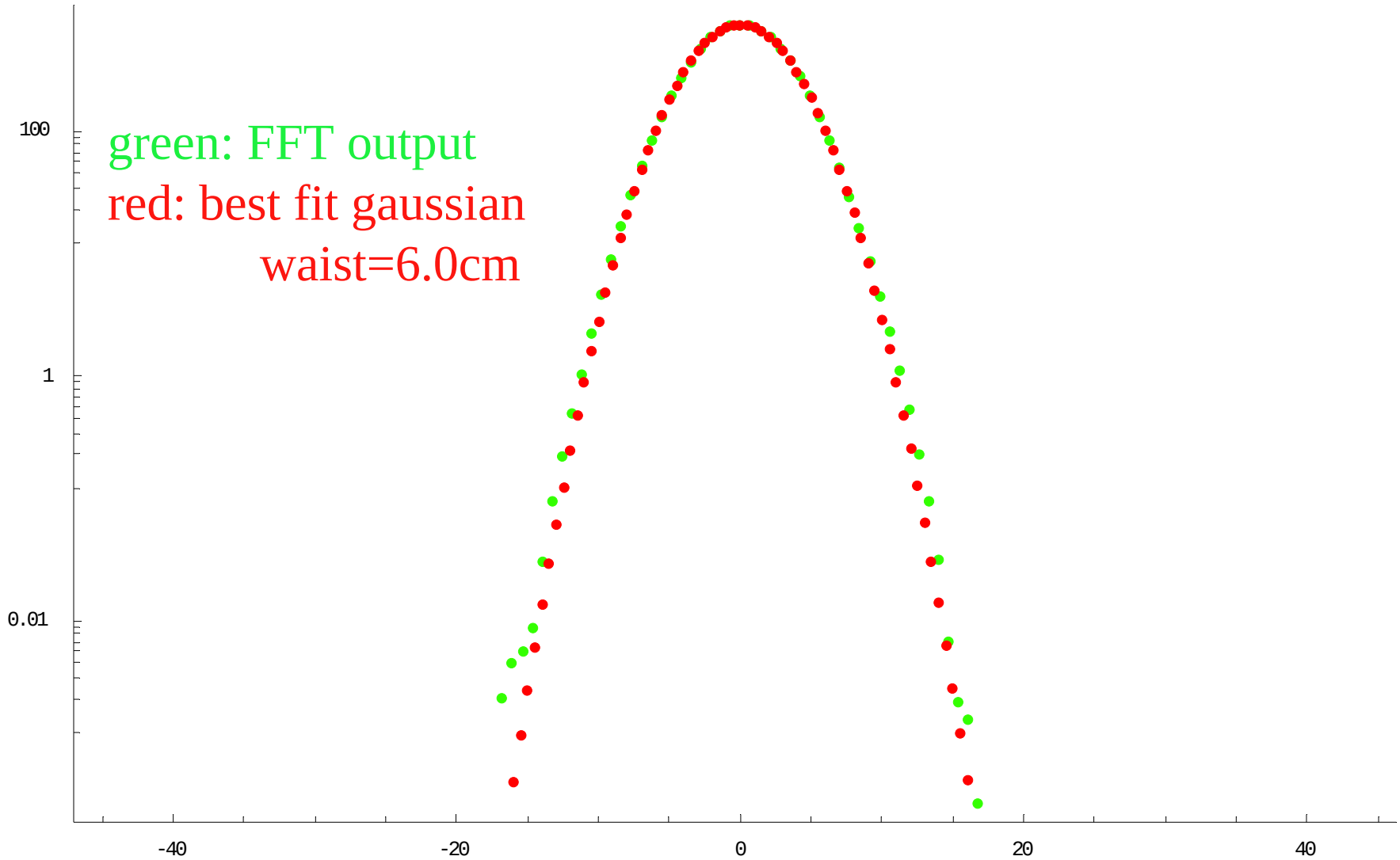
Thermally Compensated Arm Mode

green: FFT output
red: best fit gaussian
waist=6.0cm

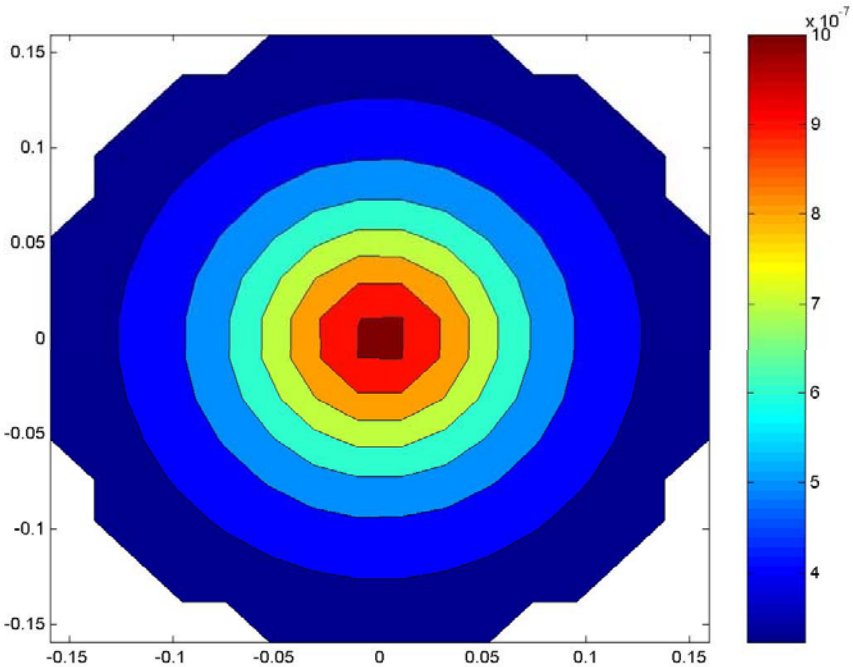


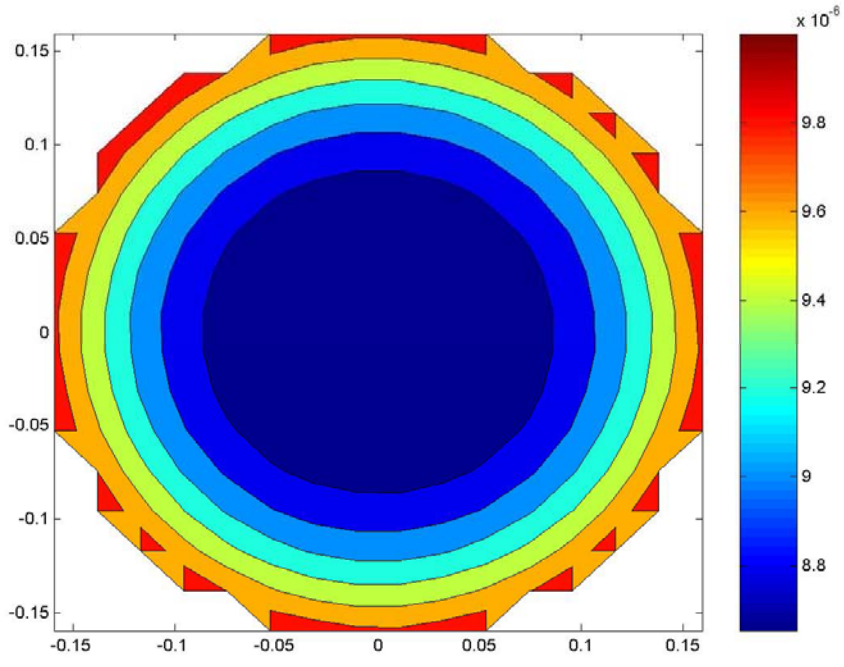
LIGO

Thermally Compensated Arm Mode



Thermal Lensing in the Substrate





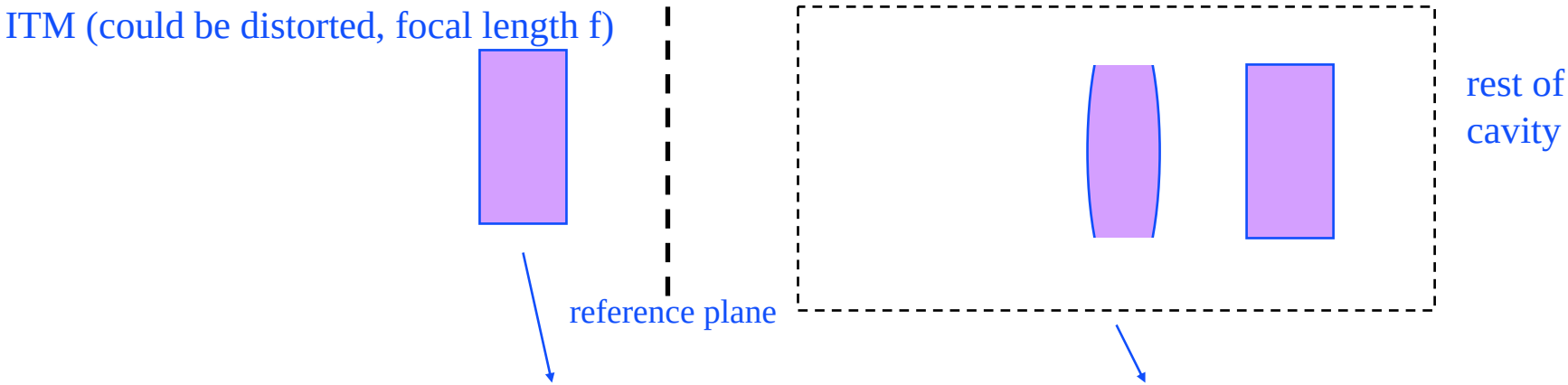
uncompensated:

4% mode overlap

compensated:

72% mode overlap

‘Thermally Invariant’ Stable Recycling Cavity



$$\begin{vmatrix} 1 & 0 \\ -1/f & 1 \end{vmatrix} \begin{vmatrix} A & B \\ C & D \end{vmatrix}$$

total cavity matrix: $\begin{vmatrix} A & B \\ -A/f+C & -B/f+D \end{vmatrix}$

Cavity Math

Stability criterion: $$\left|\frac{A - B/f + D}{2}\right| \le 1$$

Waist size at reference plane: $$w_{ref} = \sqrt{\frac{\lambda}{\pi}} \frac{\sqrt{|B|}}{\left(1 - \left(\frac{A - B/f + D}{2}\right)^2\right)^{1/4}}$$

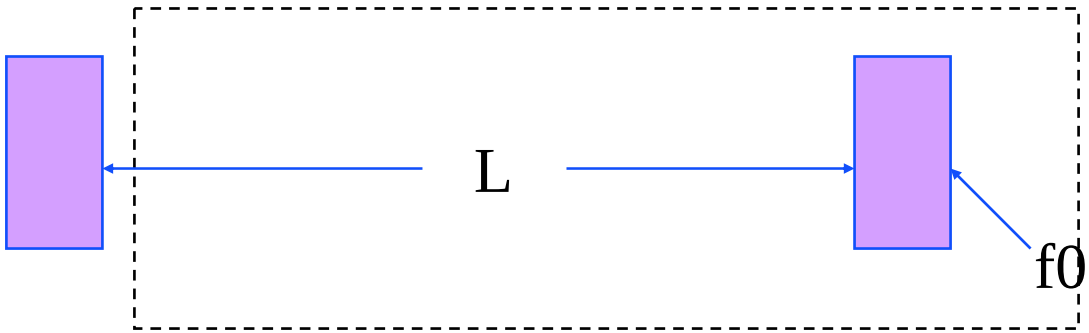
Waist size does not vary with f ($\frac{dw_{ref}}{df} = 0$) if: $$A - B/f + D = 0$$

Note: cavity stability is guaranteed by this condition.

Waist size is then $$w_{ref} = \sqrt{\frac{\lambda}{\pi}}\sqrt{|B|} = 6cm \rightarrow B = \pm 10629m$$

Apply this to a simple cavity…

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \begin{vmatrix} 1-L/f0 & 2L-L^2/f0 \\ -1/f0 & 1-L/f0 \end{vmatrix}$$

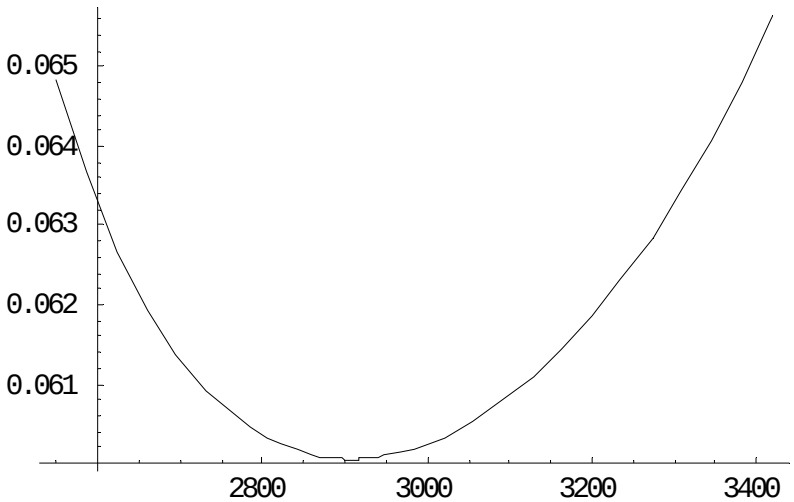


LIGO

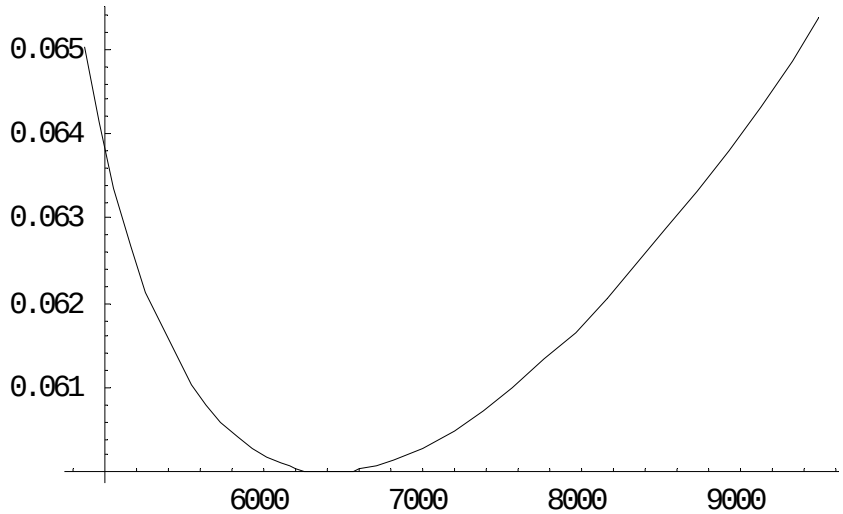
Solutions

- For L=8.3m, f0=6.5mm or f0=-6.5mm
  - » Not a good solution.
- For L=4km, two good solutions:
  - » f0=-6,086m, f=3,221m
  - » F0=858m, f=1,452m
- Both of the 4km cavity solutions are stable

Waist sizes vs. f_{ITM}

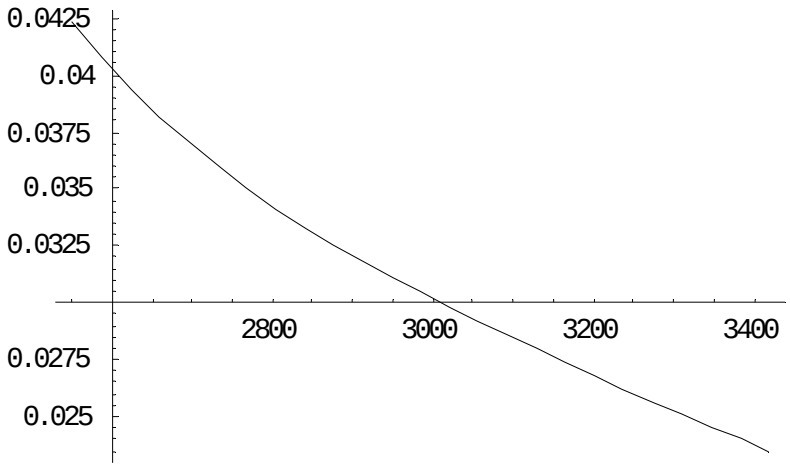


f0=858m

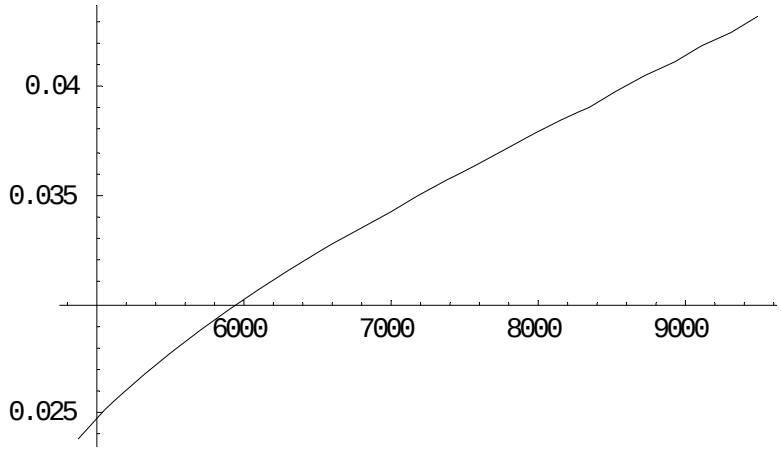


f0=-6086m

The Downside: Spot Size at SRM



f0=858m



f0=-6086m

LIGO

Comments, Future Work

- The idea of overlap integrals, used by Lawrence for unstable recycling cavities, fails for stable cavities. Problem is not as bad as naively predicted.
- 4km signal cavity is a drag, but shorter, more complex cavities might work
- Large and rapid variations in output spot size are also a drag, but at least the thermal compensation can move outside the signal cavity
- What if different arms have different heating? Fuller models are needed.