Stochastic Background Search with ALLEGRO and LIGO Science Data

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on behalf of the LIGO Scientific Collaboration

10th Gravitational Wave Data Analysis Workshop
2005 December 15

LIGO-G050633-00-Z

Outline

I Background/Motivation for LLO-ALLEGRO Search

- LLO-ALLEGRO Pair (proximity, overlap modulation)
- Technical Considerations (sampling, heterodyning)

II S4 Data Analysis Status

- Data Volume by Orientation; Expected Sensitivity
- Software Injections
- Hardware Injections; Time-Shift Analysis

Sensitivity to Stochastic GW Backgrounds

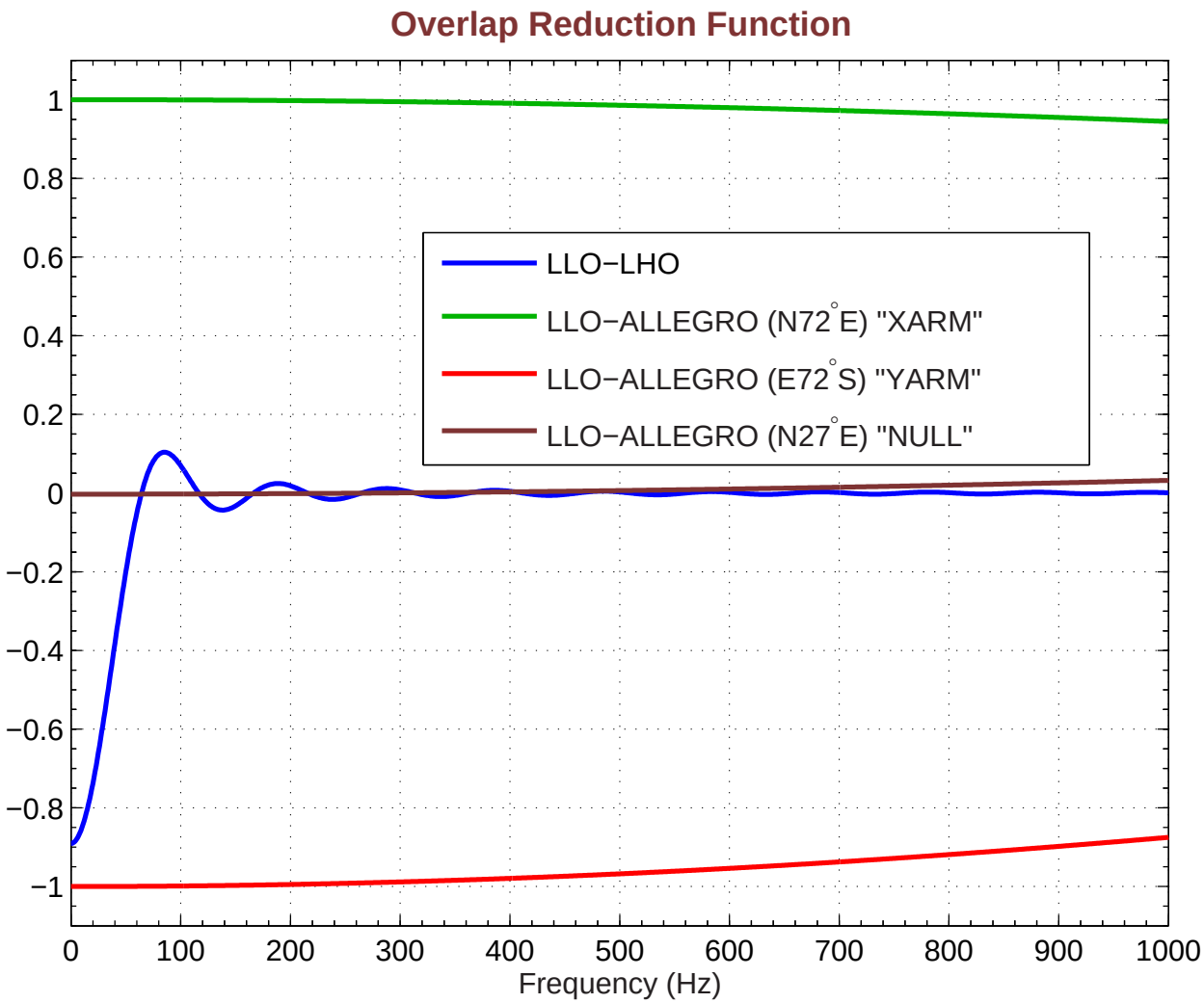
- Optimally filtered CC statistic

$$Y = \int df \underbrace{\tilde{s}_1^*(f)\,\tilde{Q}(f)\,\tilde{s}_2(f)}_{Y(f)}$$

- Optimal filter $\tilde{Q}(f) \propto \frac{S_{\rm gw}(f)\gamma_{12}(f)}{P_1(f)P_2(f)}$
  (Initial analyses assume $S_{\rm gw}(f)$ or $\Omega_{\rm gw}(f) \propto f^3 S_{\rm gw}(f)$ constant across band)
- Optimally filtered cross-correlation method has $\Omega_{\rm gw}$ sensitivity

$$\sigma_\Omega \propto \left(T \int \frac{df}{f^6} \frac{\gamma_{12}^2(f)}{P_1(f)P_2(f)} \right)^{-1/2}$$

- Significant contributions when
  - detector noise power spectra $P_1(f)$, $P_2(f)$ small
  - overlap reduction function $\gamma_{12}(f)$ (geom correction) near ± 1



LLO-ALLEGRO only ∼40 km apart ⟶ still sensitive @ 900 Hz

Response different for XARM, YARM, NULL orientations

ALLEGRO ran in all 3 orientations during LIGO S4 Run (2005 Feb 22-Mar 23)

LLO-ALLEGRO: Technical Considerations

- LIGO data digitally downsampled 16384 Hz $\rightarrow$ 4096 Hz
  ALLEGRO data heterodyned at 904 Hz & sampled at 250 Hz

- Heterodyning means CC stat complex:

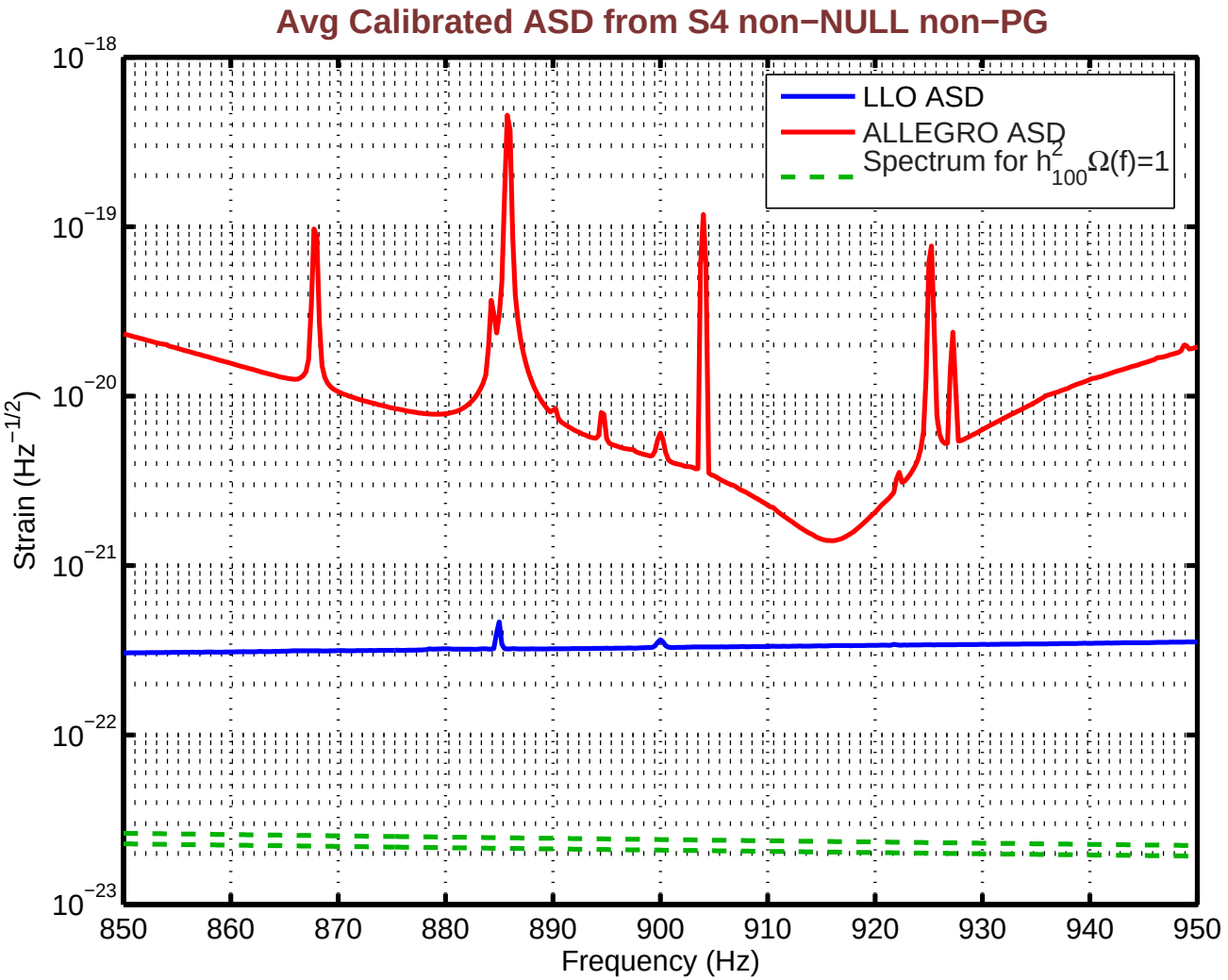
$$Y = \int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} df\, \tilde{s}_1^*(f)\, \tilde{Q}(f)\, \tilde{s}_2(f)$$

  real part Gaussian-distributed about SGWB strength;
  imag part Gaussian-distributed about 0.

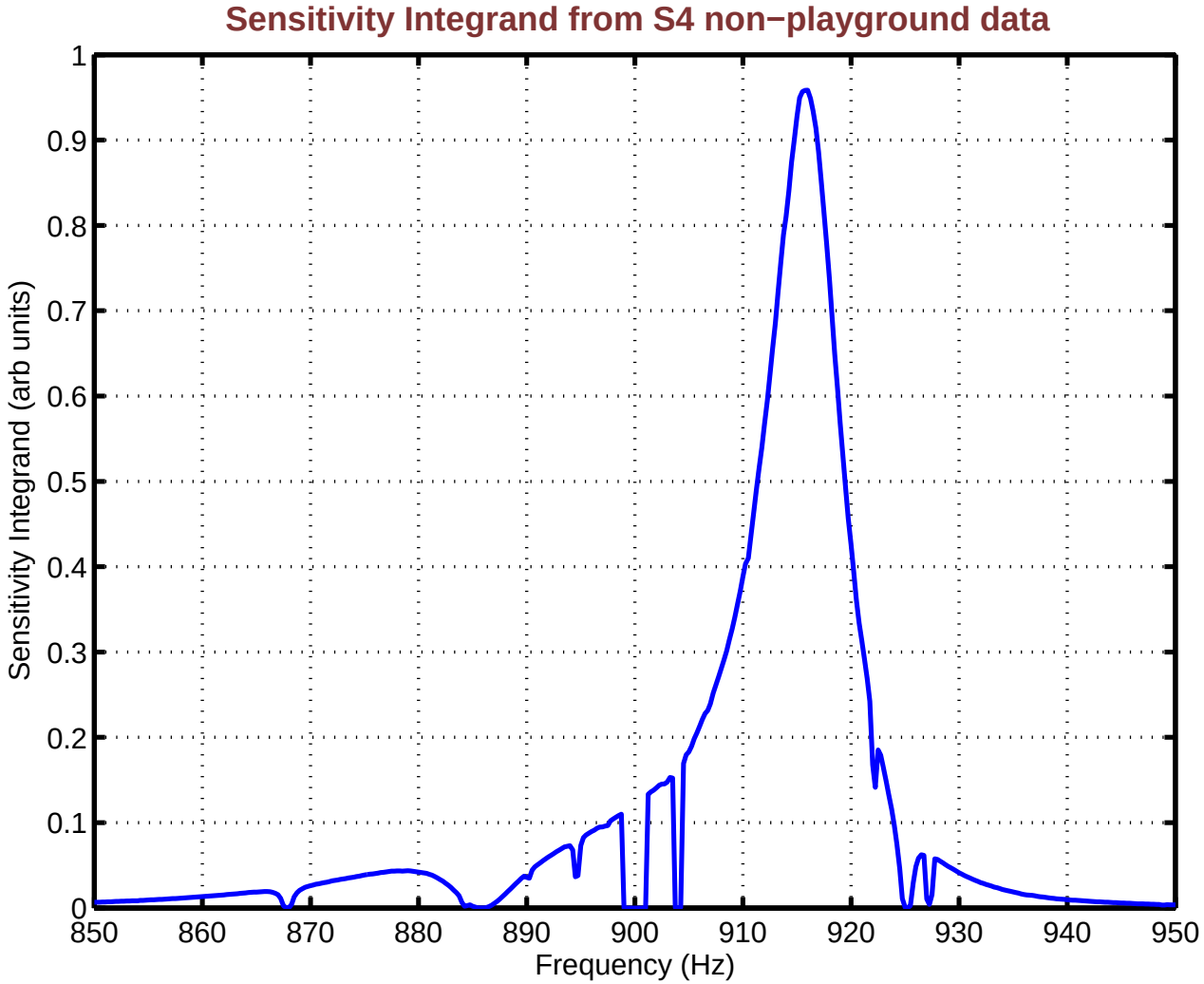
LLO-ALLEGRO data from LIGO S4 Run

- $\sim 10\%$ of data set aside as "playground";
  Non-PG data divided into 60s segments; 3 orientations:
  - "NULL" ($0.023 < \gamma(f) < 0.029$): ≈ 5200 min after cuts
    "off-source" data useful for data quality & cross-checks
  - "YARM" ($-0.89 > \gamma(f) > -0.91$): ≈ 6600 min after cuts
  - "XARM" ($0.95 < \gamma(f) < 0.96$):] ≈ 10400 min after cuts

- Projected $h_{100}^2\Omega$ sensitivity using non-playground data: ~ 0.3–1;
  Corresponds to $\sqrt{S_{\rm gw}(f = 915\,{\rm Hz})}$ of ~ 1–$2 \times 10^{-23}\,{\rm Hz}^{-1/2}$



Frequency band determined by ALLEGRO noise curve



Most of sensitivity from 905–925 Hz

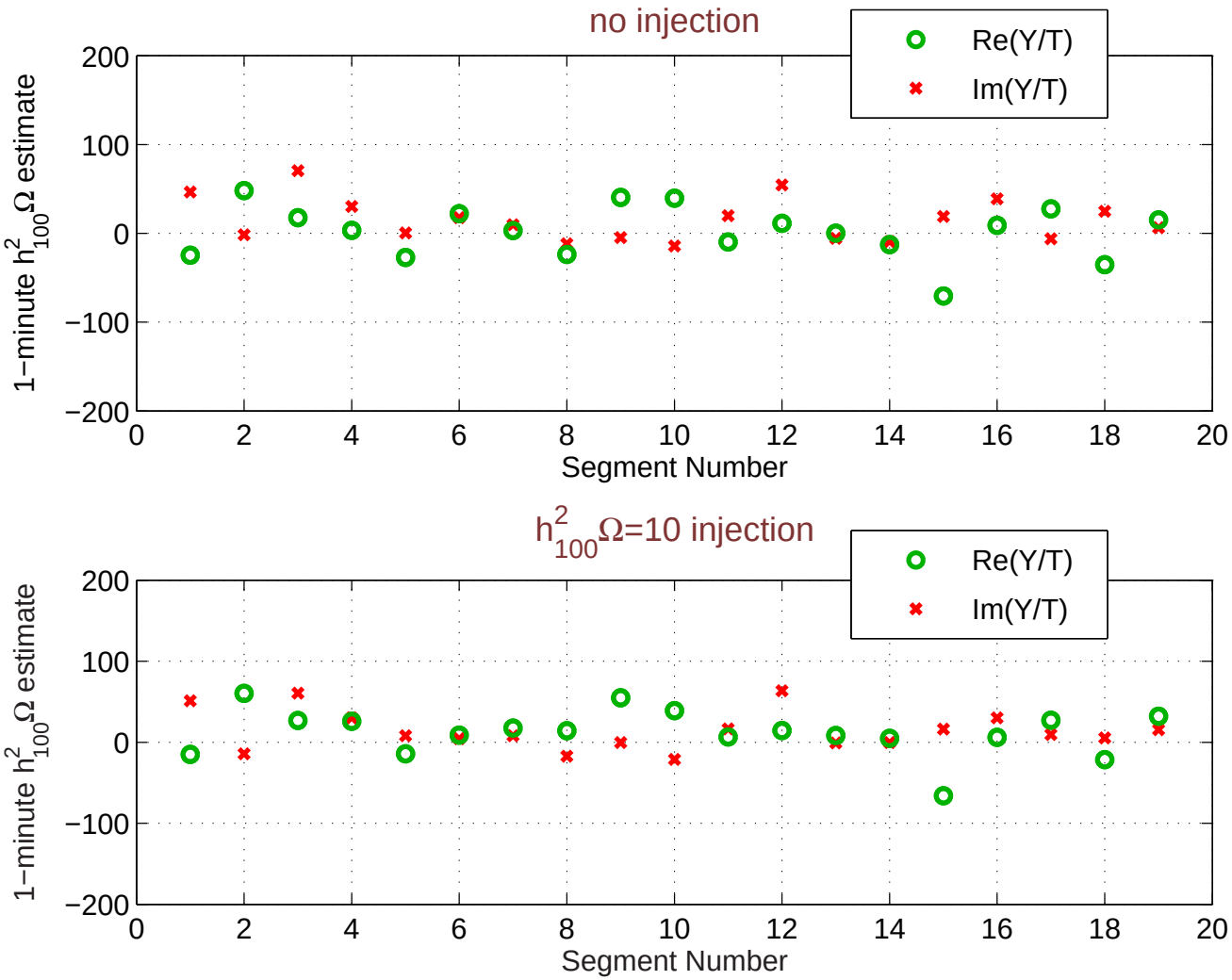
Software Injections into S4 Playground

- Combined 90% error bar for all PG data: 0.9
  (XARM-only: 1.0; YARM-only: 1.6; NULL-only 49.1)

- Inject $h_{100}^2 \Omega(f) = 1,\ 2,\ 5,\ 10$.

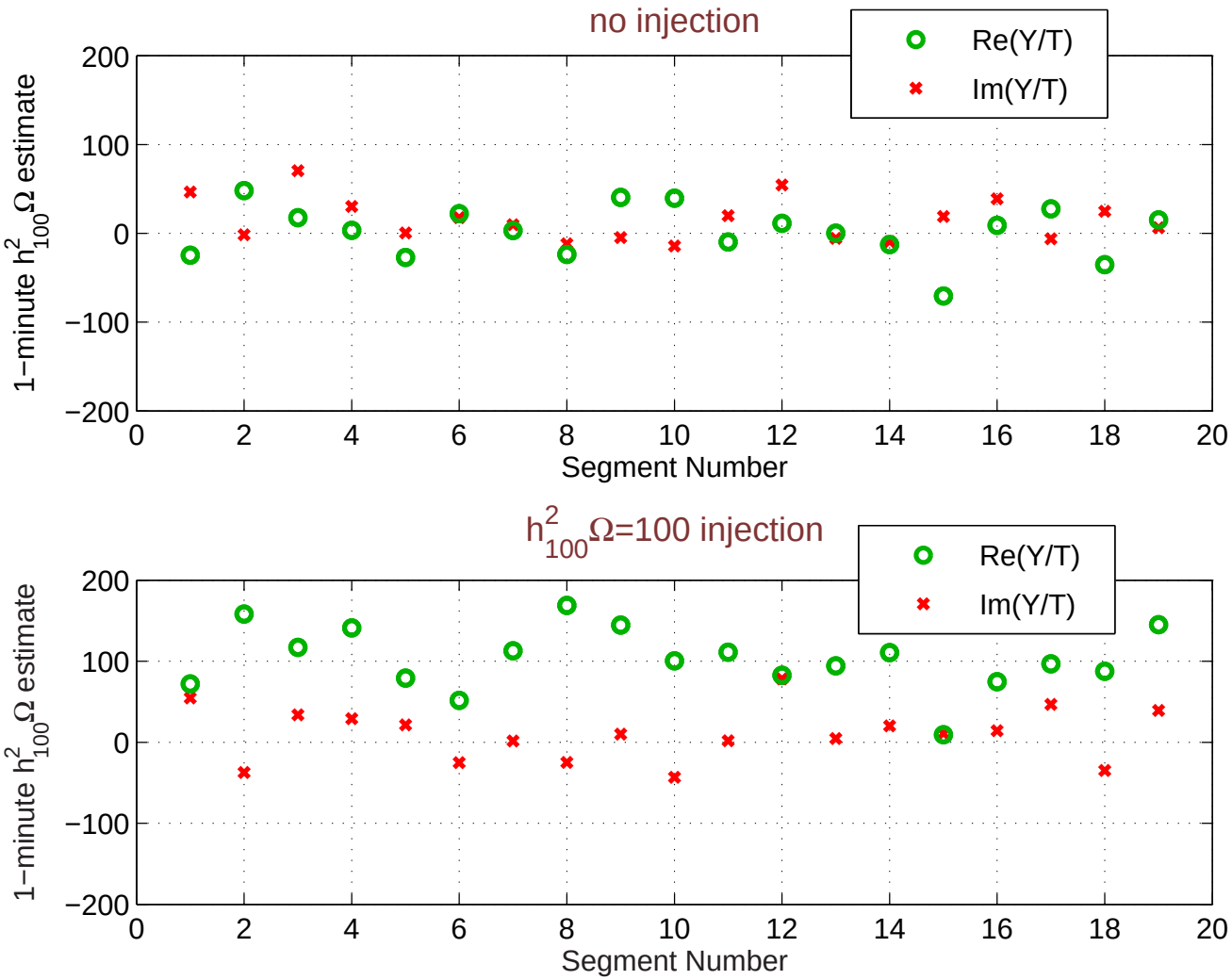
- Note: individual jobs have error bars around 60.
  SW injections only detectable over time.

Stats w/ & w/o SW Inj (19 60-sec segs)

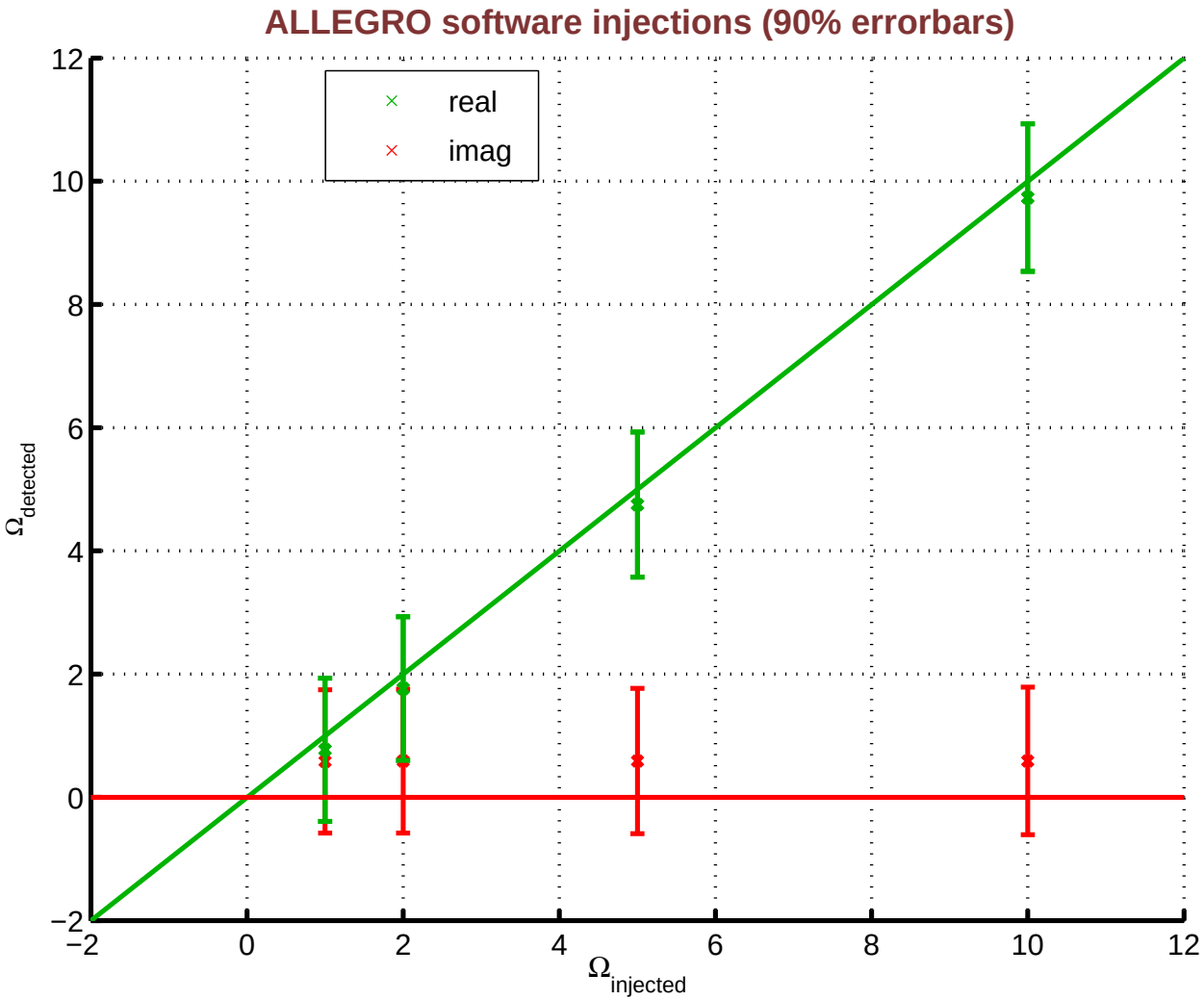


Injecting $h_{100}^2\Omega(f) = 10$ has negligible impact on minute-by-minute correlations

Stats w/ & w/o SW Inj (19 60-sec segs)



Compare $h_{100}^2\Omega(f) = 100$ injection, which is visible minute-by-minute



$h_{100}^2\Omega(f)$ =2, 5, 10 injections recovered from full PG

Note: injected same random signals w/different amplitudes into same noise

S4 Hardware Injections

- 1024-second simulated signals injected into LLO & ALLEGRO hardware a total of nine times. Simulated all three orientations.

- "second" & "third" rounds injected const $h_{100}^2 \Omega_{\rm gw}(f) = 3100$
  Sensitivity of cross-correlation to injections simulating
  XARM ("plus") and YARM ("minus") is comparable
  "null" injection less correlated b/c of simulated misalignment

- Learned about timing issues via HW injections:
  Time-shift analysis helped resolve issues w/ALLEGRO timing
  Also revealed differential sample-and-hold effects from
  different sampling rates of injection system

Time-Shift Analyses

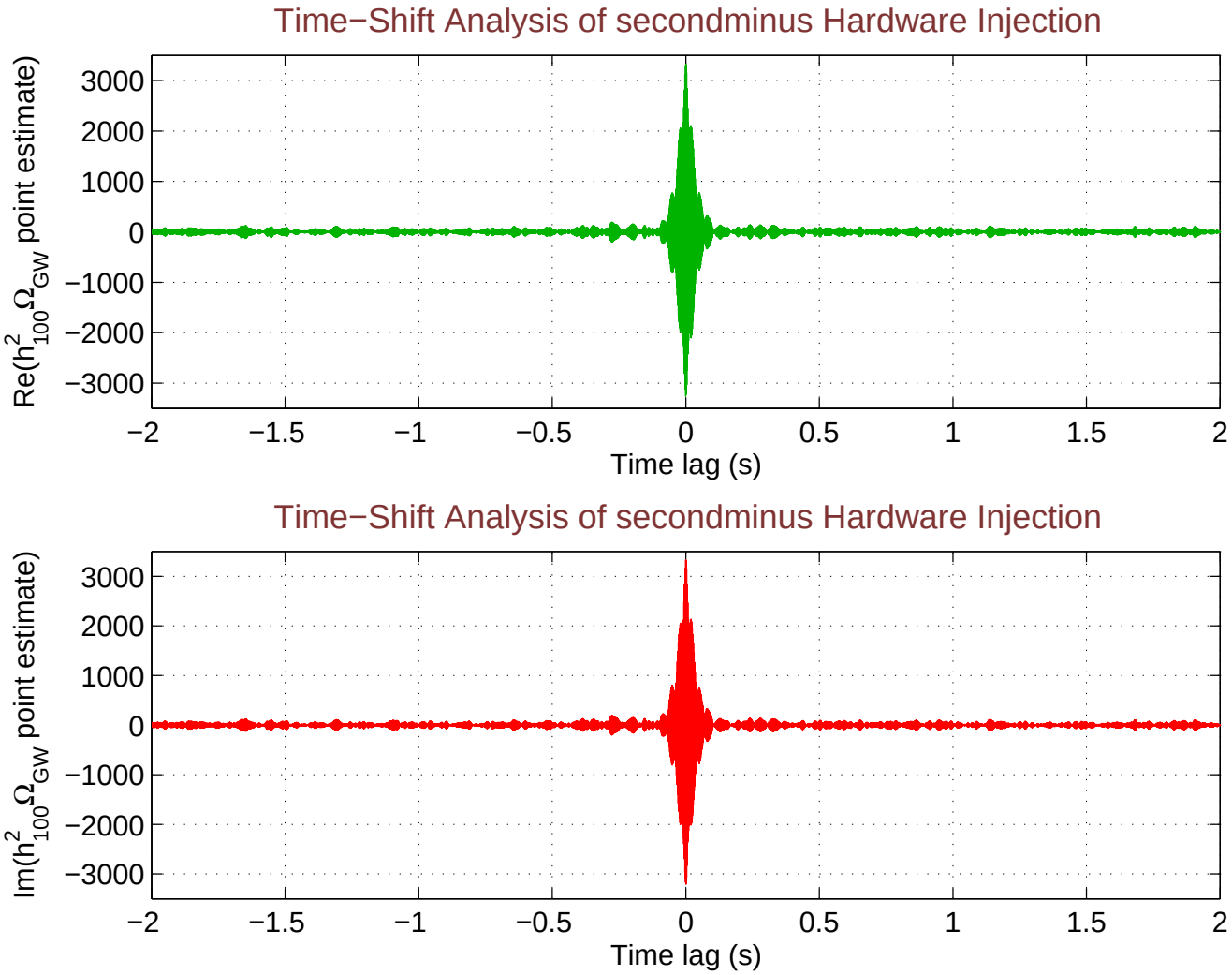
- Sample-and-hold effects mean LLO & ALLEGRO HW injections offset by $\frac{1}{2\times 4096\,\mathrm{Hz}} - \frac{1}{2\times 16384\,\mathrm{Hz}} = 92\,\mu\mathrm{s}$
  Corresponds to 30° of phase at 915 Hz.

- Simulate small timeshift w/freq-dependent phase shift

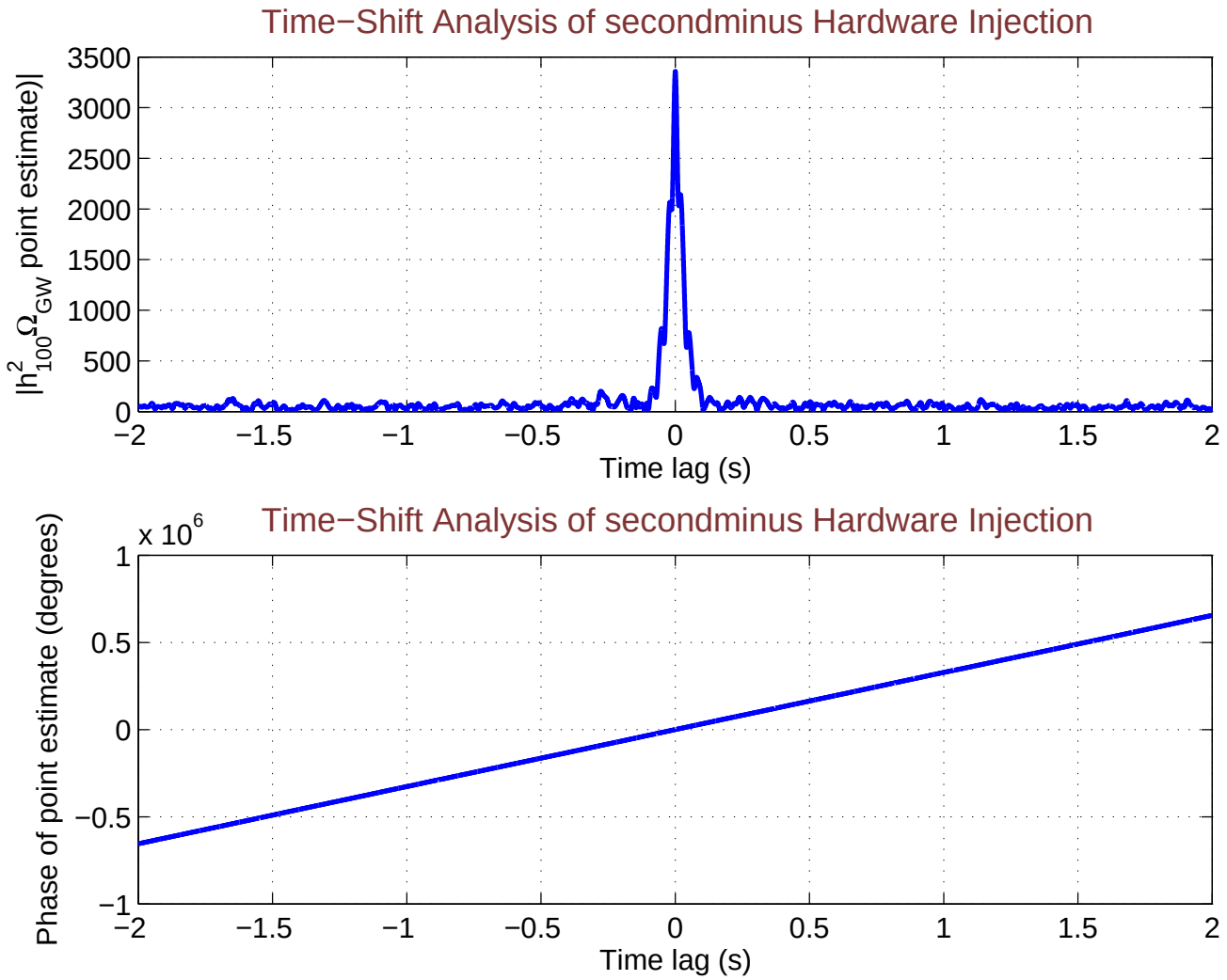
$$Y(f) \longrightarrow Y(f)\, e^{i2\pi f\tau}$$

  inv FT of CC integrand gives CC values as fcn of time-shift:

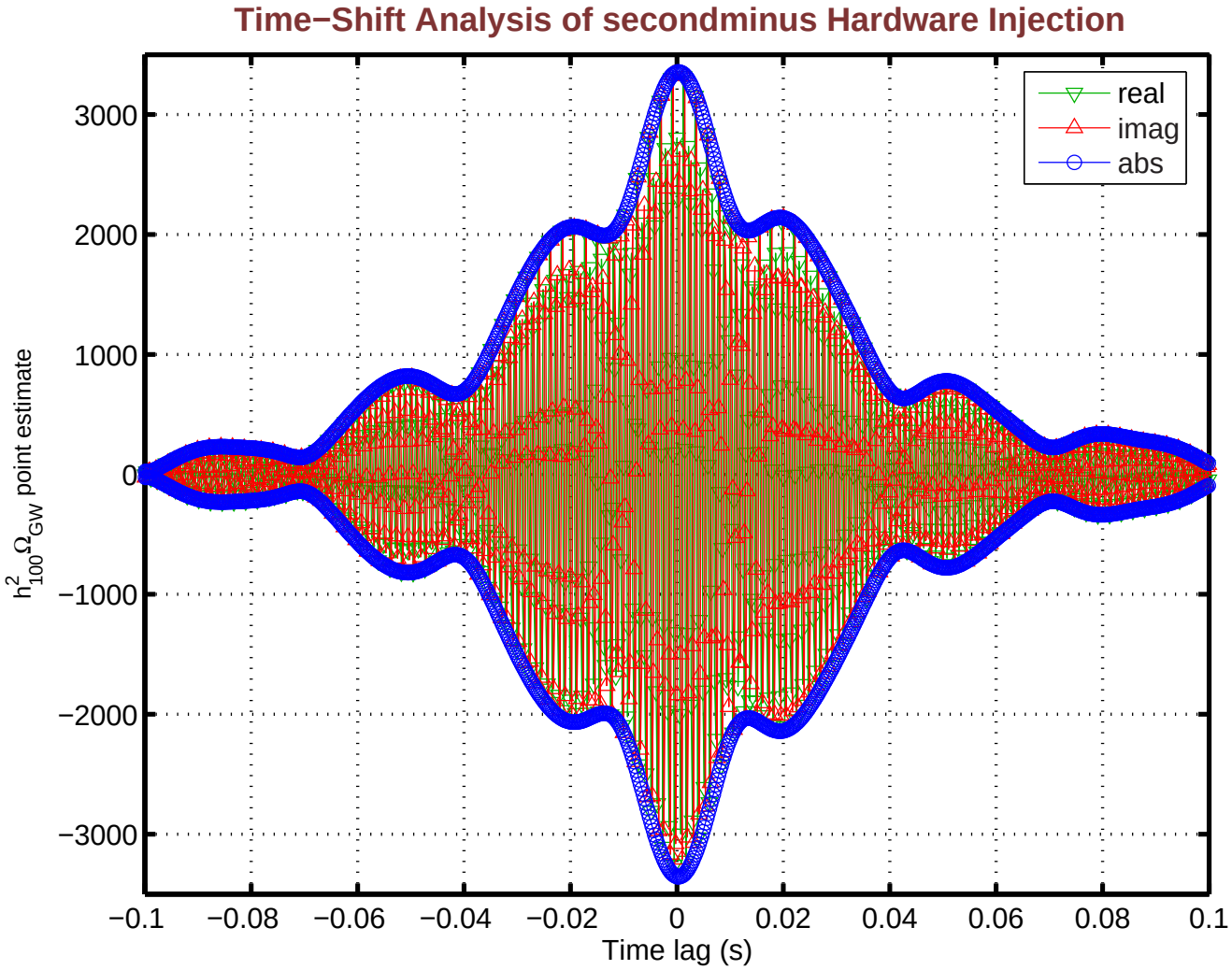
$$Y(\tau) = \int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} df\, Y(f)\, e^{i2\pi f\tau}$$



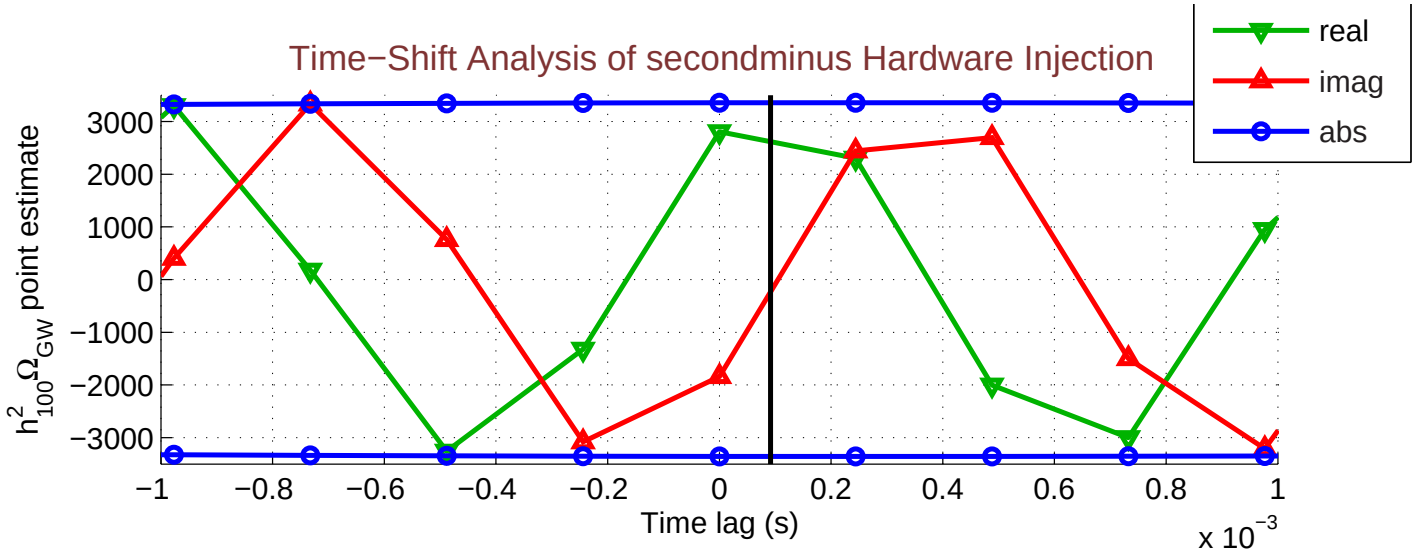
Real & imaginary parts oscillate @ $\sim 910\,\mathrm{Hz}$ due to narrow measurement band

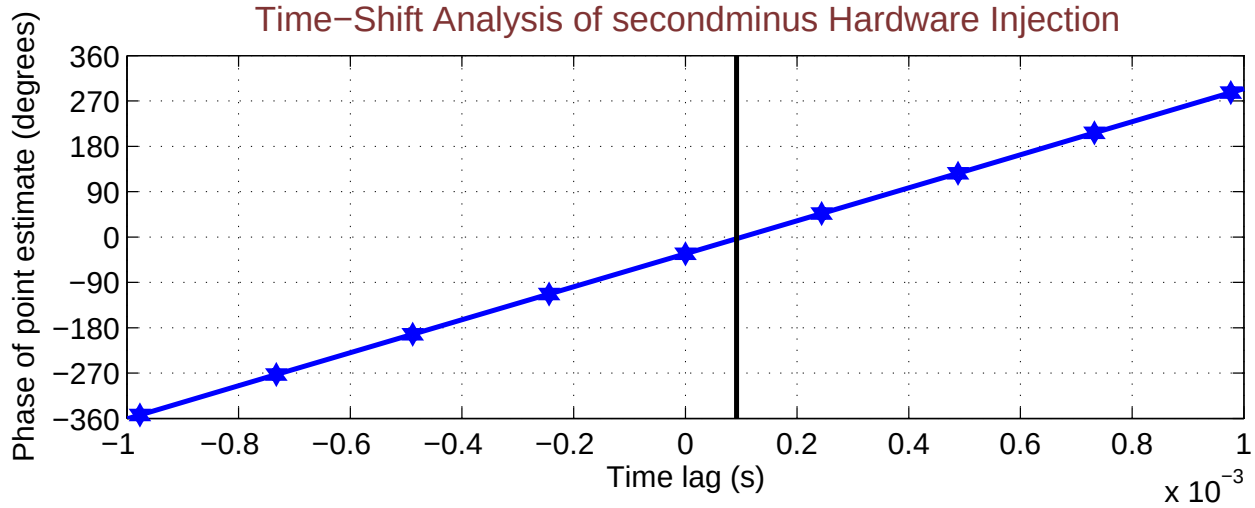


Magnitude peaked near zero time-shift; phase increases quasilinearly ($33^\circ/100\,\mu$s)



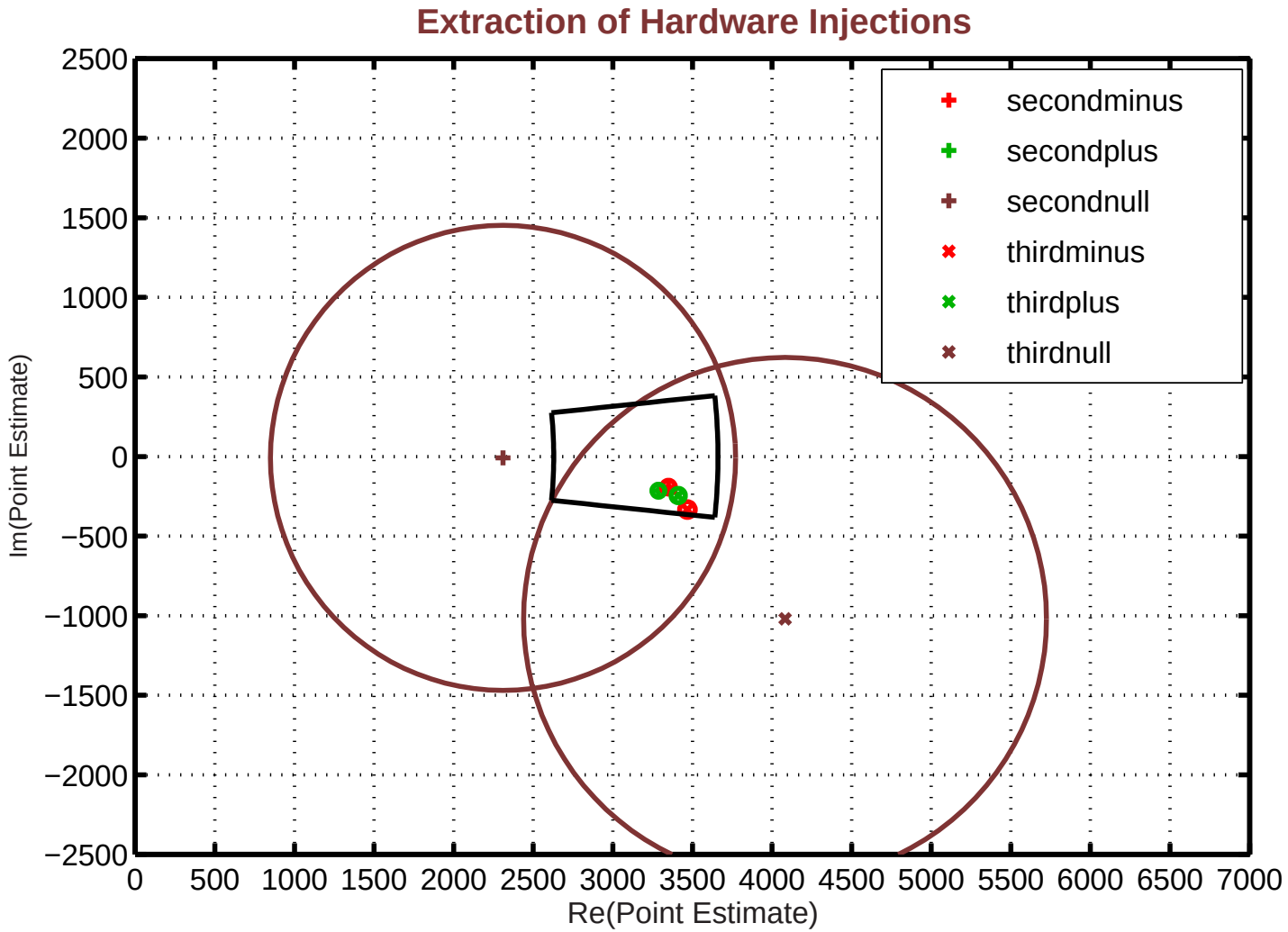
Magnitude forms envelope for real & imag oscillations





Interpolate magnitude & phase (NOT real & imag)

to get time-shifted CC measurement & correct for 92 μs sample-and-hold offset



Circles: statistical uncertainty (null measurements less sensitive)

Black calib uncertainty box (18% amp, 6° phase) around $h_{100}^2 \Omega_{\rm gw}(f) = 3100$

HW injections recovered consistent w/cal uncertainty

LLO-ALLEGRO: Summary

- First stochastic measurement correlating bar w/ifo data;
  Probes higher frequency band than LLO-LHO: $\sim 850 - 950\,\mathrm{Hz}$
- Rotation of ALLEGRO modulates stochastic response
  (data taken in 3 orientations during S4)
- Expected S4 sensitivity from $\sim 370\,\mathrm{hrs}$ of data:
  $h_{100}^2 \Omega_{\mathrm{gw}}(f) \sim 0.3$–$1$ or $\sqrt{S_{\mathrm{gw}}(f)} \sim 1$–$2 \times 10^{-23}\,\mathrm{Hz}^{-1/2}$
- Analysis extracts long-time, low-amplitude simulated signals
  (software injections)
- Hardware inj extracted consistent w/calibration uncertainty

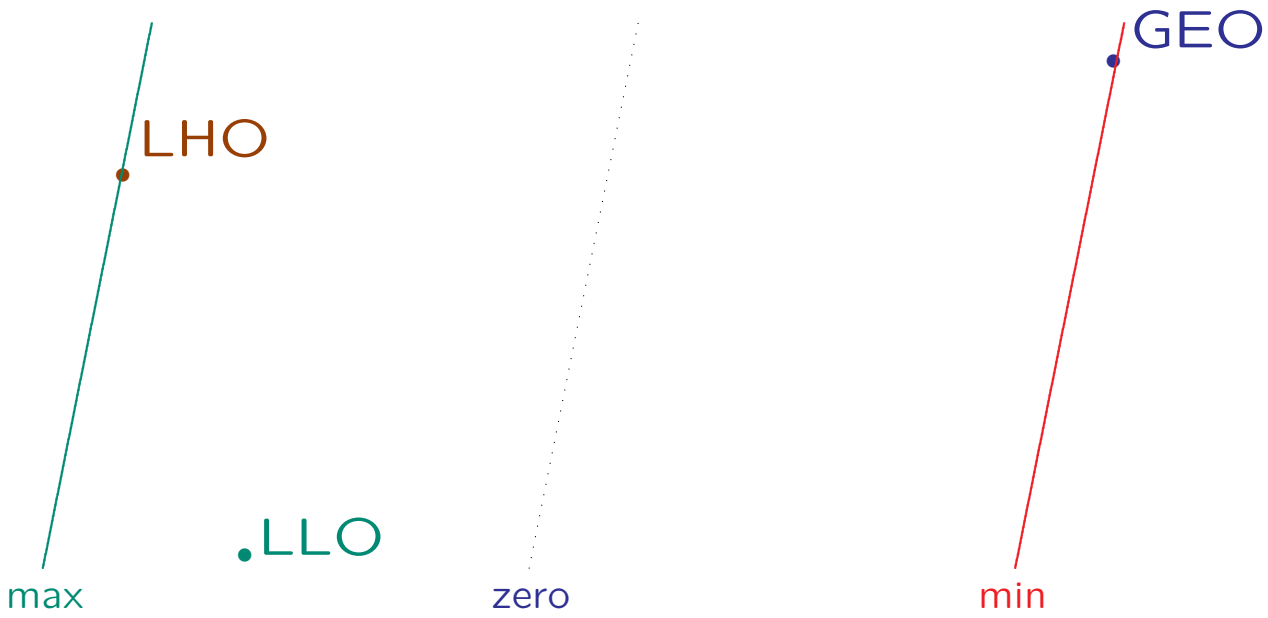
Overlap Reduction Function

$$\gamma_{12}(f) = d_{1ab} d_2^{cd} \frac{5}{4\pi} \iint_{S^2} d^2\Omega \; P^{\mathrm{TT}}{}^{ab}_{cd}(\hat{\Omega}) e^{i2\pi f \hat{\Omega}\cdot\Delta\vec{\mathbf{x}}/c}$$

Depends on alignment of detectors (polarization sensitivity)

Frequency dependence from cancellations when $\lambda \lesssim$ distance

$\rightarrow$ Widely separated detectors less sensitive at high frequencies



This wave drives LHO & GEO out of phase

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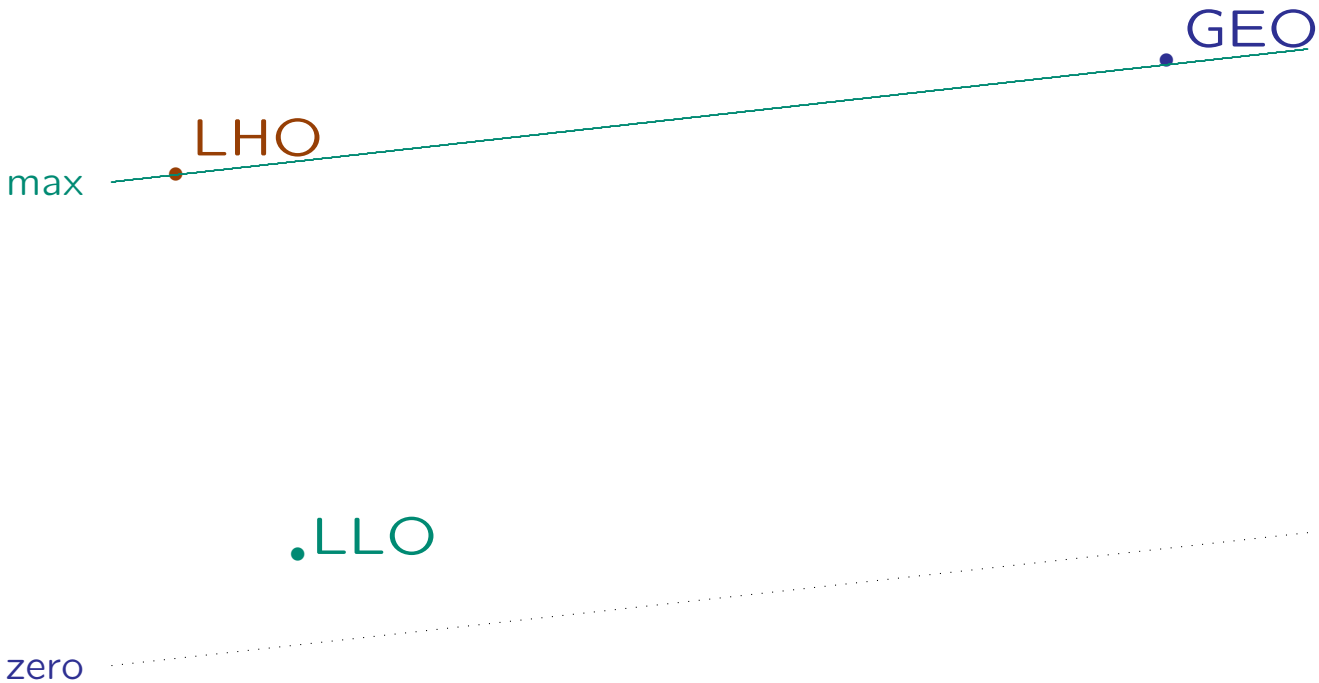
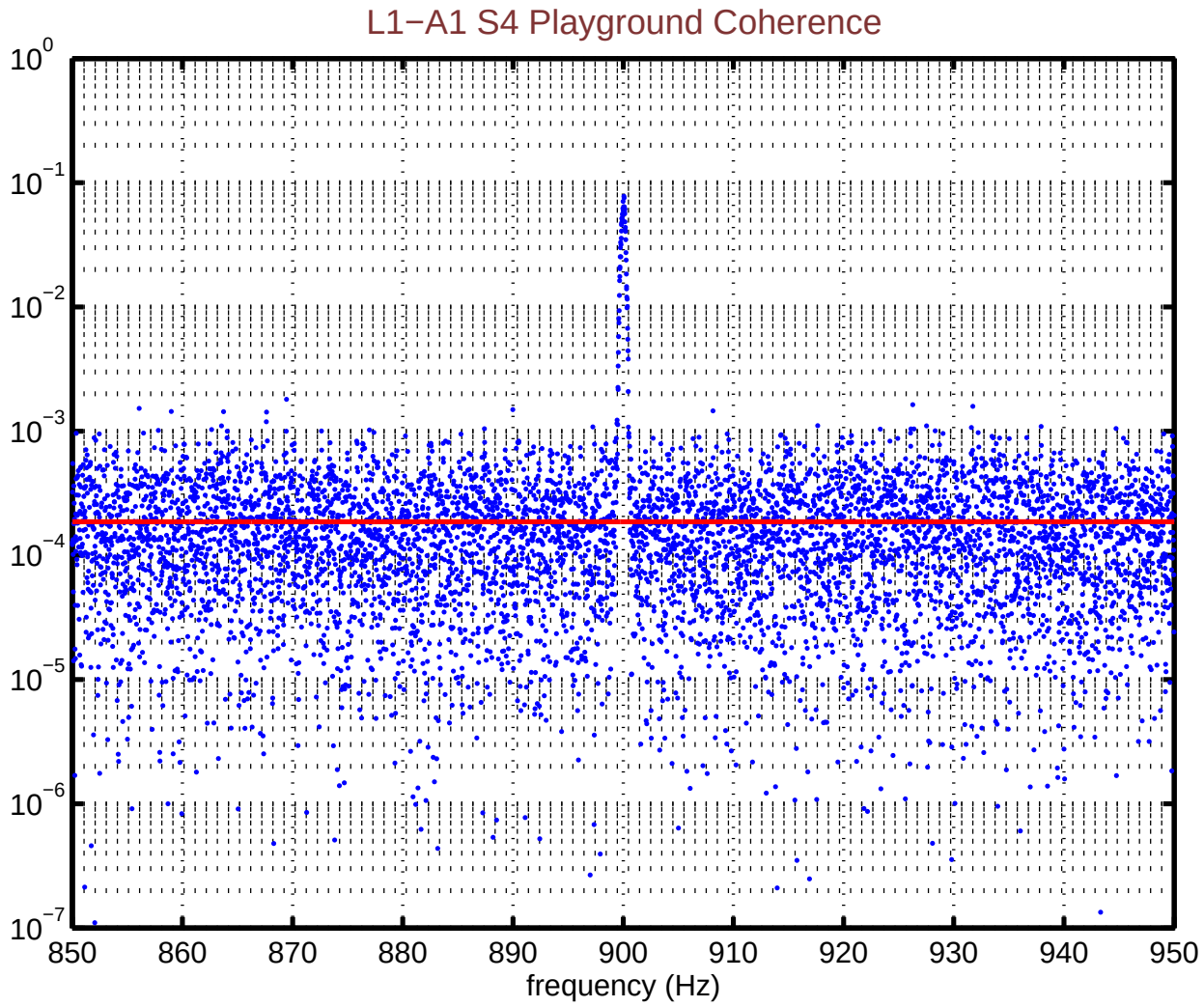


This wave (same λ) drives LHO & GEO in phase



Red line is 1/# of avgs (expected coherence)

Only significant coherence is 900 Hz power line harmonic