Search for unknown spinning Neutron Stars: Optimal Detector Network

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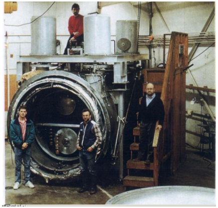
Motivation: which network should we use?



Outline

1. Coherent network detection statistic
2. Parameter-space metric
3. Optimal detector network

Continuous waves from spinning neutron stars

- NS frame: monochromatic wave, slowly varying frequency

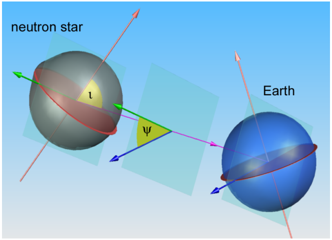
Phase $\Phi(\tau) = \phi_0 + 2\pi \left(f\,\tau + \frac{1}{2}\dot{f}\,\tau^2 + \ldots \right)$

GW frequency for triaxial NS: $f = 2\,\nu$, r-modes: $f = 4/3\,\nu$, precession: $f \approx \nu$

2 polarization amplitudes: $A_+, A_\times$

$\Longrightarrow$ Wave-components in NS frame:

$h_\times(\tau) = A_+ \cos\Phi(\tau)$
$h_+(\tau) = A_\times \sin\Phi(\tau)$



- Detector frame t: sky-position (α, δ) dependent *modulations*:
  - Phase: Doppler-effect due to earth's motion $\tau = \tau(t; \alpha, \delta)$
  - Amplitude: rotating Antenna-pattern $F_{+,\times}(t, \psi; \alpha, \delta)$

Matched Filtering: $\mathcal{F}$-statistic

GW strain at the detector: $s(t; \mathcal{A}, \lambda) = F_+(t)\, h_+(t) + F_\times(t)\, h_\times(t)$

- *Amplitude parameters:* $\mathcal{A}^\mu = \mathcal{A}^\mu\,(A_+, A_\times, \psi, \phi_0)$
- *Doppler parameters:* $\lambda = \{\alpha, \delta, f, \dot{f}, \ldots$ (+ orbital parameters) $\}$

Matched filtering with measured data $x(t)$

detection statistic: $Q(\mathcal{A}, \lambda) \equiv (x|s) - \frac{1}{2}\,(s|s)$

The "$\mathcal{F}$-statistic": partially maximized Q

$$s(t; \mathcal{A}, \lambda) = \sum_{\mu=1}^{4} \mathcal{A}^\mu\, h_\mu(t; \lambda)$$ (Jaranowski, Krolak, Schutz, PRD 1998)

$\Longrightarrow$ *analytically* maximize: ☞ $\mathcal{F}(\lambda) \equiv \max_{\mathcal{A}} Q(\mathcal{A}, \lambda)$

Multi-detector generalization (Cutler&Schutz, PRD 2005)

Single-detector scalar product (narrow-band):

$$(x|y) = S_h^{-1} \int_0^T x(t)\, y(t)\, dt = T\, S_h^{-1} \langle x\, y \rangle$$

Multi-detector scalar product (uncorrelated noises):

$$(\boldsymbol{x}|\boldsymbol{y}) = \sum_{\mathrm{X}} (x^{\mathrm{X}}|y^{\mathrm{X}}) = T\, \widehat{\mathcal{S}}\, \langle x\, y \rangle_S$$

combined "network-sensitivity": $\widehat{\mathcal{S}} \equiv \sum_{\mathrm{X}} S_{\mathrm{X}}^{-1}$

noise-weighted average: $\langle x\, y \rangle_S \equiv \sum_{\mathrm{X}} w_{\mathrm{X}} \langle x^{\mathrm{X}}\, y^{\mathrm{X}} \rangle$

$\mathcal{N}$ equal-noise detectors $\Longrightarrow$ network-sensitivity $\widehat{\mathcal{S}} = \mathcal{N}\, S_h^{-1}$

multi-detector $\mathcal{F}$-statistic: ☞ $\mathcal{F} \sim \mathrm{SNR}^2 \sim h_0^2\, T\, \widehat{\mathcal{S}}\, \langle \ldots \rangle_S$

Number of templates

mismatch $m \equiv \frac{E[\mathcal{F}(0)] - E[\mathcal{F}(\Delta\lambda)]}{E[\mathcal{F}(0)]} = g_{ij}(\mathcal{A}, \lambda)\, \Delta\lambda^i\, \Delta\lambda^j + \ldots$

☞ metric $g_{ij} = \frac{-E[\partial_i \partial_j \mathcal{F}]}{E[\mathcal{F}]} = \frac{\mathrm{Fisher}_{ij}}{\mathrm{SNR}^2}$

$E[\mathcal{F}] \propto h_0^2\, T \widehat{\mathcal{S}}\, \langle \ldots \rangle_S \implies g_{ij}$ is *independent* of $\widehat{\mathcal{S}}$! (RP gr-qc/0606088)

isolated NS: $\lambda^i \in \{\alpha, \delta, f, \dot{f}\}$:

$\partial_i \partial_j \mathcal{F}$: ☞ $\langle \partial_f \phi \rangle \propto \langle \partial_\alpha \phi \rangle \propto \langle \partial_\delta \phi \rangle \propto T$, $\langle \partial_{\dot{f}} \phi \rangle \propto T^2$

$\implies$ Number of templates: $N_p \propto \sqrt{\det g_{ij}} \propto T^5$

BUT N_p does not scale with network-sensitivity $\widehat{\mathcal{S}}$

Optimality criterion for detector network

Optimize sensitivity @ available computing power

$\mathcal{N}$ detectors (noise-floors S^{X}), computing power C_p

☞ Which combination of detectors gives the best sensitivity?

Sensitivity of the search: $\mathrm{SNR}^2 \propto h_0^2 \; T \, \widehat{\mathcal{S}}$

Number of templates: $N_p \propto T^5$ (isolated NS: $\{\alpha, \delta, f, \dot{f}\}$)

Required computing power: $C_p \propto \mathcal{N} \; T^6$

$\Longrightarrow$ longest integration time: $T \propto C_p^{1/6} \mathcal{N}^{-1/6}$

$$\mathrm{SNR} \propto h_0 \; C_p^{1/12} \underbrace{\sqrt{\mathcal{N}^{-1/6} \, \widehat{\mathcal{S}}}}_{\text{maximize}}$$

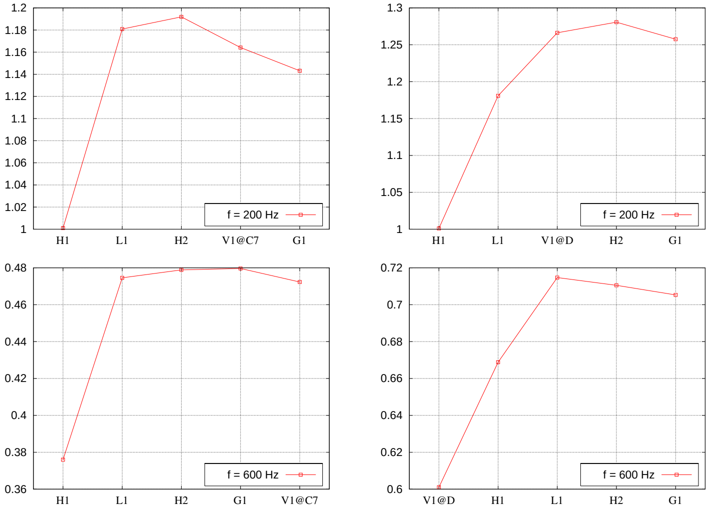
Optimal networks

$$g(\{S_{\mathrm{X}}\}) \equiv \mathcal{N}^{-1/6} \sum_{\mathrm{X}=1}^{\mathcal{N}} S_{\mathrm{X}}^{-1}$$

1. equal-sensitivity detectors: $S^{\mathrm{X}} = S_0 \implies \sum_{\mathrm{X}} S_{\mathrm{X}}^{-1} = \mathcal{N}\, S_0^{-1}$
   ☞ $g = S_0^{-1}\,\mathcal{N}^{5/6} \implies$ use as many IFOs as possible!
2. general case: different S^{X}
   - sort detectors X in order of *decreasing* sensitivity S_{X}^{-1}
   - add detectors as long as $g(\{S_{\mathrm{X}}\})$ increases
3. Example:

$10^{23}\sqrt{S_{\mathrm{X}}}$	H1	L1	H2	V1@C7	V1@D	G1
$f = 200\,\mathrm{Hz}$	3	4	8	80	5	80
$f = 600\,\mathrm{Hz}$	8	9	20	70	5	25

Optimal network: Example



Conclusions

- Metric of the network $\mathcal{F}$-statistic does not scale with $\mathcal{N}$
  (gr-qc/0606088)

- Computational cost is "only" linear in $\mathcal{N}$ (while $\propto T^6$)

- Optimality criterion for coherent network (isolated NS):
  ☞ maximize $\mathcal{N}^{-1/6}\,\widehat{\mathcal{S}}$

- Future work: optimize Hough+$\mathcal{F}$-stat hierarchical search
  number of stacks, lengths of stacks, mismatch per stack, ...