To the practical design of the optical lever intracavity topology of gravitational-wave detectors

S.L.Danilishin, F.Ya.Khalili

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Why we need intracavity topologies?

"Practical" version of the optical lever

The local meter

Conclusion

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Suppose that we have managed to overcome the SQL.

What are the next limitations?

Pumping power

$$\frac{L^2 S_h}{4} \times S_{\text{B.A.}} = \frac{\hbar^2}{4}\,,$$

$$S_{\text{B.A.}} = \frac{8\hbar\omega_p W}{\zeta^2 c L}\,\frac{\gamma}{\gamma^2 + \Omega^2}\,,$$

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$$\boxed{\xi^2 \equiv \frac{S_h}{S_h^{\rm SQL}} = \frac{\zeta^2}{2}\,\frac{W_{\rm SQL}}{W} \quad (\text{optimization: } \gamma=\Omega)}\,,$$

where

$$S_h^{\rm SQL} = \frac{4\hbar}{M\Omega^2L^2}\,,\quad W_{\rm SQL} = \frac{McL\Omega^3}{8\omega_o}\,,\quad \zeta = e^{-R}\,.$$

Optical losses

$$\xi^2_{\rm loss} = \sqrt{\zeta^2 \frac{\gamma_{\rm loss}}{\gamma_{\rm load} \approx \gamma}}\,,$$

$$\text{the half-bandwidth } \gamma = \gamma_{\rm load} + \gamma_{\rm loss}\,.$$

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Smaller ξ s require $\gamma > \Omega$, hence:

$$\boxed{\xi_{\rm sum} = \sqrt{\frac{3}{2}}\,\zeta^{2/3}\left(\frac{\gamma_{\rm loss}}{2\Omega}\,\frac{W_{\rm SQL}}{W}\right)^{1/6} \approx 0.35\,\zeta^{2/3}\left(\frac{W_{\rm SQL}}{W}\right)^{1/6}.}$$

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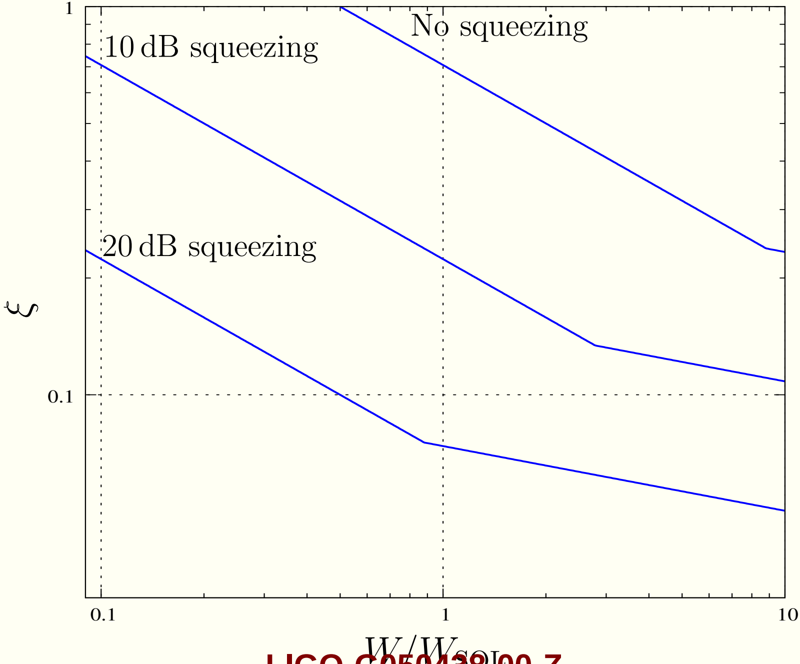
No squeezing
10 dB squeezing
20 dB squeezing
1
0.1
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ξ
W/W_{SQL}

Idea of the intracavity readout

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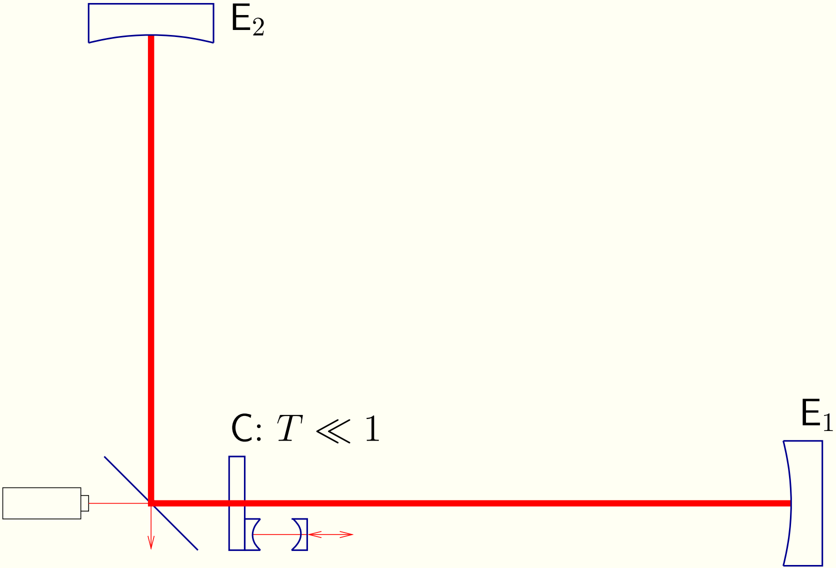
V.B.Braginsky, F.Ya.Khalili, Phys.Lett.A **218**, 167 (1996).
V.B.Braginsky, M.L.Gorodetsky, F.Ya.Khalili, Phys.Lett.A **232**, 340 (1997).

Optical bars



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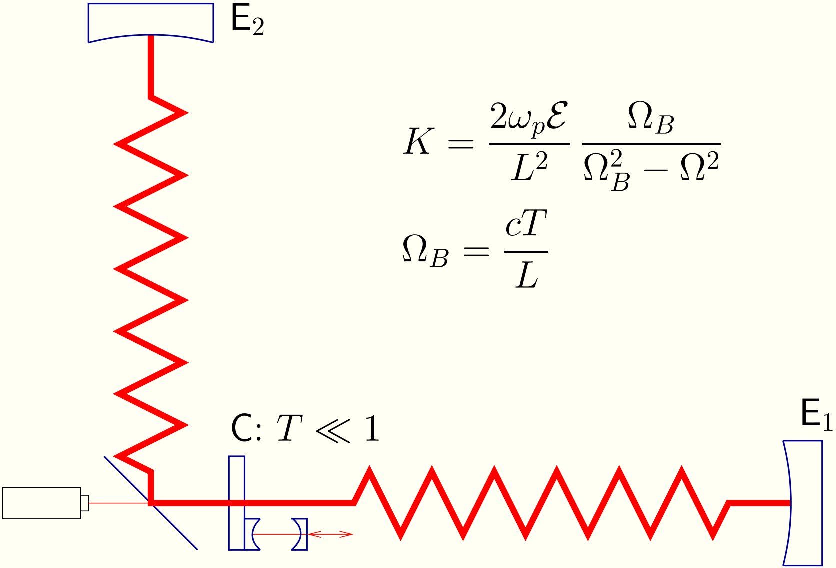
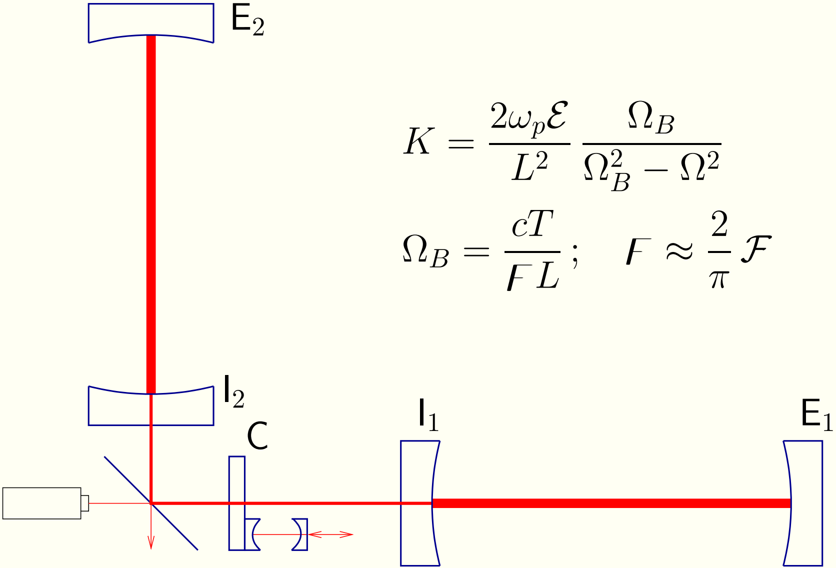


$$K = \frac{2\omega_p \mathcal{E}}{L^2} \frac{\Omega_B}{\Omega_B^2 - \Omega^2}$$

$$\Omega_B = \frac{cT}{L}$$

Optical lever



$$K = \frac{2\omega_p \mathcal{E}}{L^2} \frac{\Omega_B}{\Omega_B^2 - \Omega^2}$$

$$\Omega_B = \frac{cT}{FL}\,; \quad F \approx \frac{2}{\pi}\mathcal{F}$$

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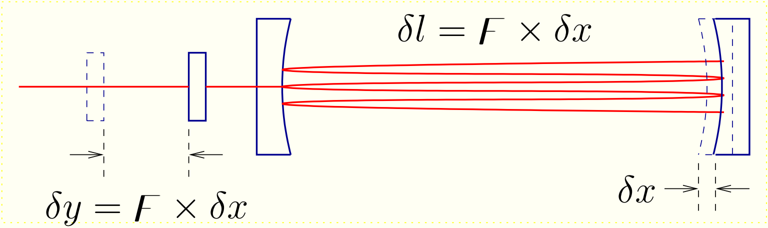
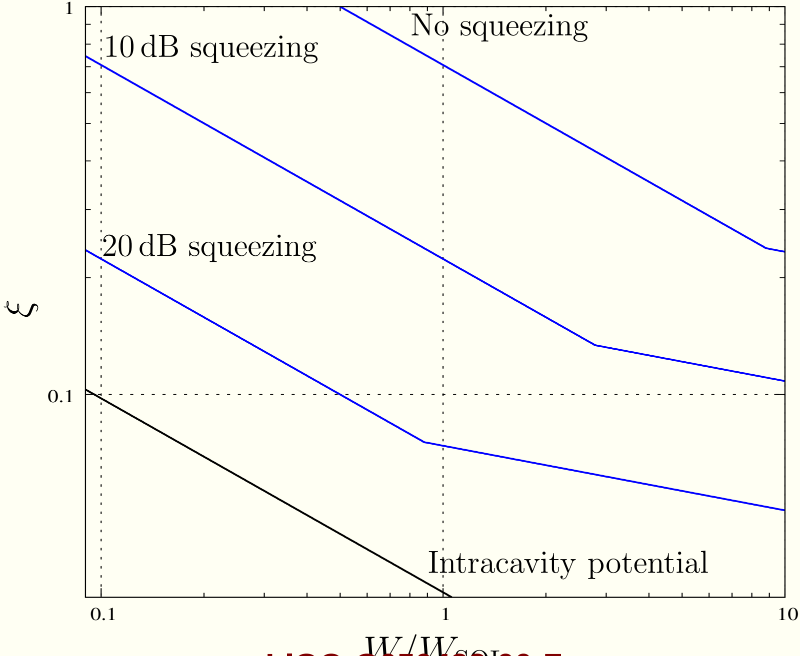


$$\frac{\delta y}{\delta x} = \mathcal{F}\,, \qquad \frac{F_{\mathrm{pond}\,y}}{F_{\mathrm{pond}\,x}} = \frac{1}{\mathcal{F}}$$

(like an ordinary mechanical lever).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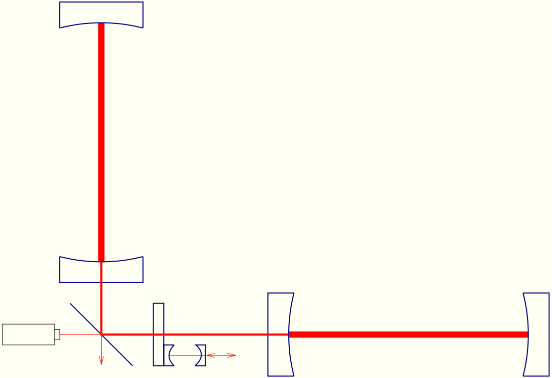

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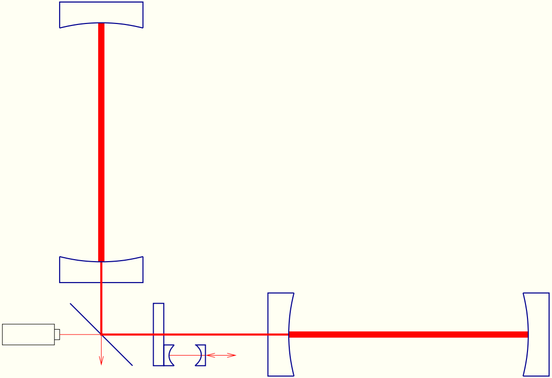
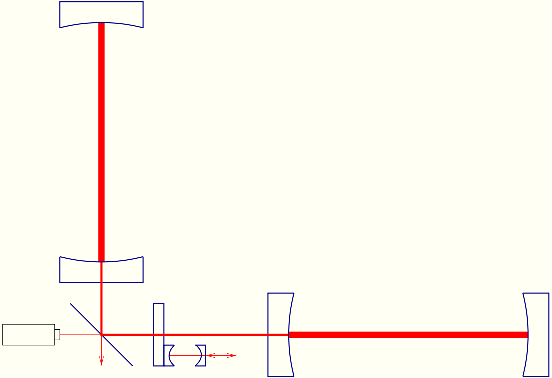


The main disadvantage: non-symmetry
$\Rightarrow$ vulnerability to the pumping power fluctuations.



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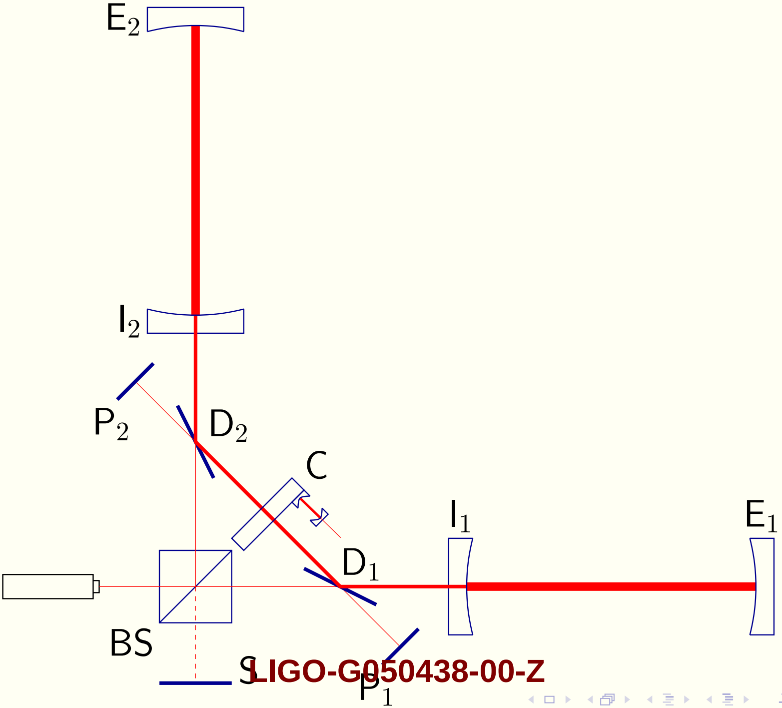
Additional disadvantages:
only half of the pumping power enters inside;
input port $\Rightarrow$ additional “hole” for noise.



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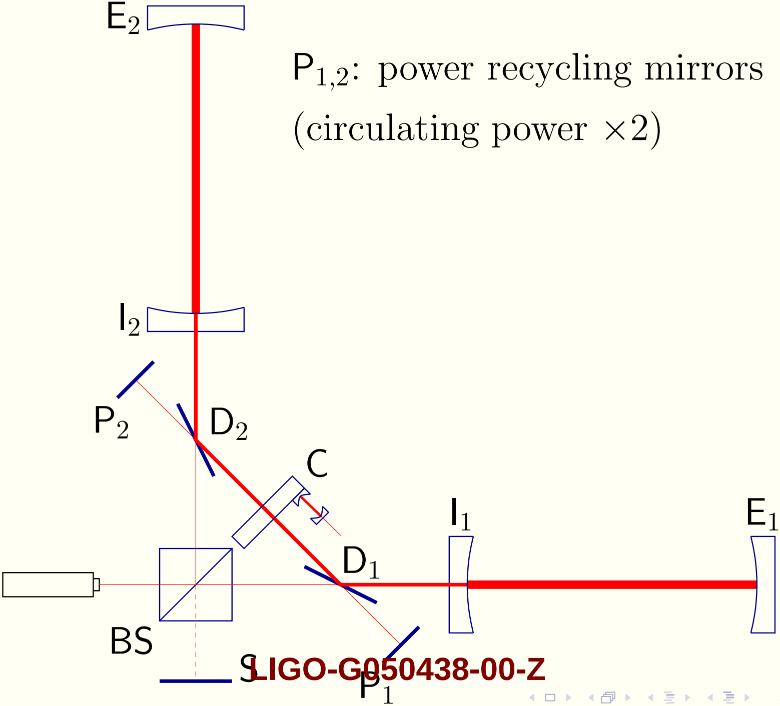
$P_{1,2}$: power recycling mirrors

(circulating power $\times 2$)



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$P_{1,2}$: power recycling mirrors

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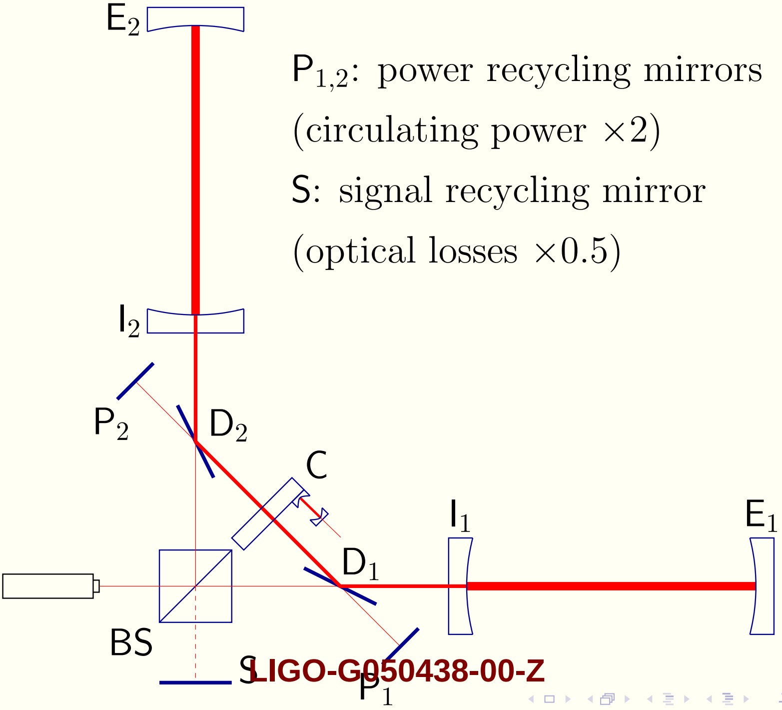
S: signal recycling mirror

(optical losses $\times 0.5$)



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- ???

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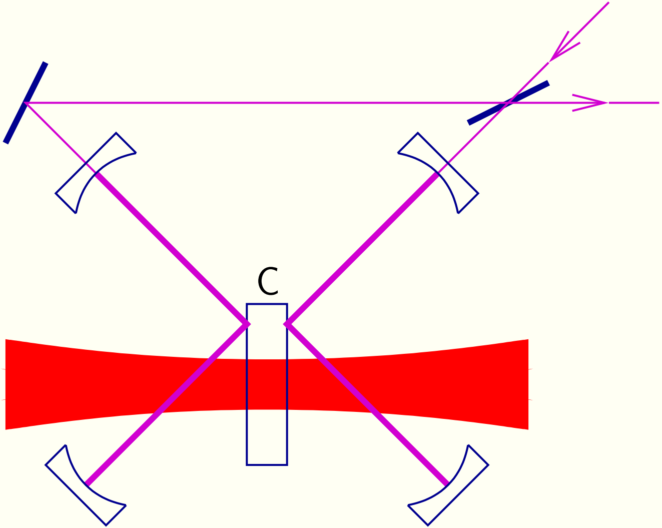
Possible design of the optical local meter



Thomas Corbitt $et\ al$, “Ponderomotive Squeezing”,

LIGO Document G040147-00 (2004)

Parameters used for the estimates

Local mirror mass	$M_{\mathsf{C}} = 1\,\mathrm{g}$
Circulating power	$w = 10\,\mathrm{kW}$
Optical losses	$A^2_{\mathrm{local}} = 5 \times 10^{-6}$

Thomas Corbitt *et al*, "Ponderomotive Squeezing",
LIGO Document G040147-00 (2004)

Spectral variation measurement

Small-scale version of the KLMTV topology.

Rather gedanken than realistic scheme, but provides a good comparison point for other schemes.

H.J.Kimble *et al*, Physical Review D **65**, 022002 (2002)

Spectral variation measurement

$$\xi^2_{\mathrm{meter}} \approx \frac{1}{2\mathsf{F}^2}\,\frac{M}{M_{\mathsf{C}}}\,\frac{w_{\mathrm{SQL}}}{w}\,,$$

$$W \geqslant \frac{\mathsf{F}^2}{8}\frac{M_{\mathsf{C}}}{M}\,W_{\mathrm{SQL}}\,,$$

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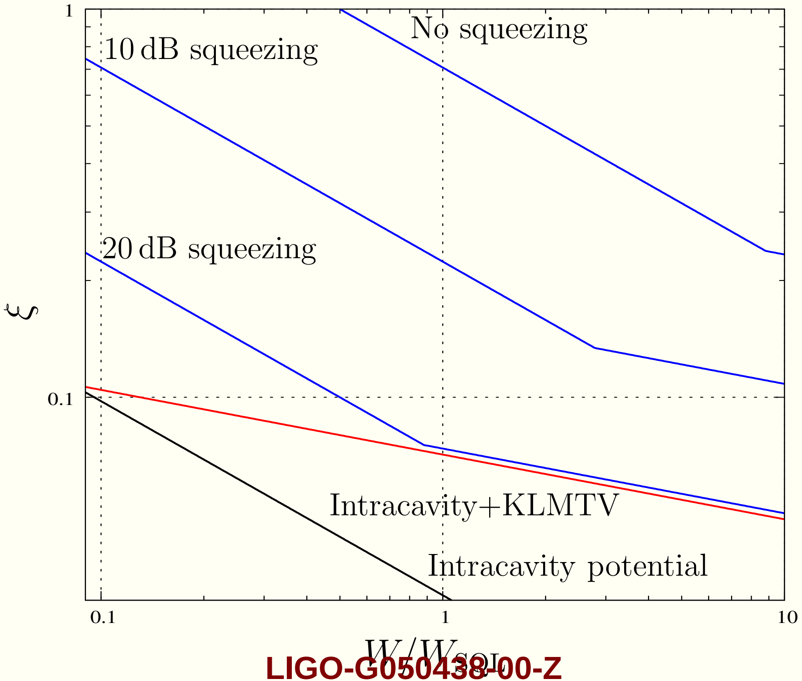
$$\Rightarrow \quad \boxed{\xi^2_{\rm meter\,loss} = \frac{\xi^2_0}{2}\left(\frac{W_{\rm SQL}}{W}\right)^{1/3}}\,,$$

$$\text{where}\quad \xi^2_0 = \frac{3}{2}\left(\frac{M_{\mathsf{C}}c^2A^2_{\rm local}\Omega^2}{32\omega_e w}\right)^{1/3} \approx 0.1\,.$$



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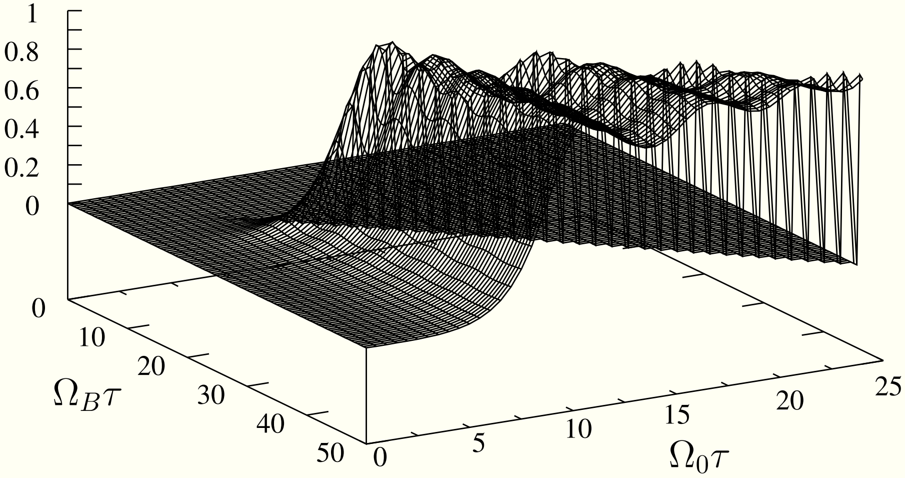
Discrete sampling variation measurement

$$\xi^2_{\mathrm{DSVM}} = \frac{720}{\pi^4} \frac{1}{\mathcal{G}(\Omega_B\tau, \Omega_0\tau)} \frac{1}{\mathsf{F}^2} \frac{M}{M_{\mathrm{C}}} \frac{w_{\mathrm{SQL}}}{w},$$

$$W \gtrsim 60\mathsf{F}^2 \frac{M_{\mathsf{C}}}{M} W_{\mathrm{SQL}}$$

(Yanbei Chen has the explanation for these weird numeric factors)

Plot of $\mathcal{G}(\Omega_B\tau, \Omega_0\tau)$



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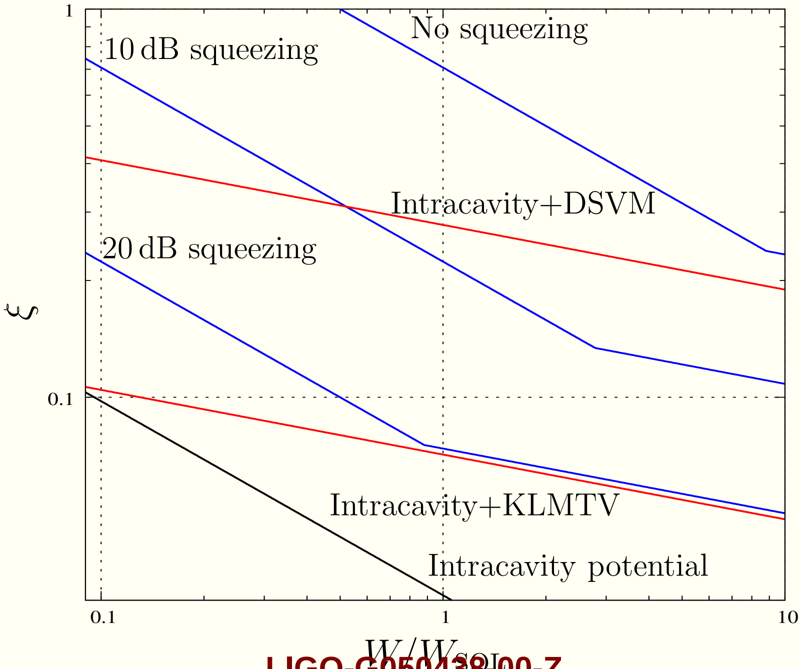

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2. Both kinds of the "supporting" devices require approximately the same level of the experimental technology.
3. However, intracavity topologies promise better sensitivity, especially if the pumping power $W < W_{\mathrm{SQL}}$.
4. Unfortunately, none of the mechanical QND schemes known today can fully realize potential sensitivity of intracavity topologies.