Near-field Radiative Coupling for Low Temperature Detectors

Stacy Wise, V. Quetschke,
D.B. Tanner, B.F. Whiting

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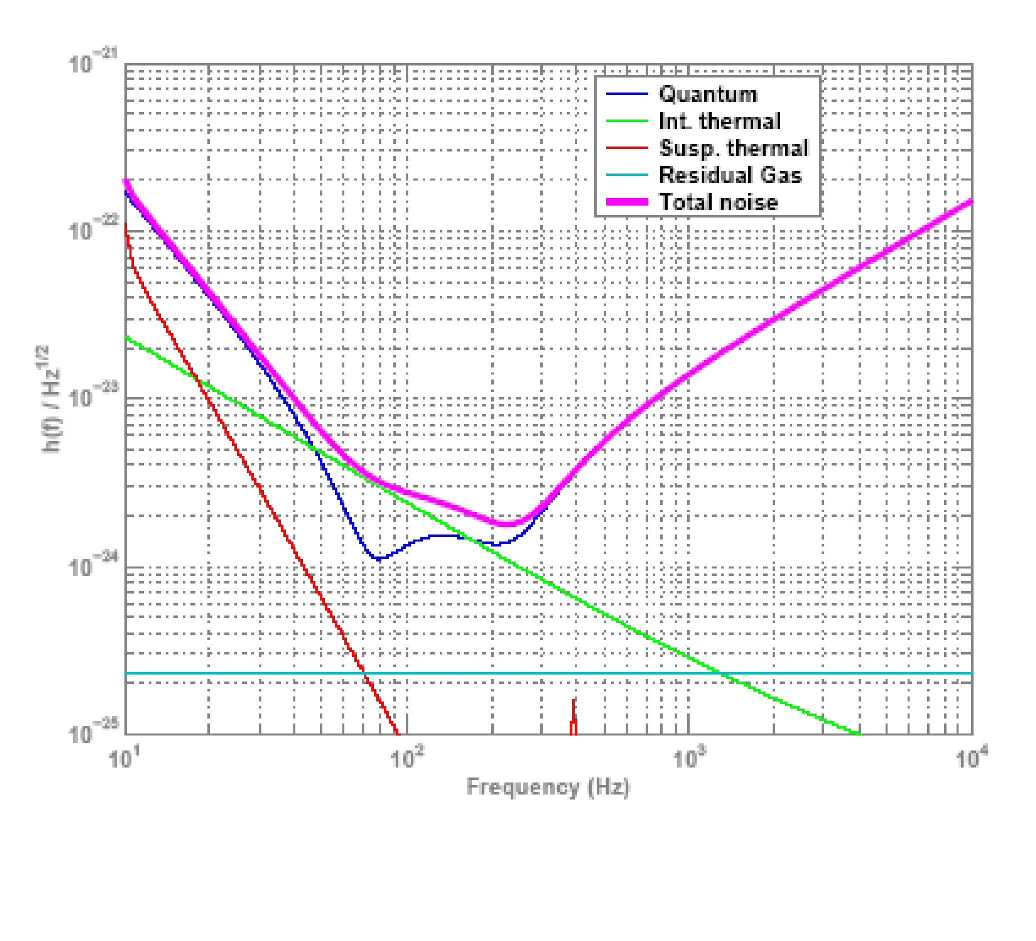
Motivation

Advanced^x LIGO test mass heat removal



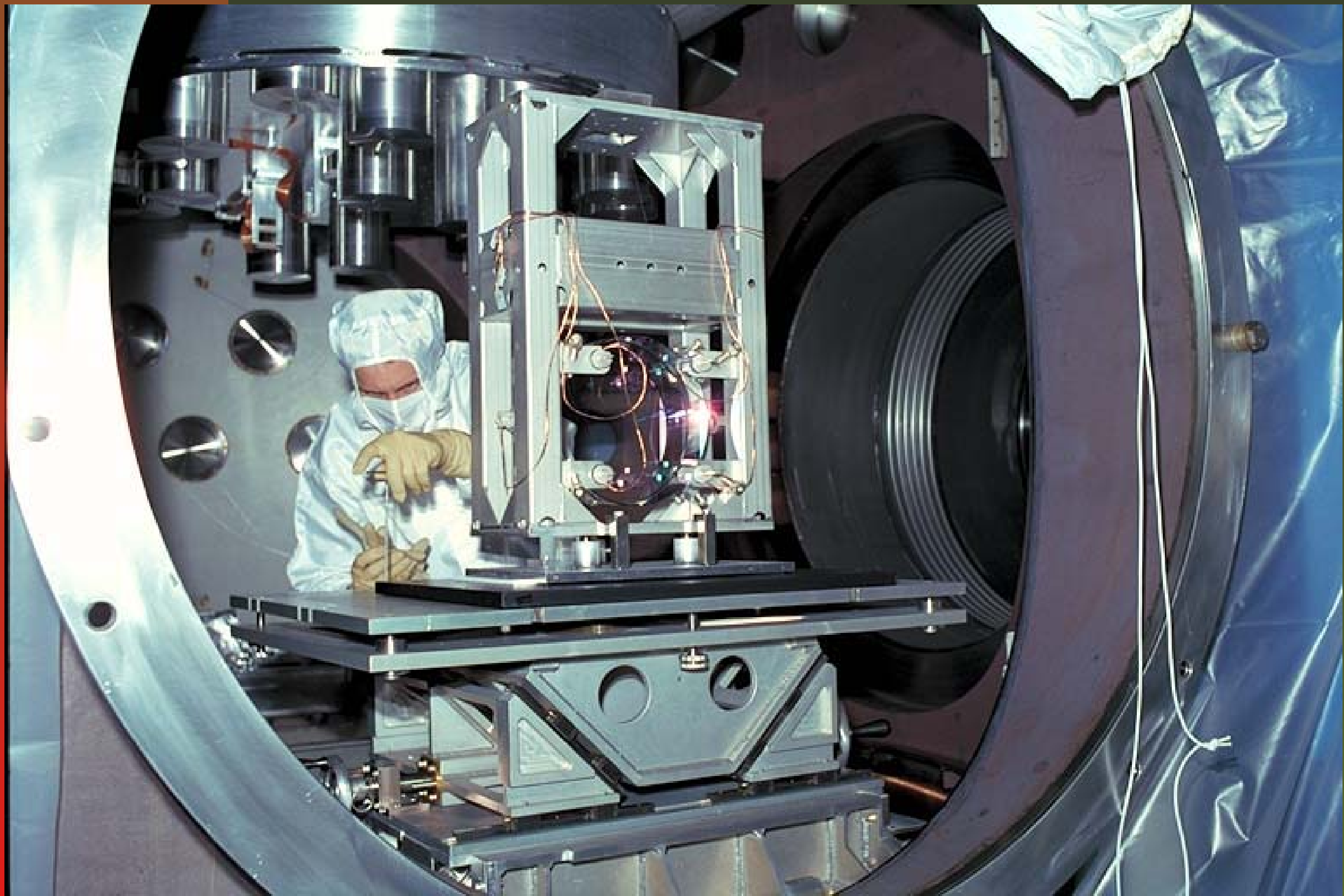
Next generation ITM's need .25 W removed. Subsequent versions of LIGO may be cryogenic.

- small $k_B T$ energy
- high quality factor
- low thermoelastic noise
- less thermal lensing (small $\frac{\partial n}{\partial T}$)

Motivation

Challenge of Low Temperature

- extract heat while maintaining isolation of test mass



BB radiation of TM

$\frac{P_{BB}}{A} = \sigma_{BB} e T^4$

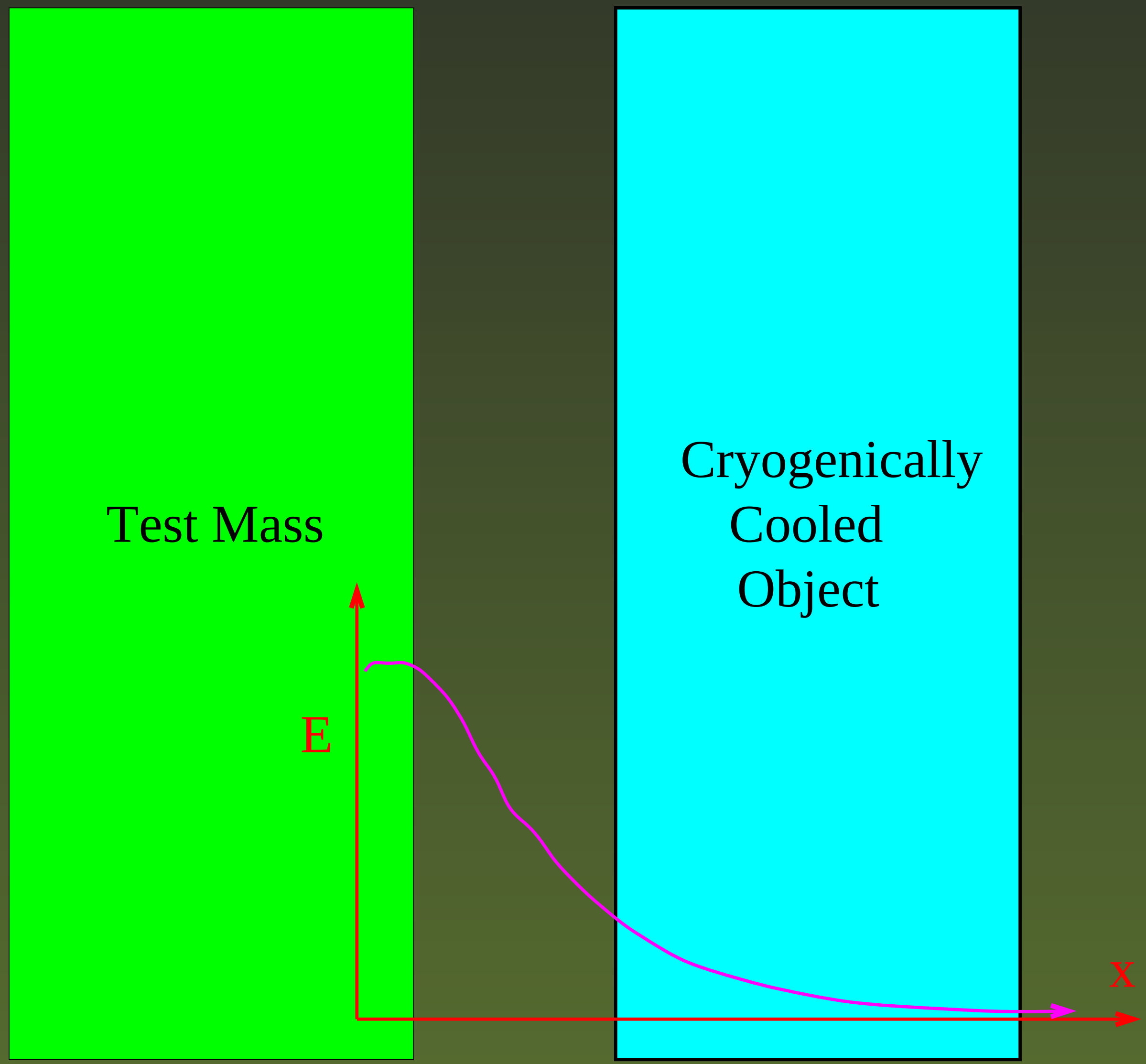
$= 130 \frac{\text{mW}}{\text{m}^2}$ for SiO_2

$=$ a mere $3 \frac{\text{mW}}{\text{m}^2}$ for Cr

SiO_2 fibers $\rightarrow \mu$W

Motivation

Photon tunneling provides heat transfer enhancement



Proximity-Enhanced Cooling

Fluctuating EM fields due to thermally-induced currents

$$\nabla \times \tilde{H} = -i\omega\epsilon_0\epsilon\left(\omega\right)\tilde{E} + \tilde{J}_{therm}$$

$$\nabla \times \tilde{E} = i\omega\mu_0\mu\tilde{B}$$

Find $\tilde{H}$ and $\tilde{E}$ in terms of $\tilde{J}_{therm}$

Assume material is nonmagnetic and isotropic

$$\epsilon(\omega) = \epsilon^{'} + i\epsilon^{''}$$

Fluctuation Dissipation Theorem

What is $\tilde{J}_{therm}$?

or rather...

What is $\langle J_a(\vec{x},\omega), J_b^*(\vec{x},\omega)\rangle$, as we will need $\left\langle \vec{S}(\vec{x}) \right\rangle$ to calculate power flux?

Fluctuation Dissipation Theorem:

$$\left\langle J_a(\vec{r},\omega), J_b^*\left(\vec{r'},\omega'\right)\right\rangle =$$

$$\frac{\omega\epsilon_0\epsilon''(\omega)}{\pi}\frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}}-1}\delta\left(\omega-\omega'\right)\delta\left(\vec{r}-\vec{r'}\right)\delta_{ab}$$

At every point, 3 orthogonal elec. dipoles uncorrelated to any other pt. (Levin et al. Sov. Phys. JETP **52**(6) 1980)

Proximity-Enhanced Cooling

$$\langle S_z(\vec{r}, T, \omega)\rangle =$$
$$\frac{\omega\Theta(\omega,T)}{16\pi^3 c}\mathrm{Re}\left[\int d^2\vec{k} e^{-2\mathrm{Im}(b1)z}\frac{b1\mathrm{Re}(b2)}{\gamma 1|b2|^2}\left(|T^s_{21}|^2 + |T^p_{21}|^2\frac{|b2|^2+k^2}{\gamma^2}\right)\right]$$

$$P(T_1, T_2) = \int_0^\infty d\omega \langle S_z(d, T_1, \omega)\rangle - \langle S_z(0, T_2, \omega)\rangle$$

where

$$\gamma_1^2 = \frac{\epsilon_1\omega^2}{c}, b_1 = \sqrt{\gamma_1^2 - k^2}, \text{ and } \Theta(\omega, T) = \frac{\hbar\omega}{exp(\frac{\hbar\omega}{k_B T})-1}$$

for $0 < k < \frac{\omega}{c} \rightarrow$ travelling waves
for $\frac{\omega}{c} < k < \infty \rightarrow$ evanescent waves

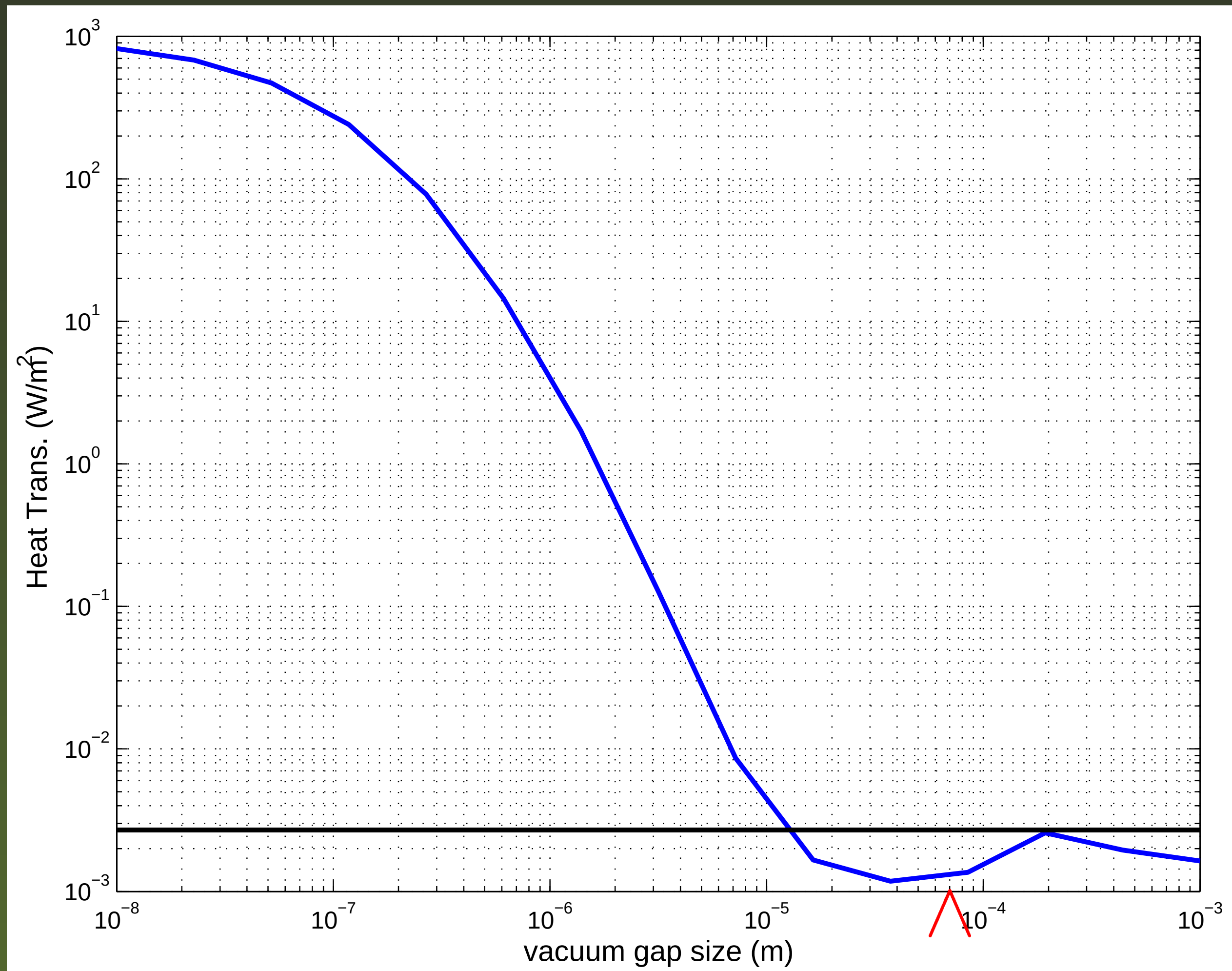
(Mulet et al. Microscale Thermophy. Eng. **6** 2002)

Advantageous Proximity

How close? Closer than dominant blackbody wavelength

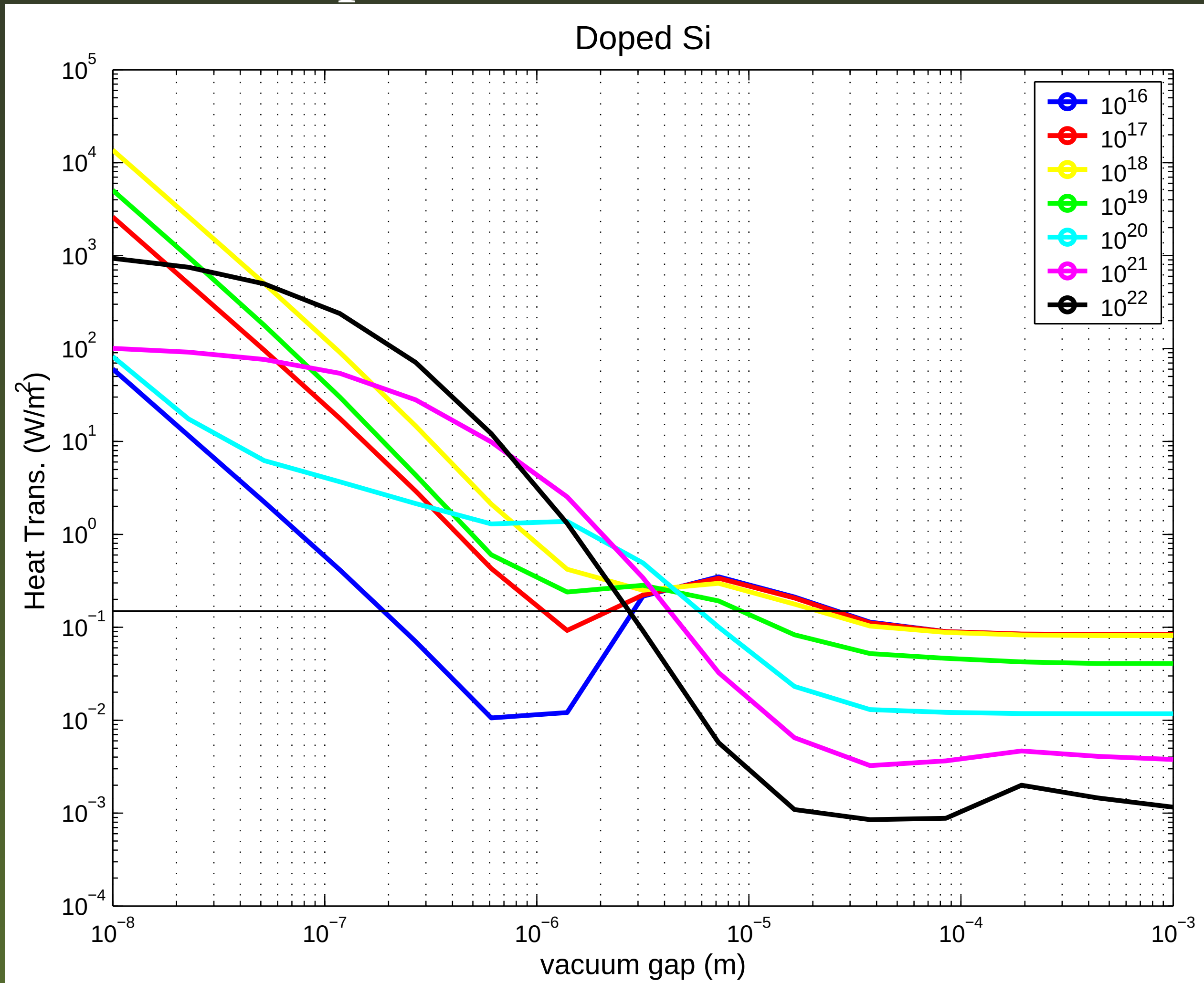
$$\lambda_{Wien} = \frac{2.9\times10^{-3}}{T}$$

$$\lambda_{T=40} = 72\mu m$$



Advantageous Proximity

Thermal coupling depends strongly on dielectric properties of material. Example: Si with various N_e



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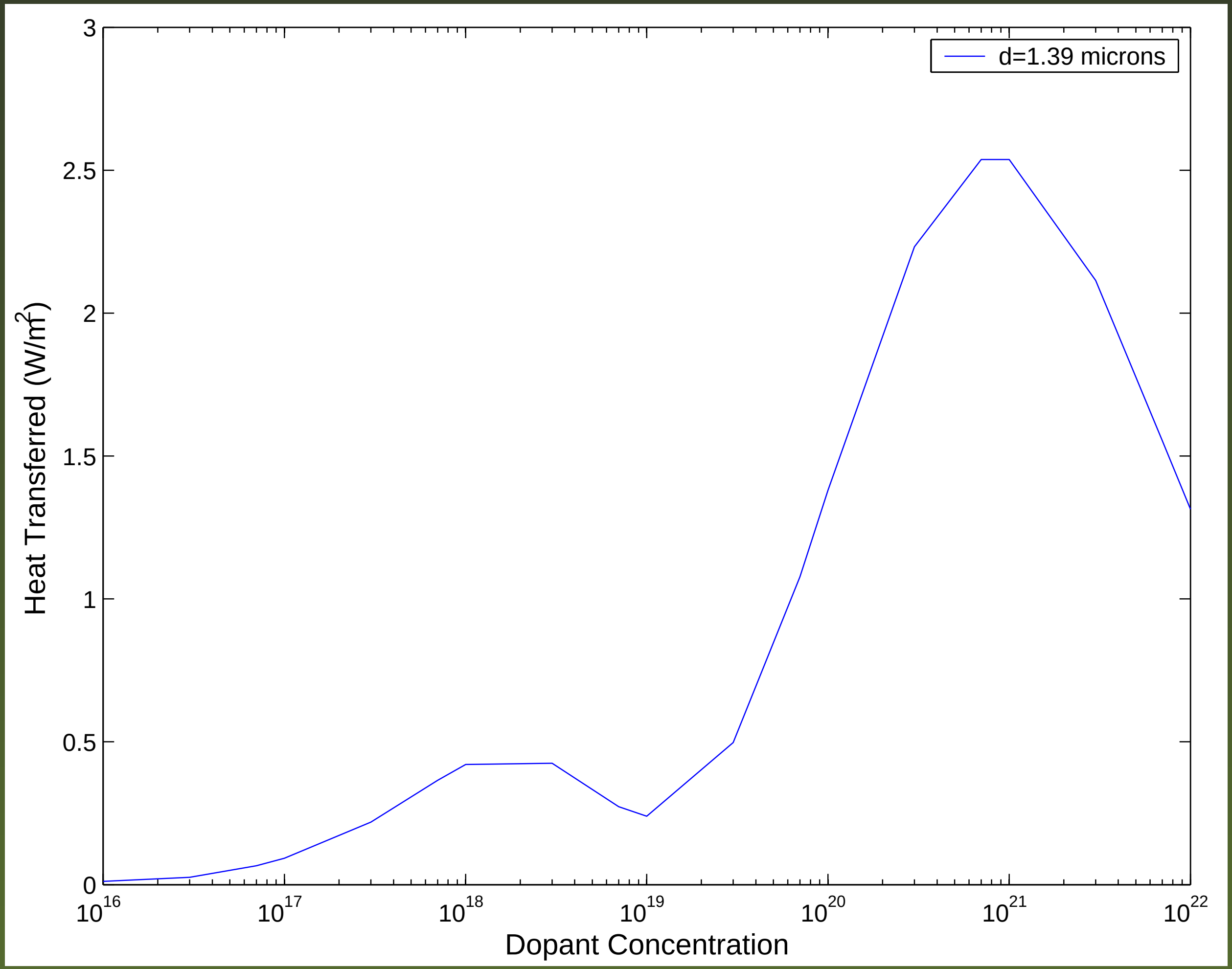
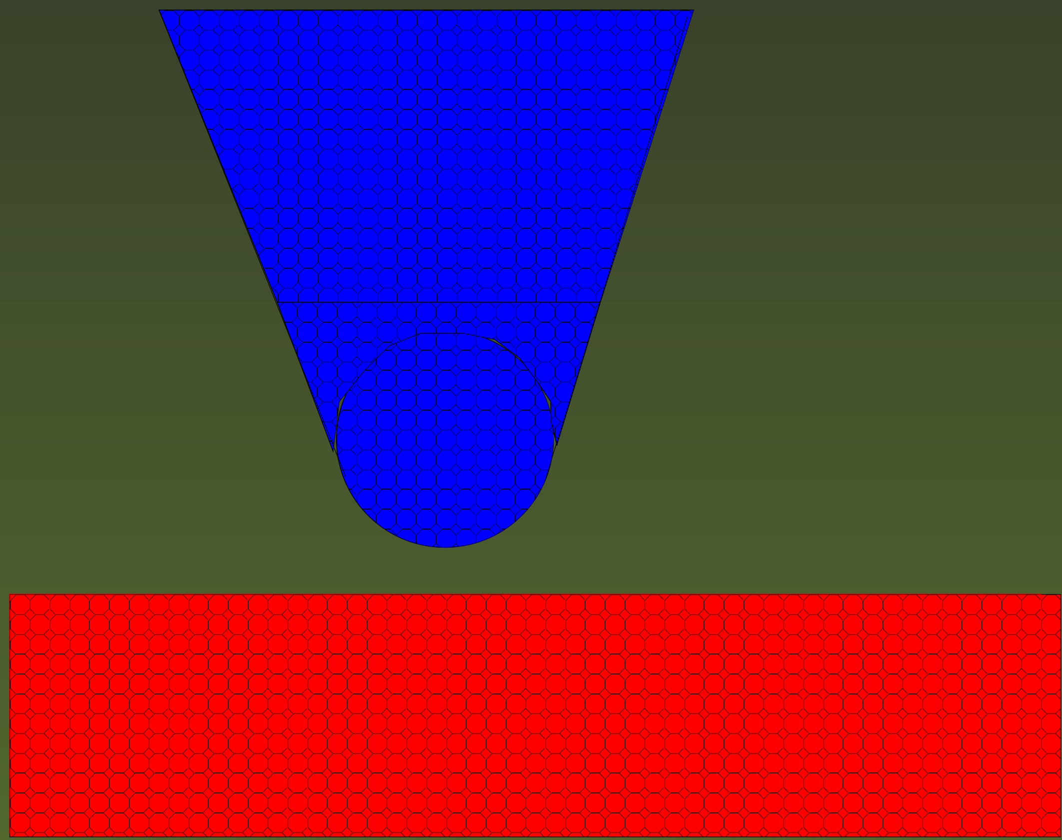


Experimental Proof

Flat-flat and flat-sphere geometries

Hargreaves 1969
Xu et al. 1994

Mueller et al. 1999



Forces due to Fluctuating Fields

Van der Waals force arises from same sources

$$F_{VdW}(d) = \frac{\hbar}{2\pi^2 c^3} \mathrm{Re} \int_0^\infty dk \int_0^\infty d\omega k^2 \omega^3 \coth(\frac{\hbar\omega}{k_B T}) \times$$
$$\left[\left(\frac{(b_1+k)(b_2+k)}{(b_1-k)(b_2-k)} e^{\frac{2ik\omega d}{c}} - 1 \right)^{-1} + \left(\frac{(b_1\epsilon 1+k)(b_2\epsilon 2+k)}{(b_1\epsilon 1-k)(b_2\epsilon 2-k)} e^{\frac{2ik\omega d}{c}} - 1 \right)^{-1} \right]$$

- between planar objects $F_{VdW}(d) = \frac{Aa}{6\pi d^3}$ for small separations

Forces due to Fluctuating Fields

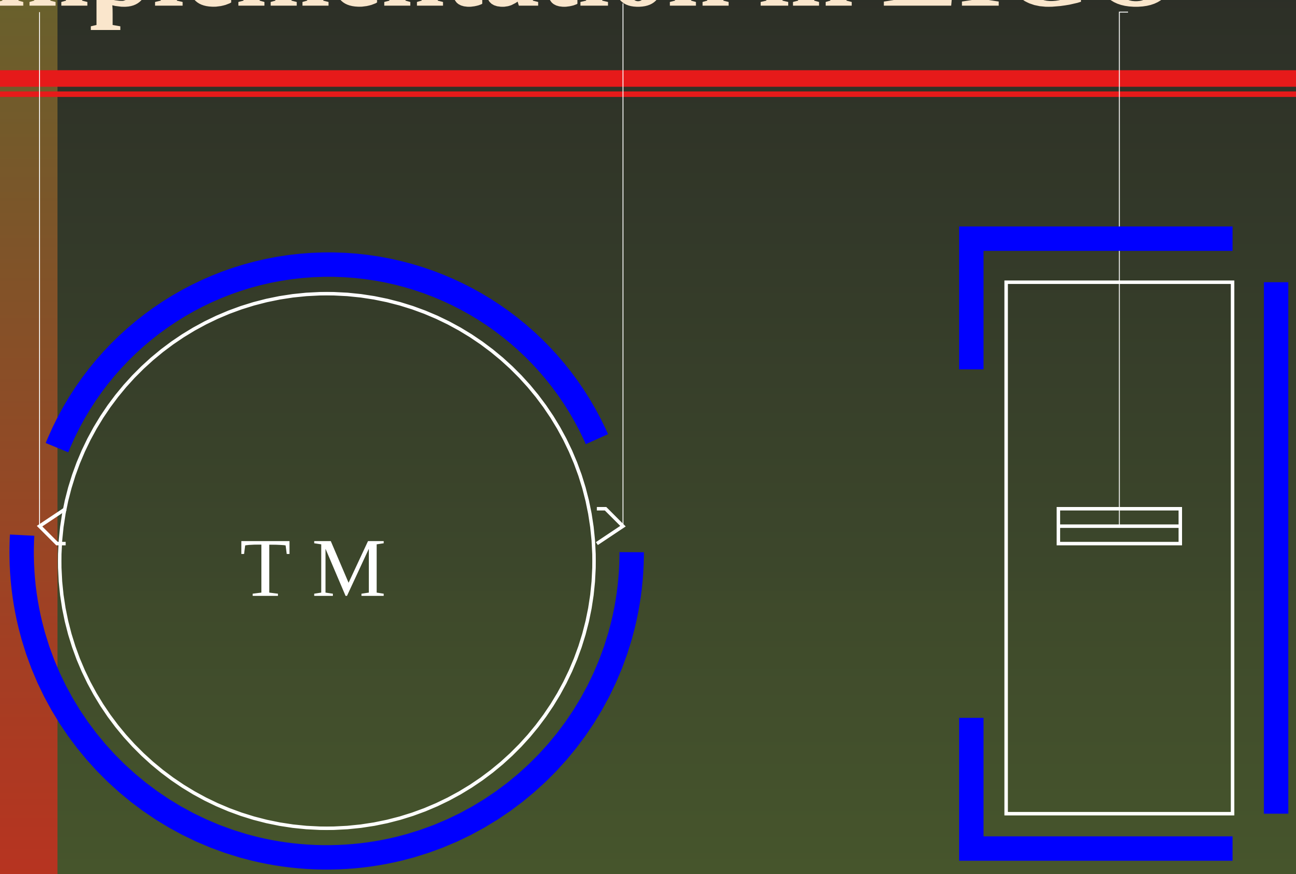
$$F_{VdW}(t) = \frac{Aa}{6\pi d^3} \text{ and } F_{Cas}(t) = \frac{\hbar\pi^2 ca}{240 d^4}$$

where A = Hamaker constant, a = area, d = separation

With Adv. LIGO requirement of $10^{-19}\frac{\text{m}}{\sqrt{\text{Hz}}}$ at 10 Hz
$\Rightarrow$ cold Si mass at $d = 2.5\mu$m (to extract 0.5W) must be stable to $\approx 1 \times 10^{-16}\frac{\text{m}}{\sqrt{\text{Hz}}}$.
Stability requirement rather independent of material

- Casimir forces between metallic objects also give 10^{-16} requirement

Implementation in LIGO



depends upon configuration:

reflective or transmissive optics and materials

To-Do List

1. Theoretical studies of variety of materials- metallic and non-
2. determine effect of dielectric layers on heat trans.
3. with optimum configuration, determine forces
4. experiments to confirm predictions
5. evaluate pertinence to GWD

Heat Transmission through Fibers

A fiber of diameter d, length L connecting heat reservoirs at T_1 and T_2 conducts:

$$P = \frac{\pi \kappa d^2}{4} \frac{T_1 - T_2}{L}$$

With silica fibers $\left(1.4\frac{W}{mK}\right)$, d=0.1 mm, L=0.2m, T1=40K and T2=10K,

$$P = 1.65 \mu W$$