Data Analysis Techniques for LIGO

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Lesson Plan

Today:

1. Introducing the problem: GW and LIGO
2. Search for Continuous Waves
3. Search for Stochastic Background

Tomorrow:

4. Search for Binary Inspirals
5. Search for Bursts
6. Network Analysis

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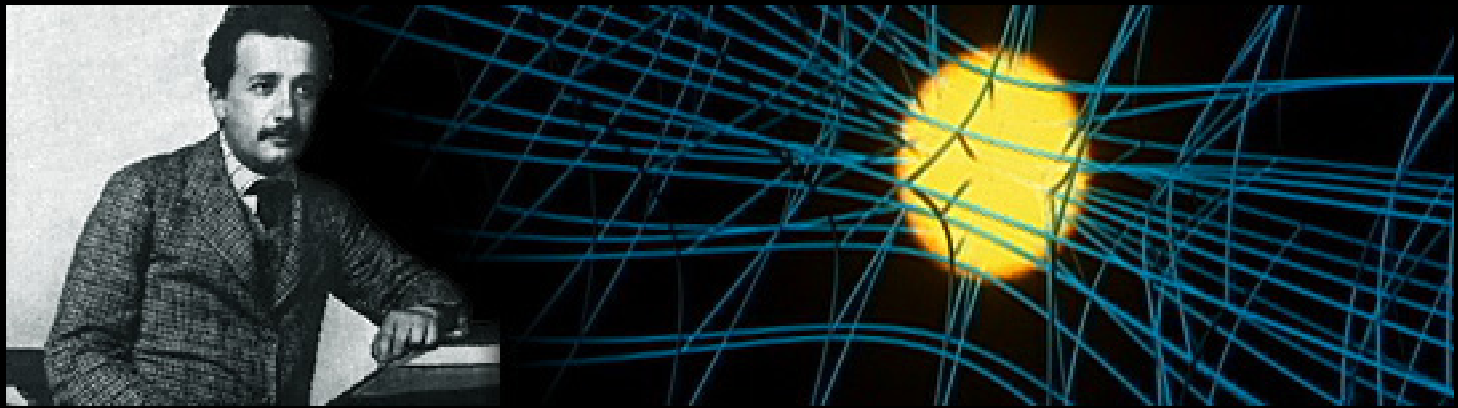
Supporting Material

- Last week's Colloquium slides are available in the LIGO Document Control Center: www.ligo.caltech.edu/dcc/G/G070030-00/
- General introductory material on LIGO:
  - American Museum of Natural History project on Gravitational Waves: http://sciencebulletins.amnh.org/astro/f/gravity.20041101/
  - "Einstein's Messengers" video: http://www.ligo.caltech.edu/einstein.ram
  - Einstein@home: www.einsteinathome.org
- Collaboration web sites: www.ligo.caltech.edu and www.ligo.org
- LIGO documents are available in the LIGO Document Control Center: http://admdbsrv.ligo.caltech.edu/dcc/
- Syllabus: www.ligo.mit.edu/~cadonati/DA-Trento07/DA.html

Gravitational Waves



Graphics from the American Museum of Natural History GW project

$$G_{\mu\nu}=8\pi T_{\mu\nu}$$

- General Relativity explains gravity in terms of the geometry of space-time
- Gravitational waves are plane-wave solution to Einstein's equations
- Gravitational waves are radiative solutions for perturbations on a background spacetime that need not be flat.
- Ripples of space-time that carry information on changes of gravitational field (i.e. of spacetime curvature)
- They travel at the speed of light

Gravitational Waves

- Curvature is small everywhere except Big Bang and black hole horizon: this is where GR gets non-linear
- Elsewhere: weak-field approximation is valid

$$\delta s^2 = g_{\mu\nu}\delta x^\mu \delta x^\nu$$

Distance between events. $g_{\mu\nu}$: metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad \text{with } |h_{\mu\nu}| \ll 1$$

Weak field approximation

$$\eta_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Minkowski metric of flat space
(special relativity)
$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$
(c=1)
$\mu,\nu = 0,1,2,3$

$$h_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

In a particular choice of coordinates: Transverse Traceless Gauge

Einstein's field equations in vacuum.

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right) h_{\mu\nu} = 0$$

Solution: plane waves

$$h_+(t - z/c) + h_\times(t - z/c) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{(i\omega t - ikx)}$$

Let's assume two free masses are placed at positions *x*1 and *x*2 (*y* = 0) and a gravitational wave with + polarization is propagating along the *z*-axis.

The free masses will stay fixed at their coordinate positions, but the space in between (and therefore the distance between *x*1 and *x*2) will expand and shrink at the frequency of the gravitational wave. Similarly, along the *y*-axis the separation of two points will decrease and increase with opposite sign.

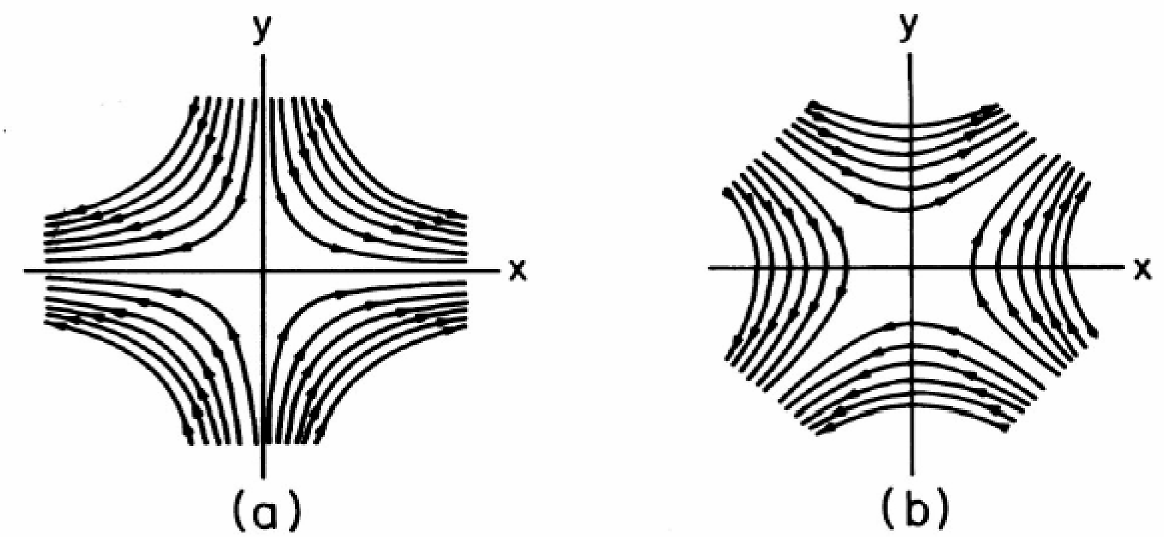
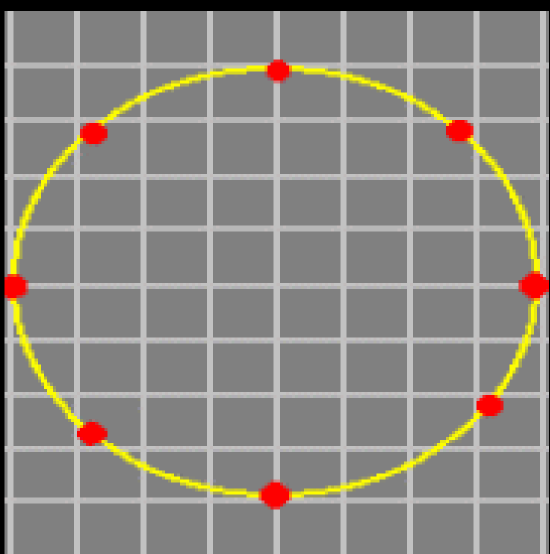


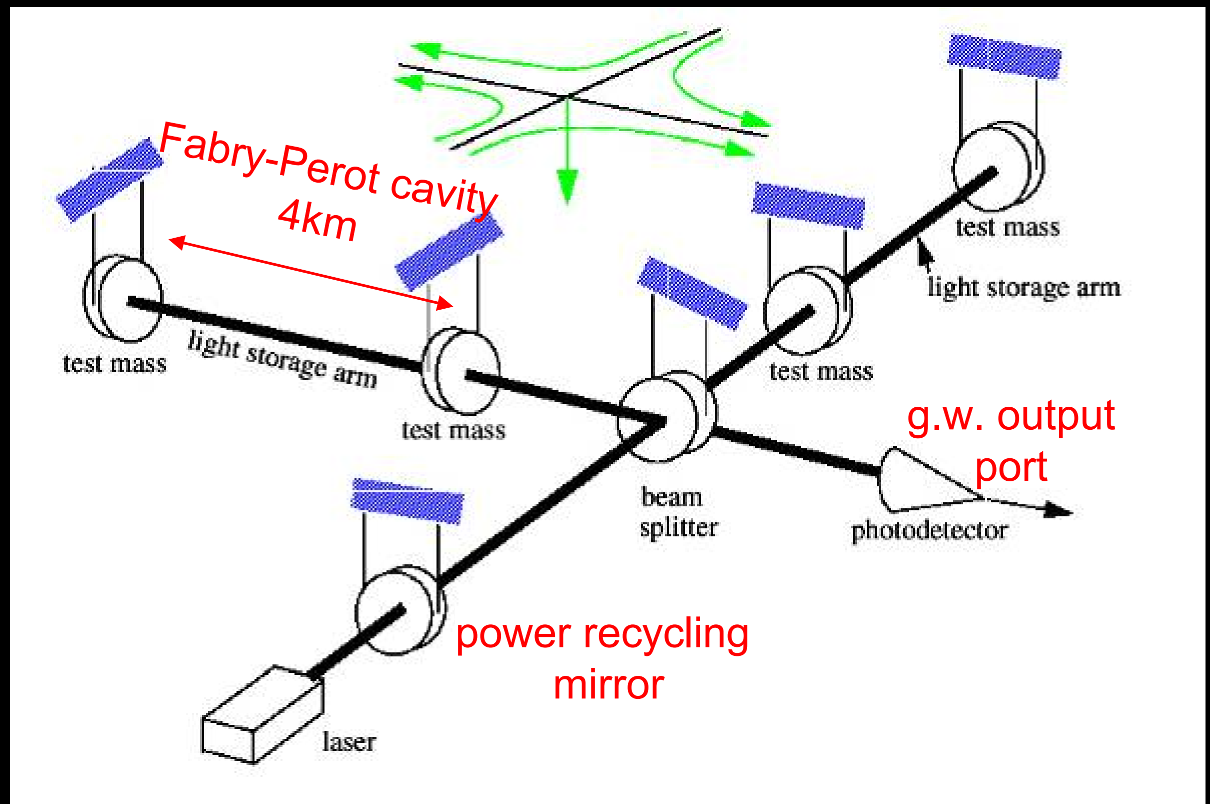
Figure 1. Direction of space deformation for a gravitational wave propagating along the z-axis, + polarization (a) and × polarization (b).

The strength of a gravitational wave is then best expressed as a dimension-less quantity, the strain *h* which measures the relative length change $\Delta L/L$.

Interferometric Gravitational Wave Detection

Suspended mirrors in “free-fall”





LIGO design length sensitivity: 10^{-18} m

Effect of a GW on the Arms of an Interferometer

Consider the round trip travel T time of a photon traveling between two freely falling test masses as measured by an observer fixed to one of the masses, in the TT gauge

$$ds^2=(\eta_{00}+h_{00})dt^2+(\eta_{11}+h_{11})dx^2=-dt^2+(1+h_+)dx^2=0$$

$$dt^2 = (1+h_+)dx^2$$

$$T = 2\int_0^L (1+h_+)^{1/2}dx$$

$$T \simeq 2(1+\frac{h_+}{2})L$$

Assuming $h_+ << 1$ and approx constant over the round trip travel time of the photon.

$$\Delta T = \frac{h_+ L}{c}$$

Additional round-trip time of the photon, due to GW

$$\Delta L = \frac{h_+}{2}L$$

Change in proper interval between test masses evaluated at fixed coordinate time. For the y coordinate: same but opposite sign. So:

$$|L_x\text{-}L_y|_+=h_+L$$

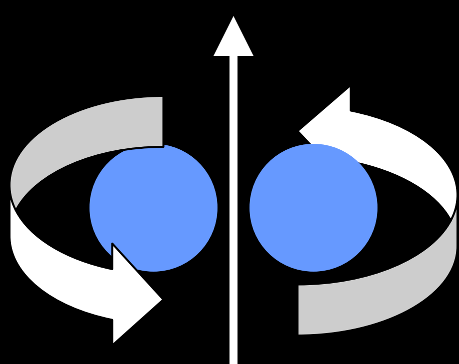
Generation of GWs

Gravitational waves are generated by the motion of matter.
Energy conservation, like charge conservation, rules out the possibility of monopolar sources of gravitational radiation.
In contrast to electromagnetic waves, there is only one type of gravitational "charge"
⇒ there is no gravitational radiation from dipolar motions of matter (conservation of linear and angular momentum).
The first radiative term in the multipole expansion of a time varying mass distribution is due to the quadrupole moment

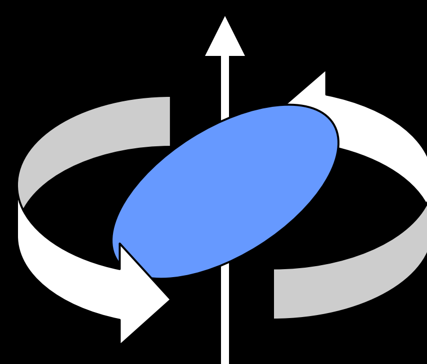
$$I_{\mu\nu} = \int_V \left(x_{\mu\nu} - \frac{1}{3}\delta_{\mu\nu} r^2 \right) \rho(\mathbf{r}) d^3 r$$

$$h_{\mu\nu} = \frac{2G}{rc^4} \frac{d^2 I_{\mu\nu}}{dt^2}$$

gravitational wave strain amplitude
Inversely proportional to the distance r from the source



Need quadrupolar motion of matter



Setting the Scale

- Strongest signal produced by relativistic motion of massive objects at distance d:

$$h \lesssim \frac{1}{d}\frac{2GM}{c^2} \lesssim 10^{-19}\left(\frac{M}{M_\odot}\right)\left(\frac{d}{\mathrm{Mpc}}\right)^{-1}$$

- A tiny effect, and these assumptions are very optimistic (all rest mass emitted as GW):

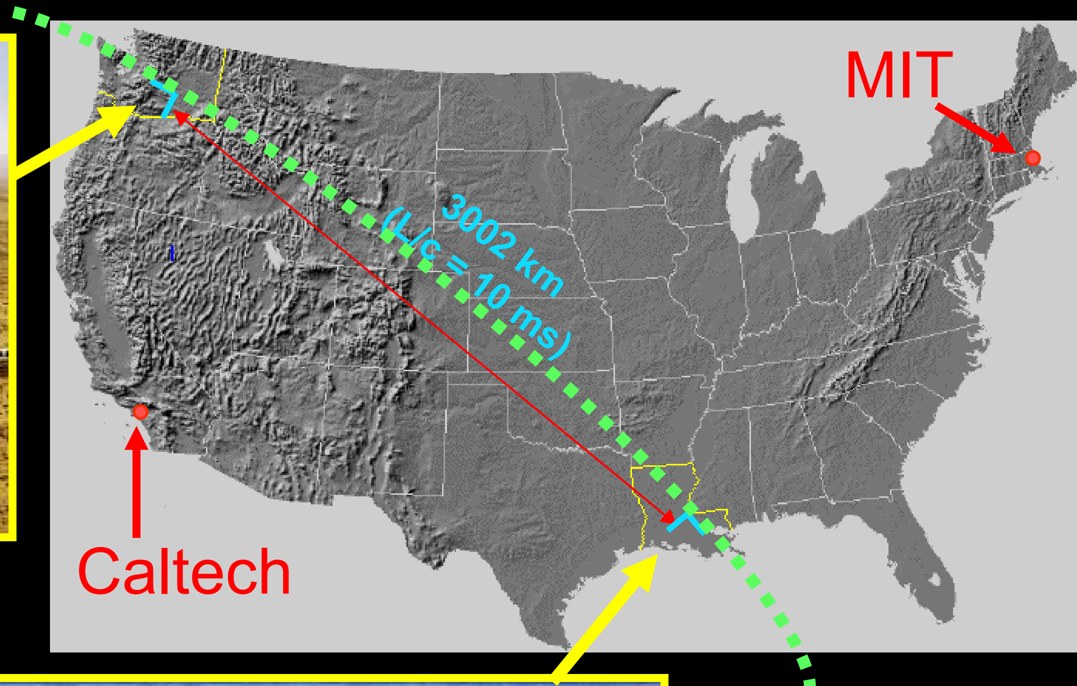
$$\frac{d^2 I_{\mu\nu}}{dt^2} \sim Mc^2$$

Sources need to be astrophysical!

- Maximum oscillation frequency of the source is limited by the round trip light travel time across its extent, limited by the Schwarzschild radius ($2GM/c^2$):

$$f \lesssim \frac{c^3}{4\pi GM} \sim 16\left(\frac{M}{M_\odot}\right)^{-1} \quad \mathrm{kHz}$$

Laser Interferometer Gravitational-wave Observatory



- Managed and operated by Caltech & MIT with funding from NSF
- Ground breaking 1995
- 1st interferometer lock 2000
- LIGO Scientific collaboration: 45 institutions, world-wide

LIGO-G070046-00

Interferometers are Giant "Ears"

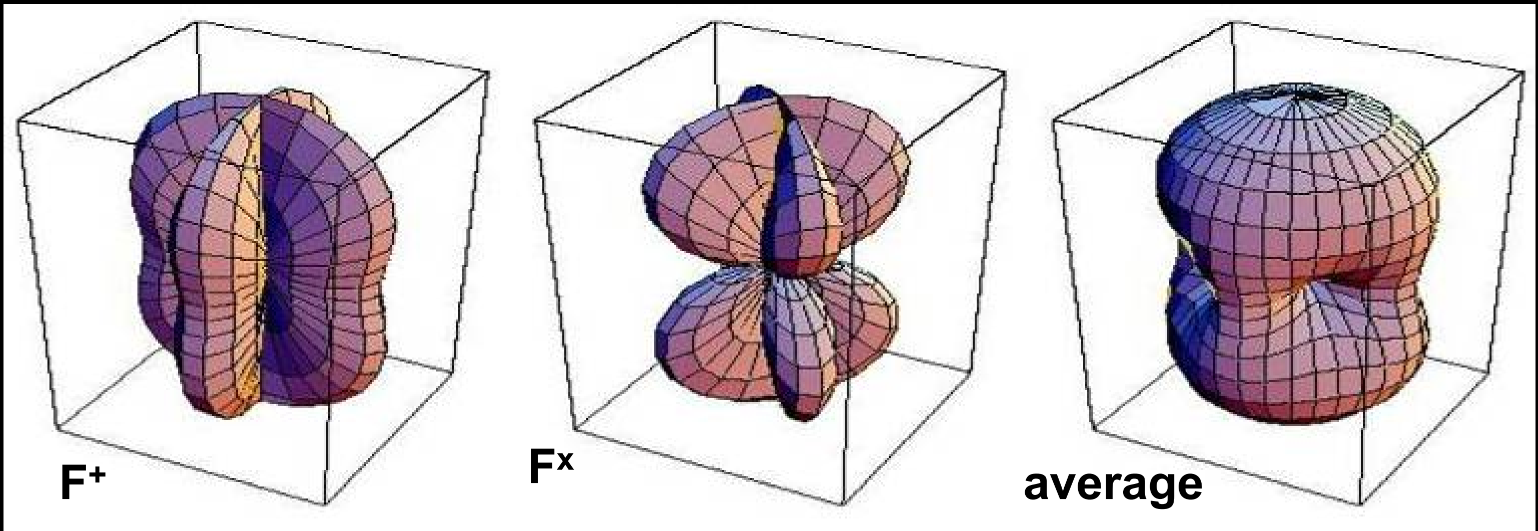
Response of x and y arms to a GW from arbitrary direction

$$h_{xx} = -\cos(\theta)\sin(2\phi)h_\times + (\cos^2(\theta)\cos\phi^2 - \sin\phi^2)h_+$$

$$h_{yy} = \cos\theta\sin 2\phi h_\times + (\cos\theta^2\sin\phi^2 - \cos\phi^2)h_+$$

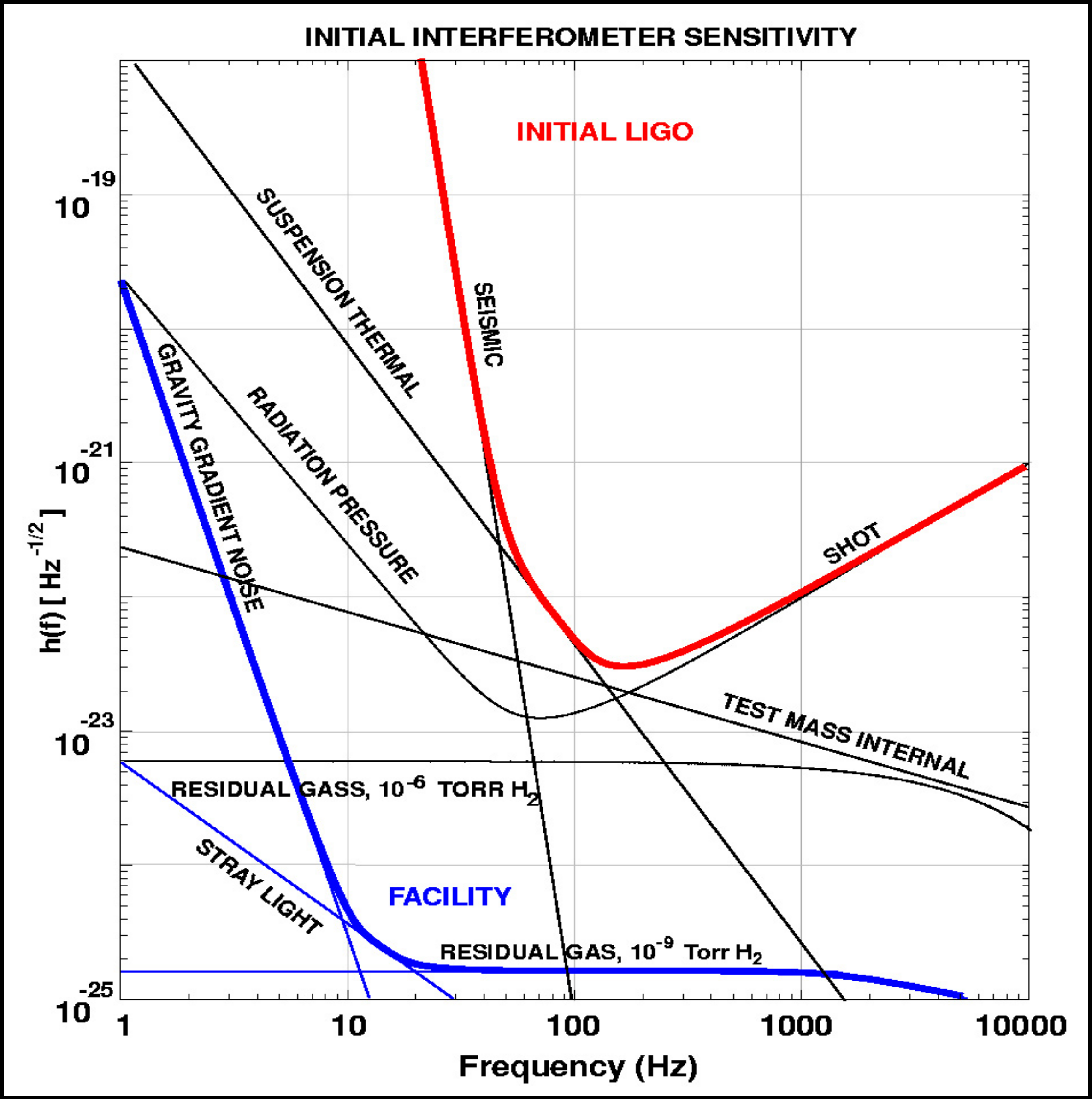
$$|h_{yy} - h_{xx}|$$

$$\frac{\delta L(t)}{L} = h(t) = F^+ h_+(t) + F^\times h_\times(t)$$



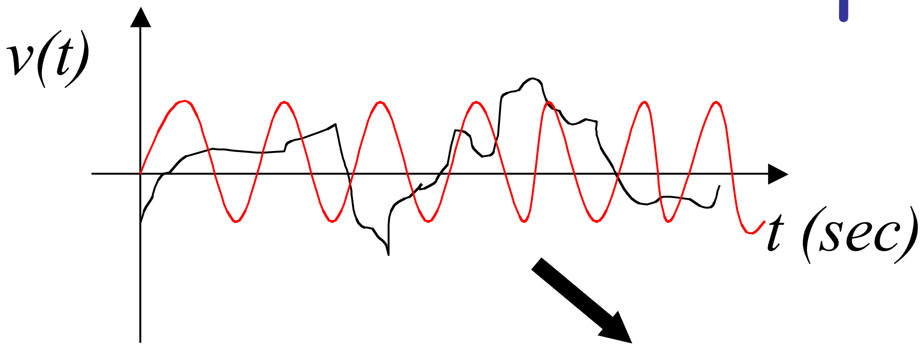
- Interferometers have a broad antenna pattern
  - Cannot locate direction of the source with a single detector
  - Can scan large portions of the sky simultaneously

Initial LIGO Sensitivity Goal



What's the vertical axis in this plot?

Power Spectra



Energy ~ $\langle v^2 \rangle \, \Delta t$

Power ~ $\langle v^2 \rangle$

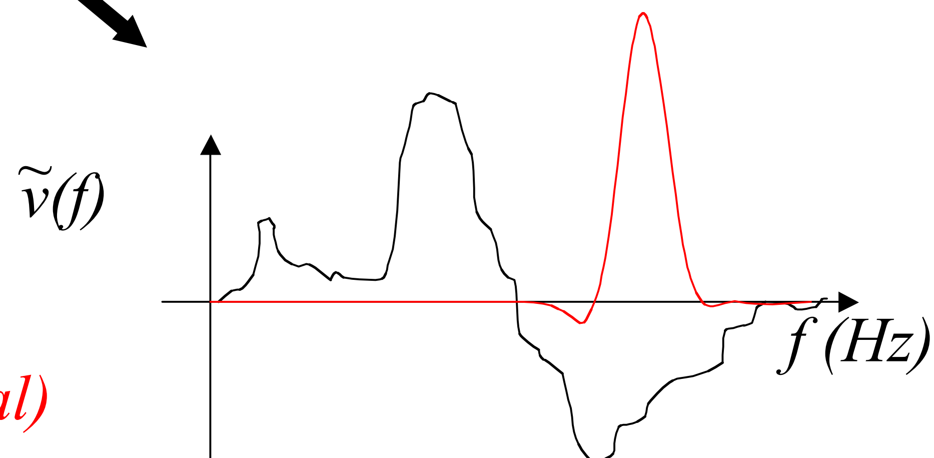
(energy per unit time)

Fourier transform

Spectral Density

$$S_v(f) = \lim_{T \to \infty} \frac{1}{T} \left| \int_{-T}^{T} v(t) e^{-i2\pi f t} dt \right|^2$$

(energy per unit frequency interval)



$$\int_0^\infty df \, S_v(f) = \int_0^\infty dt \, v^2(t)$$

Power Spectra (2)

- White noise: $S(f)$ = constant
- Sometimes we talk about the "root mean square" or RMS of a process;

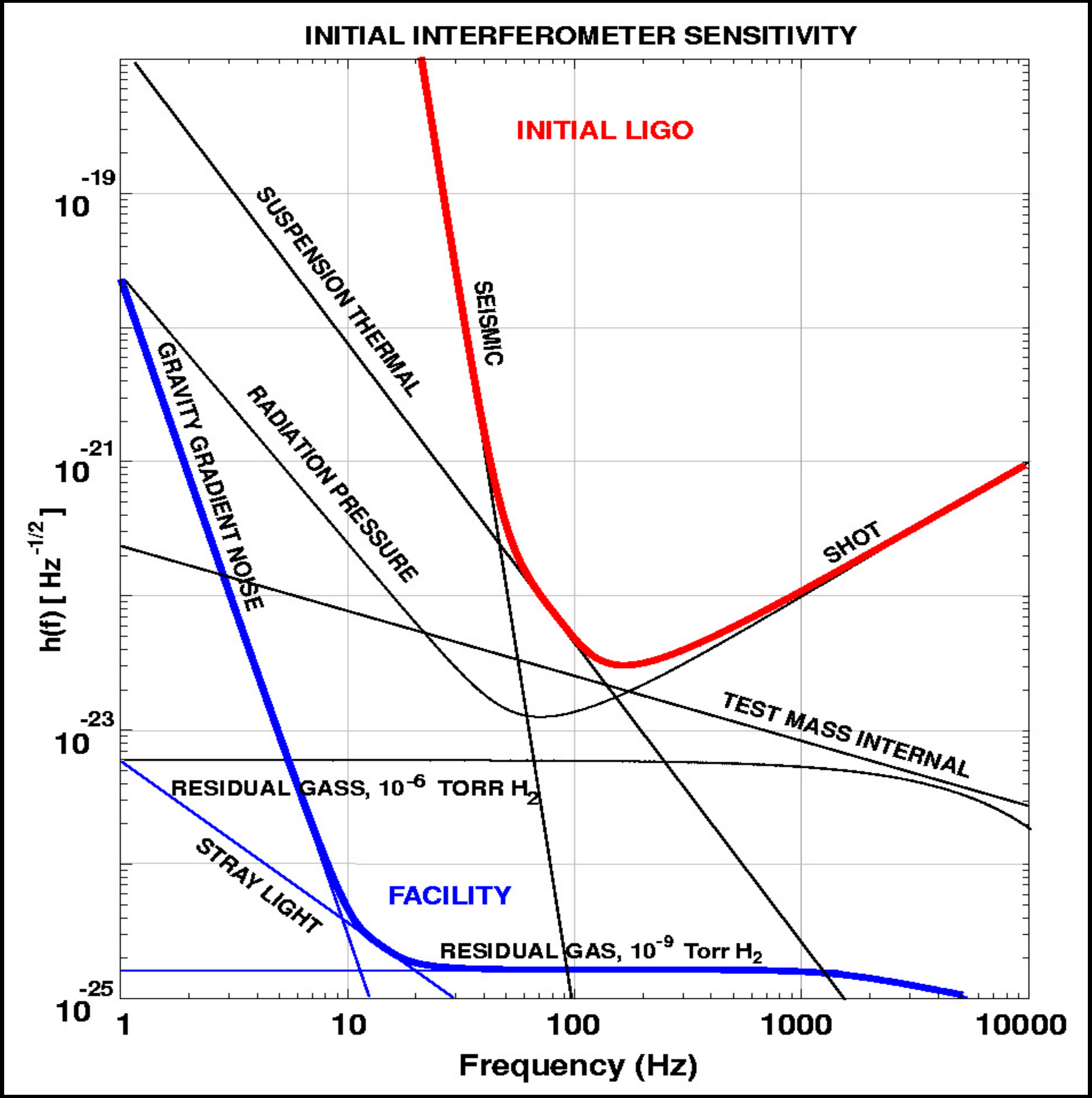
$$v_{RMS} = \sqrt{< v^2(t) >}$$

- Similarly, we can talk about an "amplitude spectral density"

$$\hat{v}(f) = \sqrt{S_v(f)}$$

- It will have units of $\dfrac{[v]}{\sqrt{\mathrm{Hz}}}$

Initial LIGO Sensitivity Goal

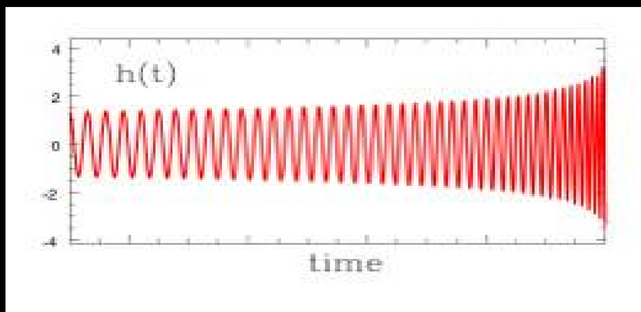


Dominant noise sources:

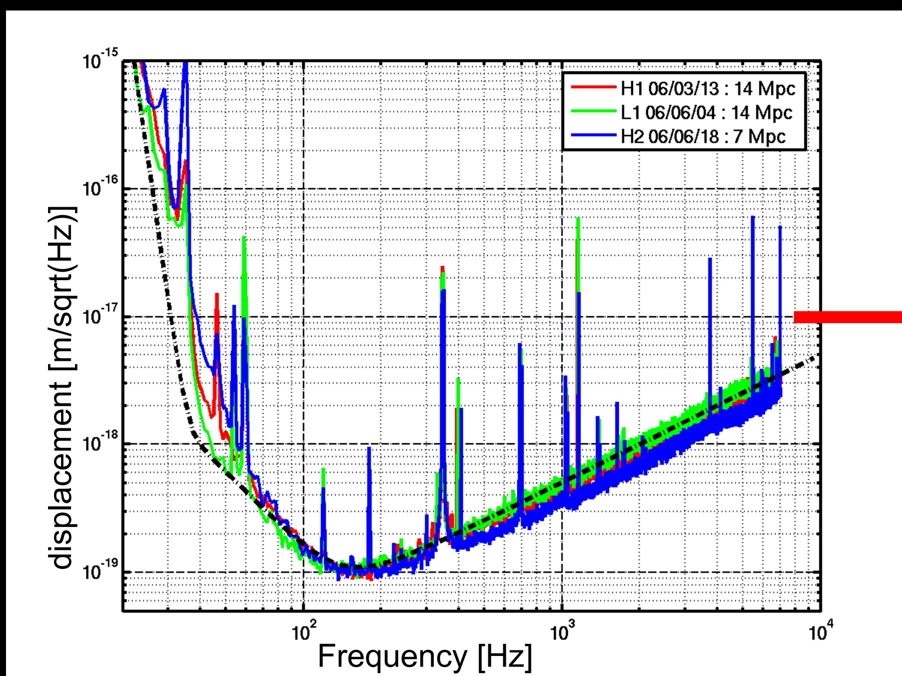
- Seismic noise at low frequencies
- Thermal fluctuations at intermediate frequencies
- Photon shot noise at high frequencies

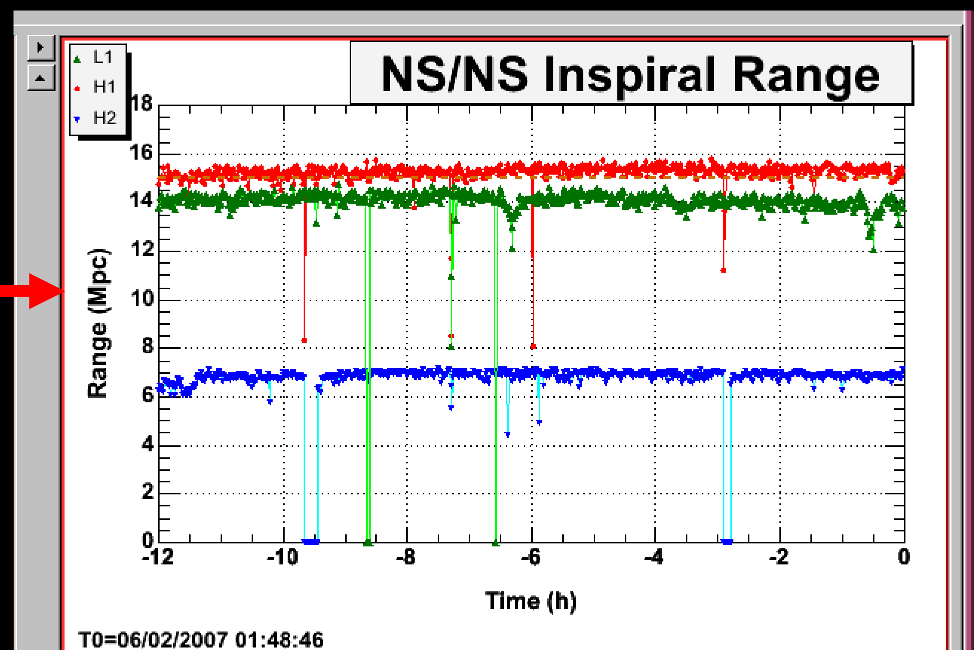
Goal: < 3×10^{-23}/sqrt(Hz) at 200 Hz (the "sweet spot")

Binary Neutron Stars: a Measure of Performance



The inspiral waveform for BNS is known analytically from post-Newtonian approximations.
We can translate strain amplitude into (effective) distance.

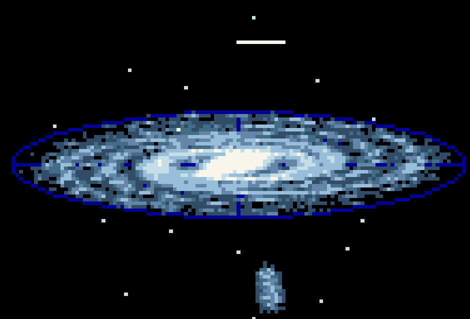




Range: distance of a 1.4-1.4 $M_{\odot}$ binary, averaged over orientation/polarization
Predicted rate for S5: 1/3year (most optimistic), 1/30years (most likely)

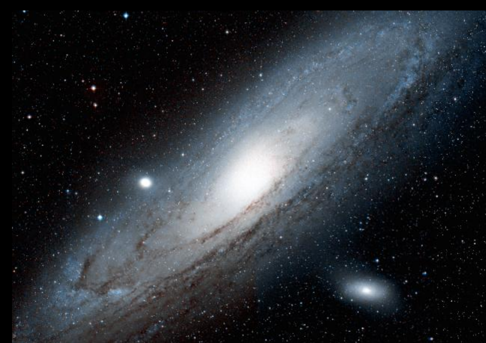
Progress in Sensitivity

Average distance for detecting a coalescing neutron-star binary:



Milky Way
(8.5 kpc)

Sept 2002
[~1 galaxy]



Andromeda
(700 kpc)

March 2003
[~2 galaxies]

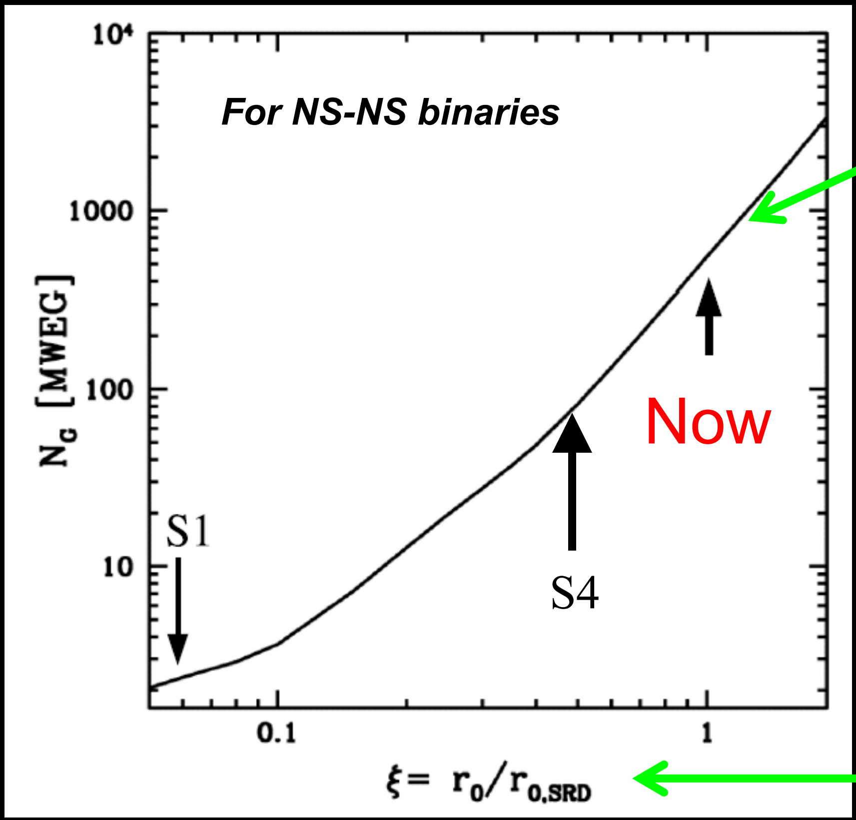
Virgo Cluster
(15 Mpc)

now
[~10^3 galaxies]

1 light year = 9.5×10^{12} km
1 pc = 30.8×10^{12} km = 3.26 light years

How does the Number of Surveyed Galaxies Increase as the Sensitivity is Improved?

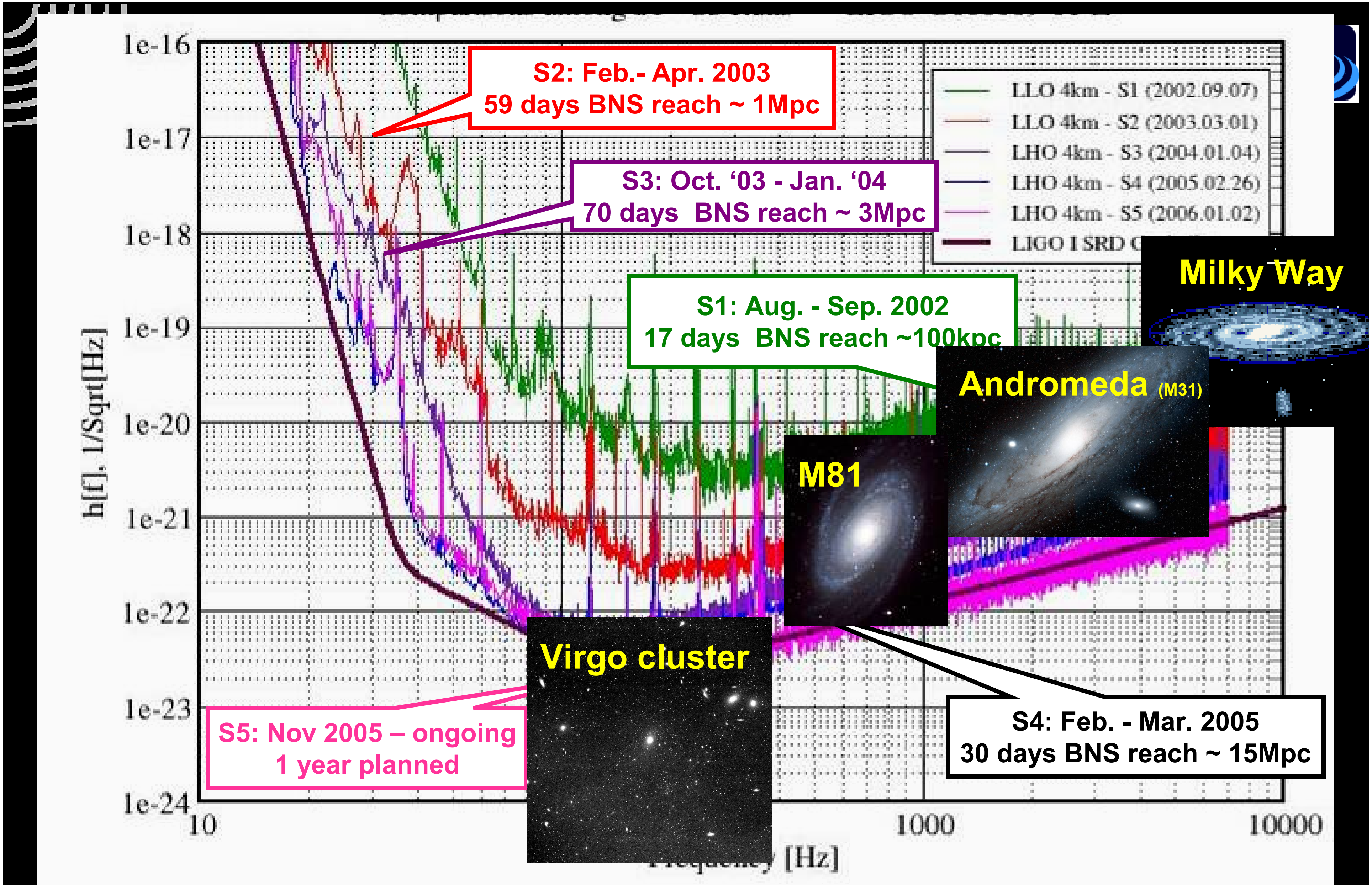
From astro-ph/0402091, Nutzman et al.



Power law: 2.7

So if we could push the strain noise down by another factor of 2, we would have a factor 6.5 increase in the number of surveyed galaxies
⇨scientific program for Enhanced LIGO (post S5)

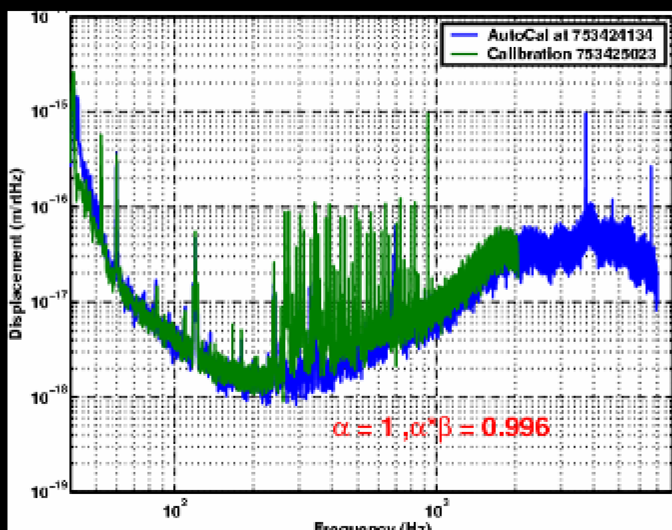
Proportional to inspiral range

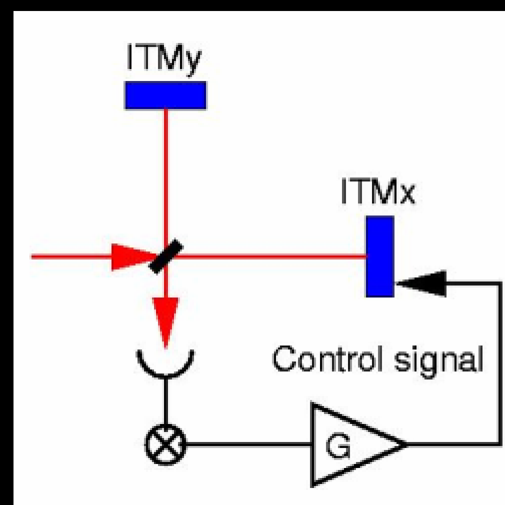


These curves are calibrated interferometer output:
spectral content of the gravity-wave channel

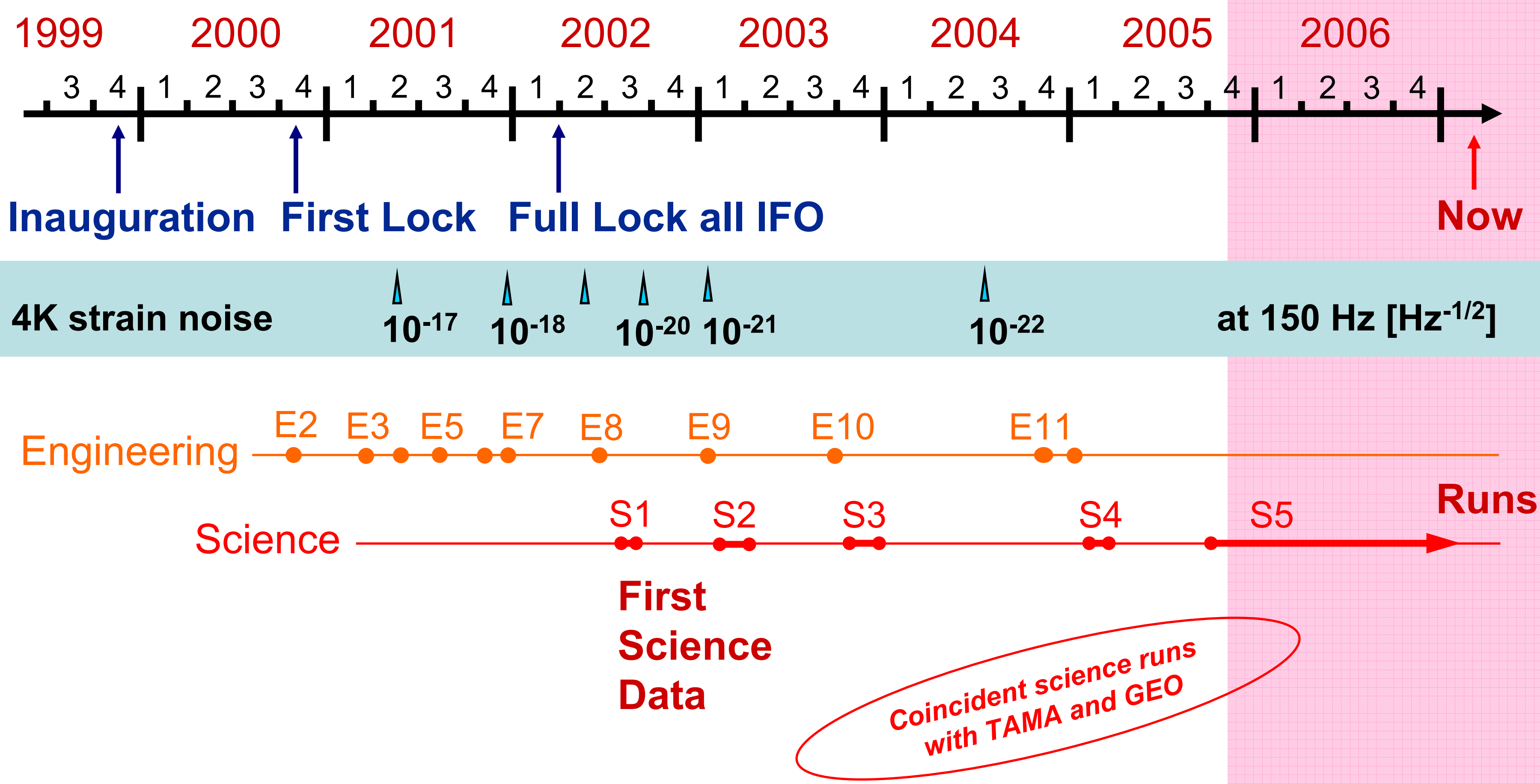
Calibration of Interferometer Output

- Combination of
  - Swept-Sine methods (accounts for gain vs. frequency) calibrate meters of mirror motion per count at digital suspension controllers across the frequency spectrum
  - DC measurements to set length scale (calibrates coil actuation of suspended mirror)
- Calibration lines injected during running to monitor optical gain changes due to drift





Timeline



1999
2000
2001
2002
2003
2004
2005
2006
3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
Inauguration
First Lock
Full Lock all IFO
Now
4K strain noise
10-17
10-18
10-20
10-21
10-22
at 150 Hz [Hz-1/2]
Engineering
E2
E3
E5
E7
E8
E9
E10
E11
Science
S1
S2
S3
S4
S5
Runs
First Science Data
Coincident science runs with TAMA and GEO

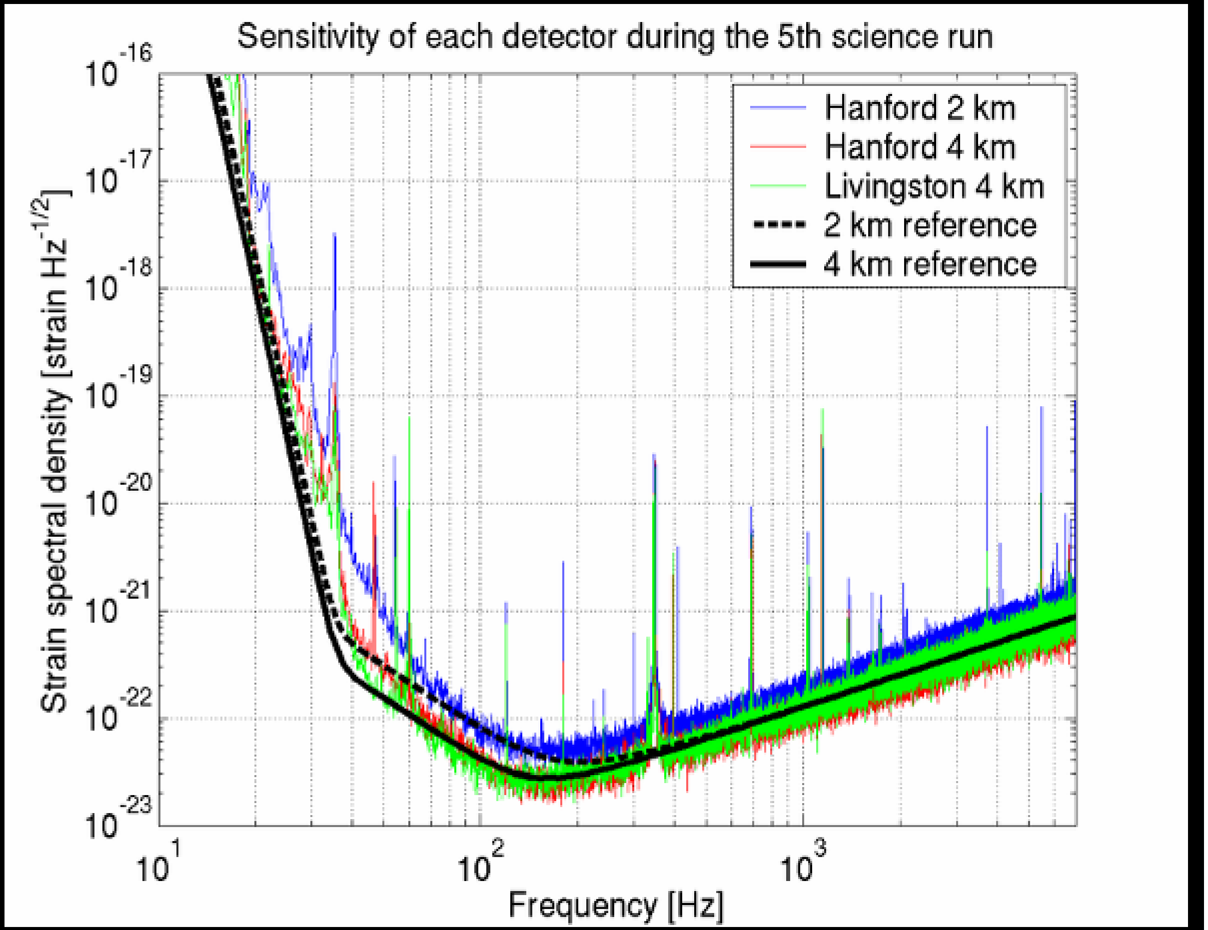
5th Science run

In the fall of 2005, LIGO reached its initial design sensitivity of 10^{-21} RMS strain in a 100 Hz band

Science Run 5 (S5) commenced in November 2005

The goal is to accumulate one year of coincident science mode data at or above design sensitivity.

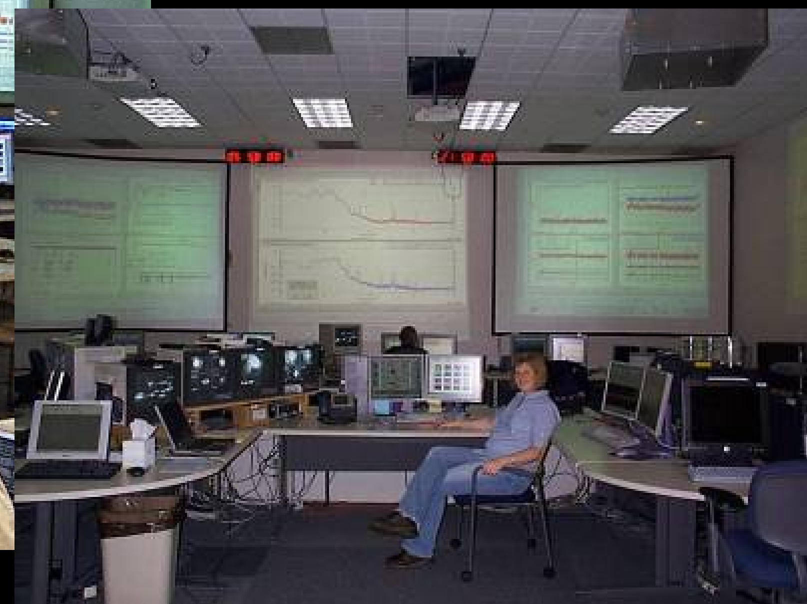
S5 is expected to last between 1.5 and 2 years



Schedule permits minor interruptions for maintenance and improvements

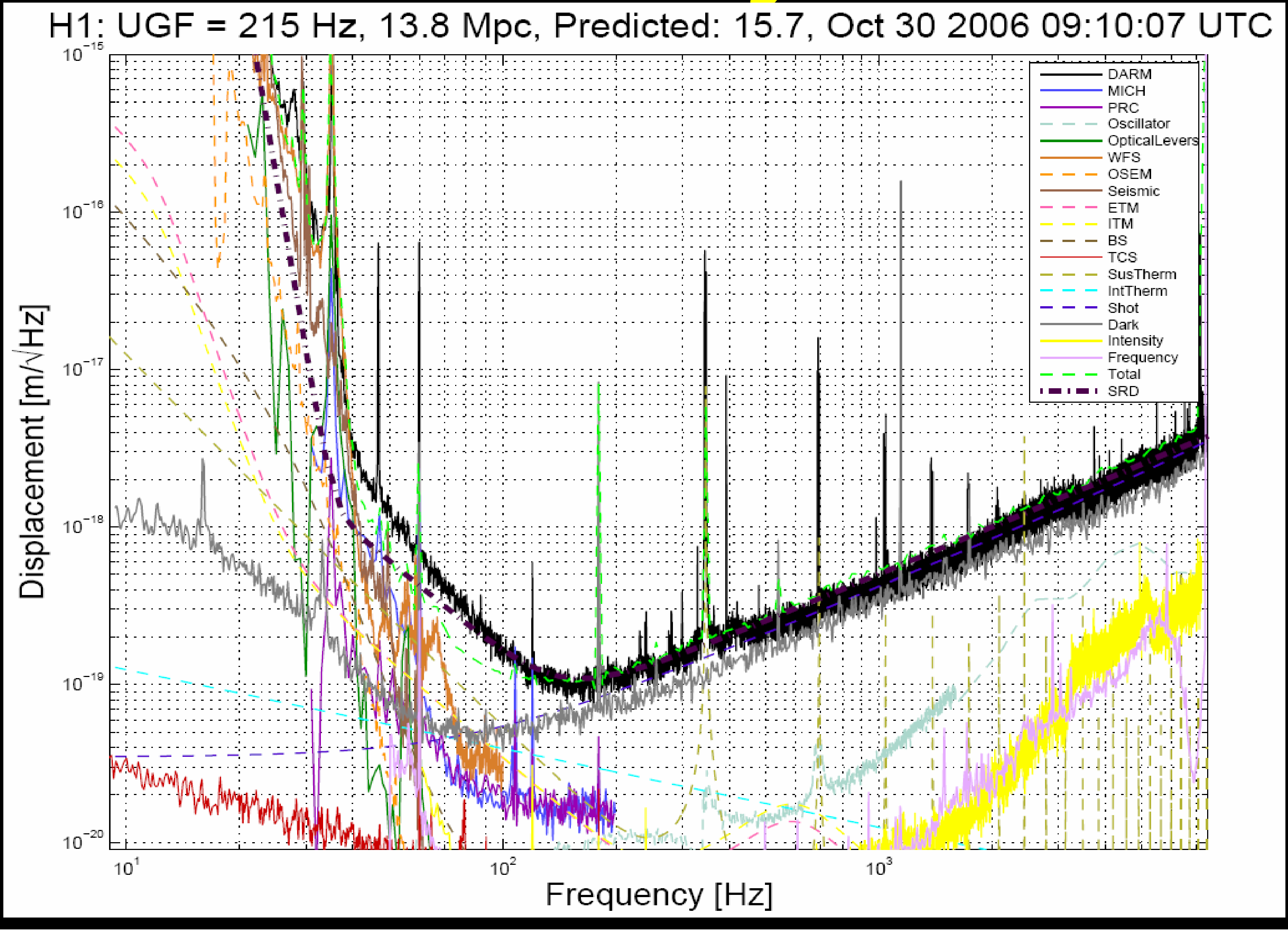
Science Runs In the Control Room

LIGO Livingston control room



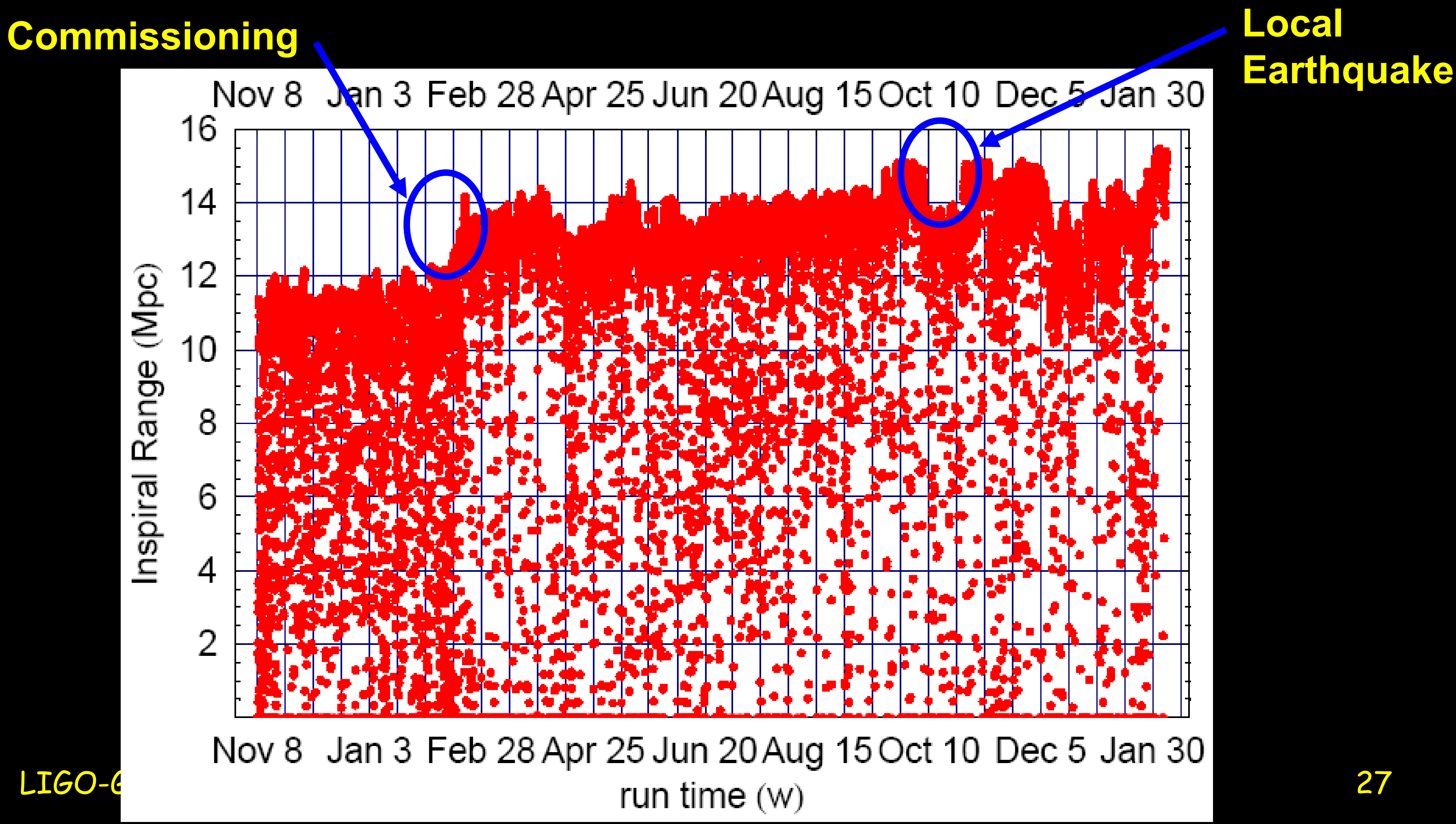
LIGO Hanford control room
31 Mar 2006 – S5

Noise budget



Hanford 4km sensitivity in S5

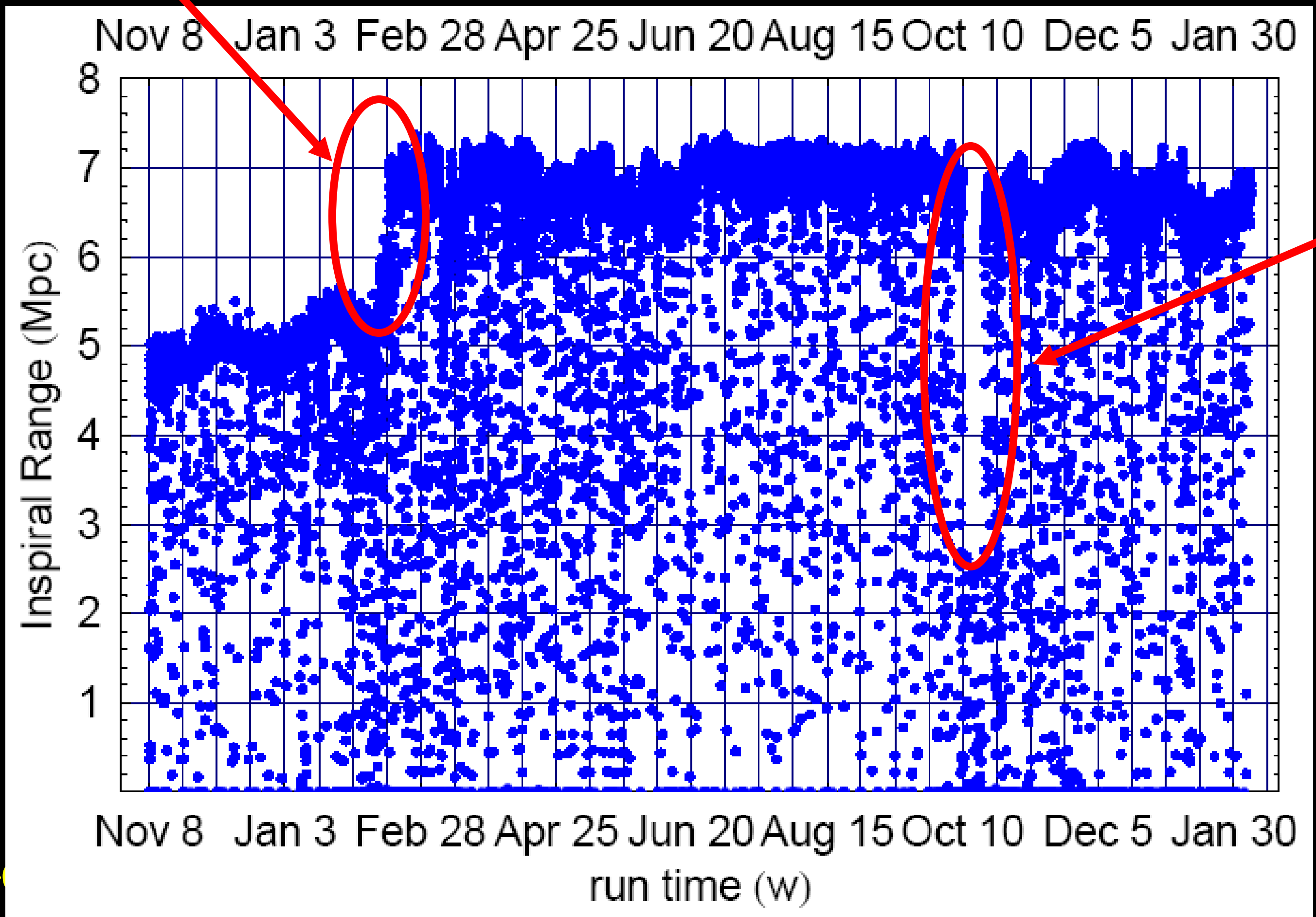
Inspiral Range = distance of a randomly oriented 1.4-1.4 $M_{\odot}$ binary neutron star inspiral detectable with SNR=8 (averaged over orientation, polarization)



Hanford 2km sensitivity in S5

Inspiral Range = distance of a randomly oriented 1.4-1.4 $M_{\odot}$ binary neutron star inspiral detectable with SNR=8 (averaged over orientation, polarization)

Commissioning



Local Earthquake

Earthquake at Hanford

- Magnitude 4.5, E of Mt Rainier, WA
- Saturday October 7, 2006 at 07:48:26.57 PM (PDT)
- Depth : 4km

- Both detectors down ~ 1Mpc after quake
- Several days later, 2km drops to ~ 2Mpc
- burped 2km ETMY to a few Torr, plus shake
- burped vertices of both detectors
- 4km restored (with tweaks) to 15Mpc
- 2km continues to ~ once/week, drop to 1Mpc

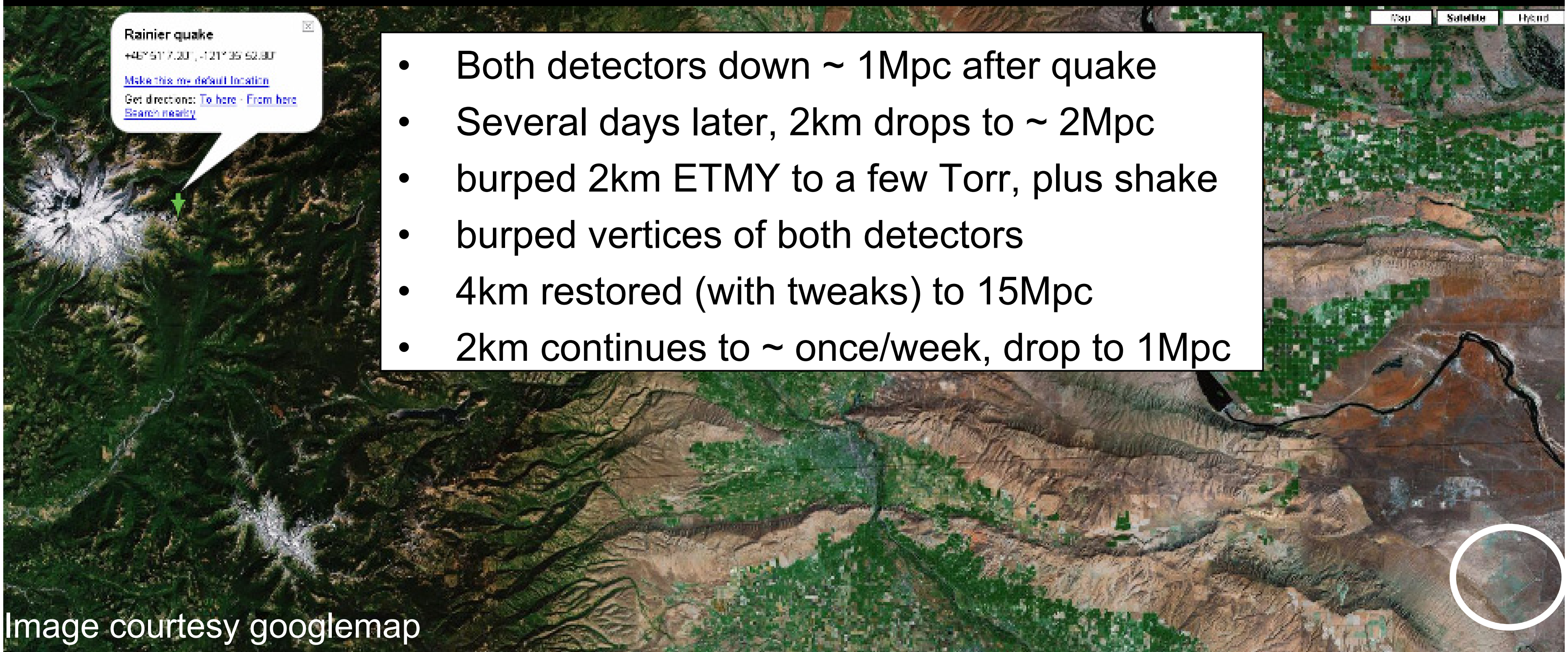
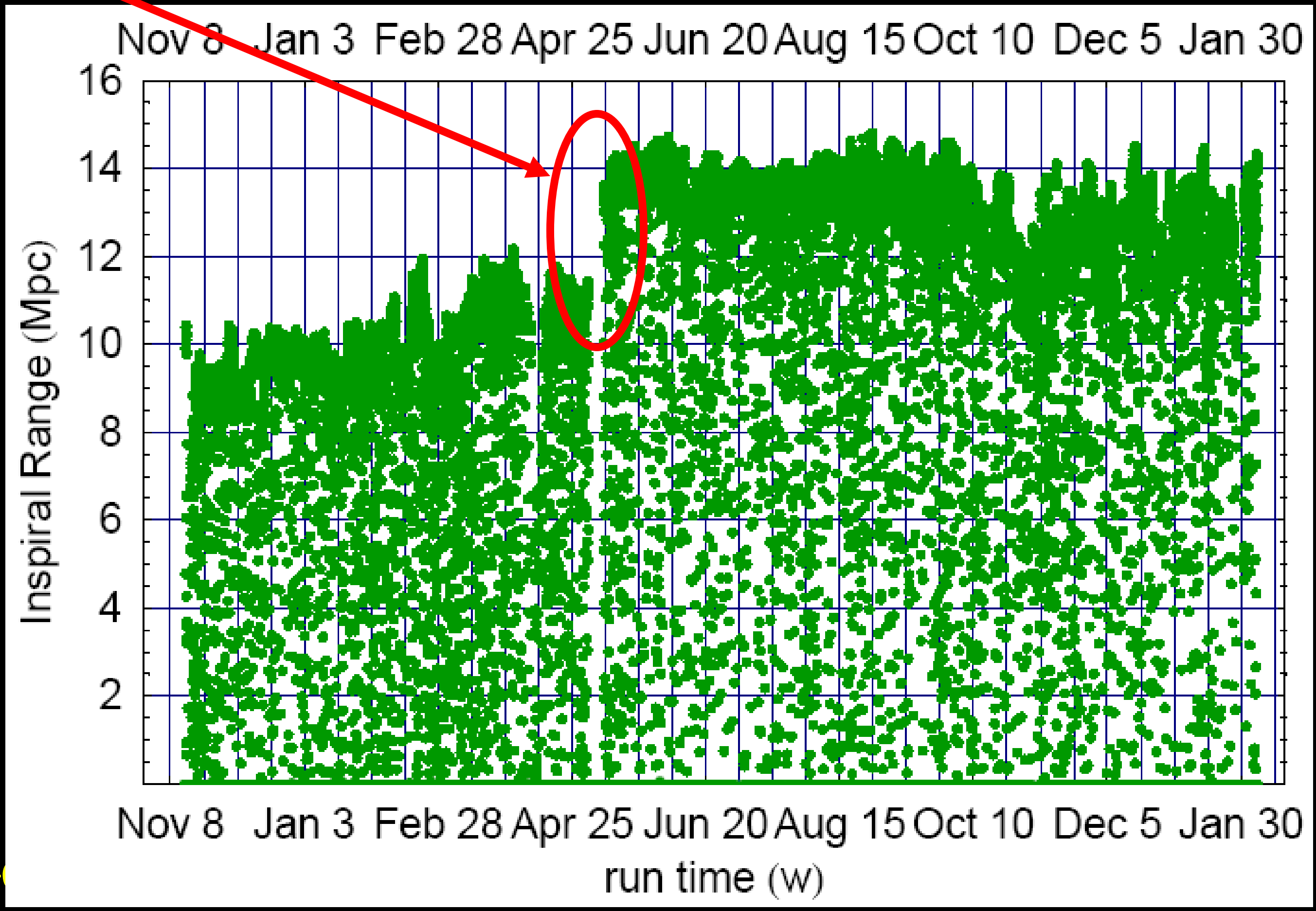


Image courtesy googlemap

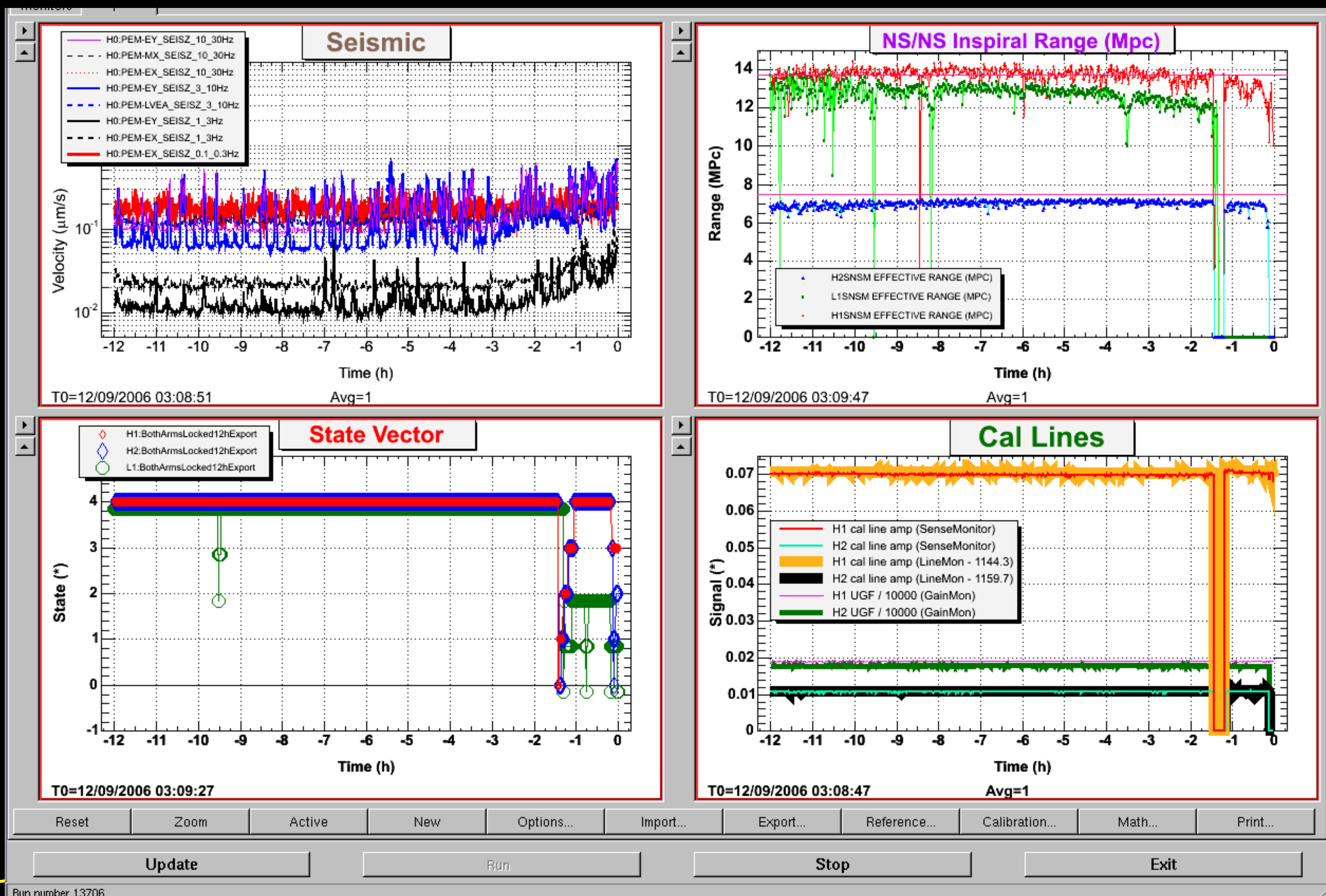
Livingston 4km sensitivity in S5

Inspiral Range = distance of a randomly oriented 1.4-1.4 $M_{\odot}$ binary neutron star inspiral detectable with SNR=8 (averaged over orientation, polarization)

Stuck optic

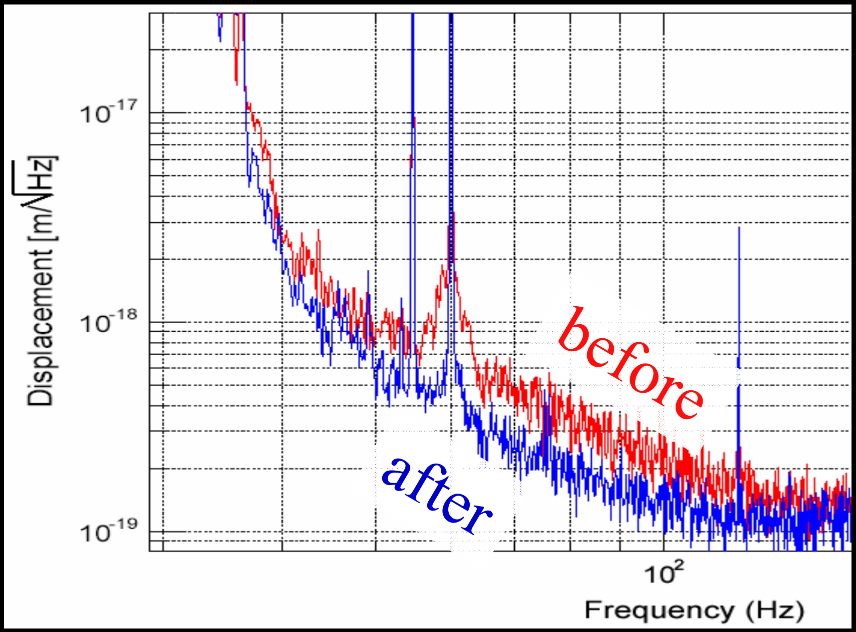


Figures of Merit



Stuck Optics and a Bit of Luck

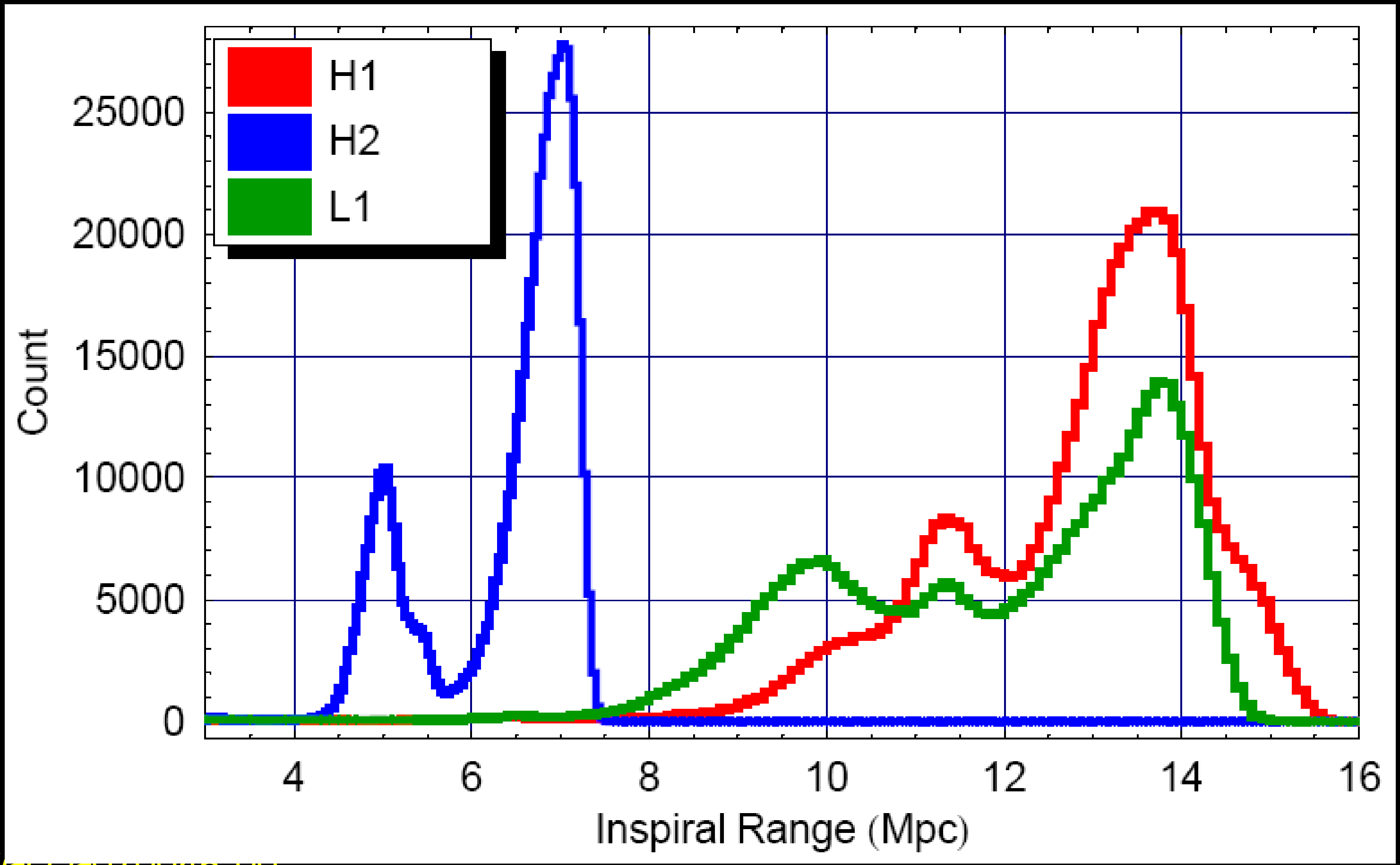
- Caused by construction activity, but after freeing the optic the range improved by 2 Mpc!
- Probably due to dissipation of static charge.



LIGO-G070046-00

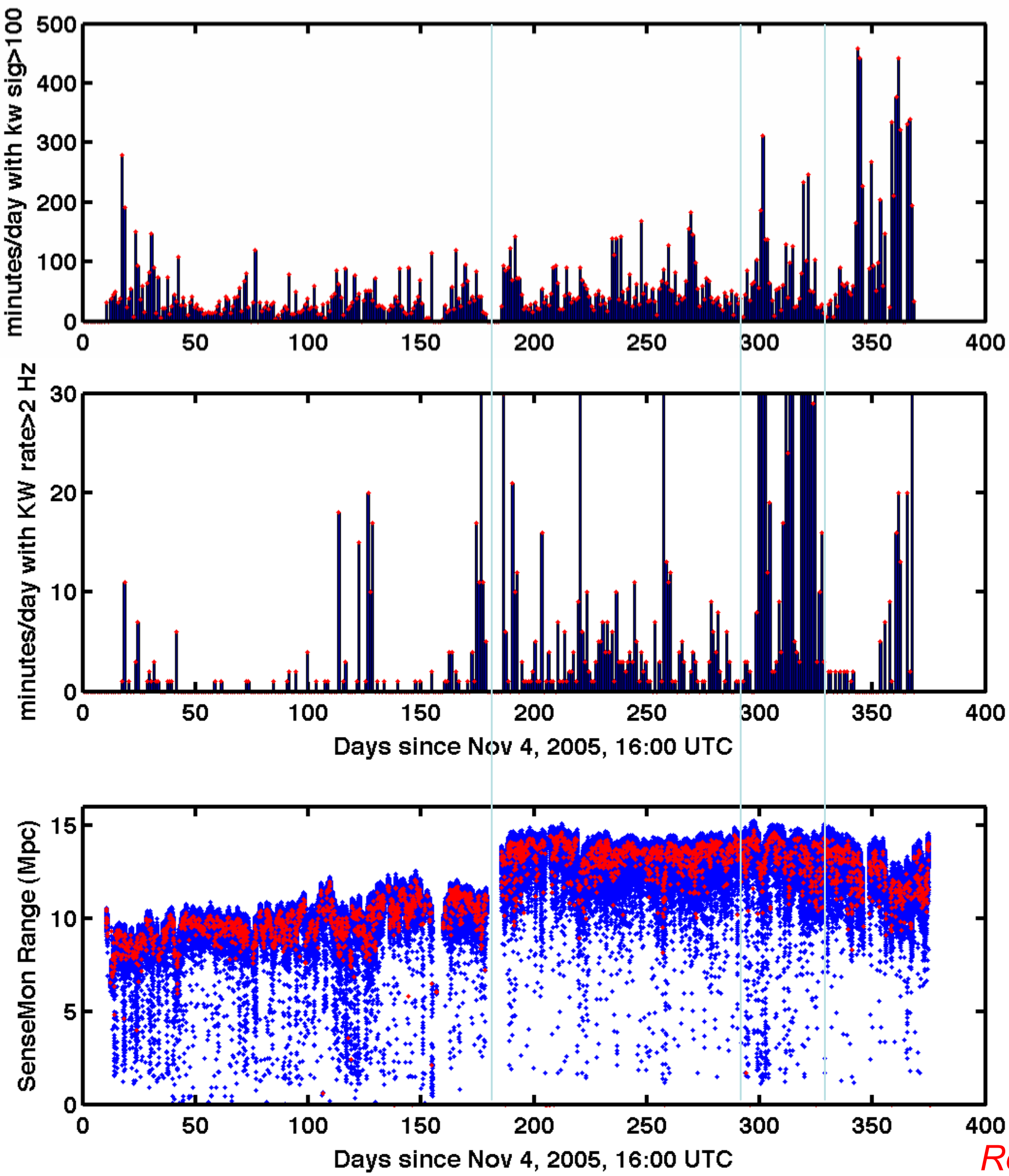
Histogram of Sensitivity in S5

Inspiral Range = distance of a randomly oriented 1.4-1.4 $M_\odot$ binary neutron star inspiral detectable with SNR=8 (averaged over orientation, polarization)



First Year of S5 in L1

Inspiral range does not tell the whole story… "glitchiness" is important too!



Minutes/day with at least one "loud" glitch

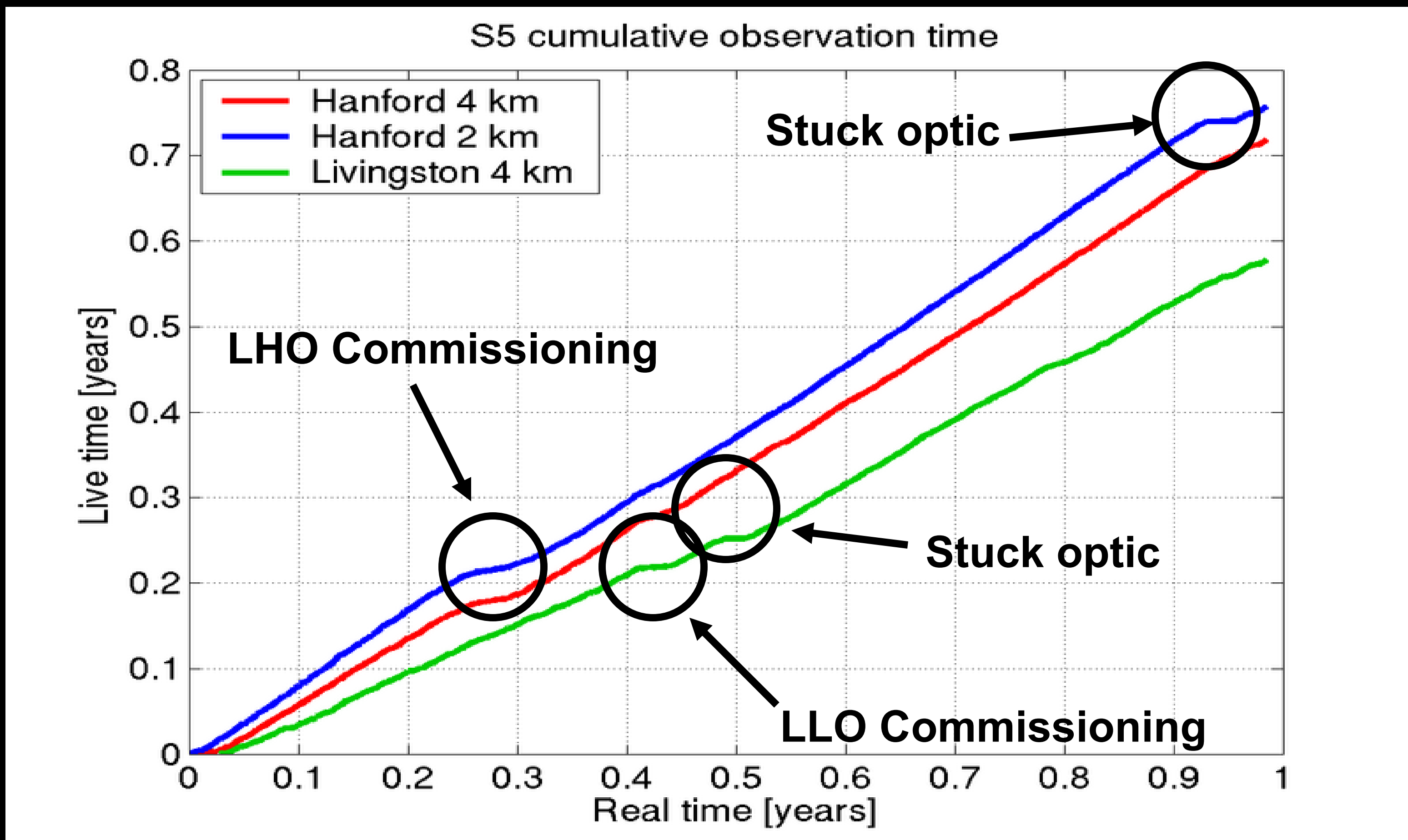
Minutes/day with >2Hz glitch rate

Inspiral Range [MPc]
- minute in science mode
- median of science segments

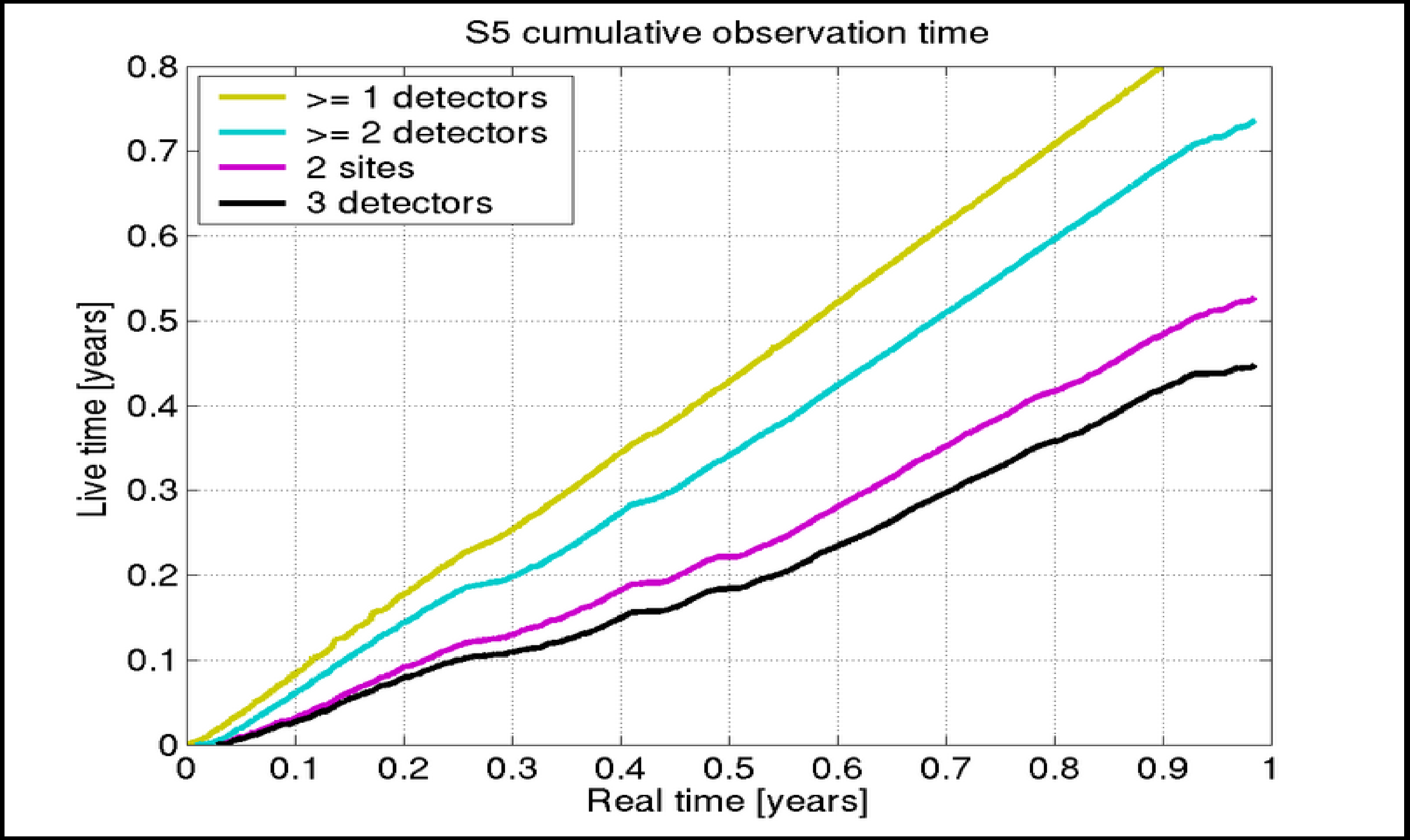
Affected by microseism, wind, instrument

Ref. LIGO-G060628-00

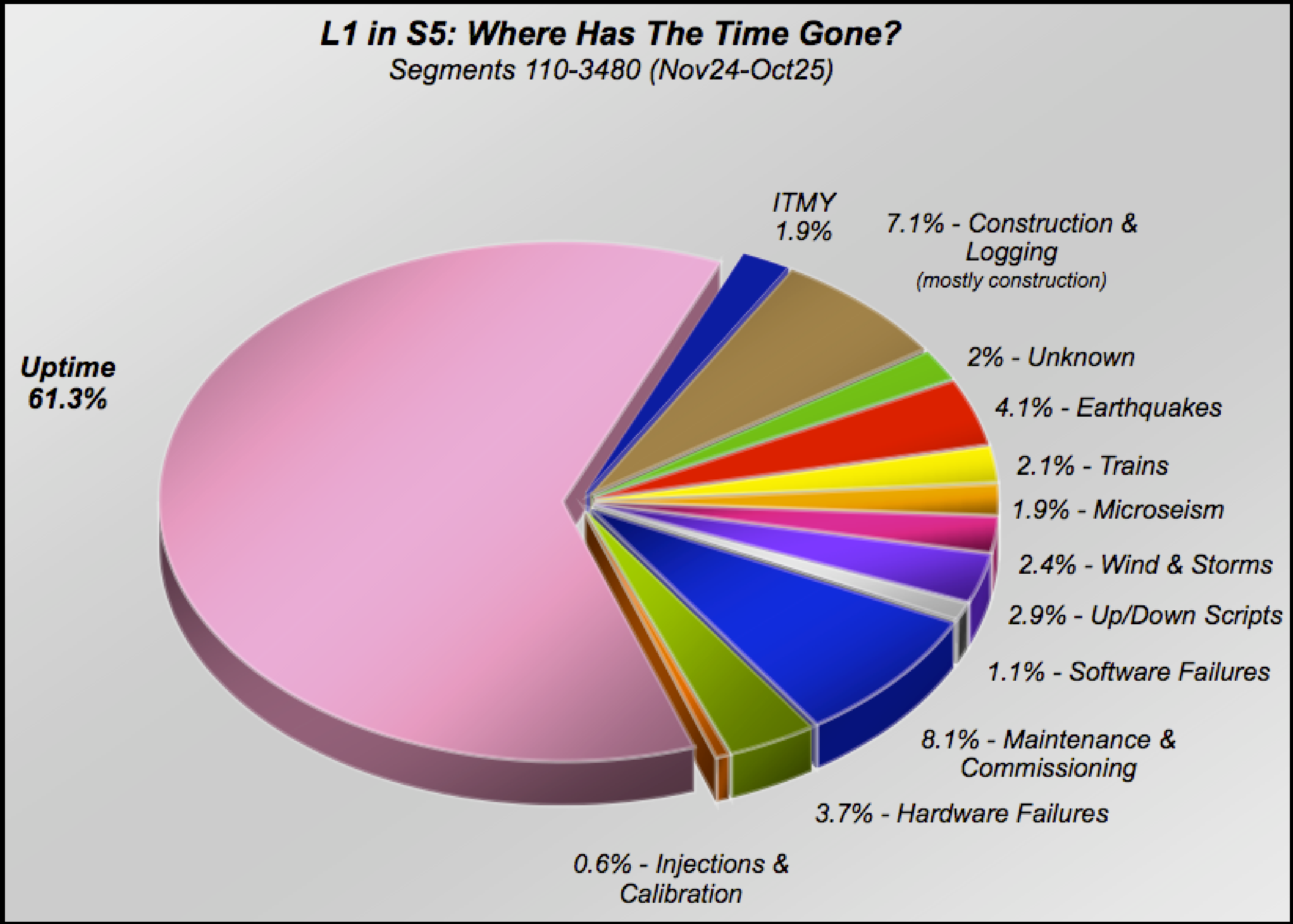
S5 Detector Observation Time



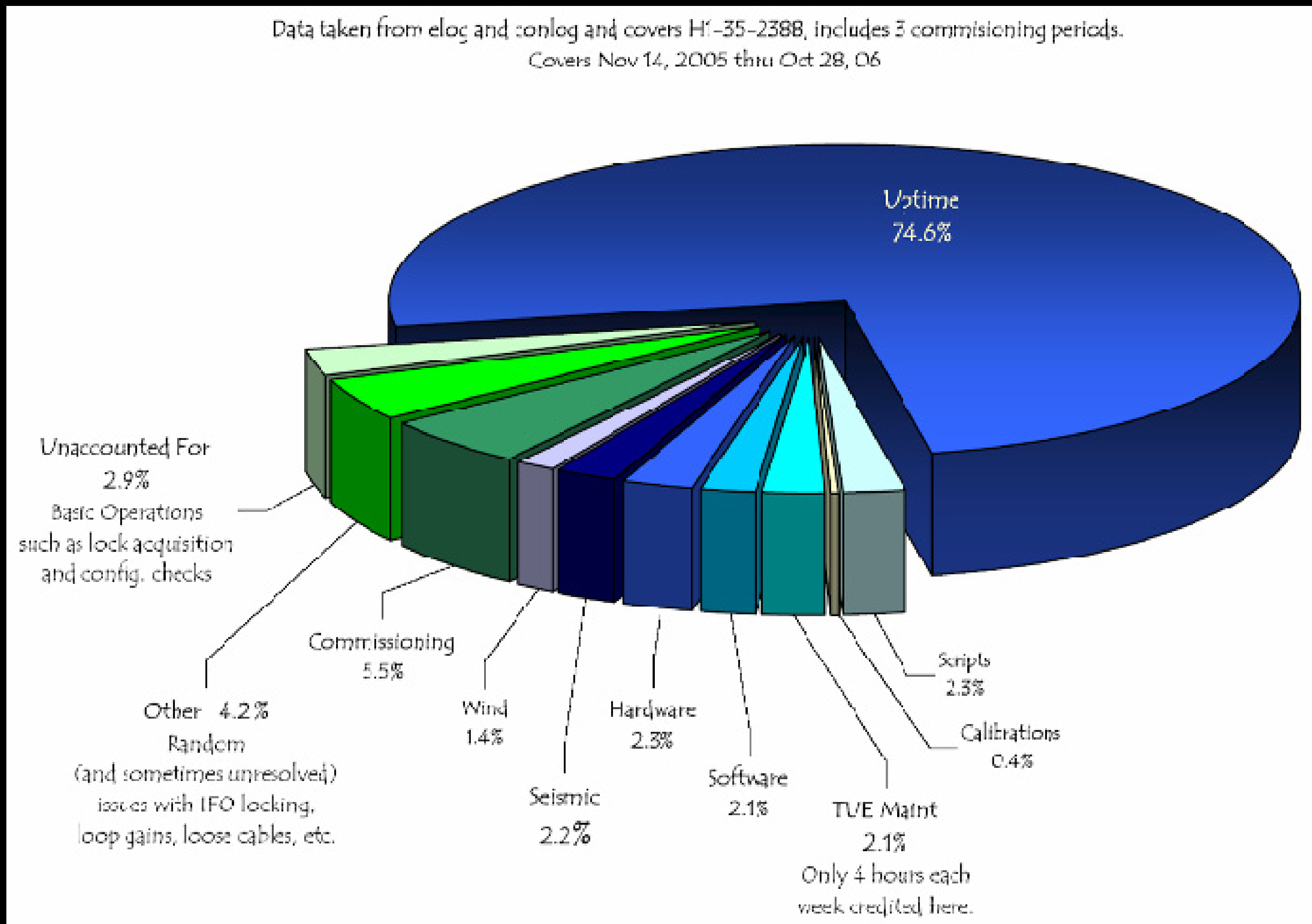
S5 network observation time



S5 Livingston 4km Uptime

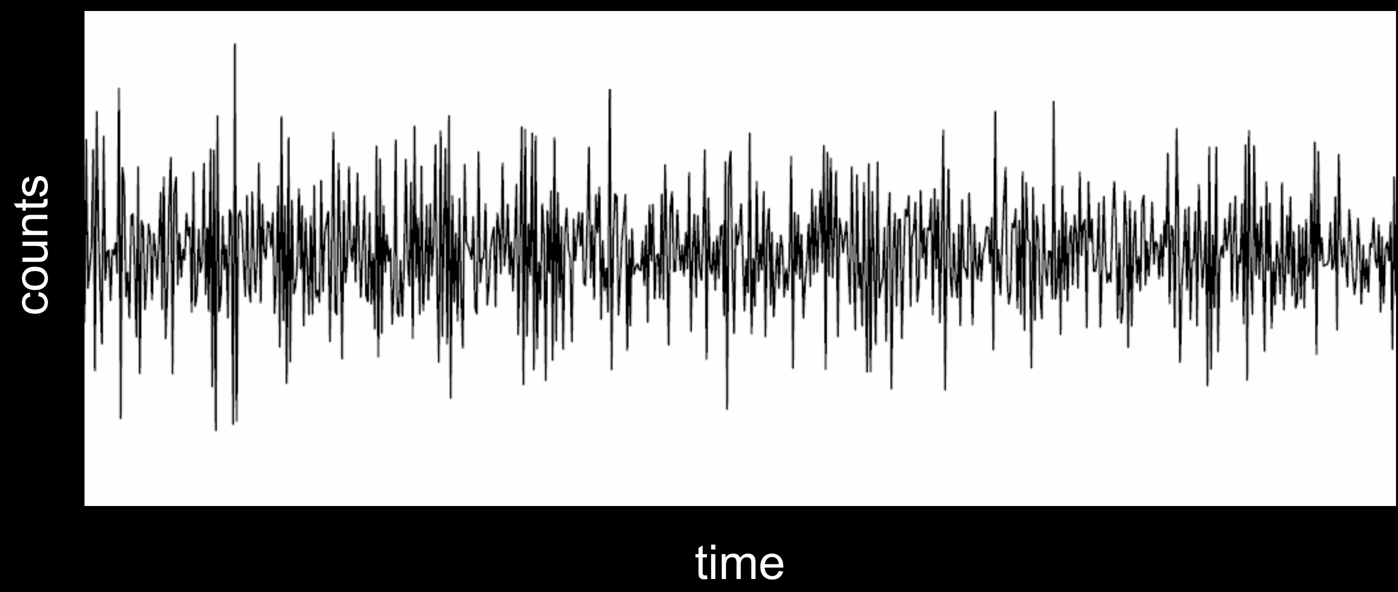


S5 Hanford 4km Uptime

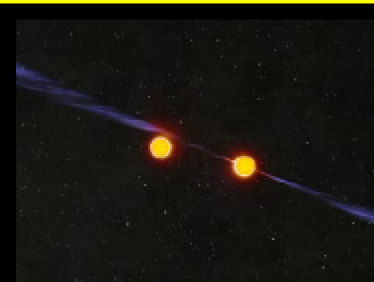
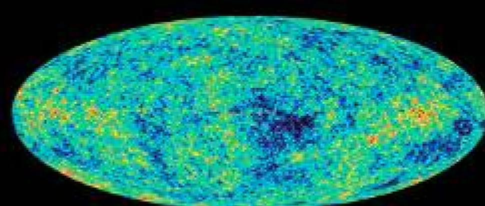


The Problem for Data Analysis

- Detectable signals at limit of detector sensitivity
- For most sources, expected waveform not known in advance
- Non-stationary detector noise
- Correlated environmental noise
- ~15 Gigabytes per day for gravitational data



Sources:
Signal Duration and Template

	Long duration	Short duration
matched filter	 CW search	 Inspiral search
no matched filter	 Stochastic search	 Burst search