Stochastic Gravitational-Wave Searches with Interferometers and Bars

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Outline

I Review Of Stochastic Background Searches

- Optimally-Filtered Cross-Correlation
- Overlap Reduction Function
- Notable Cross-Correlation Experiments

II LLO-ALLEGRO Cross-Correlations

- Overlap Modulation by Rotation of Bar
- Handling Different Sampling Rates & Heterodyning
- Status Reported at GR17

Types of Gravitational Wave Signals

Convenient classification for data analysis:

- Inspirals: “Chirp” signals (rapid decay of binary BH or NS orbit)
- Bursts: Unmodelled strong signals (e.g., Supernovae)
- Periodic: Continuous waves (e.g., rotating deformed NS)
- Stochastic: Random cosmological or astrophysical background

Stochastic Background of Gravitational Waves

- Random GW signal from superposition of unresolved sources

- Analogous to Cosmic Microwave Background, but
  - Spectrum unknown (compare CMB blackbody)
  - Component sources can be cosmological or astrophysical

- CMB comes from recombination of plasma to neutral atoms ionized plasma transparent to GWs $\longrightarrow$ Cosmological GW BGs can tell us about earlier history of universe than CMB

Cartoon courtesy of E. Coccia, NAUTILUS Group (Rome)

Stochastic Background

Backgrounds in 10–1000 Hz frequency band likely extragalactic in origin, thus isotropic, unpolarized, gaussian, & stationary.

Describe i.t.o. GW contribution to $\Omega = \frac{\rho}{\rho_{\mathrm{crit}}}$:

$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_{\mathrm{crit}}} \frac{d\rho_{\mathrm{GW}}}{d \ln f} = \frac{f}{\rho_{\mathrm{crit}}} \frac{d\rho_{\mathrm{GW}}}{df}$$

Note $\rho_{\mathrm{crit}} \propto H_0^2$, so $h_{100}^2 \Omega_{\mathrm{GW}}(f)$ is independent of

$$h_{100} = \frac{H_0}{100\,\mathrm{km/\,s/\,Mpc}}$$

How to Tell Stochastic Signal from Random Noise

- Ground-based detectors noise-dominated
  & can't be pointed "off-source"
  → identifying a GW background in a single detector impractical

- Need correlations among detectors
  - Detector 1: $s_1 = h_1 + n_1$, Detector 2: $s_2 = h_2 + n_2$
  - h=stoch GW signal, n=noise (usu. much larger)

- Assume noise uncorrelated with signal & between detectors

- Cross-correlation:

$$\langle s_1 s_2 \rangle = \langle n_1 n_2 \rangle + \langle n_1 h_2 \rangle + \langle h_1 n_2 \rangle + \langle h_1 h_2 \rangle$$

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  only surviving term is from stochastic GW signal

Sensitivity to Stochastic GW Backgrounds

- Optimally filtered CC statistic

$$Y = \int df\, \tilde{s}_1^*(f)\, \tilde{Q}(f)\, \tilde{s}_2(f)$$

- Optimal filter $\tilde{Q}(f) \propto \frac{f^{-3}\Omega_{\mathrm{GW}}(f)\gamma_{12}(f)}{P_1(f)P_2(f)}$
  (Initial analyses assume $\Omega_{\mathrm{GW}}(f)$ constant across band)
- Optimally filtered cross-correlation method sensitive to

$$\Omega_{\mathrm{GW}} \propto \left(T \int \frac{df}{f^6} \frac{\gamma_{12}^2(f)}{P_1(f)P_2(f)} \right)^{-1/2}$$

- Significant contributions when
  - detector noise power spectra $P_1(f)$, $P_2(f)$ small
  - overlap reduction function $\gamma_{12}(f)$ (geom correction) near ± 1

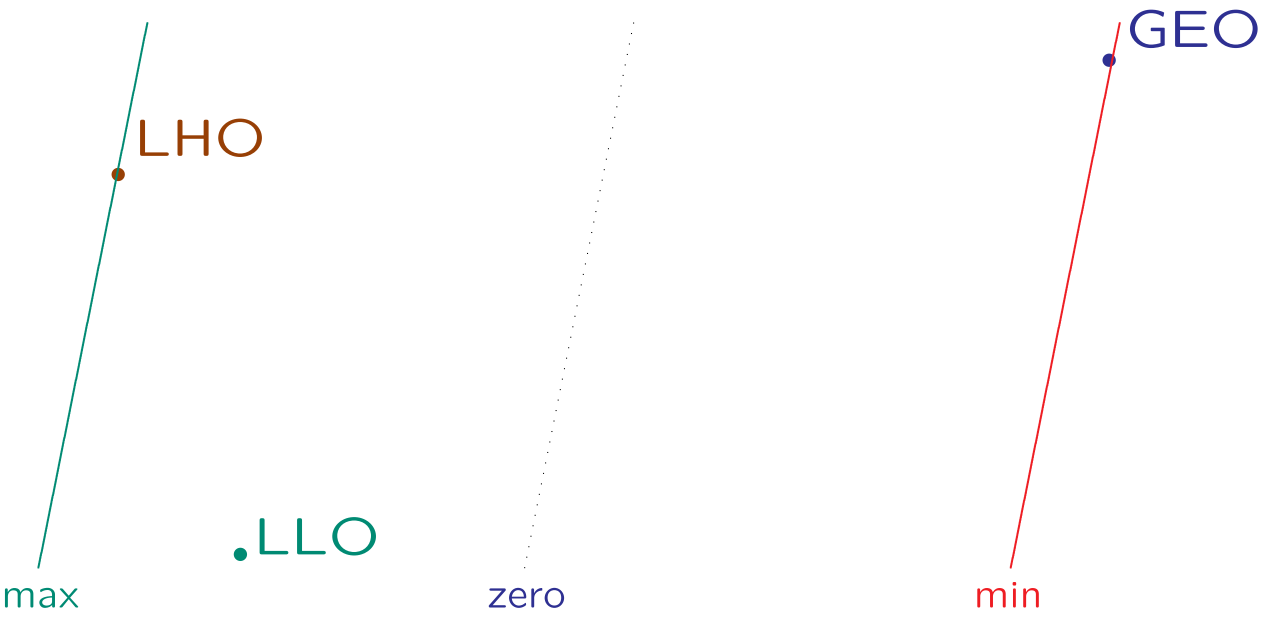
Overlap Reduction Function

$$\gamma_{12}(f) = d_{1ab} d_2^{cd} \frac{5}{4\pi} \iint_{S^2} d^2\Omega \; P^{\mathrm{TT}ab}{}_{cd}(\widehat{\mathbf{\Omega}}) e^{i2\pi f \widehat{\mathbf{\Omega}}\cdot\Delta\vec{\mathbf{x}}/c}$$

Depends on alignment of detectors (polarization sensitivity)

Frequency dependence from cancellations when $\lambda \lesssim$ distance

→ Widely separated detectors less sensitive at high frequencies



This wave drives LHO & GEO out of phase

Overlap Reduction Function

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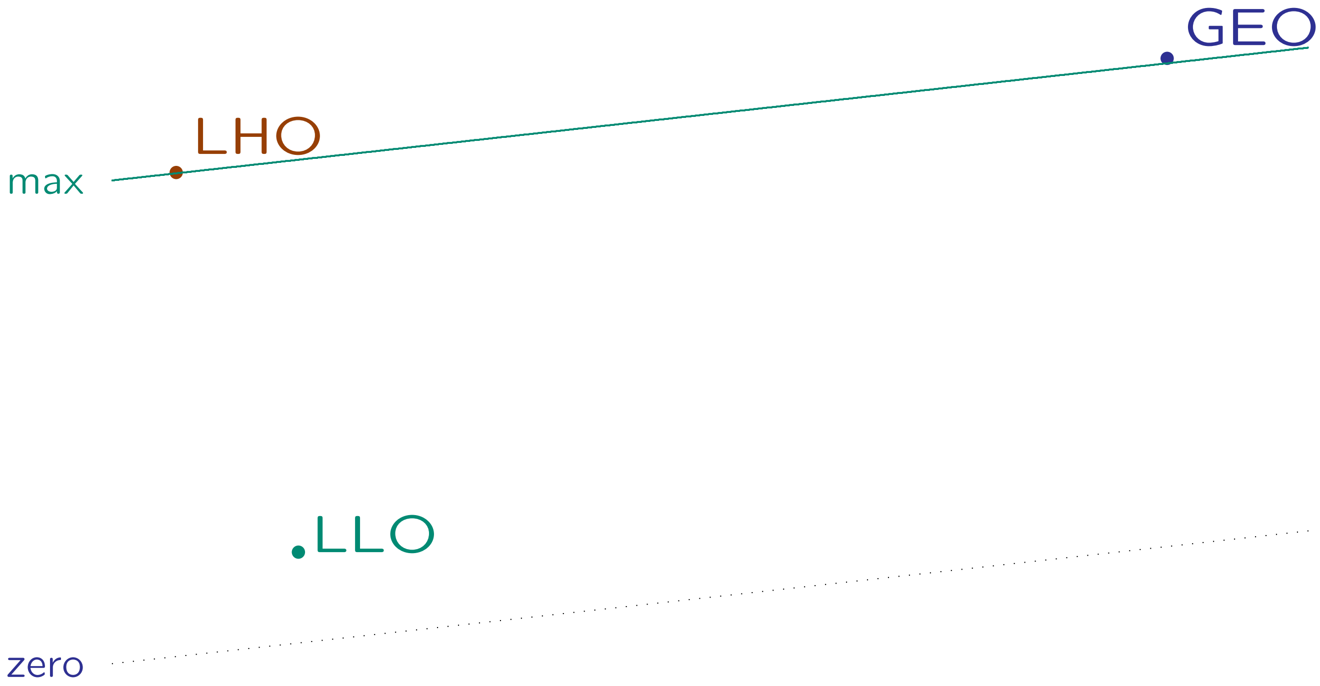
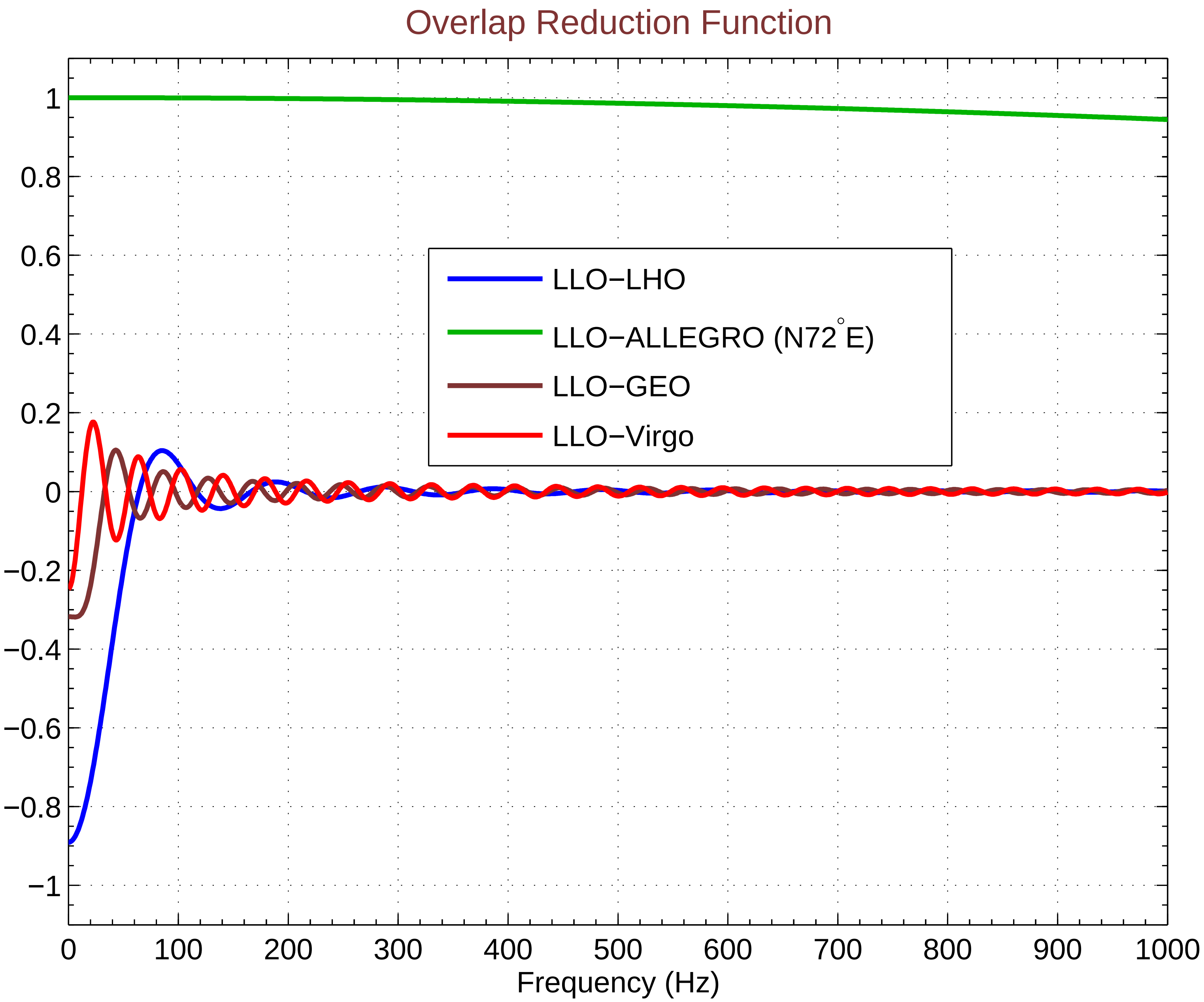
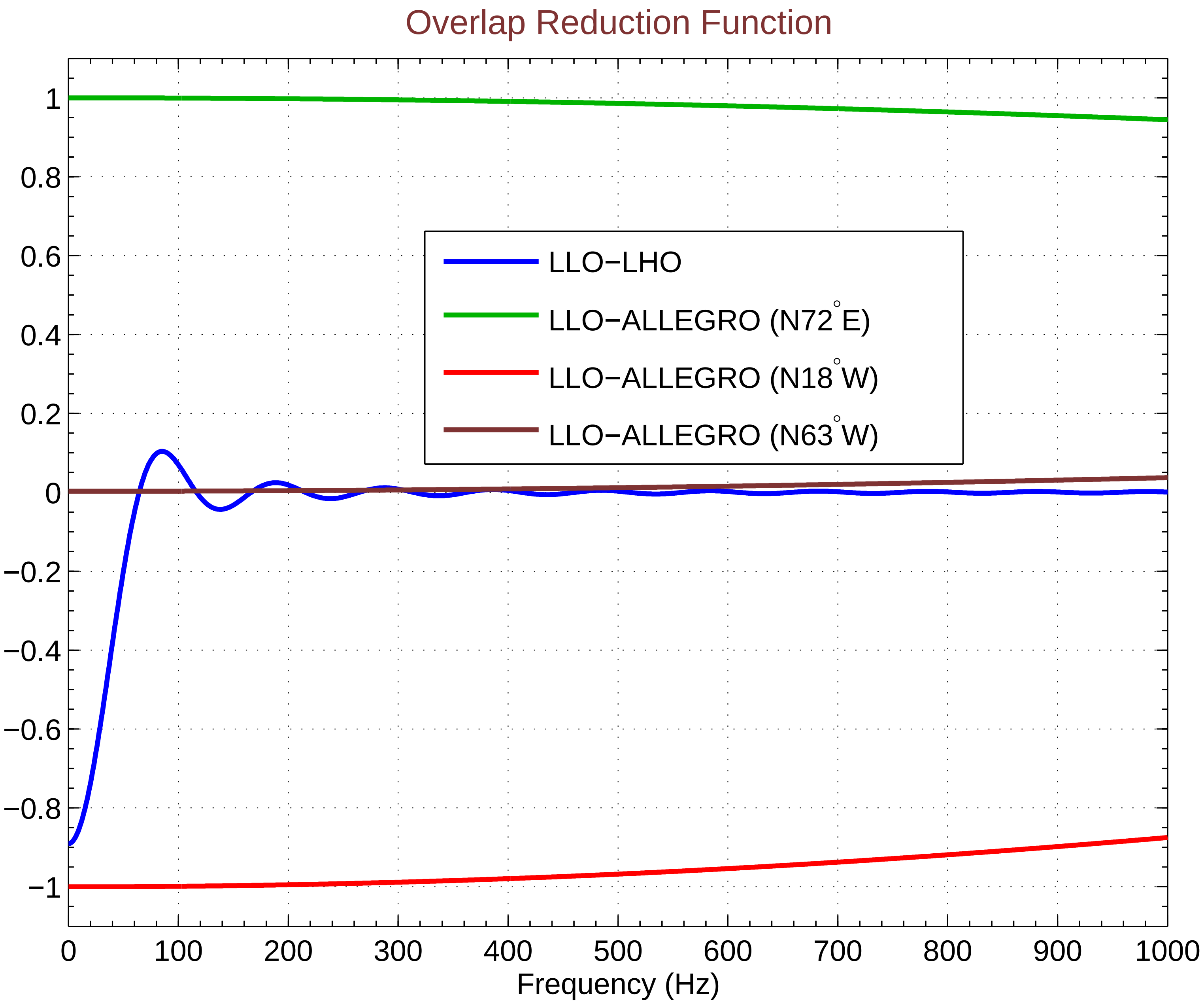


This wave (same λ) drives LHO & GEO in phase


Overlap Reduction Function
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
100
200
300
400
500
600
700
800
900
1000
Frequency (Hz)
LLO−LHO
LLO−ALLEGRO (N72° E)
LLO−GEO
LLO−Virgo

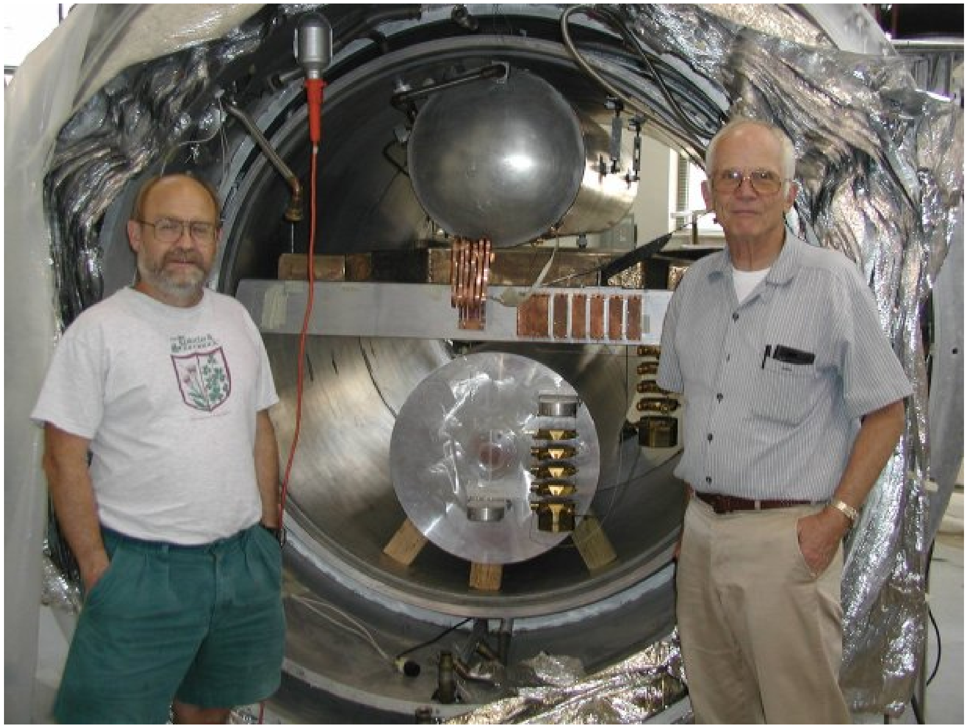


Overlap Reduction Function
LLO−LHO
LLO−ALLEGRO (N72°E)
LLO−ALLEGRO (N18°W)
LLO−ALLEGRO (N63°W)
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
100
200
300
400
500
600
700
800
900
1000
Frequency (Hz)

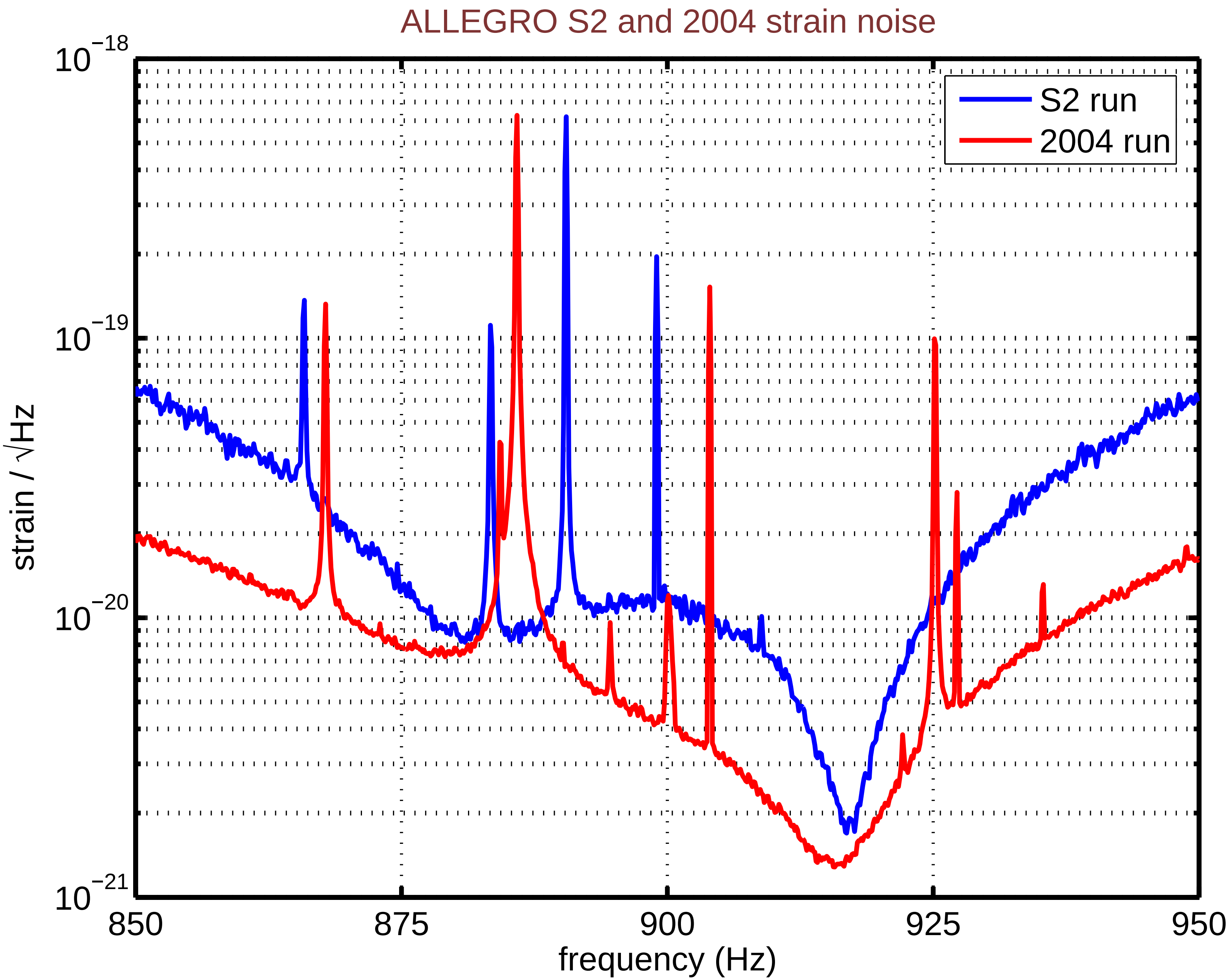
Upper Limits

- Correlation between
  EXPLORER & NAUTILUS bars (Astone et al, 1999):
  $h_{100}^2\Omega_{\rm GW}(907\,{\rm Hz}) \leq 60$
- Correlation between LIGO Hanford & Livingston
  S1 data (LSC, Abbott et al, 2004):
  $h_{100}^2\Omega_{\rm GW}(f) \leq 23$ at $64 < f < 265$
- Correlation between LIGO Hanford & Livingston
  S2/S3 Data Analysis Ongoing;
  Preliminary S2 result reported at GR17:
  $h_{100}^2\Omega_{\rm GW}(f) \leq 0.018^{+0.007}_{-0.003}$ at $50 < f < 300$
- Correlations between LIGO Livingston & ALLEGRO data
  This talk focusses on methods
  Status was reported at GR17

ALLEGRO Detector (Baton Rouge, LA)



W. Johnson, ALLEGRO & W. Hamilton from LSU Website


ALLEGRO S2 and 2004 strain noise
S2 run
2004 run
10^{-18}
10^{-19}
10^{-20}
10^{-21}
strain / √Hz
850
875
900
925
950
frequency (Hz)

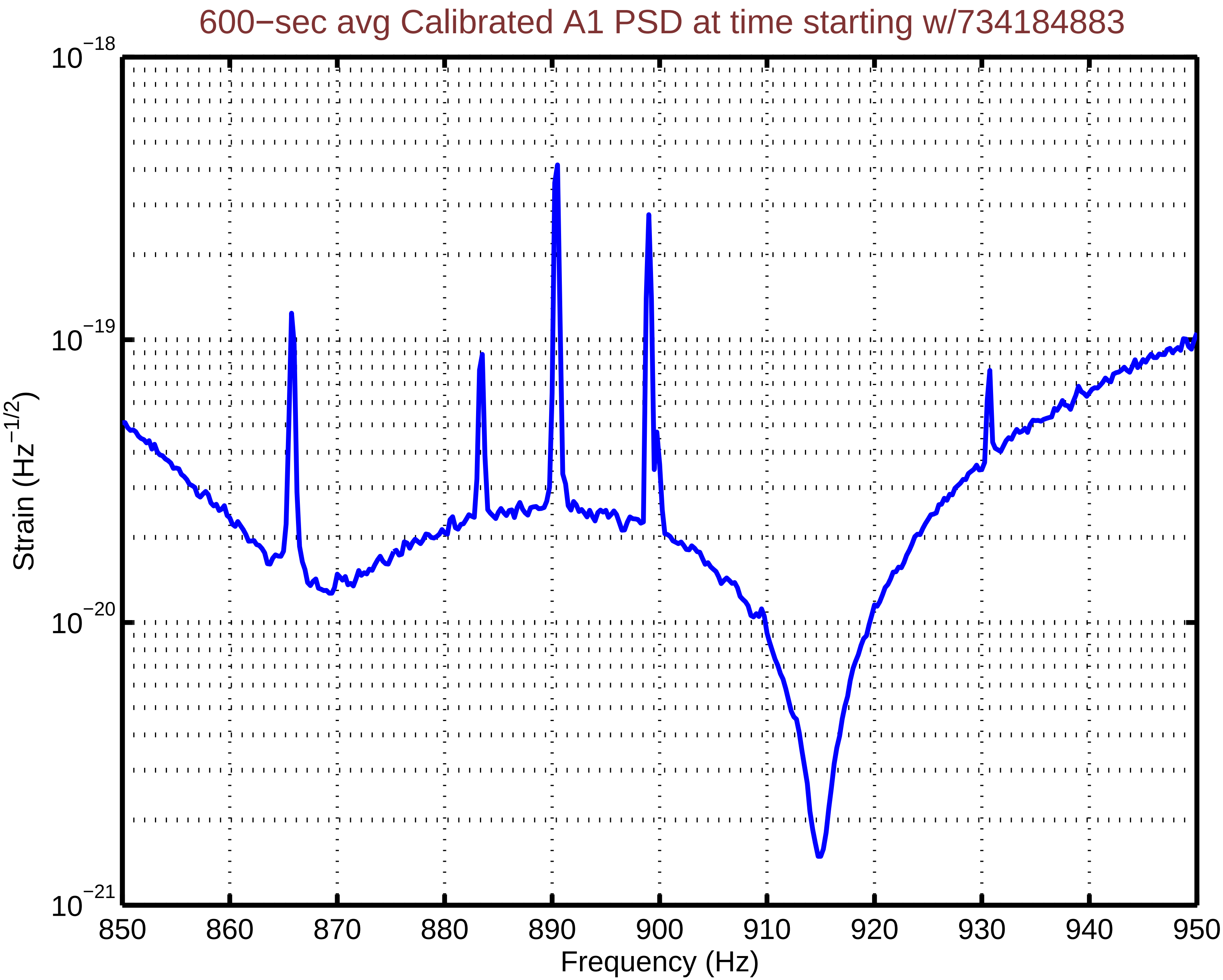
LLO-ALLEGRO Correlations

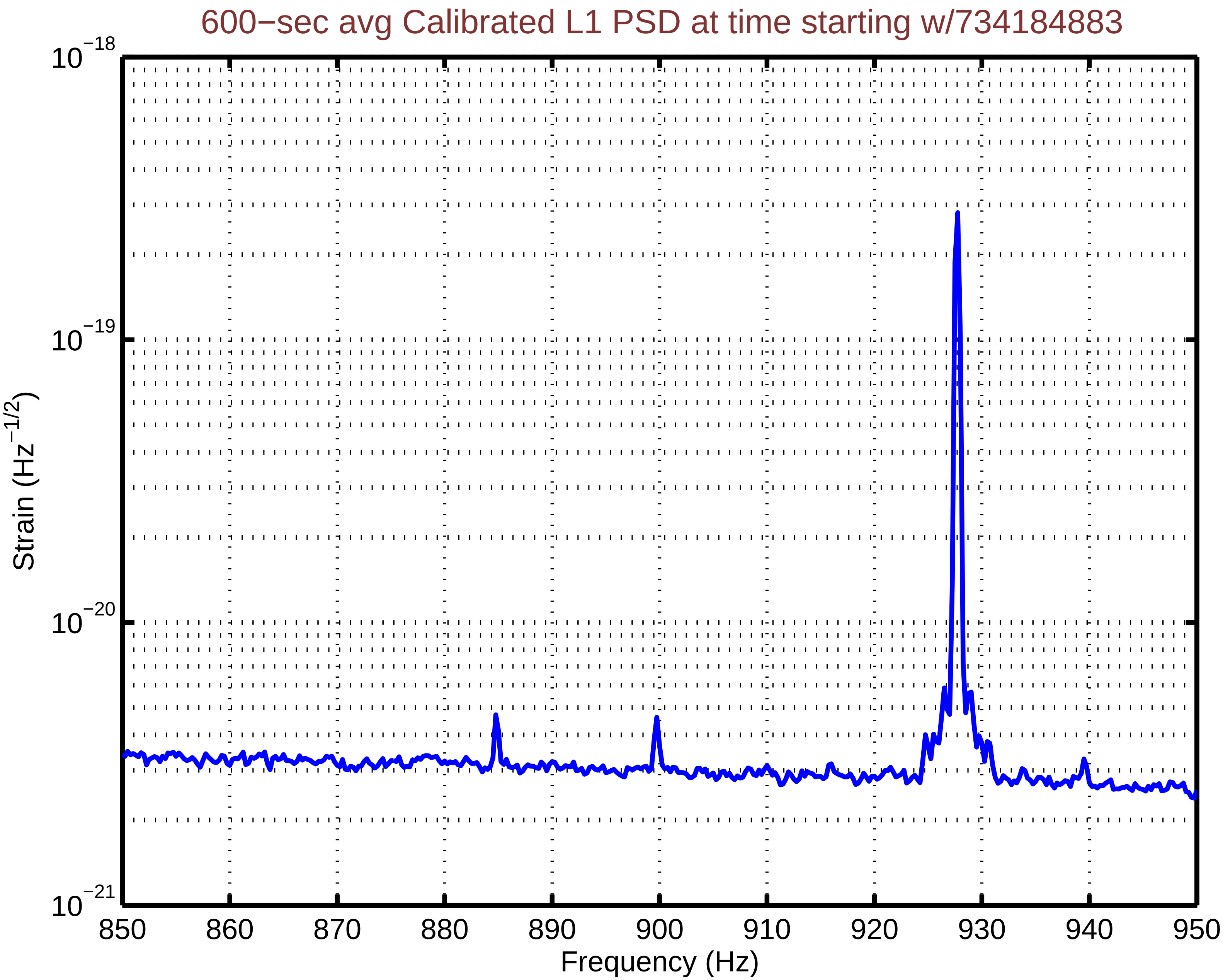
- Only $\sim$40 km apart → $\gamma(900\,\mathrm{Hz}) \approx 95\%$ for best alignment

- Sensitive in different freq band from LLO/LHO pair

- New experimental technique: rotate ALLEGRO to callibrate cross-correlated noise (Finn & Lazzarini)
  - Aligned & Anti-aligned orientations have opposite GW sign
    $\longrightarrow$ can "cancel" out CC noise by subtracting results
  - Null orientation has no expected GW signal
    $\longrightarrow$ "off-source" measurement of CC noise

- Currently analyzing S2 (2003 Feb 14-Apr 14) data; ALLEGRO was offline for S3 (2003 Oct 31-2004 Jan 9), now running again; Further work planned for S4 & beyond


600−sec avg Calibrated A1 PSD at time starting w/734184883
10^{-18}
10^{-19}
10^{-20}
10^{-21}
Strain (Hz$^{-1/2}$)
850
860
870
880
890
900
910
920
930
940
950
Frequency (Hz)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940
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LLO-ALLEGRO: Technical Considerations

- ALLEGRO data heterodyned at 899 Hz & sampled at 250 Hz
  LIGO data digitally downsampled 16384 Hz → 2048 Hz
  Time domain resampling undesirable: $2^{10}/5^3$ sampling ratio
  → work in freq domain w/overlapping frequencies

- Uncalibrated ALLEGRO data have sharper spectral features
  → Work w/calibrated heterodyned strain "$h(t)$" for ALLEGRO

- Calibrating ALLEGRO data is major undertaking
  (McHugh + Johnson & LSU)
  (Coherent analysis requires more precise calibration than before)
  See McHugh GR17 talk for more details

Crash Course on Heterodyning (base-banding)

Think in terms of continuous Fourier transform

$$\widetilde{G}(f) = \int_{-\infty}^{\infty} dt\, e^{-i2\pi f(t-t_0)} G(t)$$

Analog heterodyne: multiply by exp oscillating @ base freq f_b:
$G_h(t) = e^{-i2\pi f_b(t-t_0)} G(t)$ so that Fourier transform is

$$\widetilde{G}_h(f) = \widetilde{G}(f_b + f)$$

Low-pass anti-aliasing filter on G_h is then band-pass filter on G;

$$\widetilde{g}_h(f) = \begin{cases} \widetilde{G}_h(f) & |f| \le \frac{1}{2\,\delta t} \\ 0 & |f| > \frac{1}{2\,\delta t} \end{cases}$$

$g_h(t)$ then sampled @ $\frac{1}{\delta t}$ so $f_{\text{Ny}} = \frac{1}{2\,\delta t}$; range of physical freqs

$$f_b - f_{\text{Ny}} \le f_{\text{phys}} \le f_b + f_{\text{Ny}}$$

Working in Frequency Domain

- LLO & ALLEGRO data are FFTed to produce freq series (normalized to approximate CFT)
  $\widetilde{s}^L[f]: \quad 0 \le f \le f^L_{\mathsf{Ny}}$
  $\widetilde{s}^A_h[f]: \quad -f^A_{\mathsf{Ny}} \le f < f^A_{\mathsf{Ny}}$
  If duration is T, zero-padded to $2T$, each has freq res $\delta f = \frac{1}{2T}$

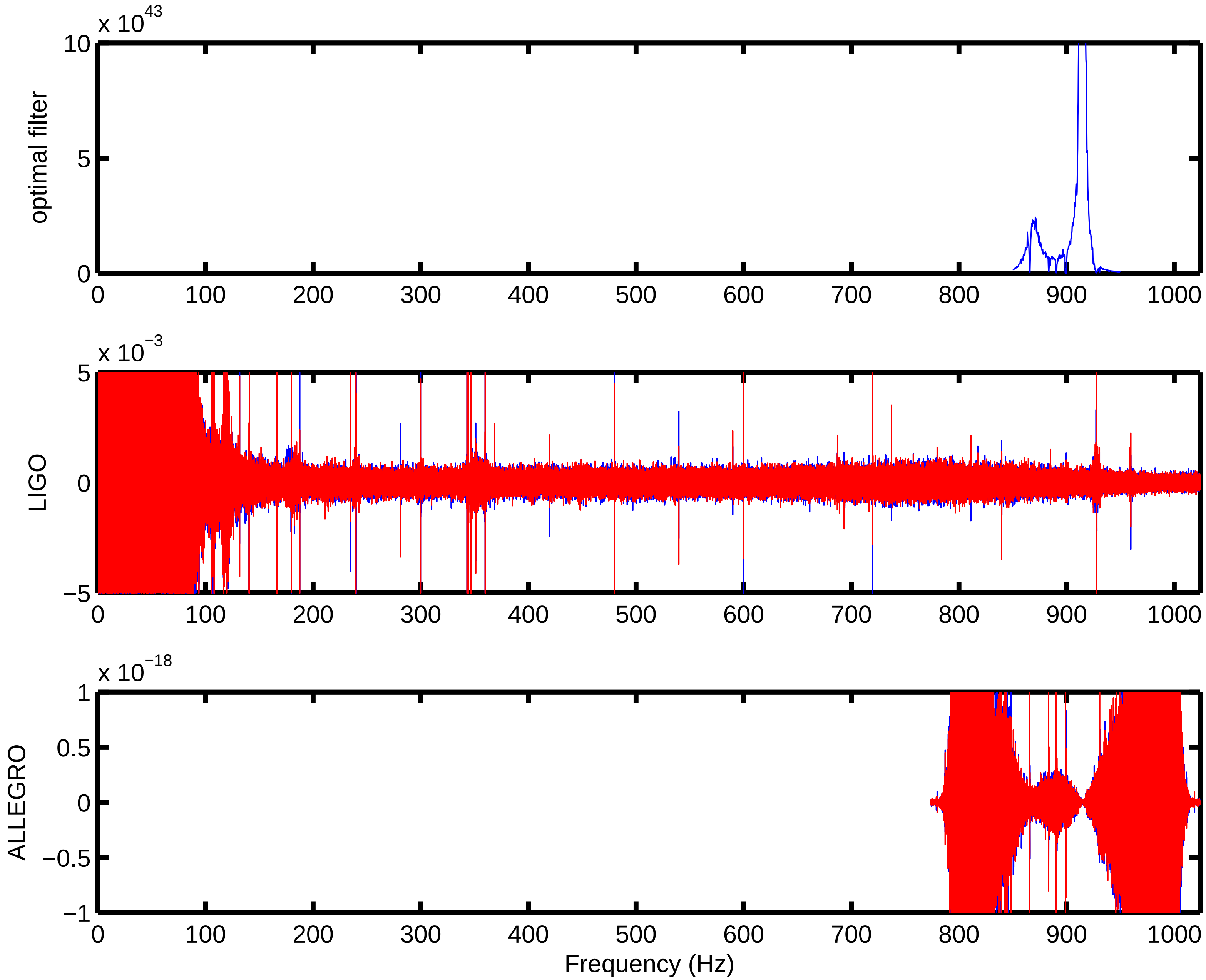
- Optimal filter created in freq domain w/same freq resolution

$$\widetilde{Q}[f]: \quad f_{\mathsf{min}} \le f \le f_{\mathsf{max}}$$

- Cross-correlation statistic is

$$Y = \sum_{f=f_{\mathsf{min}}}^{f_{\mathsf{max}}} \delta f \left(\widetilde{s}^L[f]\right)^* \widetilde{Q}[f]\, \widetilde{s}^A_h[f-f_b] \approx \int_{f_{\mathsf{min}}}^{f_{\mathsf{max}}} df \left[\widetilde{s}^L(f)\right]^* \widetilde{Q}(f)\, \widetilde{s}^A(f)$$

  So long as $[f_{\mathsf{min}}, f_{\mathsf{max}}]$ a subset of LLO & ALLEGRO freq ranges & $\frac{f_b}{\delta f} \in \mathbb{Z}$, freq bins “line up”


x 10^43
10
5
0
optimal filter
0
100
200
300
400
500
600
700
800
900
1000
x 10^−3
5
0
−5
LIGO
x 10^−18
1
0.5
0
−0.5
−1
ALLEGRO
Frequency (Hz)

Example of Frequency Domain Method

- Assume $T = 50\,\text{sec}$;
  after zero-padding $\delta f = .01\,\text{Hz}$ for both ALLEGRO & LLO

- FFT real LLO data, sampled at $2048\,\text{Hz}$
  102401 bins: DC to $1024\,\text{Hz}$ (Nyquist)

- FFT cmplx heterodyned ALLEGRO data, sampled at $250\,\text{Hz}$
  25000 bins:
  $774\,\text{Hz}$ $(f_b - f^A_{\text{Ny}})$ to $1023.99\,\text{Hz}$ $(f_b + f^A_{\text{Ny}} - \delta f)$

- Correlate only the bins from (say) $850\,\text{Hz}$ to $950\,\text{Hz}$
  ALLEGRO & LLO bins “line up”

LLO-ALLEGRO: Summary

- Probes higher frequency band: $\sim 850 - 950\,\mathrm{Hz}$
- Rotate ALLEGRO to modulate stochastic response (data taken in 3 orientations during S2)
- Freq-domain method seems to solve sampling rate problems
  $\exists$ more careful analytic demonstration
- Analyzing S2 data; next coïncident run is S4
- Reported at GR17:
  Expected S2 sensitivity of $\Omega_{\mathrm{GW}}(f) \sim 10$
  Projected "S4" sensitivity of $\Omega_{\mathrm{GW}}(f) \sim 0.07$ (for 1 yr @ design)

References

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3. B. Allen, Les Houches lecture: gr-qc/9604033
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5. P. Astone et al, A&A **351**, 811 (1999)
6. P. Astone et al, PLB **385**, 421 (1996)
7. LIGO S1 Paper: B. Abbott et al, PRD: gr-qc/0312088
8. Finn & Lazzarini, PRD: gr-qc/0104040;
   Poster: LIGO graphical presentation LIGO-G010246-00-E
9. LSC Stochastic BG Page: `http://www.ligo.org/sgwb/`
10. JTW GWDAW talk: LIGO-G030692-00-Z
11. LHO-LLO GR17 talk: LIGO-G040312-00-Z
12. LLO-ALLEGRO GR17 talk: LIGO-G040304-00-Z