Modeling of the AdvLIGO Quad Pendulum Controls Prototype

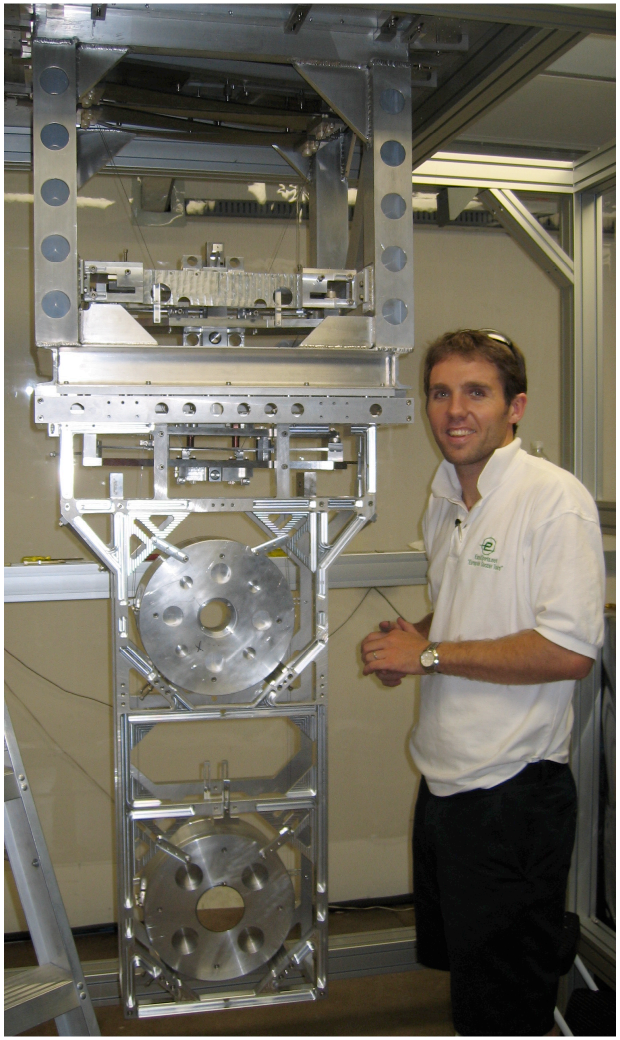
Mark Barton (LIGO-Caltech)

SWG Session
LSC Meeting 3/21/06

G060086-00-D

Quad Controls Prototype

- Controls prototype is a quad pendulum with a full (quad) reaction chain.
- It was assembled at Caltech from July 2005, and reassembled clean at LASTI in February 2006.
- Two significant anomalies relative to design model:
  - » Fundamental pitch mode was unstable.
  - » Despite careful mass budgeting, blade tips sat too low.
- New physics had to be added to design model in three areas.



Toolkit

- A Mathematica pendulum modeling toolkit by Mark Barton has been used extensively to understand the performance of the AdvLIGO prototypes.

  http://www.ligo.caltech.edu/~mbarton/SUSmodels/

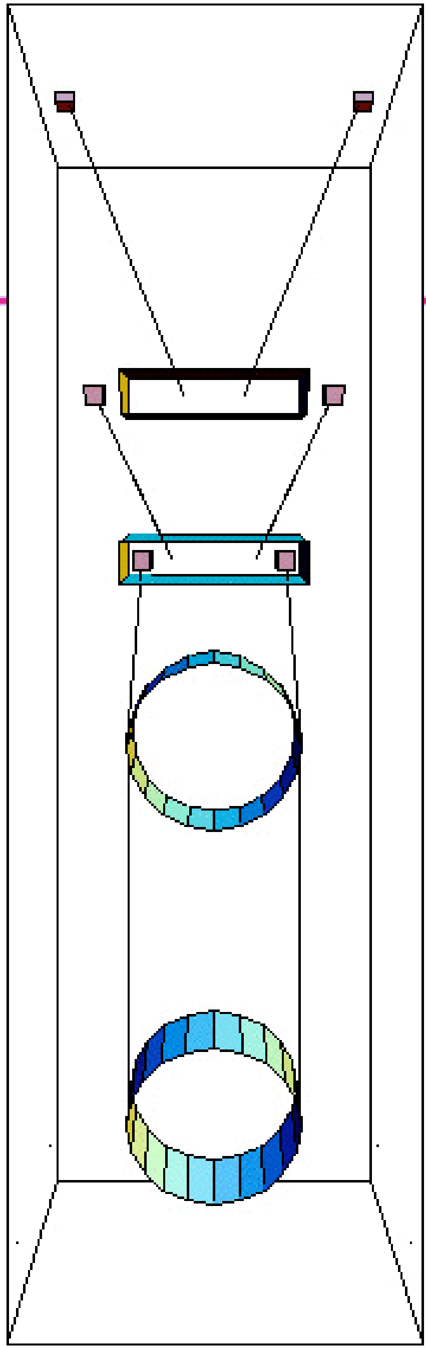
- Models for GEO-style triple and AdvLIGO-style quad have been developed using toolkit elements

- MATLAB model by Torrie et al. is still used for some applications but core code has been replaced by improved matrix elements generated symbolically in the Mathematica and exported as MATLAB code.

Quad Model

- **Features:**
  - Fully 3D with provision for arbitrary asymmetries
  - Rigid-body masses (no internal modes)
  - Realistic wires with longitudinal and bending elasticity
  - Arbitrary frequency dependent damping on all sources of elasticity
  - Optional violin-mode modeling using 5 mass beads in each fibre to approximate distributed mass

- **Configuration:**
  - Two blade springs
  - Two wires
  - Top mass
  - Two blade springs
  - Four wires (two per spring)
  - Upper intermediate mass
  - Four wires
  - Intermediate mass
  - Four wires (fibres)
  - Test mass



Fundamental Pitch Mode

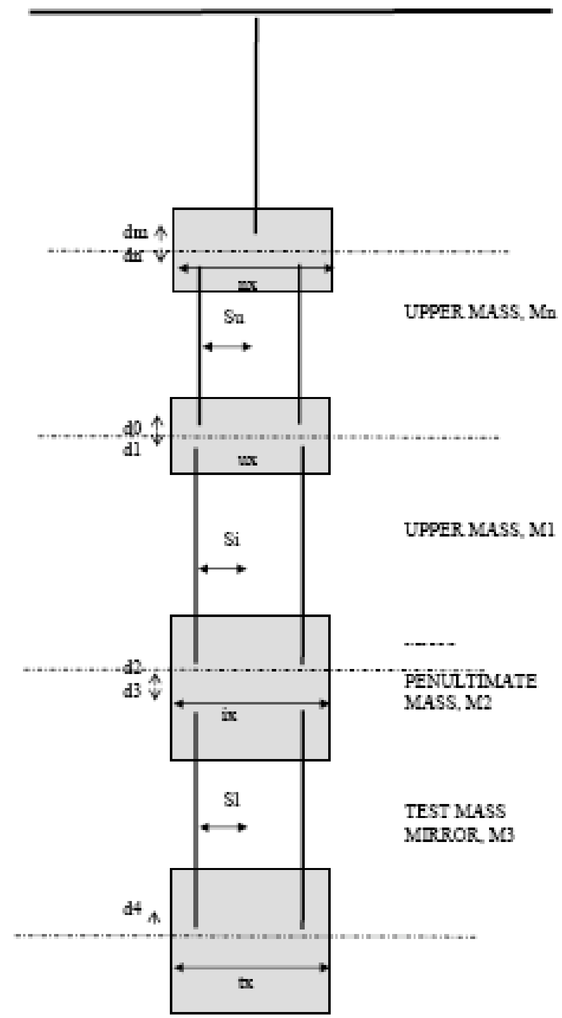
- Initial suspect for pitch problem was wire flexure correction.
- Fundamental pitch mode has masses moving in phase, ganged together by lower pairs of wires
- For ideally flexible wire as in Matlab model, frequency is set by gravitational restoring torque (weighted sum of d's) and sum of MOIs:

```
kk = (
      (mn+m1+m2+m3) g dm
      +(m1+m2+m3) g (dn+d0)
      +(m2+m3) g (d1+d2)
      +m3 g (d3+d4)
)

II = Iny+I1y+I2y+I3y

ff = Sqrt[kk/II]/2/Pi
```

- However real wire is stiff, especially under tension -> pitch frequencies increase -> decrease d's to compensate

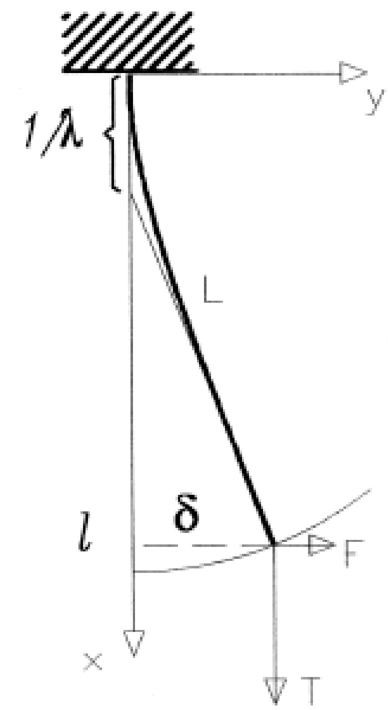


Wire Flexure Correction

- Conceptual design has flexible wire attached at d = +1 mm (i.e., “stable” side of COM).
- Real wire has effective flexure point 2 to 5 mm from end -> set d = -1 to -4 mm to compensate.
- Delicate cancellation -> need to get it right.
- Theory for simple pendulum case given in

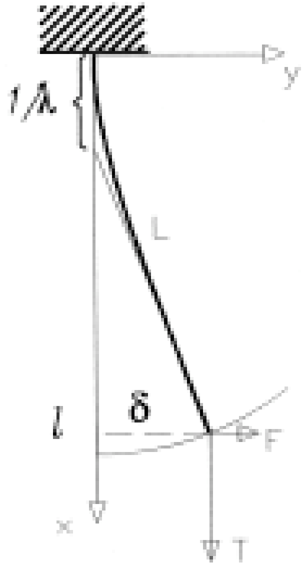
  G. Cagnoli et al., Physics Letters A 272 (2000): 39 – 45

- Applicable for more complicated cases?



Wire Stiffness - Simple Pendulum Case

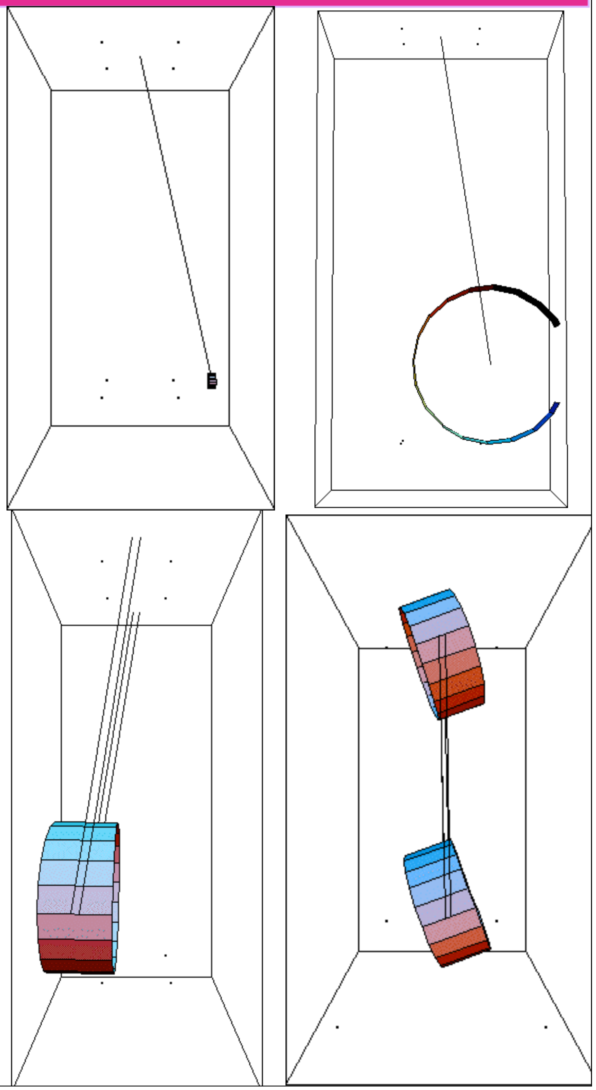
- The wire can be modeled as an elastic beam under tension. At low frequency, $y(x) = \frac{F}{T\lambda}[e^{-\lambda x} + \lambda x - 1]$ $\lambda = \sqrt{T/Y_0 I}$



- Pendulum frequency increases due to two effects:
  - » Wire is effectively shorter by 0.5/lambda -> extra gravitational restoring force
  - » Wire is stiff -> elastic restoring force equivalent to another 0.5/lambda of length change
- Perhaps one effect doesn't apply for multiple wires, and/or with flexure at both ends?
- Reworked pendulum to halve flexure correction -> pendulum stable! But was that really the culprit?

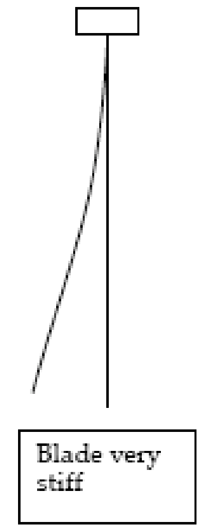
Multiple/Angled wires

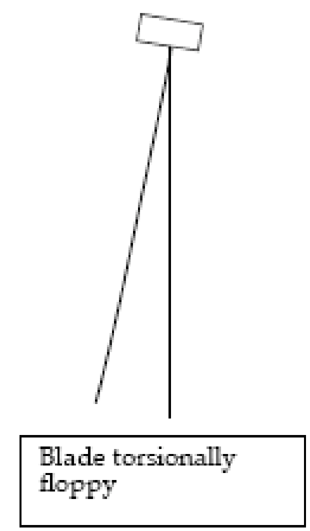
- The Mathematica pendulum toolkit has independent implementation of theory from Matthew Husman's thesis.
- Lots of comparisons were done using toy models.
- Conclusions:
  - » Cagnoli theory still valid for multiple wires *provided* wires are vertical
  - » For angled wires, get factor of $(\cos\theta)^{1/2}$ from increased tension, $\cos\theta$ from geometry near clamp
  - » Net $(\cos\theta)^{3/2}$ -> smaller correction
  - » Only accounts for about 25% of observed instability.



Blade Torsional Compliance

- OK then, how about blades?
- Blades are compliant torsionally.
- Norna Robertson had previously considered this and dismissed it, but double-check was made:
  - » add torsional flexure to model
  - » use torsional compliance values from CAD
  - » tiny effect on higher pitch frequencies, none on fundamental
- See also T050255-06 by Ian Wilmut and Justin Greenhalgh.
- Indeed, not the answer.



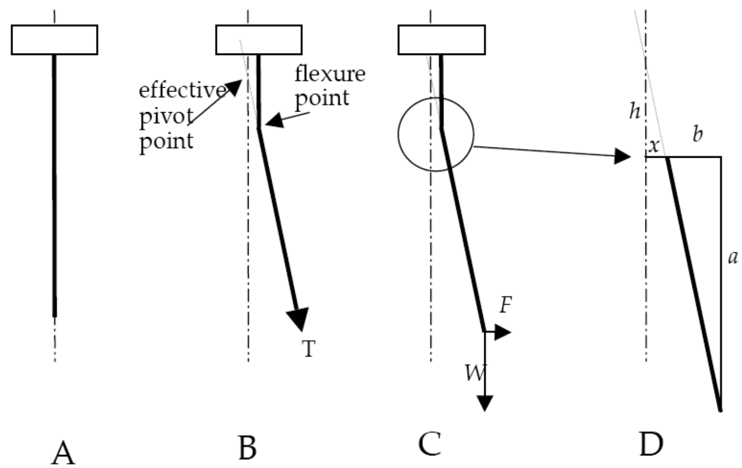


Blade Lateral Compliance

- How about lateral compliance?
- Add to model, using CAD values for stiffness
- BINGO! - good agreement with measured pitch frequencies
- For future design, use the fact that compliance mimics a wire flexure correction of

$$h = \frac{W}{k_l}$$

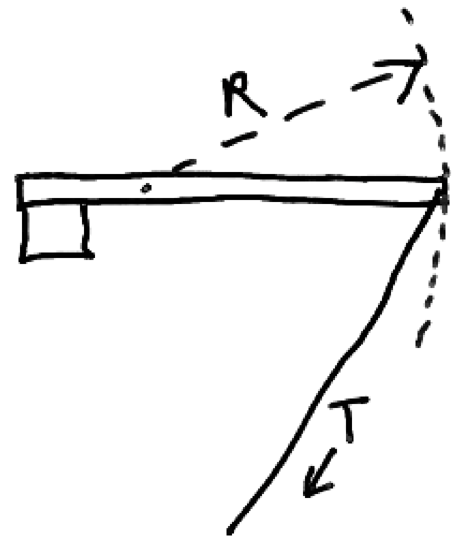
  where W is blade's share of payload force and k_l is lateral stiffness



- In retrospect, prototype has the right total amount of wire flexure correction but distributed wrongly
- Not critical - don't fix

Vertical Sag

- Angled wire also turns out to explain drooping blades.
- Curve-fitting to CAD data from Ian Wilmut showed softening proportional to horizontal component of load -> suggests inverse pendulum effect
- Magnitude of effect implies radius of curvature of tip locus just less than length of blade
- Accounts for blade tip position and vertical frequencies



Conclusions (i)

- Combining all three effects gives good predictions with all as-built numbers.
- We're confident that we know how to design a quad and have it work when first assembled.

ID	f (theory)	f (exp)
pitch	0.395	0.403
x	0.443	0.440
y	0.464	0.464
z	0.595	0.549
yaw	0.685	0.684
roll	0.810	0.794
x	0.987	0.989
y	1.043	1.038
pitch	1.167	1.355
yaw	1.428	1.428
x	1.981	1.978
y	2.095	2.075
z	2.362	2.222
yaw	2.538	2.515
pitch	2.818	2.576
roll	2.762	2.734
yaw	3.167	3.149
pitch?	3.228	3.162
roll	3.332	3.333
x	3.401	3.381
z	3.793	3.589
roll	5.120	5.029
z	17.700	?
roll	25.741	?

Conclusions (ii)

- Other activities not related to the quad:

  » A symmetrical case of the improved quad model with blade lateral compliance has been exported to Matlab code (which runs faster) for rapid testing, as well as for controller design with Simulink.

  » It has also been exported to numerical state-space form for use in E2E.

  » A version with 5-bead violin modes has been produced, and Ken Strain has been using it to investigate the feasibility of various strategies for damping the modes. See T050267-00.