Mirror Thermal Noise: Gaussian vs Mesa beams

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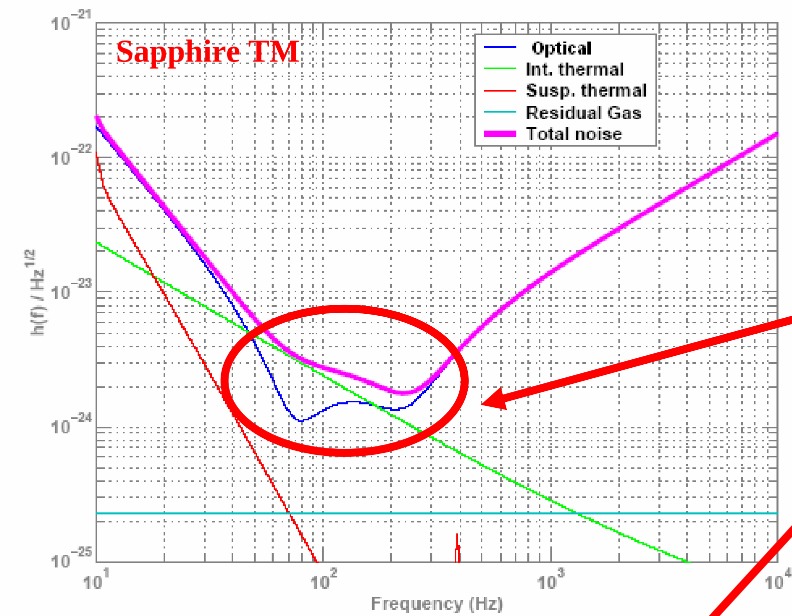
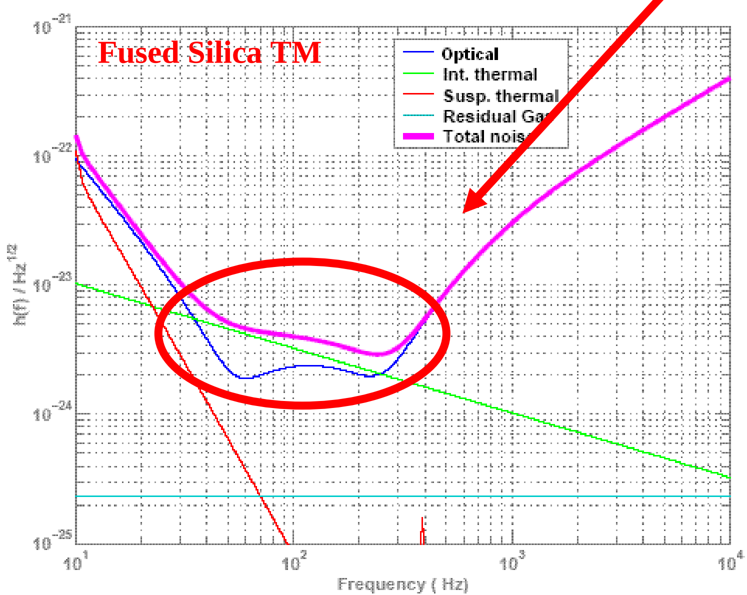
LIGO-Virgo Thermal Noise Workshop

Mirror thermal noise problem:





Advanced-Ligo sensitivity

Dominated by test-masses thermoelastic (S-TM) or coating (FS-TM) thermal noises.

Can we reduce the influence of thermal noise on the sensitivity of the interferometer?

Without drastic design changes

Mirror Thermal Noise:

Thermoelastic noise

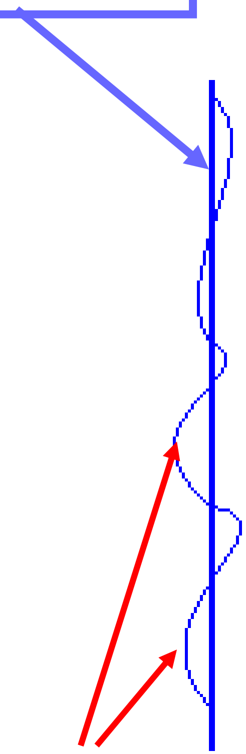
Created by stochastic flow of heat within the test mass

↓

Fluctuating hot spots and cold spots inside the mirror

↓

Expansion in the hot spots and contraction in the cold spots creating fluctuating bumps and valleys on the mirror's surface



Brownian noise

Due to all forms of intrinsic dissipations within a material (impurities, dislocations of atoms, etc..)

Interferometer output: proportional to the test mass average surface position, sampled by to the beam's intensity profile.

Indicative thermal noise trends

Noise spectral densities in the **Gaussian beam** case

(infinite semi-space mirror)

$$S_X^{TE-s} \propto \frac{1}{w^3}$$

Substrate thermoelastic noise

$$S_X^{TE-c} \propto \frac{1}{w^2}$$

Coating thermoelastic noise

$$S_X^{B-s} \propto \frac{1}{w}$$

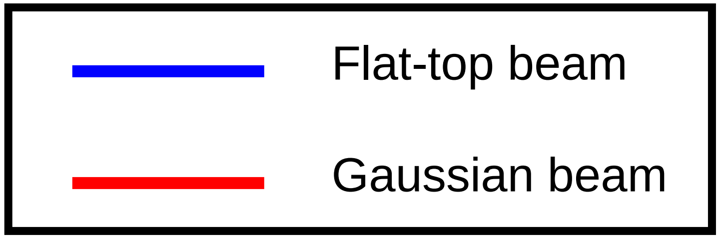
Substrate Brownian noise

$$S_X^{B-c} \propto \frac{1}{w^2}$$

Coating Brownian noise

Exact results require accurate information on material properties and finite size effects must be taken in account.

Diffraction prevents the creation of a beam with a rectangular power profile…but we can build a nearly optimal flat-top beam:

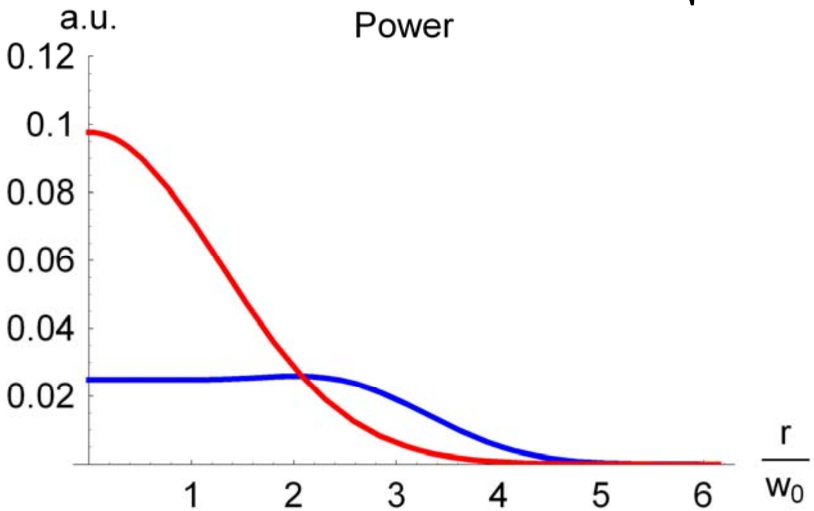


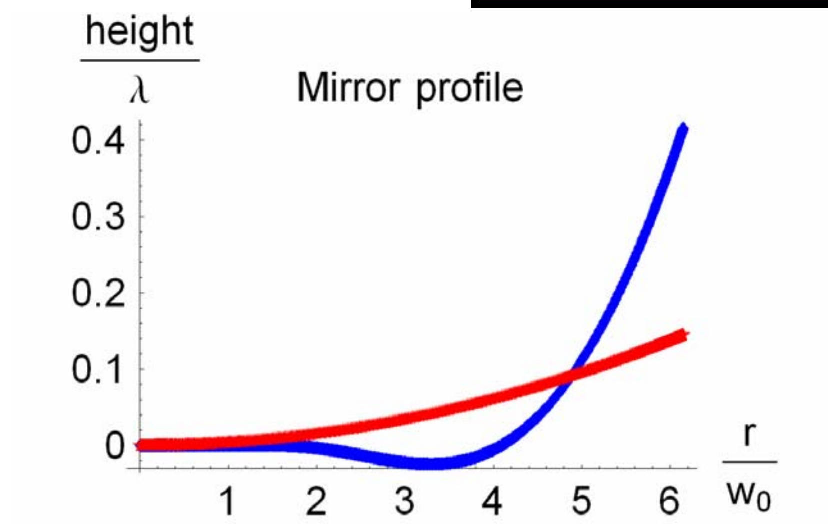
$$u_{FT}(r) \propto \int_{r' \leq D} d^2\vec{r}'\, e^{-\frac{(\vec{r}-\vec{r}')^2(1-i)}{2w_0^2}}$$

$$u_G(r) \propto e^{-\frac{r^2}{w^2}+\frac{ikr^2}{2R}}$$

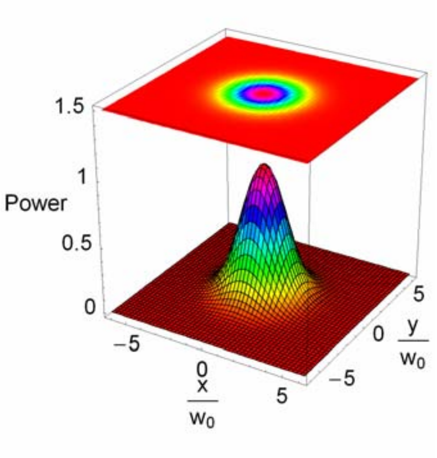
$$w_0 = \sqrt{\frac{L}{k}}$$

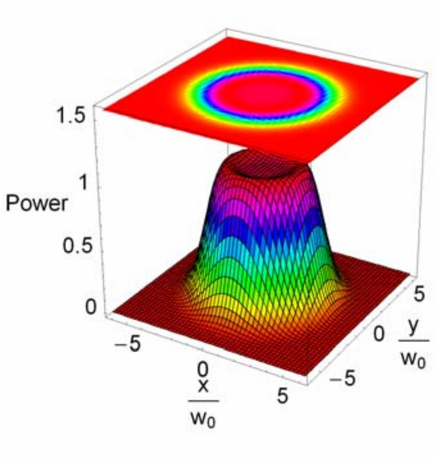
The mirror shapes match the phase front of the beams.





Thermal noise for finite sized mirrors:





1. **Precise comparative estimation of the various thermal noise contributions for finite test masses (design optimization).**
2. **Noise suppression using Mesa beam.**

Thermal noise calculations

Interferometer is sensitive to the test mass surface displacement

$$X(t) = \int_{Mirror} d^2\vec{r}\; u_z(\vec{r},t)\; f(\vec{r})$$

Levin's approach to Fluctation Dissipation Theorem

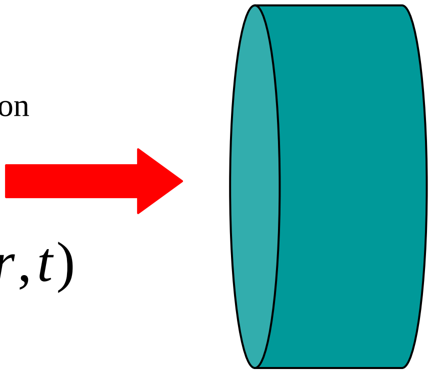
$$S_X(\omega) = \frac{8\,k_B\,T}{\omega^2}\frac{W_{diss}}{F_0^2}$$

W_{diss} **Is the energy dissipated by the mirror in responce to the oscillating pressure**

$$P(\vec{r},t) = F_0\; f(\vec{r})\cos(\omega\,t)$$

Assumptions in our analysis

BHV+LT (accurate) approximate analyical solution of elasicity equations for a cylindrical test mass



Quasistatic approximation for the oscillations of stress and strain induced by P.

$$\tau_{sound} << \tau_{GW}$$

Adiabatic approximation for the substrate thermoelastic problem (negligible heat flow during elastic deformation).

$$r_{heat} << r_{beam}$$

Brakes down for coating thermoelastic problem → **Perturbative approach**

Coating is an isotropic and homogeneous thin film

Material properties:

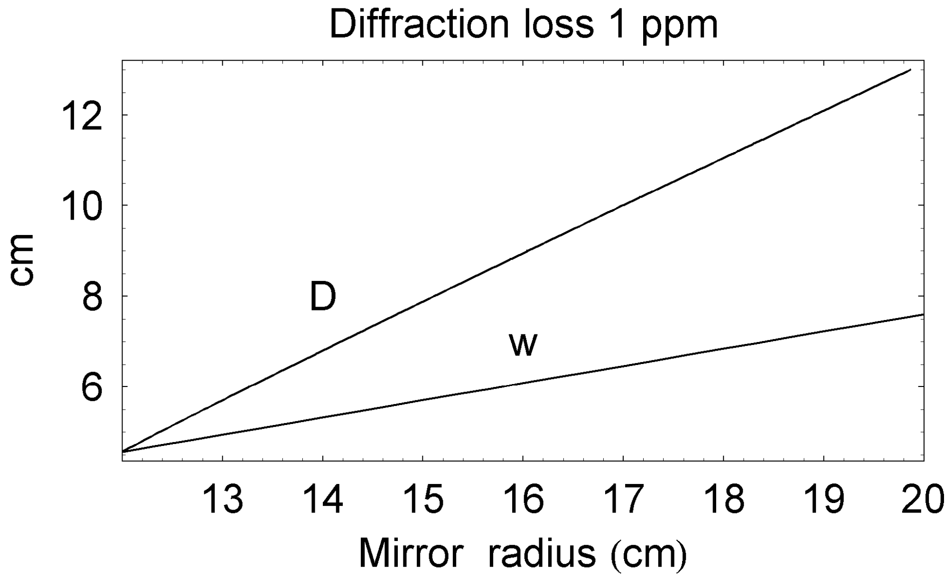
Parameters : (c.g.s. units)	**Fused Silica:**	**Sapphire:**	**Coating**	
			Ta2O5	**SiO2**
Density (g/cm3)	2.2	4	6.85	2.2
Young modulus (erg/cm^3)	7.2 10^{11}	4 10^{12}	1.4 10^{12}	7.2 10^{11}
Poisson ratio	0.17	0.29	0.23	0.17
Loss angle	5 10^{-9}	3 10^{-9}	10^{-4} (total)	
Lin. therm. expansion coeff. (K^{-1})	5.5 10^{-7}	5 10^{-6}	3.6 10^{-6}	5.1 10^{-7}
Specific heat per unit mass (const. vol.) (erg/(g K))	6.7 10^6	7.9 10^6	3.06$10^6$	6.7 10^6
Thermal conductivity (erg/(cm s K))	1.4 10^5	4 10^6	1.4 10^5	1.4 10^5
Total thickness (cm)	variable	variable	$19\,\lambda/4n_1$	$19\,\lambda/4n_2$

Ideas behind calculations

- **Fixed total mirror mass = 40 Kg.**
- **The beam radius is dynamically adjusted to maintain a fixed diffraction loss = 1ppm (clipping approximation).**
- **The mirror thickness is also dynamically adjusted as a function of the mirror radius in order to maintain the total 40 Kg mass fixed.**
- **Calculation at the frequency 100 Hz**

$$\text{Noise}_{\text{TE-s}} \propto 1/f$$

$$\text{Noise}_{\text{B-s / B-c}} \propto 1/\sqrt{f}$$



Substrate Brownian noise

$$W_{diss} = 2\omega\phi_s \langle U \rangle$$

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \varepsilon_{\phi\phi} = \frac{u_r}{r}, \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z}, \quad \varepsilon_{rz} = \tfrac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right) \quad ,$$

$$U = \int_{test\ mass} \tfrac{1}{2}\varepsilon_{ij}\sigma_{ij}\,dV$$

$$\sigma_{ii} = \lambda\varepsilon + 2\mu\varepsilon_{ii}, \quad \sigma_{rz} = 2\mu\varepsilon_{rz}, \quad \varepsilon = \varepsilon_{rr} + \varepsilon_{\phi\phi} + \varepsilon_{zz}$$

Substrate thermoelastic noise

$$W_{diss} = \left\langle \int_{test\ mass} \frac{\kappa}{T}\left(\vec{\nabla}\delta T\right)^2 dV \right\rangle$$

$$r_{beam} >> r_t \qquad r_t = \sqrt{\frac{\kappa}{\rho C\omega}}$$

$$\delta T = -\frac{\alpha Y T}{C\rho(1-2\sigma)}\varepsilon$$

Coating Brownian noise

$$W_{diss} = 2\omega\phi_c < U_c >$$

$$U_c \approx \delta U_c\, d$$

$$\delta U_c = \int_S \tfrac{1}{2}\varepsilon_{ij}^c \sigma_{ij}^c \, dS$$

Boundary condition

$$\varepsilon^c{}_{rr} = \varepsilon_{rr}(z=0) \qquad \varepsilon^c{}_{\phi\phi} = \varepsilon_{\phi\phi}(z=0) \qquad \sigma^c{}_{zz} = \sigma_{zz}(z=0)$$

$$\sigma^c{}_{ii} = \lambda_c \varepsilon^c + 2\mu_c \varepsilon^c{}_{ii}, \quad \sigma^c{}_{rz} = 2\mu_c \varepsilon^c{}_{rz}, \quad \varepsilon^c = \varepsilon^c{}_{rr} + \varepsilon^c{}_{\phi\phi} + \varepsilon^c{}_{zz}$$

$$\sigma_{rz}^c = 0$$

Coating thermoelastic noise

$$d << r_t << r_{beam}$$

$$\left(\frac{\partial}{\partial t} - K_\beta \frac{\partial^2}{\partial z^2}\right)\delta T_\beta = -\left(\frac{Y\alpha T}{(1-2\sigma)C\rho}\frac{\partial \varepsilon}{\partial t}\right)_\beta = -\mathrm{B}_\beta \qquad \beta = s, c$$

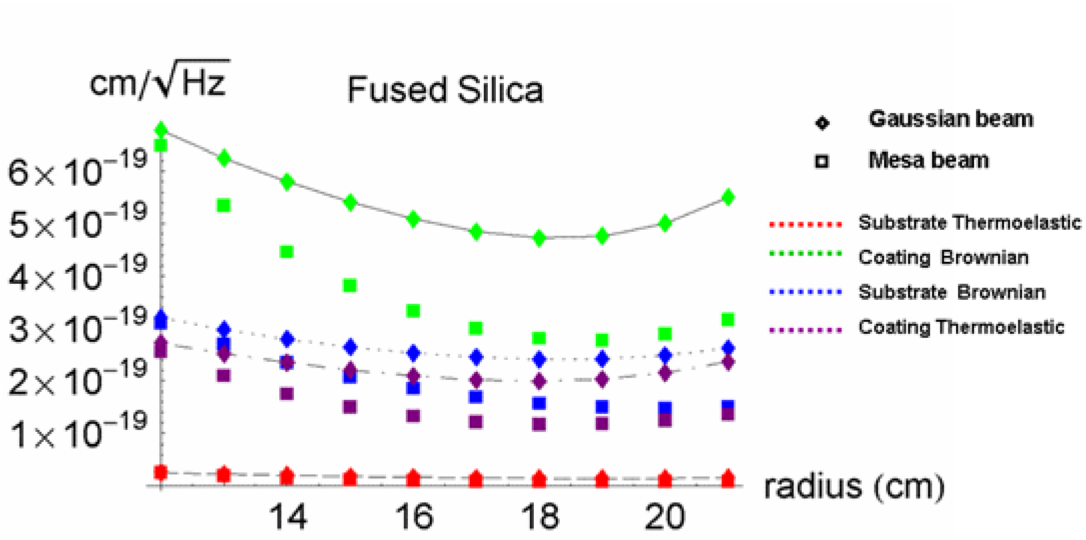
at the surface

$$(i\omega - K_\beta)\delta T_\beta = -i\omega \mathrm{B}_\beta$$

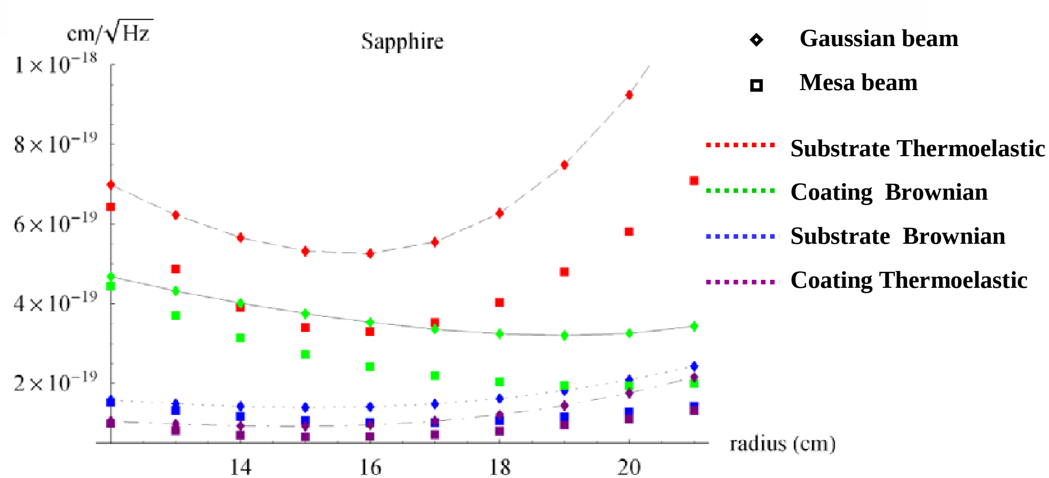
Boundary condition $\left.\frac{\partial \delta T_c}{\partial z}\right|_{z=0} = 0, \quad \left.\frac{\partial \delta T_s}{\partial z}\right|_{z=H} = 0, \quad \delta T_c = \delta T_s\big|_{z=d}, \quad K_c \frac{\partial \delta T_c}{\partial z} = K_s \left.\frac{\partial \delta T_s}{\partial z}\right|_{z=d}$

$$W_{diss} = \left\langle \int_{V_s} \frac{\kappa_s}{T}\left(\frac{\partial \delta T_s}{\partial z}\right)^2 dV_s \right\rangle + \left\langle \int_{V_c} \frac{\kappa_c}{T}\left(\frac{\partial \delta T_c}{\partial z}\right)^2 dV_c \right\rangle$$

Results for Gaussian and Mesa beam

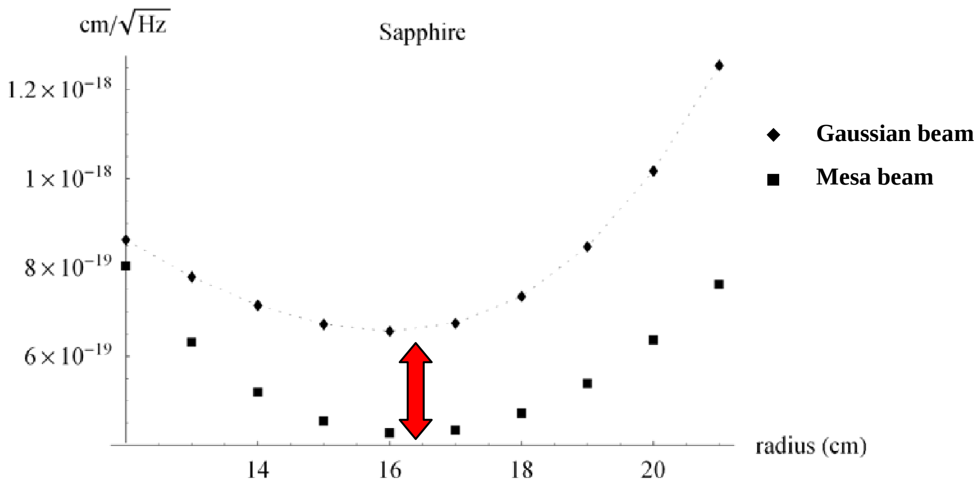


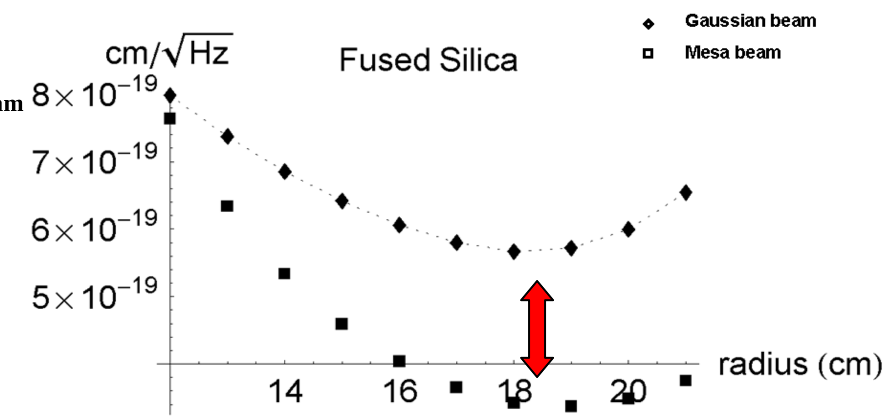
FS	$\sqrt{S_X^{GB}/S_X^{MB}}$
CB	1.7
CT	1.7
SB	1.55
ST	1.92



S	$\sqrt{S_X^{GB}/S_X^{MB}}$
CB	1.6
CT	1.5
SB	1.4
ST	1.4

Comparison between Gaussian and Mesa beam





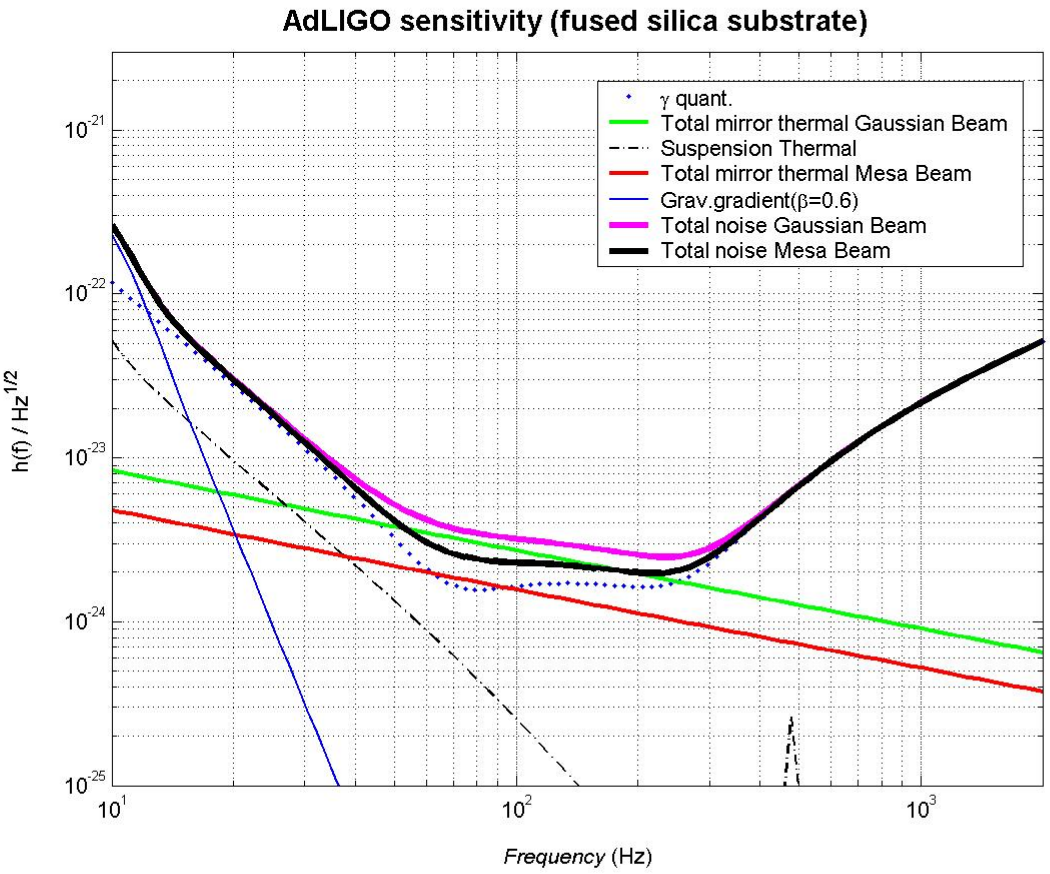
Gain factor ≈ 1.6

$2a/H \approx 2.6$

Gain factor ≈ 1.7

$2a/H \approx 2-2.4$

Sensitivity improvement



	GB	MB
NS-NS range	**177** Mpc	**228** Mpc

Coating Thermo-refractive noise estimation

$$\beta = \frac{dn}{dT}$$

- Infinite mirrors
- Perfect square beam

$$S_X(\omega) = \lambda^2 \beta_{eff}^2 \frac{4k_b T^2 K}{\rho C} \int_{-\infty}^{\infty} dq_z \int_0^{\infty} \frac{q_\perp dq_\perp}{(2\pi)^2} \frac{2q^2}{K^2 q^4 + \omega^2} \frac{1}{1 + q_\perp^2 d^2} \left|\tilde{g}(q_\perp)\right|^2$$

$$\tilde{g}(q_\perp) = 2\pi \int_0^{\infty} r\,dr\, f(r) J_0(q_\perp r)$$

$$\beta_{eff} = \frac{n_1 n_2 (\beta_1 + \beta_2)}{4(n_1^2 - n_2^2)}$$

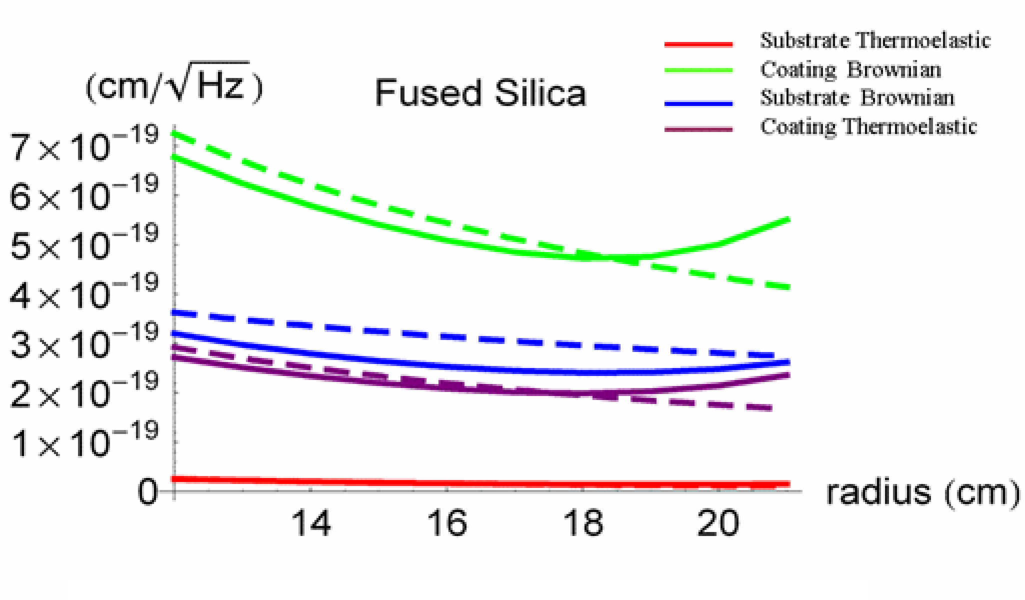
$$f_{FT}(r) = \frac{1}{\pi D^2} \quad for \quad r \le D, \quad 0 \quad for \quad r > D$$

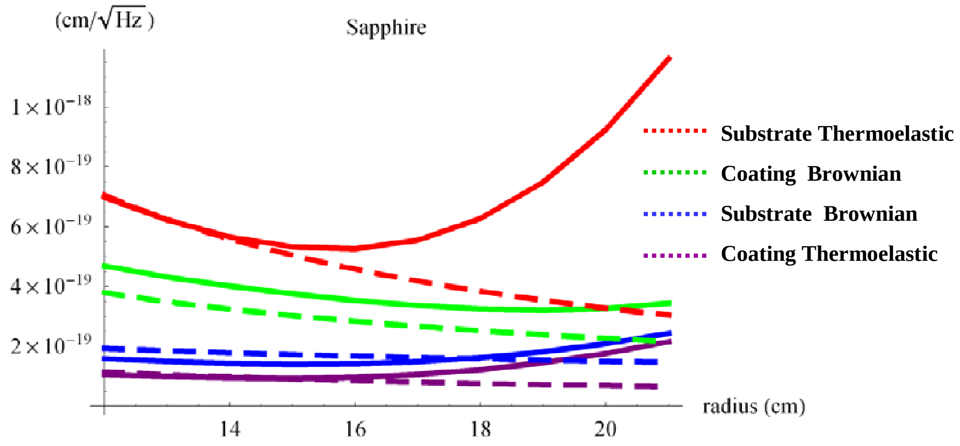
$$\sqrt{\frac{S_X^{GB}}{S_X^{FT}}(f = 100 Hz)} \approx \sqrt{3}$$

$D = 4 w_0$ $\qquad w_0 = 2.6\,cm$

$w = 6\,cm$

Finite size test mass correction for Gaussian Beam





5ppm Diff. loss