The Q transform search for gravitational-wave bursts with LIGO

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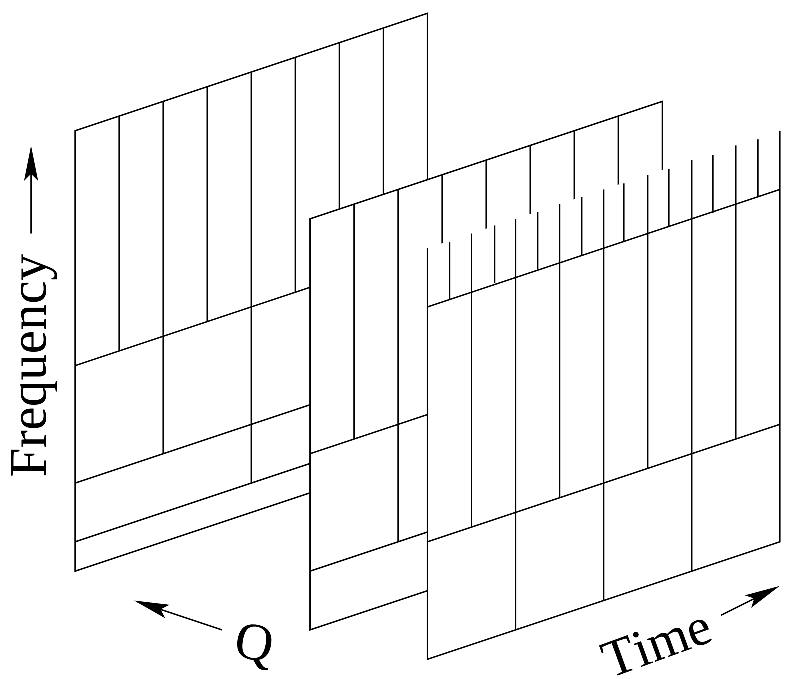
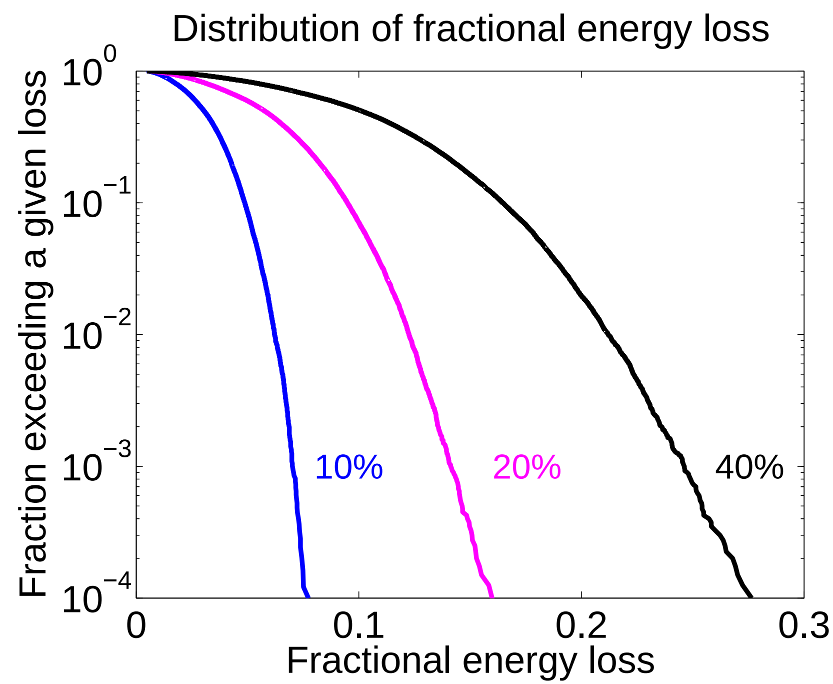
Tiling in time, frequency, and Q

- Multi-resolution time-frequency search for excess power.
- Targets minimum uncertainty waveforms in time, frequency, and Q space.
- Space tiled for a worst case fractional energy loss due to mismatch.
- Fractional energy loss due to mismatch represented as a metric

$$\delta s^2 = \frac{4\pi^2\phi^2}{Q^2}\,\delta\tau^2 + \frac{2+Q^2}{4\phi^2}\,\delta\phi^2 + \frac{1}{2Q^2}\,\delta Q^2 - \frac{1}{\phi Q}\delta\phi\,\delta Q,$$

- Yields logarithmic tiling in frequency and Q and linear tiling in time.



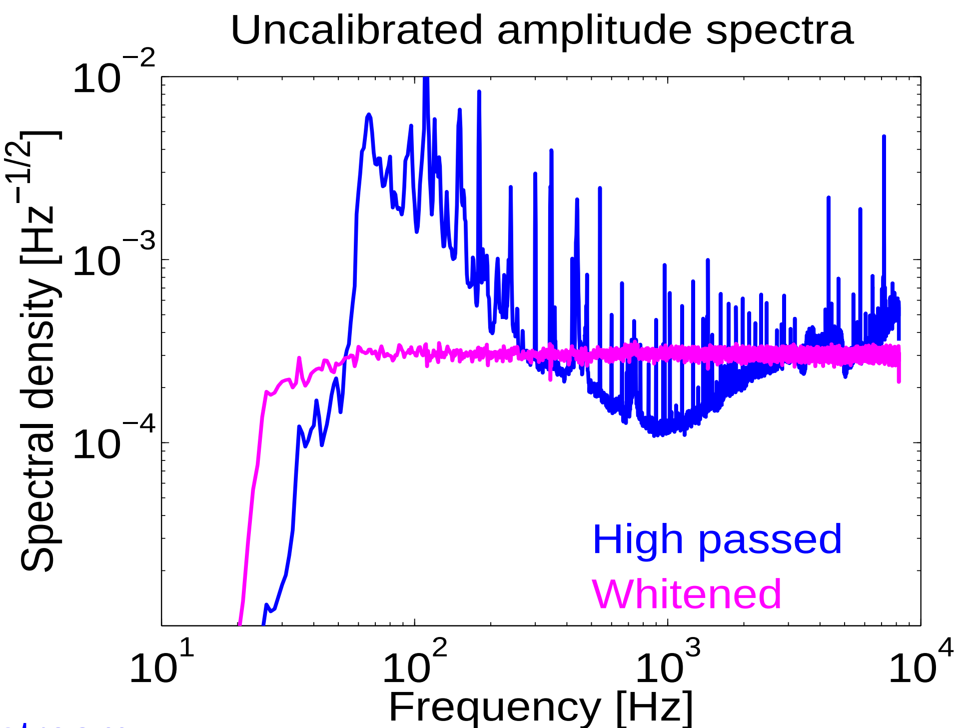


Whitening by linear prediction

- Data first whitened by zero-phase linear prediction.
- Whitening greatly simplifies the statistical analysis.
- Define the prediction error,

$$e[n] = x[n] - \sum_{m=1}^{M} c[n]x[n-m]$$

- Coefficients $c[m]$ trained to minimize $e[n]$ in the least squares sense.



- Prediction error is the whitened data stream.
- Consists of uncorrelated noise and transients non-stationarities on time scales shorter than M.
- Choose M greater than longest signal in the search space.
- Whitening filter introduces unknown group delay.
- Construct zero-phase filter from transfer function magnitude (and increased filter order).
- Projection onto complex waveforms obeys Rayleigh statistics.

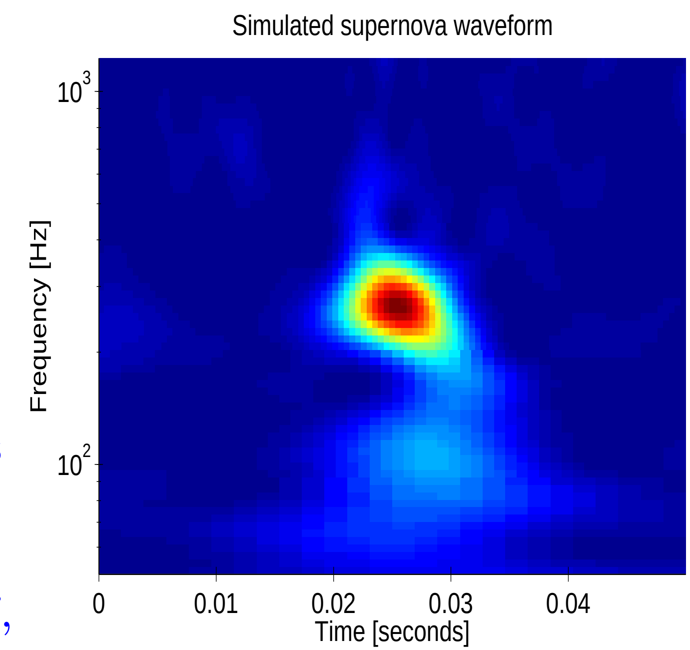
The Q transform

- Data $x(t)$ projected onto windowed complex exponentials

$$X(\tau,\phi,Q) = \int_{-\infty}^{+\infty} x(t)\, w(t-\tau,\phi,Q)\, e^{-i2\pi\phi t}\, dt,$$

- Window $w(t)$ has minimum time-frequency uncertainty and bandwidth ϕ/Q.
- Alternative frequency domain computation resembles heterodyne detector and allows efficient computation.

$$X(\tau,\phi,Q) = \int_{-\infty}^{+\infty} \tilde{x}(f+\phi)\, \tilde{w}^*(f,\phi,Q)\, e^{+i2\pi f\tau}\, df,$$



- Window normalized to recover energy $\|h\|^2$ of minimum uncertainty waveforms or power spectral density $S_n(\phi)$ of detector noise.

$$|X(\tau,\phi,Q)|^2 = \|h\|^2 + |N(\tau,\phi,Q)|^2 + 2\|h\|\,|N(\tau,\phi,Q)|\cos\theta$$

$$\|h\|^2 = \int_{-\infty}^{+\infty} |h(t)|^2\, dt \qquad \left\langle |N(\tau,\phi,Q)|^2 \right\rangle = \frac{1}{2}\int_0^{\infty} S_n(f)\, |\tilde{w}(\phi-f)|^2\, df$$

- Alternative normalization recovers energy of non-localized bursts.

$$\int_0^{\infty}\int_{-\infty}^{+\infty} |X'(\tau,\phi,Q)|^2\, d\tau\, d\phi = \|h\|^2 + \text{noise terms}$$

LIGO

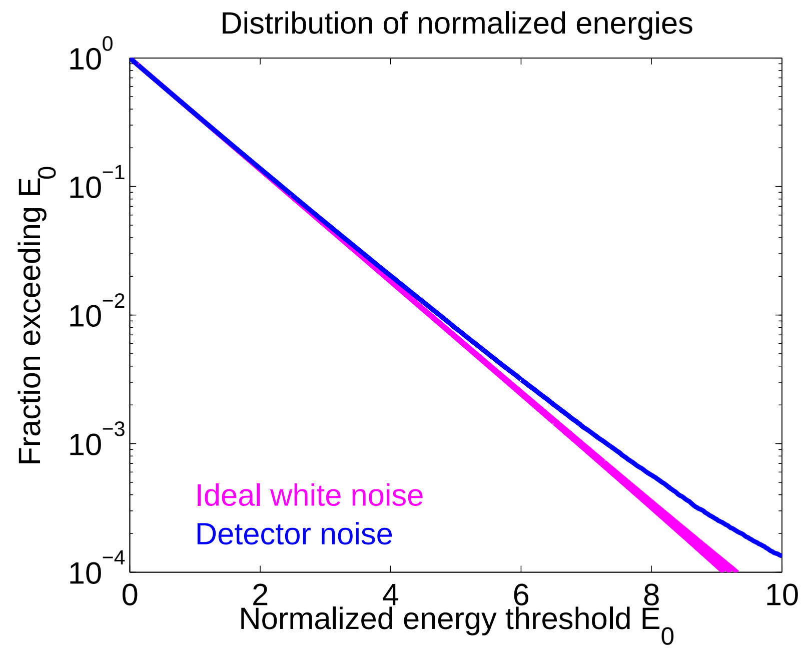
Significant events

- Identify significant events assuming white noise statistics.
- Normalized energy of Q transform coefficients

$$E = \frac{|X(\tau, \phi, Q)|^2}{\left\langle |X(\tau, \phi, Q)|^2 \right\rangle},$$

  is exponentially distributed with unity mean.
- White noise significance at energy E_0 is

$$P(E > E_0) = \exp(-E_0).$$

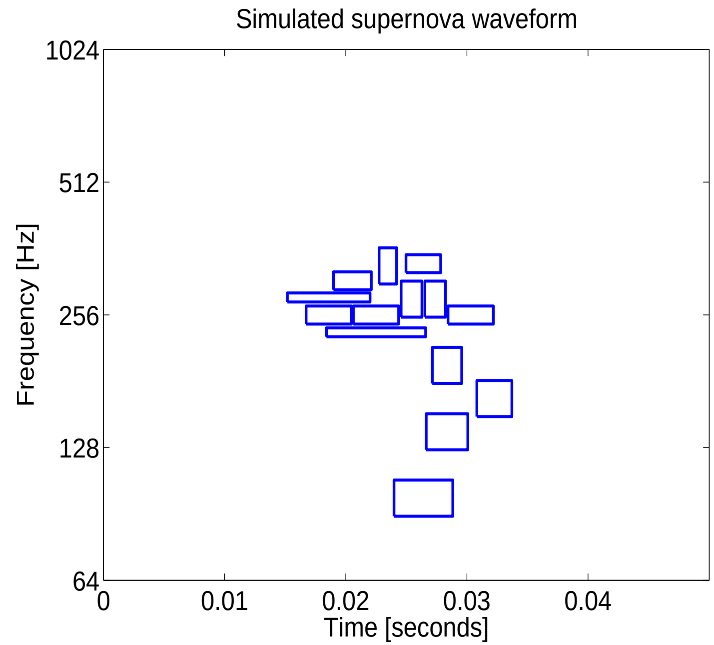


- Optimal matched filter signal to noise ratio

$$\rho = \left[\int_0^\infty \frac{4|\tilde{h}(f)|^2}{S_n(f)}\, df \right]^{1/2} \quad \text{is well estimated by} \quad \hat{\rho} = \sqrt{2(E-1)}$$

  for minimum uncertainty waveforms in white noise.
- The Q pipeline is equivalent to an optimal matched filter search for minimum uncertainty waveforms of unknown phase in the whitened data stream.
- Optimal performance predictable by Monte Carlo.
- Maximum false rate at threshold E_0 is $f_s \exp(-E_0)$.

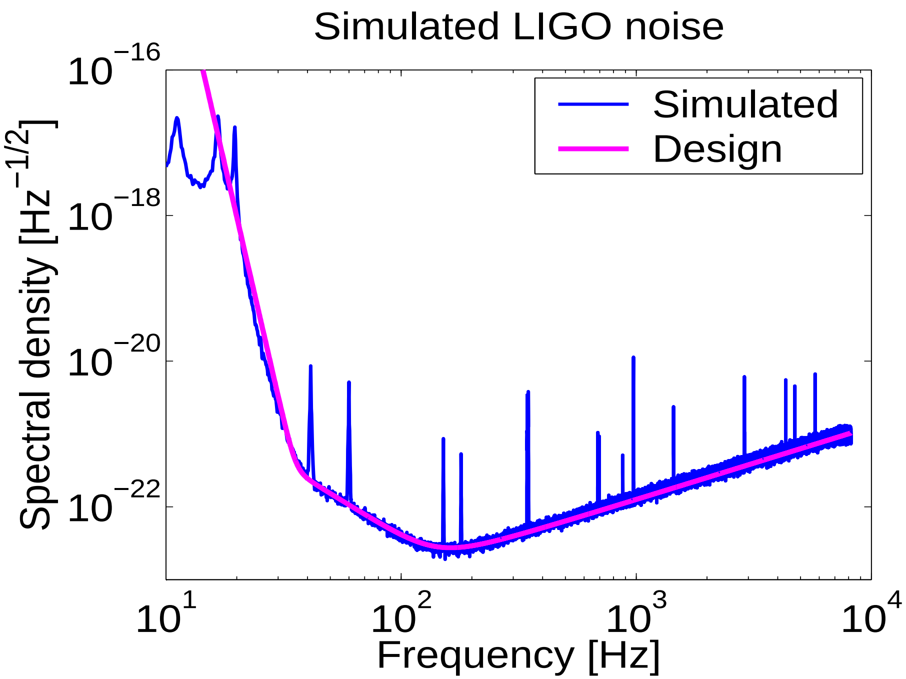
Candidate events



- Threshold on single detector significance.
- Produces multiple overlapping time-frequency tiles.
- Identify most significant non-overlapping tiles
  - Best match parameterization of minimum uncertainty bursts.
  - Resolves most significant features of arbitrary bursts.
  - Isolate signal energy in minimum number of tiles
  - Optimal signal to noise ratio for minimum uncertainty bursts.
- Test for time-frequency coincidence between detectors
  - Overlap in time (accounting for time delay between detectors)
  - Overlap in frequency
- Threshold on joint detector significance.
  - Sum of N normalized energies χ^2 distributed with 2N degrees of freedom.
- Test for consistency in $||h||$ between co-located detectors.
- Waveform consistency test (r-statistic) not yet applied.
- Veto events coincident with environmental transients.

Simulated data

- Three hours of single detector $h(t)$ noise have been simulated
- Useful for benchmarking and validating search algorithms.
- Stationary Gaussian noise shaped to initial LIGO 4km design sensitivity.
- A subset of resonant line sources are approximated.
- Spectrum inaccurate below 20 Hz.
- Non-stationarities of real detectors are not modeled.



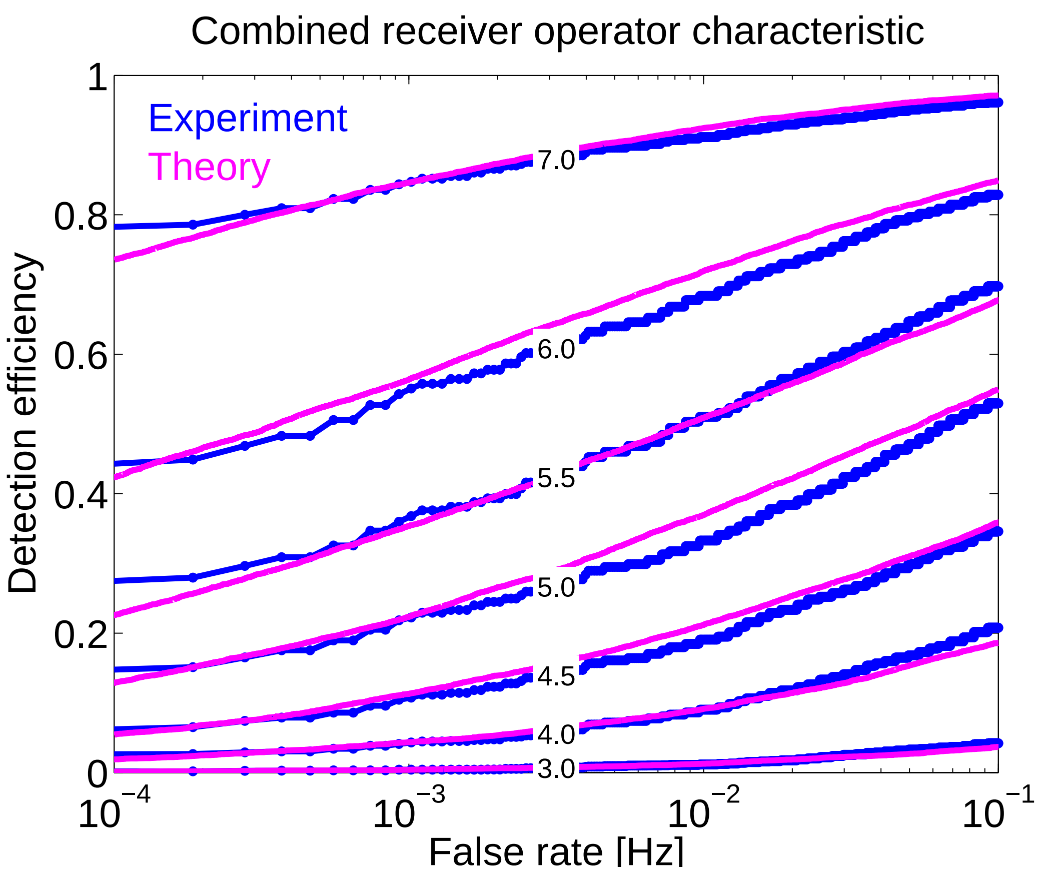
- Sine-Gaussian bursts injected at random time every 60 seconds.

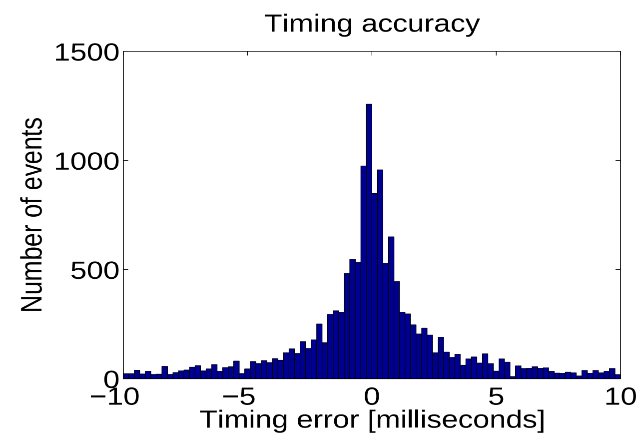
$$h(t) = \|h\| \left(\frac{32\pi f^2}{Q^2} \right)^{1/4} \exp\left[-\frac{4\pi^2 f^2 (t - t_0)^2}{Q^2} \right] \sin\left[2\pi f (t - t_0)\right],$$

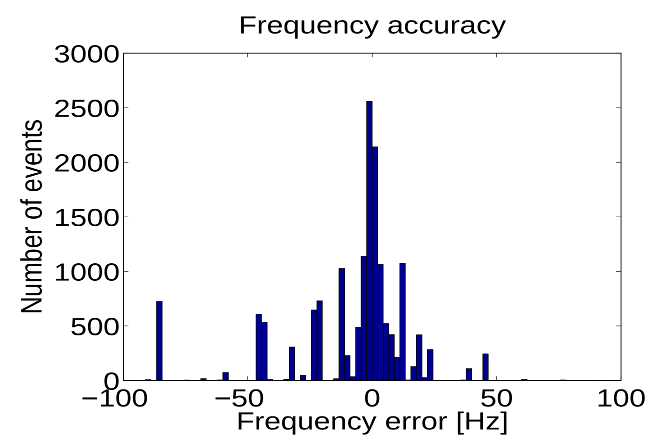
- Central frequencies of 64, 128, 256, 512, and 1024 Hz.
- Qs of 4, 8, 16, 32, and 64.
- Optimal matched filter signal to noise ratios of 3.0, 4.0, 4.5, 5.0, 5.5, 6.0, and 7.0.

Performance on simulated data

- Aggregate performance in good agreement with Monte Carlo predictions of optimal matched filter performance for minimum uncertainty waveforms of unknown phase in stationary white noise.
- Similar results for all waveforms with minor degradation at edge of search space.







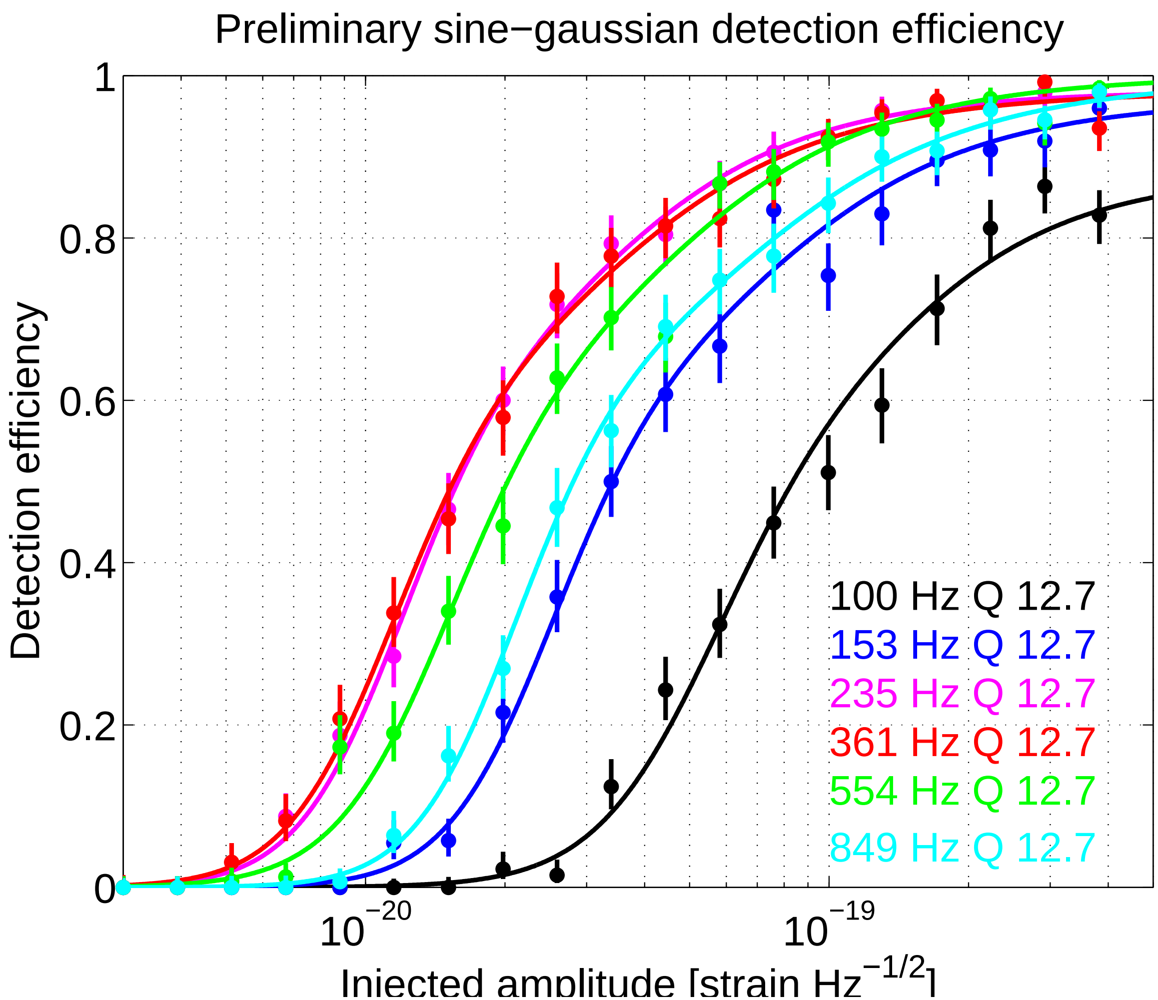
- Further information is available at http://ligo.mit.edu/∼shourov/q/validation.

LIGO S2 H1–H2 efficiency study

- A preliminary efficiency study has been performed for the H1–H2 double coincident data set from the second LIGO science run.
  - Twice the observation time of triple coincident search.
  - Co-located detectors permit $\|h\|$ consistency check.
  - Increased detection threshold necessary for similar event rate?
  - Excess foreground events due to common environment?
- Q Pipeline applied to search for bursts
  - Frequency range of 64 to 1024 Hz
  - Q range of 4 to 64
  - Worst case 20% energy loss due to mismatch
  - Normalized energies $E_{\mathrm{H1,H2}}$ greater than 20
  - Coincidence window of 5 milliseconds.
  - Joint normalized energy $E_{\mathrm{H1}} + E_{\mathrm{H2}}$ greater than 60
  - $\|h\|$ consistency within a factor of 2
  - Remove events coincident with acoustic transients ($\sim 1\%$ deadtime).
- Preliminary detection efficiencies for simulated sine-Gaussian bursts.
  - Isotropic all-sky distribution with random linear polarization.
  - Central frequencies of 100, 153, 235, 361, 554, and 849 Hz.
  - Q of 12.7 (9 according to S1 definition).

Preliminary detection efficiency

- Preliminary efficiency curves indicate comparable performance to existing triple coincident analysis.
- However, a thorough analysis of foreground and background event rates is not complete.



Summary and outlook

- We have presented a minimal analysis pipeline that is equivalent to an optimal matched filter search for minimum uncertainty waveforms of unknown phase in whitened data.
- A validation of the pipeline has been performed using simulated data that yields results consistent with theoretical expectation.
- Preliminary detection efficiencies for the S2 H1–H2 double coincident study are very promising.
- A number of future improvements are under consideration.
  - Clustering of time-frequency tiles to improve the detection of bursts which are non-localized in the time-frequency plane.
  - Evaluating performance using a larger variety of simulated waveforms.
  - Testing of candidate events for waveform consistency using the r-statistic.
  - Thresholding based on the sensitivity and performance of individual detectors.
- A number of alternative applications are under consideration.
  - Detector characterization and the identification of vetoes.
  - Parameter estimation and waveform reconstruction.
  - A targeted sky search for bursts.
- For further information, visit the Q Pipeline web page at http://ligo.mit.edu/∼shourov/q/ or contact shourov@ligo.mit.edu.