Coating Design Optimization for Advanced Interferometers : Minimizing the Total Noise Budget

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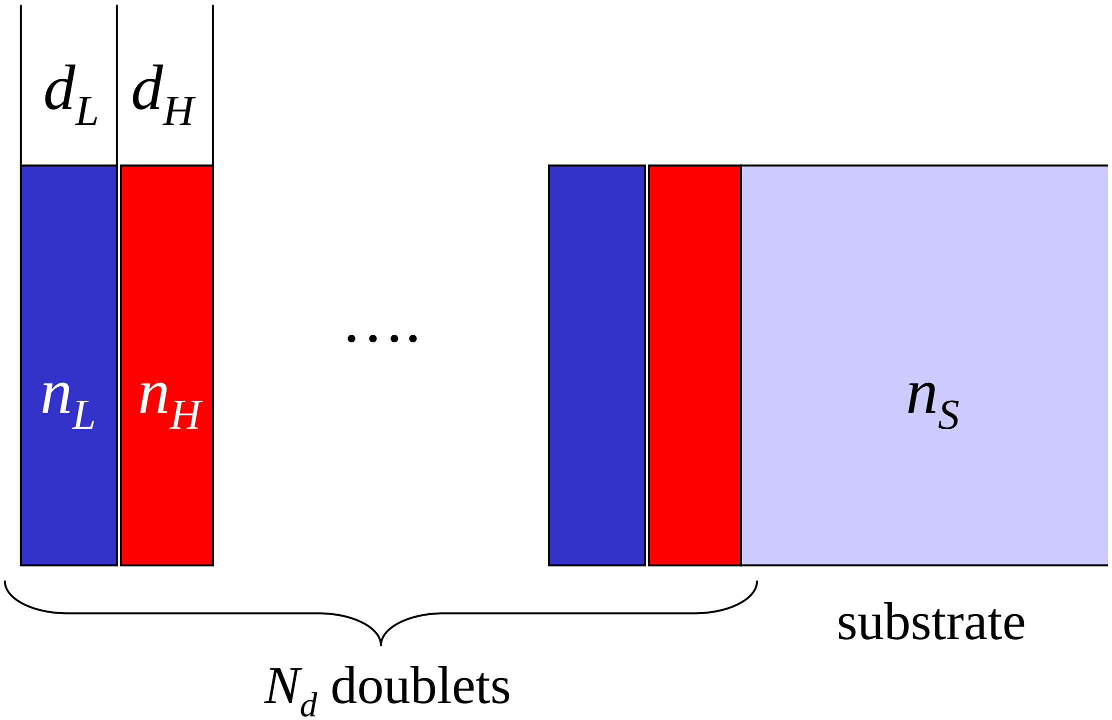
Outlook

- Iso-reflective Stacked-Doublet Coatings;
- Minimizing the Coating Brownian Noise;
- Doped Tantala;
- Total Coating Noise Budget Ingredients;
- Results;
- Conclusions.

Stacked Doublet Coatings Geometry and Notation



$$d_{L,H} = \left(\frac{\lambda_0}{n_{L,H}}\right) z_{L,H}$$

$$z_{L,H} = \frac{1}{4} \pm \xi$$

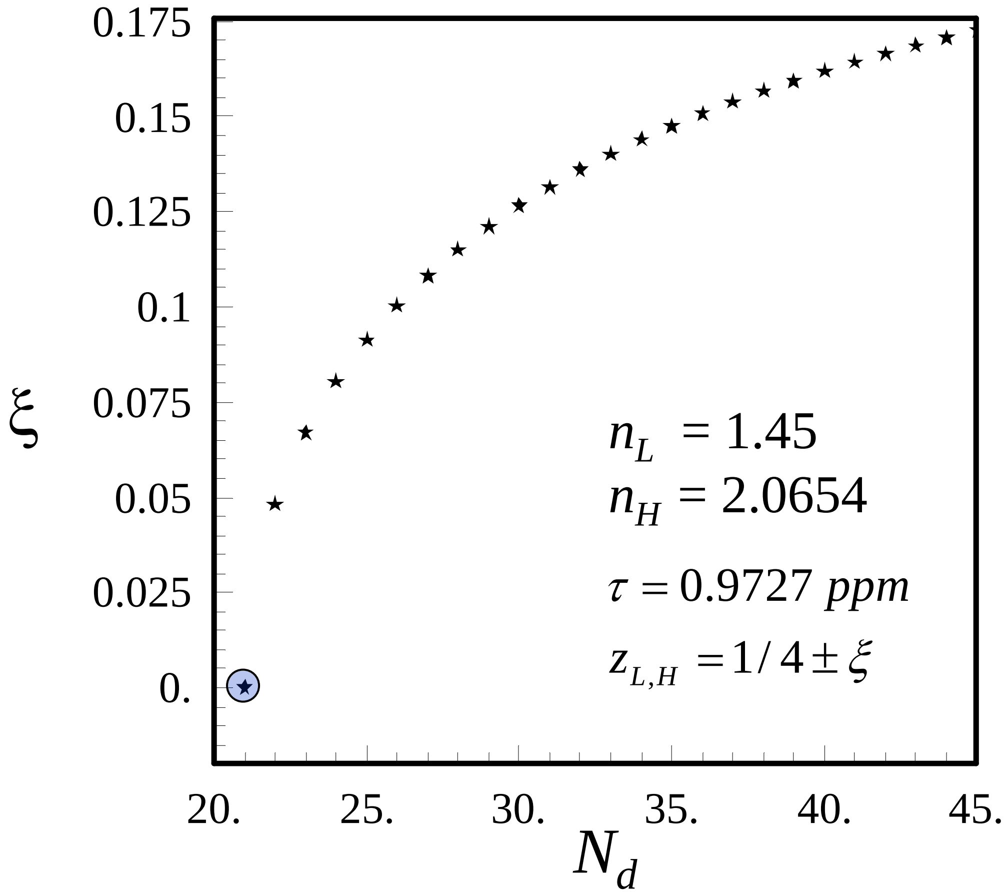
Alternative, Isoreflective Non QWL Coatings

For

$$n_L = 1.45$$
$$n_H = 2.0654$$

the QWL (ξ=0) design which goes closest to the 1ppm Adv LIGO design goal has

$$N_D = 21$$
$$\tau = 0.9727 \text{ ppm}$$



Coating Brownian Noise

Boltzmann · Poisson ratio · Coating loss angle

$$S_{\Delta x}^{(B)}(f) = \frac{\sqrt{2} k_B T}{\pi^{3/2} f} \frac{(1-\nu_s^2)}{r_0 E_s} \phi_c \quad , \qquad \phi_c = N_d (b_L z_L + b_H z_H)$$

Beam spot radius · Young modulus

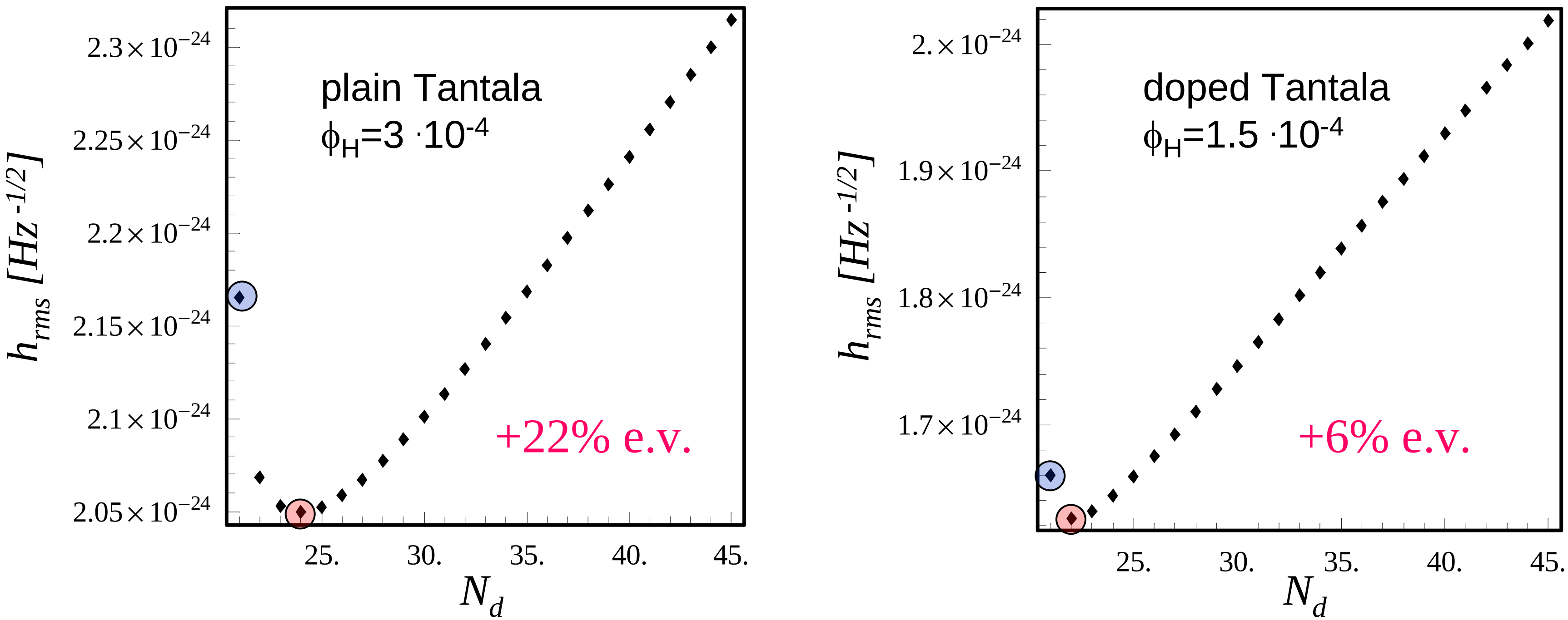
$$b_{L,H} \approx \frac{\lambda_0}{\sqrt{2\pi} r_0} \frac{\phi_{L,H}}{n_{L,H}} \left(\frac{E_{L,H}}{E_s} + \frac{E_s}{E_{L,H}} \right) \qquad \nu_{L,H} \ll 1$$

$$b_H / b_L = 5.149$$ [Tantala (plain) – Silica coatings]

Plain vs. Doped Tantala Brownian Noise Only

Brownian Noise Only. τ = 0.9727ppm, f = 100Hz

QWL coating   Optimized coating



Thermal Fluctuations of Test Mass Face Position

Effective fluctuations of the test-mass (coated mirror) front - face position with respect to the mirror center of mass may occur as an effect of

-Thermal expansion of the coating layers (thermoelastic effect),

$$\Delta x^{(TE)} = \alpha_{eff} d_{tot} \Delta T$$

effective coating expansion coeff. — coating thickness

-Thermal variations of the refraction indexes $n_{H,L}$ of the coating materials (thermorefractive effect),

$$\Delta x^{(TR)} = \beta_{eff} \lambda_0 \Delta T$$

thermorefractive coefficient — optical wavelength (vacuum)

Temperature Fluctuation Driven Displacement Noises

Power spectral density (PSD) :

Wiener-Khinchin th.

$$S_{\Delta x}(f) = \mathcal{F}_{\tau \to f} \langle \Delta x(t) \Delta x(t+\tau) \rangle_t = \left(\frac{\Delta x}{\Delta T} \right)^2 \mathcal{F}_{\tau \to f} \langle \Delta T(t) \Delta T(t+\tau) \rangle_t =$$

$$= \left(\frac{\Delta x}{\Delta T} \right)^2 S_{\Delta T}(f)$$

PSD of T - fluctuations in the coating

$$S_{\Delta T}(f) = S_{\Delta T}^{(\Theta)}(f) + S_{\Delta T}^{(\Phi)}(f)$$

Intrinsic fluctuations of thermodynamic origin

add in-coherently

Photo-thermal fluctuactions arising from laser shot noise through optical absorption

Temperature Fluctuations in the Coating

$$S_{\Delta T}^{(\Theta)}(f) = \frac{k_B T^2}{\pi^{3/2} r_0^2 \sqrt{f \kappa_s C_s \rho_s}}$$

[V. Braginsky, Phys. Lett A264 (1999) 1]

mass density
specific heat capacity
thermal conducivity
} of substrate

single photon energy
power abs. in coating

$$S_{\Delta T}^{(\Phi)}(f) = \frac{P_{\mathrm{abs}} E_\lambda}{4\pi^3 r_0^4 \kappa_s \rho_s C_s f}$$

[S. Rao, PhD Thesis, Caltech, 2003, etd-05092003-153759]

$E_\lambda \cong 1.867 \cdot 10^{-19} J @ \lambda = 1064 nm$

$P_{abs} = 0.4 W$ for Adv LIGO

(a *different* formula for $S_{\Delta T}^{(\Phi)}$ applies for sapphire substrates)

Coating Thermal Noise Budget

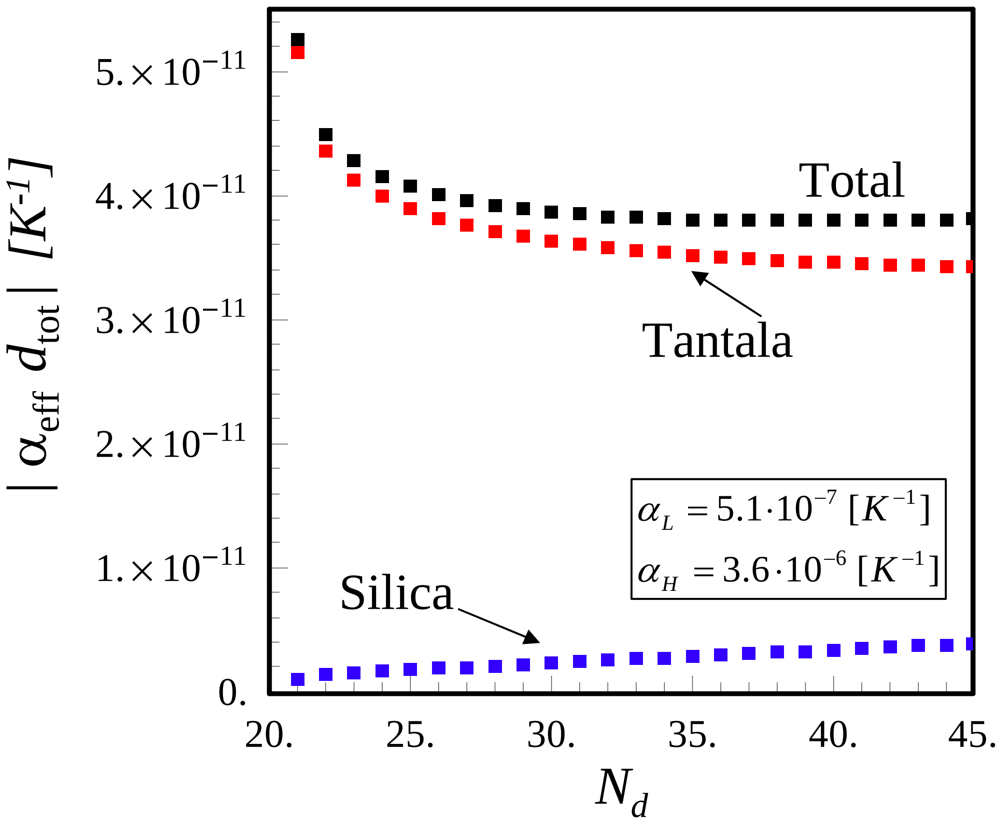
Total Coating Noise PSD

$$S_{\Delta x}^{(tot)}(f) = S_{\Delta x}^{(B\ )}(f) + \left(\frac{\Delta x^{(TE)}}{\Delta T} + \frac{\Delta x^{(TR)}}{\Delta T} \right)^2 S_{\Delta T}(f)$$

The thermal - driven elastic and refractive fluctuations should add *coherently* . Indeed, the temperature in the coating does *not* fluctuate

- on the space-scale (thickness) of the coating,
- on the time scales whereby the field in the coating builds up.

Thermoelastic Noise Coefficient



[V.B. Braginsky and S.A. Vyatchanin, Phys. Lett. A312 (2003) 244; idem, cond-mat/0302617 contains important corrections]

[M. Fejer et al., PRD-70 (2004) 082003]

General formula available, OK for general SD coatings (also in the form of linear combination of z_L, z_H)

Thermorefractive Coefficient β_{eff} in a Nutshell

$$\overline{Y}_{in} = \overline{Y}_{in}^{(0)} + \Delta\overline{Y}_{in}$$

(photorefractive change in coating input admittance)

$$\Gamma_{in} = \frac{1-\overline{Y}_{in}}{1+\overline{Y}_{in}} \approx \frac{1-\overline{Y}_{in}^{(0)}}{1+\overline{Y}_{in}^{(0)}}\left(1-\frac{2\Delta\overline{Y}_{in}}{1-(\overline{Y}_{in}^{(0)})^2}\right) = \Gamma^{(0)}\left(1-\frac{2\Delta\overline{Y}_{in}}{1-(\overline{Y}_{in}^{(0)})^2}\right)$$

$$\Gamma(\Delta x) = \Gamma(0)\exp\left[\imath\frac{4\pi}{\lambda_0}\Delta x\right] \approx \Gamma(0)\left[1+\imath\frac{4\pi}{\lambda_0}\Delta x\right]$$

(Transport equation for reflection coeff.)

$$\Delta x = \beta_{eff}\lambda_0$$

⇓

$$\beta_{eff} = -\frac{1}{2\pi\imath}\frac{\Delta\overline{Y}_{in}}{1-(\overline{Y}_{in}^{(0)})^2}$$

Braginsky's Ansatz

- High reflectivity coatings, thus N_d *very large*;
- For N_d very large ($N_d \to \infty$), addition of a further doublet *does not* change the coating input admittance;
- For this *single added* doublet we accordingly have

$$\frac{E_{in}}{Z_0 H_{in}} = \bar{Y}_{in} = \frac{E_{out}}{Z_0 H_{out}}$$

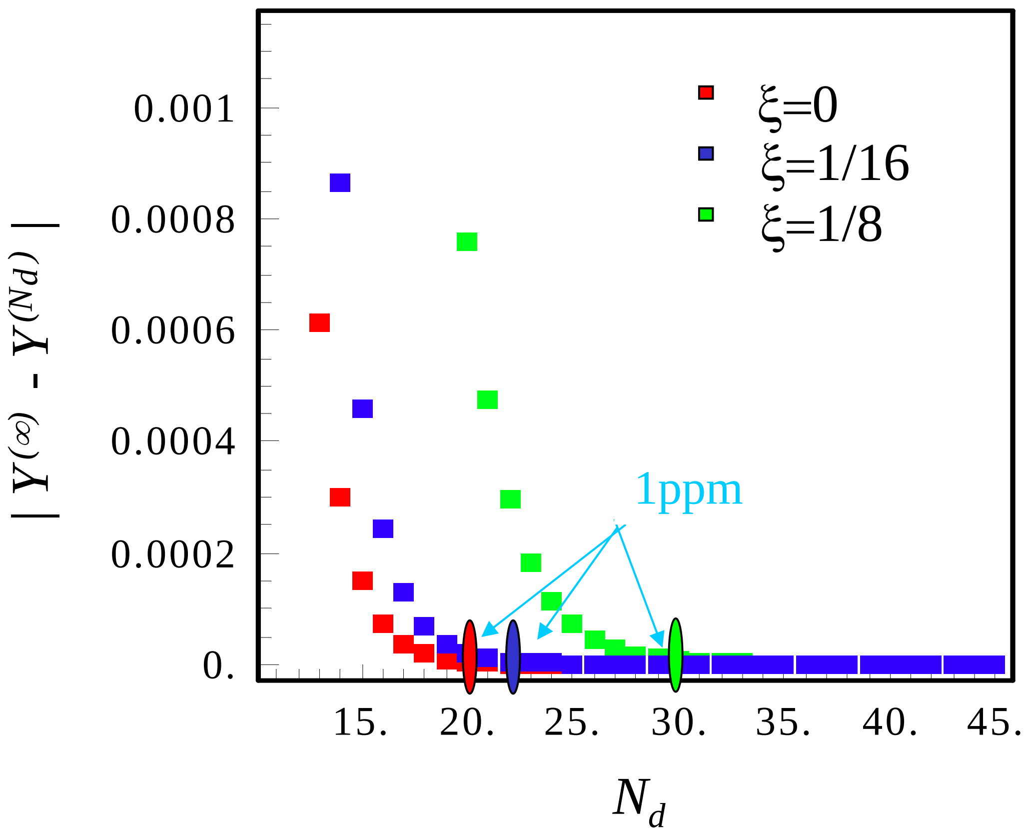
combined with single-doublet transmission matrix equation

$$\begin{pmatrix} E_{out} \\ Z_0 H_{out} \end{pmatrix} = \begin{pmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{pmatrix} \cdot \begin{pmatrix} E_{in} \\ Z_0 H_{in} \end{pmatrix}$$

gives an equation in Y_{in} . Yields formula "1" for QWL case.

[V. Galdi and I.M. Pinto, 2007]

Validating Braginsky's Ansatz



Thermorefractive Coefficient : Which Formula ?

$$\beta_{\text{eff}} = \frac{n_H^2 \beta_L + n_L^2 \beta_H}{4(n_L^2 - n_H^2)}$$

[Braginsky, Gorodetsky and Vyatchanin, Phys. Lett. A 271 (2000) 303]

- deduced for QWL coatings only
- deduction based on a consistency argument
- β_{eff} does not vanish in the limit $n_H \to \infty$

$$\beta_{\text{eff}} = \frac{n_L n_H (\beta_L + \beta_H)}{4(n_L^2 - n_H^2)}$$

[Braginsky and Vyatchanin, Phys. Lett. A312 (2003) 244]

- claimed to fix an error in previous formula
- no details given about deduction
- β_{eff} vanishes in the limit $n_H \to \infty$

Coating Input Admittance Alternative Derivation (QWL)

[V. Galdi and I.M. Pinto, 2007]

$$\begin{pmatrix} E_{in} \\ Z_0 H_{in} \end{pmatrix} = \begin{pmatrix} T_{11}^{(N_d)} & T_{12}^{(N_d)} \\ T_{21}^{(N_d)} & T_{22}^{(N_d)} \end{pmatrix} \cdot \begin{pmatrix} E_{out} \\ Z_0 H_{out} \end{pmatrix}$$

N_d - doublets coating QWL transmission matrix

$$T_{11}^{(N_d)} = \left(-\frac{n_H^{(0)}}{n_L^{(0)}}\right)^{N_d} \left[1 + \frac{2N_d}{\pi}\left(\Delta\psi_H - \Delta\psi_L\right)\right],$$

$$T_{12}^{(N_d)} = \imath \left(\frac{\Delta\psi_H}{n_L^{(0)}} + \frac{\Delta\psi_L}{n_H^{(0)}}\right) S(N_d),$$

$$T_{21}^{(N_d)} = -\imath \left(n_L^{(0)}\Delta\psi_H + n_H^{(0)}\Delta\psi_L\right) S(N_d),$$

$$T_{22}^{(N_d)} = \left(-\frac{n_L^{(0)}}{n_H^{(0)}}\right)^{N_d} \left[1 - \frac{2N_d}{\pi}\left(\Delta\psi_H - \Delta\psi_L\right)\right],$$

(proven by complete induction)

where:

$$\Delta\psi_{L,H} = \frac{\pi}{2}\frac{\beta_{L,H}^{(0)}}{n_{L,H}^{(0)}}\Delta T$$

$$S(N_d) = \begin{cases} \displaystyle\sum_{m=-P}^{P} \left(\frac{n_L^{(0)}}{n_H^{(0)}}\right)^{2m} & N_d = 2P \text{ (even)} \\ \displaystyle -\sum_{m=-P}^{P-1} \left(\frac{n_L^{(0)}}{n_H^{(0)}}\right)^{2m+1} & N_d = 2P+1 \text{ (odd)} \end{cases}$$

Coating Input Admittance Alternative Derivation (QWL), contd.

$$E_{out} = n_S^{-1} Z_0 H_{out} \implies \bar{Y} = \frac{Z_0 H_{in}}{E_{in}} = \frac{T_{21}^{(N_d)} + n_S T_{22}^{(N_d)}}{T_{11}^{(N_d)} + n_S T_{12}^{(N_d)}}.$$

$$\bar{Y} = n_S \left(\frac{n_L}{n_H}\right)^{2N_d} \left[1 - \frac{4N_d}{\pi}(\Delta\psi_H - \Delta\psi_L)\right]$$

$$- \imath \frac{n_L^2 n_H \Delta\psi_H + n_H^2 n_L \Delta\psi_L}{n_L^2 - n_H^2}$$

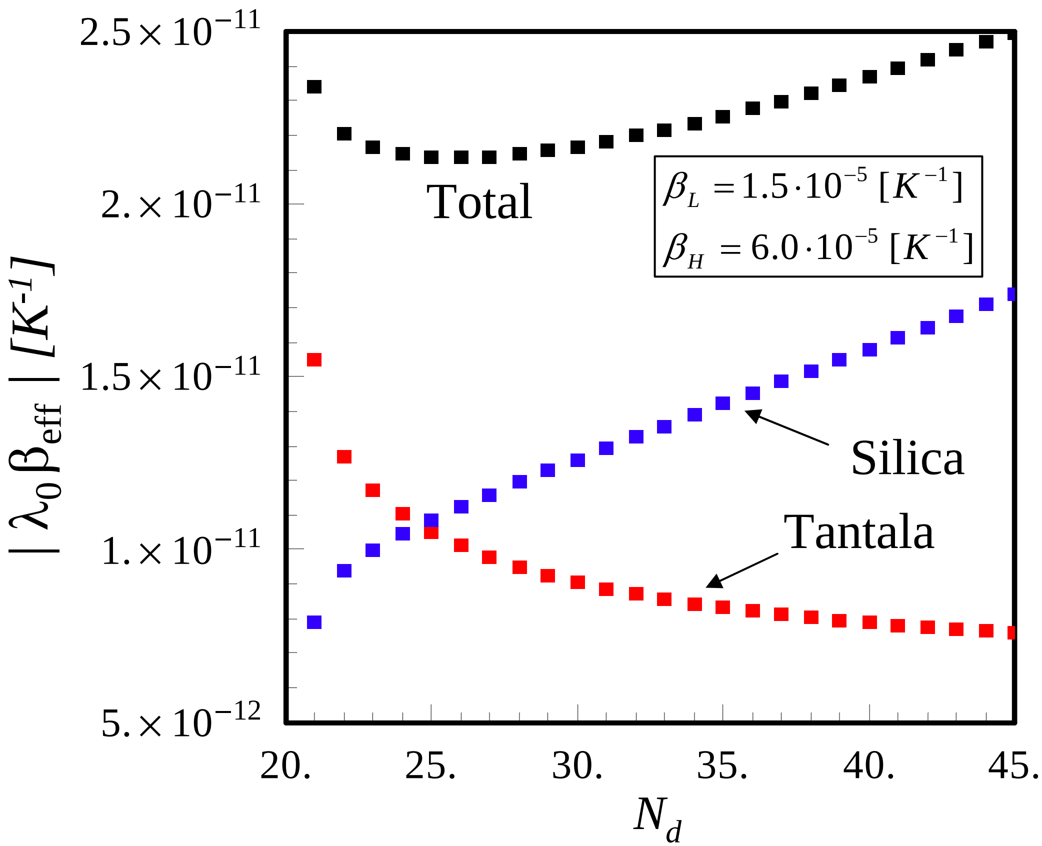
$$+ \imath \frac{(n_L^2 + n_S^2) n_H \Delta\psi_H + (n_H^2 + n_S^2) n_L \Delta\psi_L}{n_L^2 - n_H^2} \left(\frac{n_L}{n_H}\right)^{2N_d}$$

$$- \imath \frac{n_S^2 (n_H \Delta\psi_H + n_L \Delta\psi_L)}{n_L^2 - n_H^2} \left(\frac{n_L}{n_H}\right)^{4N_d}$$

returns Braginsky's formula "1"

These terms vanish as $N_d \to \infty$, since $n_H > n_L$.

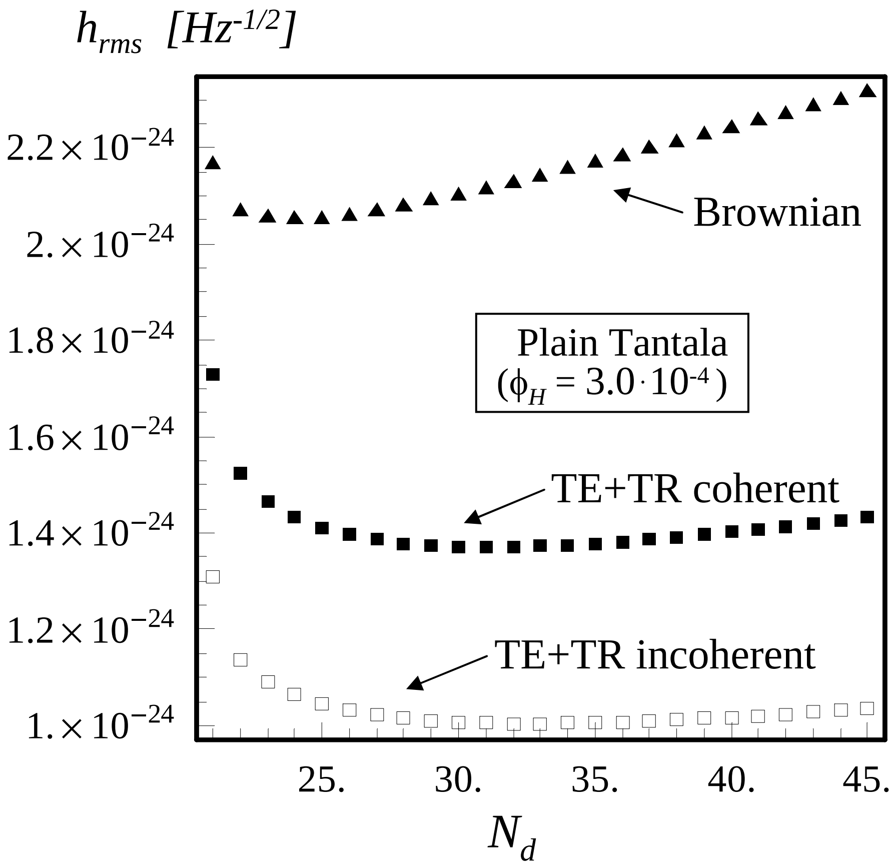
Thermorefractive Noise Coefficient



A non - QWL, minimum thermorefractive coeff-icient stacked doublet design exists, featuring the lowest combination of the low - high index material contributions

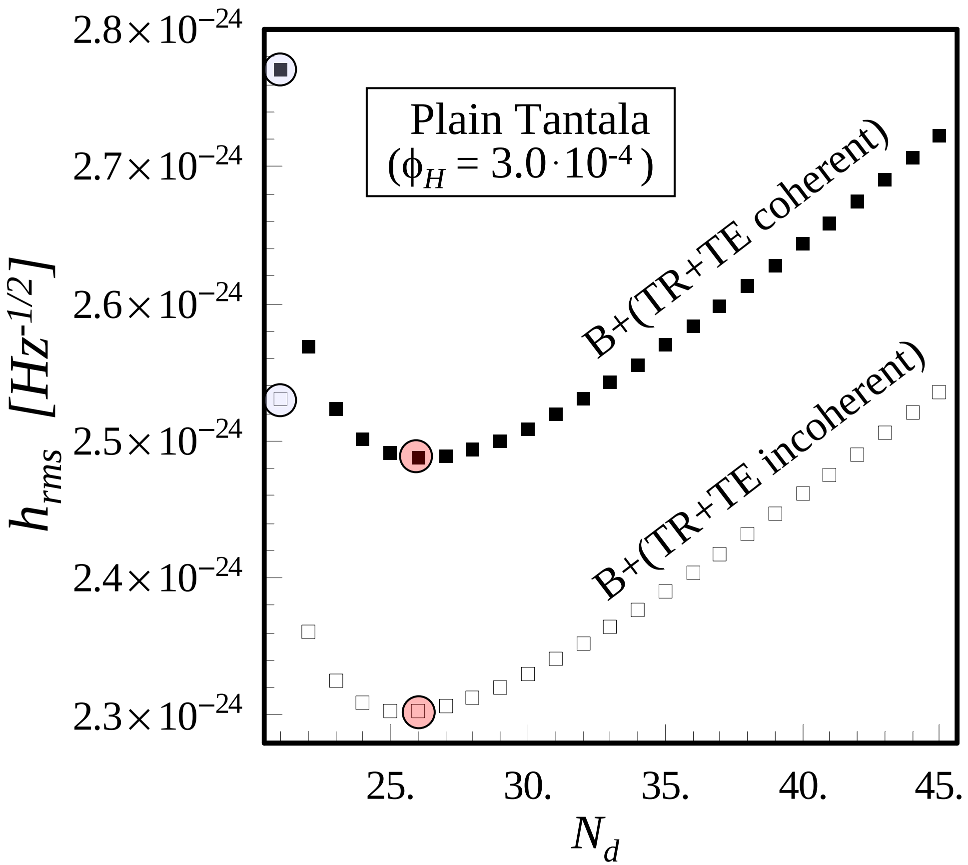
Non-Brownian vs Brownian Coating Noise Components

$\tau = 0.9727\text{ppm}, \; f = 100\text{Hz}$



When using *plain* Tantala, in the standard QWL design, the *total* (TE+TR, coherent) non-Brownian coating noise term is *comparable* to the Brownian one…

Plain Tantala
Total Coating Noise Budget



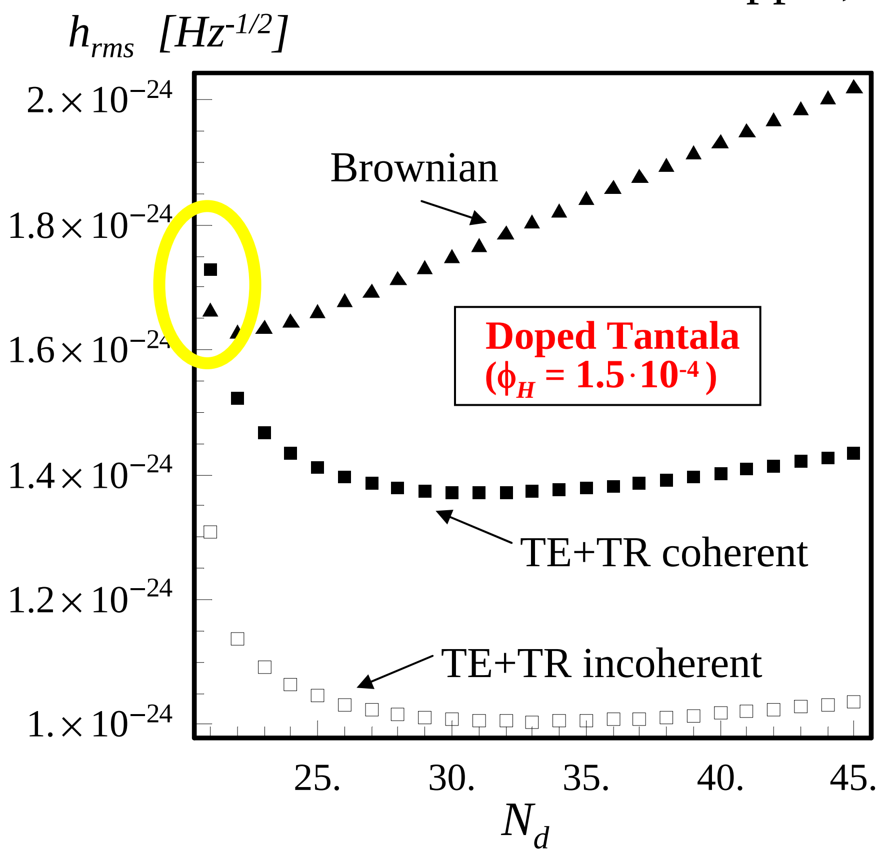
$\tau = 0.9727$ppm, $f = 100$Hz

- QWL coating
- Optimized coating

...The optimal SD coating design is *distinctly different* from QWL, and the related event rate boost is sensible (+ 38%)…

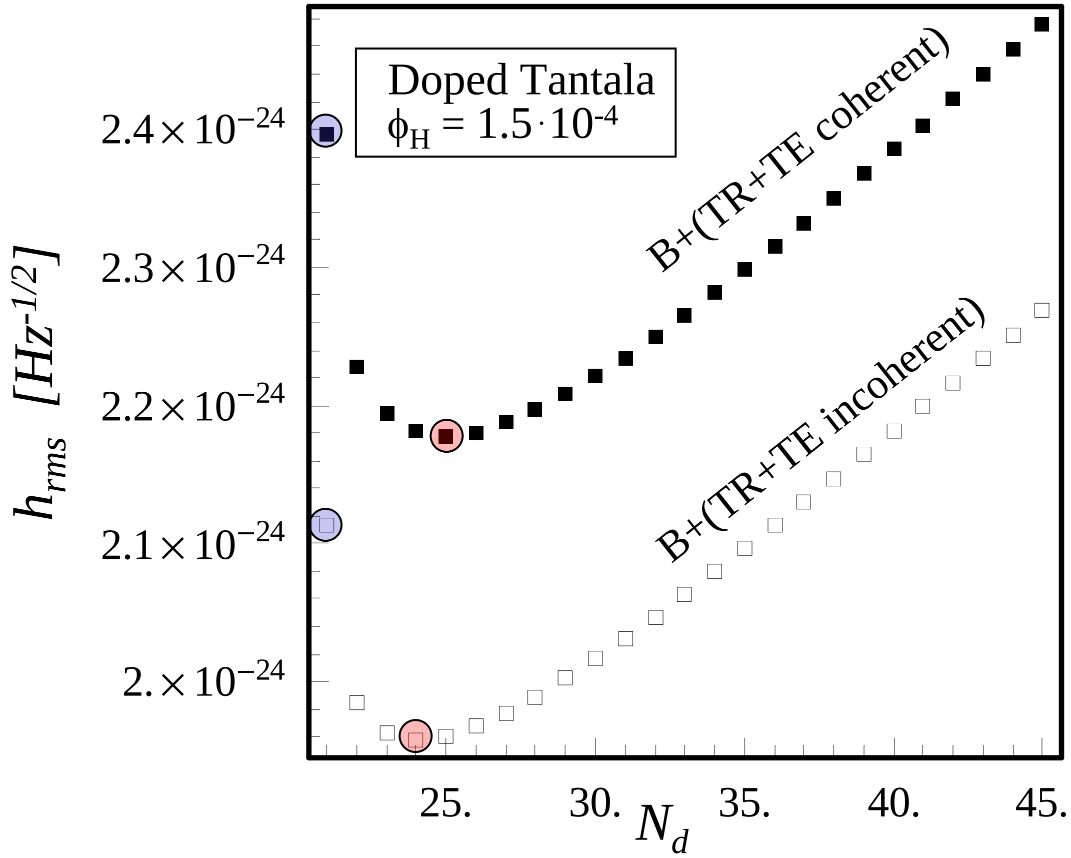
Non-Brownian vs Brownian Coating Noise Components TiO_2 - Doped Tantala

$\tau = 0.9727\text{ppm}, \; f = 100\text{Hz}$



When using doped Tantala, in the standard QWL design, the *total* (TE+TR, coherent) non-Brownian coating noise term turns out to be *larger than* the Brownian one…

Doped Tantala
Total Coating Noise Budget



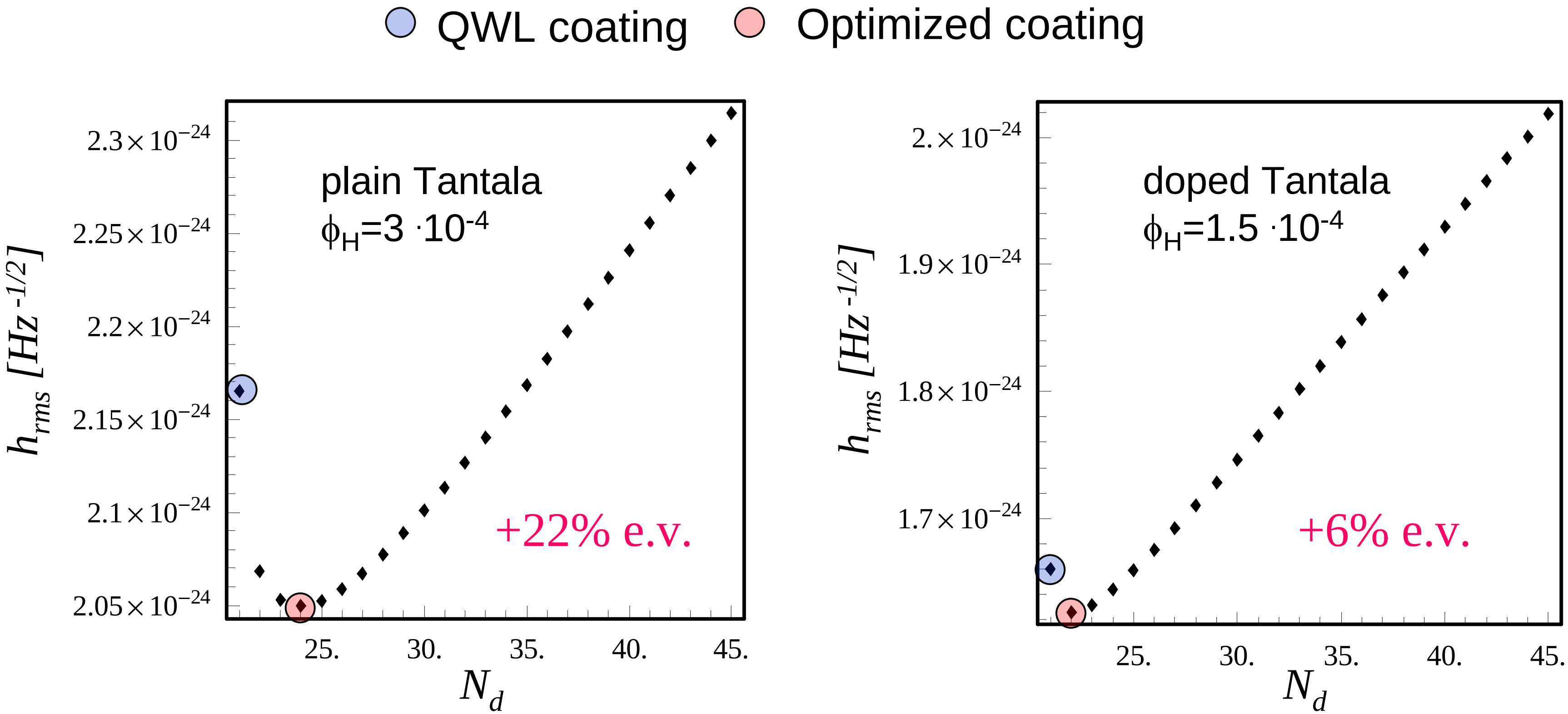
$\tau = 0.9727$ppm, $f = 100$Hz

- QWL coating
- Optimized coating

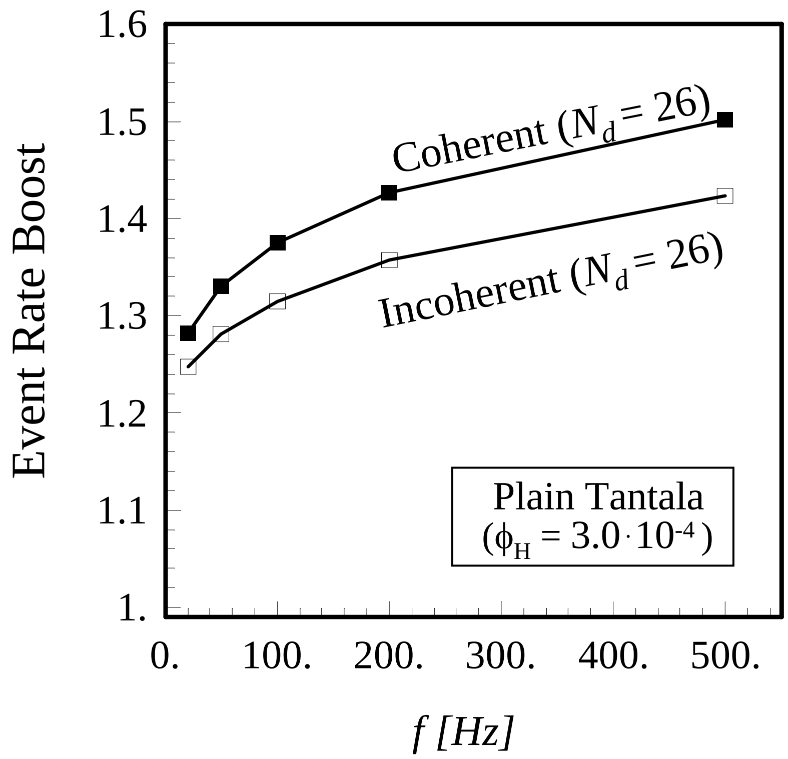
...The optimal SD coating design is *distinctly different* from QWL, and the related event rate boost is sensible (+ 33%)…

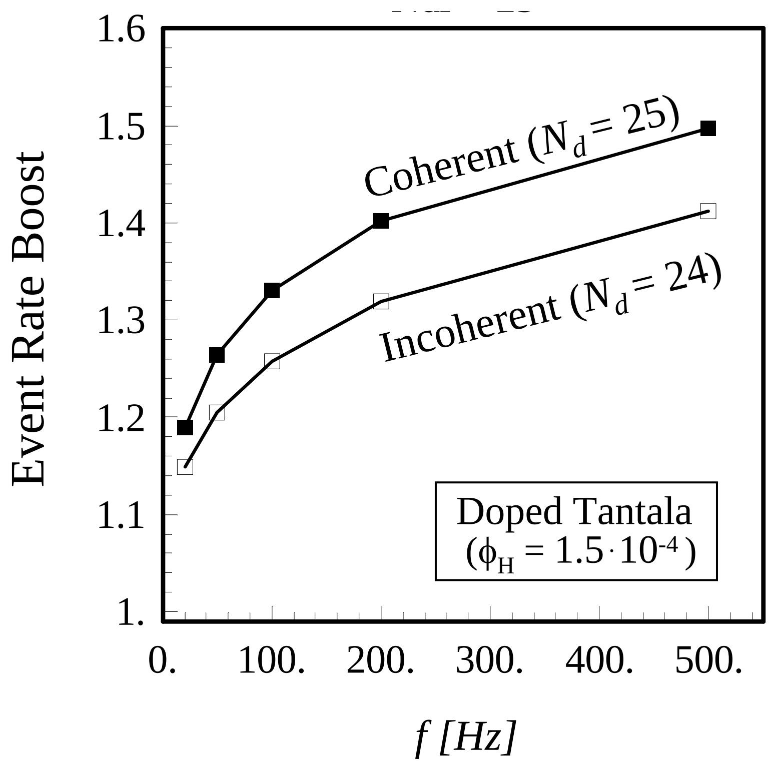
Back to Plain vs. Doped Tantala Brownian Noise Only

Brownian Noise Only. τ = .9727ppm, f = 100Hz



Event Rate Boost Vs. Frequency





Event Rate Boost (Isotropic/Homogeneous Source Distribution)

(Total Noise Budget))

$\tau = 0.9727\, ppm$	Event rate boost@100Hz		
Plain Tantala, QWL	1		
Plain Tantala, OPT	1.38	1	
Doped Tantala, QWL	1.54	1.11	1
Doped Tantala, OPT	2.05	1.48	1.33

Conclusions

- Coating thickness optimization should be considered as *almost mandatory* to minimize coating noise, even more so when using doped Tantala, yielding in all cases a substantial increase (> 30%@ \00Hz) in the expected event rate, as compared to the QWL design.

- Among all proposed coating noise reduction techniques (new materials, cryogenic mirrors, flat-top beams) thickness optimization offers a cheapest reliable addition

- Coating thickness optimization has been shown to be *effective* in reducing the total coating noise even when using the controversial Inci's values for α_H, β_H.

- Optimized coating prototypes are scheduled for testing at Caltech TNI; and (still) waiting for delivery from LMA.

Acknowledgements

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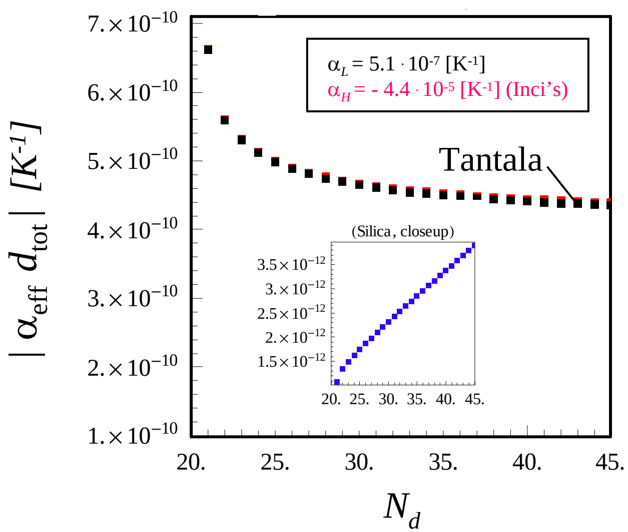
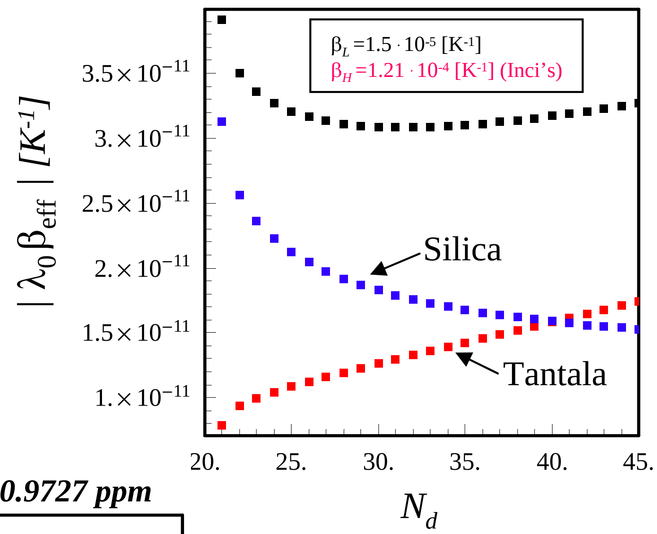
Inci's Numbers

[Naci-Inci, J. Phys. D37 (2004) 3151.]



(TE Coefficient)



(TR Coefficient)