# Observing Gravitational Waves from Spinning Neutron Stars

**LIGO-G060662-00-Z**

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for the LIGO Scientific Collaboration

Toulouse, 6 July 2006

# Outline

# Orders of Magnitude

Quadrupole formula (Einstein 1916).
GW luminosity ($\epsilon$: deviation from axisymmetry):

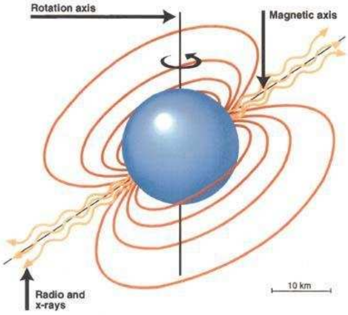
$$L_{\mathrm{GW}} \sim \frac{G}{c^5}\epsilon^2 \left(\frac{M\,V^3}{R}\right)^2 \qquad \frac{G}{c^5}: \mathcal{O}(10^{-53})$$

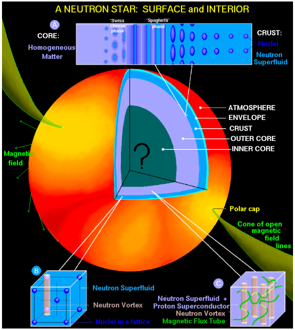
$$= \frac{c^5}{G}\epsilon^2 \left(\frac{R_s}{R}\right)^2 \left(\frac{V}{c}\right)^6 \qquad \frac{c^5}{G}: \mathcal{O}(10^{59})\ \frac{\mathrm{erg}}{\mathrm{s}}$$

Schwarzschild radius $R_s = 2GM/c^2$

☞ Need compact objects in relativistic motion:
Black Holes, Neutron Stars, White Dwarfs

# What is a neutron star?




Mass: $M \sim 1.4\, M_\odot$ Rotation: $\nu \lesssim 700\ s^{-1}$

Radius: $R \sim 10$ km Magnetic field: $B \sim 10^{12} - 10^{14}$ G

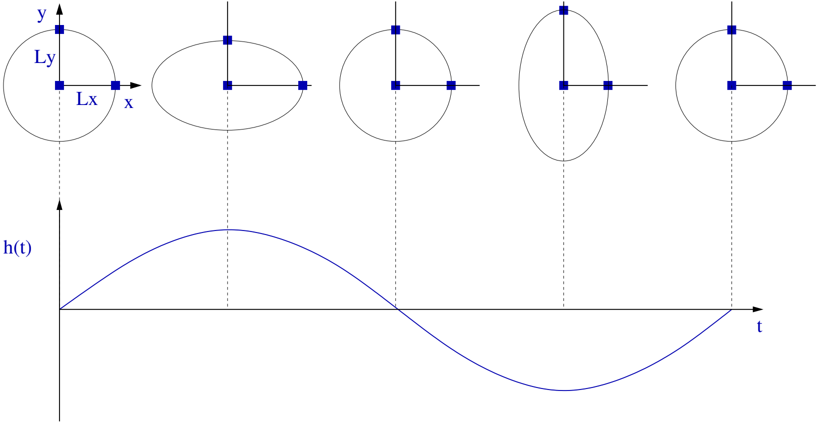
$\Longrightarrow$ density: $\bar{\rho} \gtrsim \rho_{\rm nucl}$

$\Longrightarrow$ relativistic: $\frac{R_s}{R} = \frac{2GM}{c^2 R} \sim 0.4$

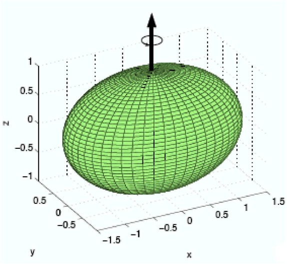
# Gravitational Wave Strain $h(t)$

Plane gravitational wave $h^{+}_{\mu\nu}$ along $z$-direction:

Strain $h(t) \equiv \frac{L_x - L_y}{2L}$:



# Triaxial Spinning Neutron Stars




Rotating neutron star:

- non-axisymmetric $\epsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$
- rotation rate $\nu$

☞ GW with frequency $f = 2\,\nu$

Strain-amplitude $h_0$ on earth:

$$h_0 = \frac{16\pi^2\, G}{c^4}\, \frac{\epsilon\, I_{zz}\, \nu^2}{d}$$

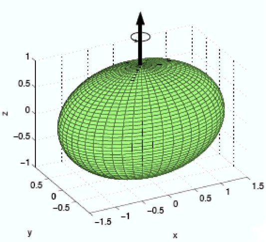
$$= 4 \times 10^{-25} \left(\frac{\epsilon}{10^{-6}}\right) \left(\frac{I_{zz}}{10^{45}\,\mathrm{g\,cm^2}}\right) \left(\frac{\nu}{100\ \mathrm{Hz}}\right)^2 \left(\frac{100\,\mathrm{pc}}{d}\right)$$

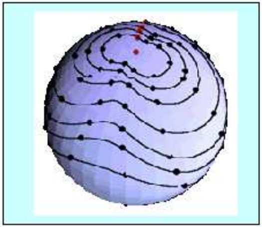
Current LIGO sensitivity (S5): $\sqrt{S_n} \sim 4 \times 10^{-23}\,\mathrm{Hz}^{-1/2}$

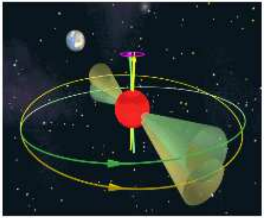
☞ NS signals buried in the noise $\Longrightarrow$ need "matched filtering"

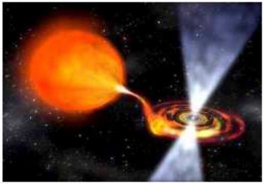
# Possible Emission Mechanisms

- “Mountains”
- Oscillations
- Free precession
- Accretion (driver)



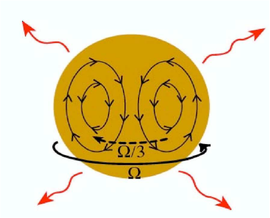




# Neutron Star "Mountains"

- Conventional NS crustal shear mountains:
  ☞ $\epsilon_{\rm crust} \lesssim 10^{-7} - 10^{-6}$ (Ushomirsky, Cutler, Bildsten)
- Superfluid vortices: Magnus-strain deforming crust
  ☞ $\epsilon_{\rm Magnus} \sim 5 \times 10^{-7}$ (D.I. Jones; Ruderman)
- Exotic EOS: strange-quark solid cores
  ☞ $\epsilon_{\rm strange} \lesssim 10^{-5} - 10^{-4}$ (B. Owen)
- Magnetic mountains:
  - large toroidal field $B_t \sim 10^{15}\,\rm G \perp$ to rotation:
    ☞ $\epsilon_{\rm toroidal} \sim 10^{-6}$ (C. Cutler)
  - accretion along $B$-lines $\Longrightarrow$"bottled" mountains
    ☞ $\epsilon_{\rm bottle} \lesssim 10^{-6} - 10^{-5}$ (Melatos, Payne)
  - non-aligned poloidal magnetic field $B \sim 10^{13}\,\rm G$, type-I or type-II superconducting interior,
    $\epsilon_B \lesssim 10^{-6}$ (Bonazzola&Gourgoulhon)

# Oscillation Modes

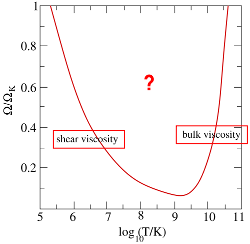



Chandrasekhar-Friedman-Schutz instability:

counter-rotating mode "dragged forward"
$\Longrightarrow$ negative energy and angular momentum

- ☞ emission of GW amplifies the mode
- ☞ counteracted by dissipation

r-mode instability window:

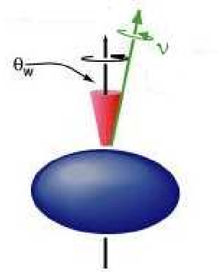



Open questions:

- Dissipation mechanisms: vortex friction, hyperons, crust-core coupling,...
- saturation amplitude, mode-mode coupling, evolution timescales

# Free Precession

"Most general motion of a rigid body" (Landau&Lifshitz 1976)

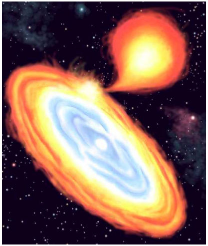


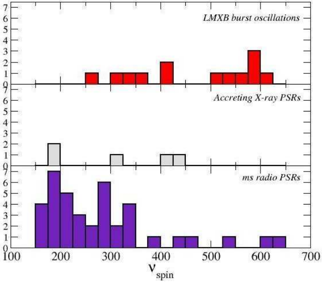

NS are not rigid: coupled crust - core (viscosity + superfluid vortex pinning)

- likely to be damped rapidly
- no obvious instability or "pumping mechanism"

$$h_0 \sim 10^{-26} \left(\frac{\theta_w}{0.1}\right) \left(\frac{100\,\mathrm{pc}}{d}\right) \left(\frac{\nu}{500\,\mathrm{Hz}}\right)^2$$

# Accretion






Breakup-limit $\nu_K \sim 1.5$ kHz ☞ What limits the NS-spin?

Bildsten, Wagoner: Accretion-torque = GW torque ($\propto \nu^5$)

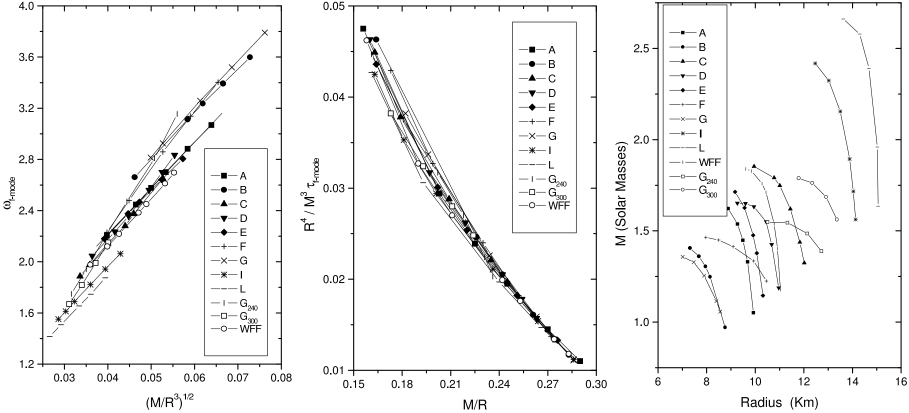
Observed X-ray flux ☞ Sco X-1: $h_0 \sim 3 \times 10^{-26}\,(270\,\mathrm{Hz}/\nu)^{1/2}$

# Astrophysics Summary

- NS are plausible sources for LIGO I, II or VIRGO
- Whether or not they are detectable depends on many poorly-understood aspects of NS physics
- ☞ Any GW-detection from rotating NS will be extremely valuable for NS physics
- ☞ Even the absence of detection can yield astrophysically interesting information (crust deformation, $B$, instabilities)
- NS physics producing GWs is very different and complementary to electromagnetic emission (bulk-mass motion vs magnetosphere-electron motion)

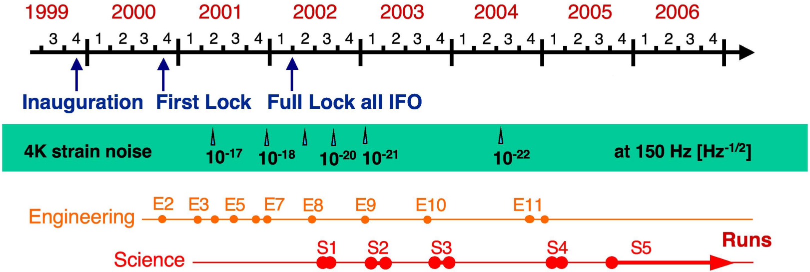
# Gravitational Wave Astronomy

## "Astero-Seismology" (Andersson, Kokkotas 1998): f-mode

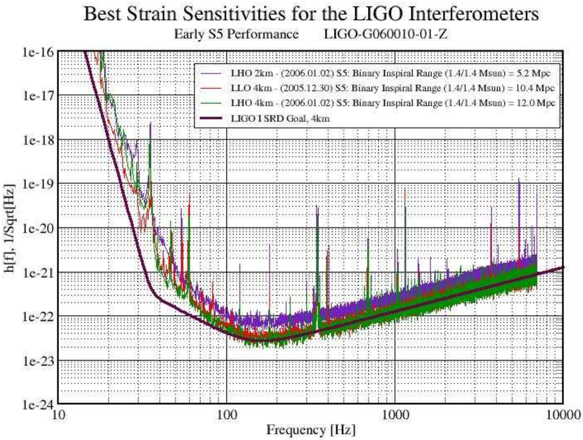



☞ Measurement of $(\omega_f, \tau_f) \Longrightarrow$ deduce $(M, R) \Longrightarrow$ EOS

# LSC detectors: LIGO + GEO600



# Current LIGO noise performance




$$h_0 = \frac{\Delta L}{L} \sim 3 \times 10^{-23} \Longrightarrow \Delta L \sim 10^{-19}\ m = 10^{-4}\ fm!!$$

# LSC Data Analysis

LIGO (H1, H2, L1) and GEO600 data analyzed within the LIGO Scientific Collaboration (LSC):
$\sim$ 40 institutions, $\sim$ 320 authors (S3)

4 major search groups (different targets and methods):

- Binary inspirals: short inspiral signals (*modeled*)
- Bursts: short *unmodeled* signals (supernovae, merger)
- Stochastic background: cosmological background GWs
- "Continuous waves": spinning NS signals (long-lived)

# Nature of GW from Rotating Neutron Stars

- NS frame: monochromatic wave, slowly varying frequency

Phase $\Phi(\tau) = \phi_0 + 2\pi \left( f\,\tau + \frac{1}{2}\dot{f}\,\tau^2 + \ldots \right)$

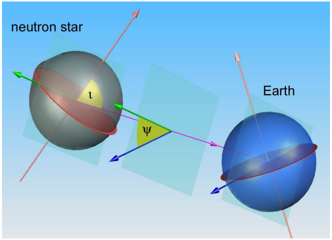
GW frequency for triaxial NS: $f = 2\,\nu$ , r-modes: $f = 4/3\,\nu$, precession: $f \approx \nu$

2 polarization amplitudes: $A_+, A_\times$

$\Longrightarrow$ Wave-components in NS frame:

$h_\times(\tau) = A_+ \cos\Phi(\tau)$

$h_+(\tau) = A_\times \sin\Phi(\tau)$




- Detector frame $t$: sky-position $(\alpha, \delta)$ dependent *modulations*:
  - Phase: Doppler-effect due to earth's motion $\tau = \tau(t; \alpha, \delta)$
  - Amplitude: rotating Antenna-pattern $F_{+,\times}(t, \psi; \alpha, \delta)$

# Signal Received at the Detector

GW strain at the detector:
$h(t) = F_+(t)\, h_+(t) + F_\times(t)\, h_\times(t)$

## Signal dependencies

$$h(t) = F_+(t, \psi; \alpha, \delta)\ A_+\ \cos\left[\phi_0 + \phi(t; \alpha, \delta, f, \dot{f}, ..)\right]$$
$$+ F_\times(t, \psi; \alpha, \delta)\ A_\times\ \sin\left[\phi_0 + \phi(t; \alpha, \delta, f, \dot{f}, ..)\right]$$

Signal parameters:

- 4 *"Amplitude parameters"*: $\mathcal{A}^\mu = \mathcal{A}^\mu\,(A_+, A_\times, \psi, \phi_0)$
- *"Doppler parameters"*: $\lambda = \{\alpha, \delta, f, \dot{f}, \ldots$ (+ orbital parameters) $\}$

# Optimal detection statistic: "Matched filtering"

Measured strain: $\overbrace{x(t)}^{\text{data}} = \overbrace{n(t)}^{\text{noise}} + \overbrace{s(t; \mathcal{A}, \lambda)}^{\text{signal}}$

scalar product: $$(x|y) \equiv \int \frac{\widetilde{x}(f)\, \widetilde{y}^*(f)}{S_n(f)}\, df$$

pdf for Gaussian noise $n(t)$: $P(n(t)|S_n) = k\, e^{-\frac{1}{2}(n|n)}$

$\Longrightarrow$ likelihood of $x(t)$ in presence of signal $s(t; \mathcal{A}, \lambda)$:

$$P(x(t)|\mathcal{A}, \lambda; S_n) = k\, e^{-\frac{1}{2}(x|x)}\, e^{(x|s)-\frac{1}{2}(s|s)}$$

Bayesian posterior probability for signal $\{\mathcal{A}, \lambda\}$ in data $x(t)$:

$$P(\mathcal{A}, \lambda|x(t); S_n) = k' \quad \underbrace{P(\mathcal{A}, \lambda)}_{\text{"prior" probability}} \quad e^{(x|s)-\frac{1}{2}(s|s)}$$

# Matched filtering II: the $\mathcal{F}$-statistic

detection statistic: $Q(\mathcal{A}, \lambda) \equiv (x|s) - \frac{1}{2}(s|s)$

☞ find **maximum** of $Q$ in *parameter space* $\{\mathcal{A}, \lambda\}$.

$$s(t; \mathcal{A}, \lambda) = \sum_{\mu=1}^{4} \mathcal{A}^\mu \, h_\mu(t; \lambda) \quad \text{(Jaranowski, Krolak, Schutz, PRD 1998)}$$

$\Longrightarrow$ analytically maximize $Q$ over $\mathcal{A}^\mu$: $\frac{\partial Q}{\partial \mathcal{A}^\mu} = 0 \Longrightarrow \mathcal{A}^\mu{}_{\mathrm{MLE}}$

**Definition of the "$\mathcal{F}$-statistic": $\mathcal{F} = Q(\mathcal{A}_{\mathrm{MLE}}, \lambda)$**

$$2\mathcal{F}(\lambda) = x_\mu \, \mathcal{M}^{\mu\nu} \, x_\nu$$

where $x_\mu(\lambda) \equiv (x|h_\mu(\lambda))$, and $\mathcal{M}^{\mu\nu}(\lambda) = (h_\mu(\lambda)|h_\nu(\lambda))^{-1}$

☞ find maximum of $\mathcal{F}$ in reduced parameter-space $\{\lambda\}$.

# Matched filtering III: multi-detector generalization

multi-detector vector $\{\boldsymbol{x}(t)\}^{\mathrm{X}} = x^{\mathrm{X}}(t)$ with $\mathrm{X} \in \{\mathrm{H1}, \mathrm{L1}, \mathrm{V1}...\}$

$$(\boldsymbol{x}|\boldsymbol{y}) = \int \widetilde{x}^{\mathrm{X}}(f)\, S^{-1}_{\mathrm{XY}}\, \widetilde{y}^{\mathrm{Y}*}(f)\, df$$

$x_\mu(\lambda) = (\boldsymbol{x}|\boldsymbol{h}_\mu), \qquad \mathcal{M}^{\mu\nu}(\lambda) = (\boldsymbol{h}_\mu|\boldsymbol{h}_\nu)^{-1}$
$\Longrightarrow \; 2\mathcal{F}(\lambda) = x_\mu\, \mathcal{M}^{\mu\nu}\, x_\nu$ (Cutler&Schutz, PRD 2005)

## Signal-to-noise ratio @ perfect match

$$\mathrm{SNR} = \sqrt{(\boldsymbol{s}|\boldsymbol{s})} \propto \frac{h_0}{\sqrt{S_n}} \sqrt{T\,\mathcal{N}}$$

$T$ ... observation time
$\mathcal{N}$ ... equal-noise detectors

$h_0/\sqrt{S_n} \ll 1$ ☞ need long $T$ (and many detectors $\mathcal{N}$)

# Matched filtering IV: parameter-space covering

**The *covering problem***

Choose a finite number $N_p$ of "templates" $\lambda_{(k)}$, such that

1. never lose more than a fraction $m$ at closest template $\lambda_{(k)}$
2. $N_p$ is the smallest possible number satisfying 1

Relative loss in mismatched $\mathcal{F}(\lambda)$ at $\lambda = \lambda_{\rm sig} + \Delta\lambda$:

$\mathcal{F}(\lambda) = \mathcal{F}(\lambda_{\rm sig})\left(1 - g_{ij}\,\Delta\lambda^i\Delta\lambda^j + ..\right) \implies$ "metric" $g_{ij}$

$$N_p \propto \int_{\{\lambda\}} \sqrt{\det g_{ij}}\; d^n\lambda$$

☞ isolated NS $\lambda^i = (\alpha, \delta, f, \dot{f})$:

$N_p \propto T^5$... but NO scaling with $\mathcal{N}$! (R. Prix, gr-qc/0606088)

Computing "cost": $C_p \propto \mathcal{N}\, T^6$

# Cost-benefit example: LIGO + VIRGO

Assume similar sensitivity H1 $\sim$ L1 $\sim$ V1

$N_p \propto T^5$    $C_p \propto \mathcal{N}\, T^6$    $\mathrm{SNR} \propto \sqrt{\mathcal{N}}\, T$

| Det | T | SNR | $C_p$ |
|---|---|---|---|
| H1+L1 | $T_0$ | $\rho_0$ | $C_0$ |
| H1+L1+V1 | $T_0$ | $1.22\,\rho_0$ | $1.5\,C_0$ |
| H1+L1 | $\frac{3}{2}\,T_0$ | $1.22\,\rho_0$ | $11.4\,C_0$ |
| V1 | $2\,T_0$ | $\rho_0$ | $32\,C_0$ |
| V1 | $3\,T_0$ | $1.22\,\rho_0$ | $364\,C_0$ |

Combining (similar-sensitivity) detectors is the computationally cheapest way to increase sensitivity!

(at fixed computing power $\Longrightarrow$ highest sensitivity)

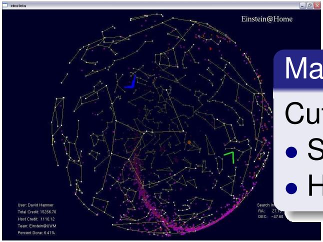
# Search Strategies

- Wide-parameter searches for unknown NS:
  Need to scan space of Doppler-parameters $\lambda$ (but not $\mathcal{A}$)
  e.g. isolated NS $(\alpha, \delta, f, \dot{f})$: number of templates $N_p \propto T^5$

1. Fully coherent: $\mathcal{F}$-statistic (Einstein@Home $T \lesssim 30$ hours)
   ☞ optimal sensitivity @ *infinite* computing power
2. Semi-coherent: Hough, StackSlide, PowerFlux ($T \sim$ data)
   ☞ sub-optimal but fast
3. Hierarchical search: combine 1 + 2, will run on E@H
   ☞ optimal sensitivity @ *finite* computing power

- Targeted searches for known pulsars ($f = 2\nu$)
  ☞ only *one* template $\lambda_0 = \{\alpha, \delta, f, \dot{f}, ..\}$ from radio/X-ray
  Fully coherent, not computationally limited ($T \sim$ data),
  $\Longrightarrow$ most sensitive search!

# Einstein@Home: Search for Unknown NS



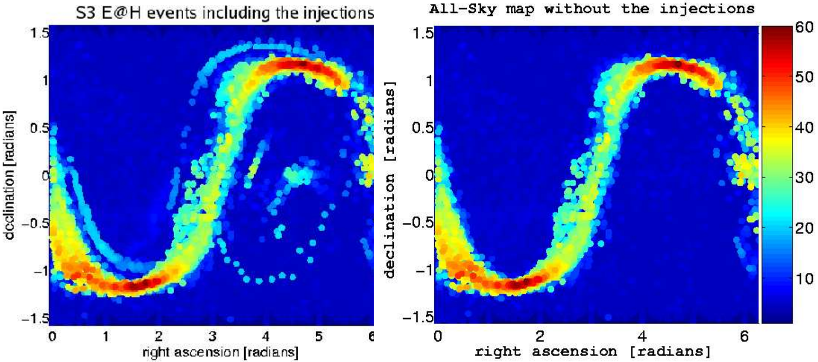

**Maximize available computing power**

Cut parameter-space $\lambda$ in small pieces $\Delta\lambda$

- Send workunits $\Delta\lambda$ to participating hosts
- Hosts return finished work and request next

- Public distributed computing project, launched Feb. 2005
- Currently $\sim$120,000 active participants, $\sim$50Tflops
- runs on GNU/Linux, Mac OSX, Windows,..
- Search for isolated neutron stars $f \in [50, 1500]$ Hz
- Aiming for detection, not upper limits
- Analyzed data from S3, S4, just started: S5

# Einstein@Home S3 results




- correctly identified injections ($h_0 \sim 10^{-23}$)
- all "outliers" either on $\boldsymbol{r}(t) \cdot \boldsymbol{n} = 0$ circles (☞ stationary lines), or ruled out by follow-up studies (S4)

# Wide-Parameter Searches: (Best) Upper Limits

- Fully coherent ($\mathcal{F}$-statistic) searches `[gr-qc/0605028]`:

S2 Sco X-1 (unknown $f, a_p, \bar{T}$), using $T = 6\ h$ of S2
☞ $h_0^{95\%} \sim 2 \times 10^{-22}$

S2 All-sky, isolated NS, ($f \in [160, 728]\ \mathrm{Hz}$), using $T = 10\ h$ of S2
☞ $h_0^{95\%} \sim 7 \times 10^{-23}$

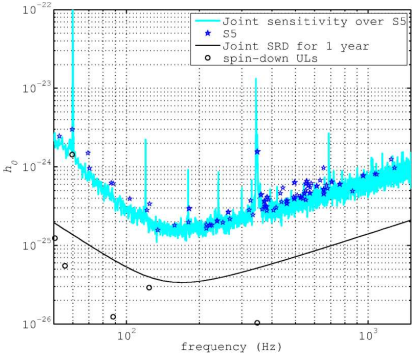
- Semi-coherent searches:

S2 Hough-transform: all-sky, isolated NS ($f \in [200, 400]\ \mathrm{Hz}$)
☞ $h_0^{95\%} \sim 4.5 \times 10^{-23}$

S4 StackSlide: all-sky, isolated NS ($f \in [50, 225]\ \mathrm{Hz}$)
☞ $h_0^{95\%} \sim 4.5 \times 10^{-24}$ *(preliminary)*

Early S5 PowerFlux: all-sky, isolated NS ($f \in [40, 700]\mathrm{Hz}$)
☞ $h_0^{95\%} \sim 2 \times 10^{-24}$ *(preliminary)*

# Targeted Pulsar Search: Early S5 (*preliminary*)




- Targeted 73 pulsars ($f = 2\nu$):
  32 isolated, 41 binary (29 in GCs)
- first 2 months of S5
- all 3 detectors: H1, H2, L1
- Best 95% upper limits:
  $h_0 \lesssim 2 \times 10^{-25}$ (PSR J1603-7202)
  $\epsilon \lesssim 4 \times 10^{-7}$ (PSR J2124-3358)

Upper-limits well above *spindown-limit* (except in GCs)

*But:* Crab-pulsar is only a factor 2.1 away from spindown-limit

☞ will (most likely) be able to beat spindown-limit during S5!

# Published results

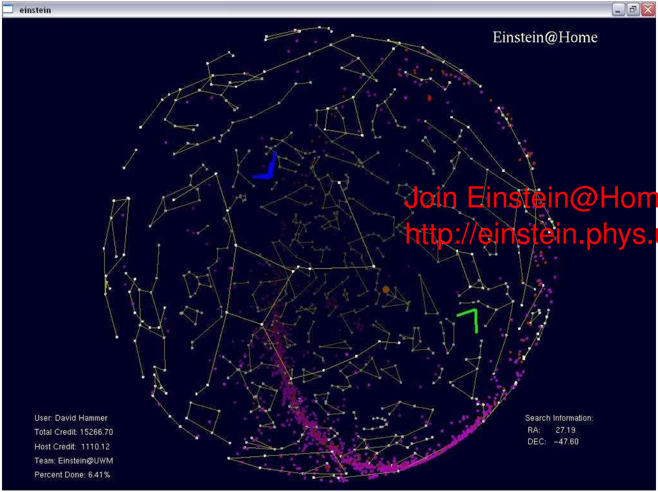
## Published LSC results of neutron-star searches:

- S1 Setting upper limits on the strength of periodic gravitational waves from PSR J1939 + 2134 using the first science data from the GEO 600 and LIGO detectors, B. Abbott et al. (LSC), Phys. Rev. D 69, 082004 (2004)
- S2 Limits on gravitational wave emission from selected pulsars using LIGO data, B. Abbott et al. (LSC), Phys. Rev. Lett. 94, 181103 (2005)
- S2 First all-sky upper limits from LIGO on the strength of periodic gravitational waves using the Hough transform, B. Abbott et al. (LSC), Phys. Rev. D 72, 102004 (2005)
- S2 Coherent searches for periodic gravitational waves from unknown isolated sources and Scorpius X-1: results from the second LIGO science run, to be submitted, `[gr-qc/0605028]`
- S3 Online report on Einstein@Home results for S3 search: `http://einstein.phys.uwm.edu/PartialS3Results/`

# Summary and outlook

- No GW detection so far, but none expected
  ☞ setting upper limits on $h_0$ and $\epsilon$
- S5 upper-limits are approaching astrophysically relevant regimes (☞ Crab, EOS-limits on $\epsilon$)
- LIGO S5 operating at design-sensitivity, will collect one year's worth of data (duration $\sim$1.5 years)
- Einstein@Home: Started analyzing S5.
  Developing a fully hierarchical search ☞ most sensitive possible search for unknown NS
- NS detection with LIGO-I not very likely, but not impossible ("Expect the unexpected!")
- The future is bright: S6, VIRGO, LIGO-II, GEO-HF, ...

# You can help us find Gravitational Waves!




Join Einstein@Home at:
http://einstein.phys.uwm.edu

## The "spindown-limit" (for known pulsars)

Energy lost in GW: $\frac{dE_{\mathrm{GW}}}{dt} \propto \nu^6 \, I_{zz}^2 \, \epsilon^2$

Rotational energy: $\frac{dE_{\mathrm{rot}}}{dt} \propto I_{zz} \underbrace{\nu \, \dot{\nu}}_{\text{observed}}$

### Spindown limit

$$\frac{dE_{\mathrm{GW}}}{dt} \leq \frac{dE_{\mathrm{rot}}}{dt} \quad \Longrightarrow \quad \text{upper limit on } \epsilon \text{ and } h_0$$

☞ limit on deformation $\epsilon$ and amplitude $h_0$:

$$\epsilon_{\mathrm{sd}}^2 \propto \frac{1}{I_{zz}} \frac{\dot{\nu}}{\nu^5} \quad , \qquad h_{\mathrm{sd}} \propto \frac{\sqrt{I_{zz}}}{d} \sqrt{\frac{\dot{\nu}}{\nu}}$$