

# Modeling of fused silica optics and coatings

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*LSC meeting*

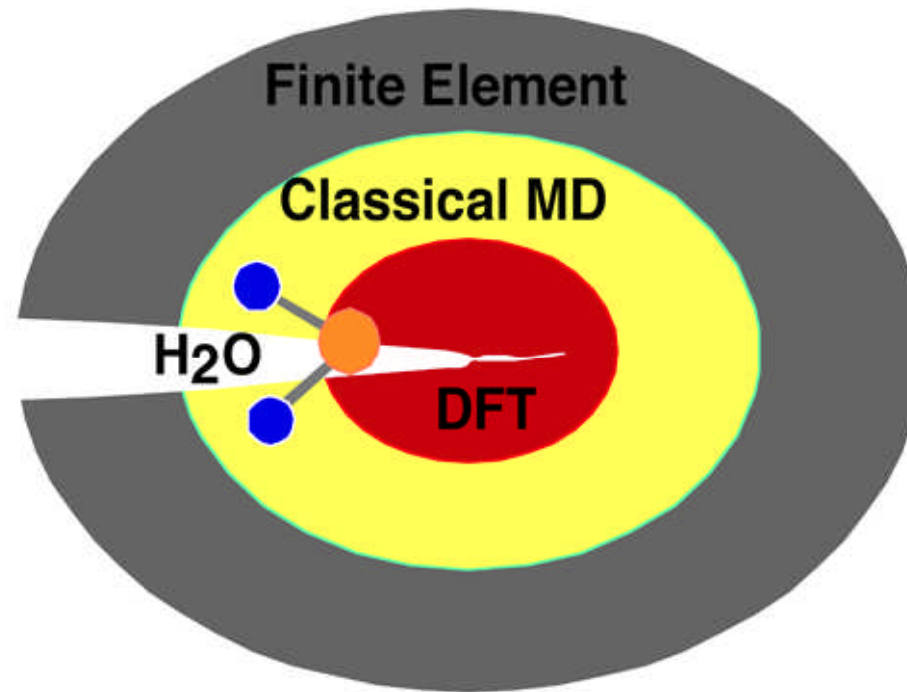
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*MIT*

*LIGO-G070567-00-Z*

*Acknowledgment:* *NSF/KDI-ITR Program and DOE/NERSC  
Super Computer Center,  
UF/HPC*

*Multi-scale simulation of material behaviors  
including chemical reactions*



- *Thermal Noise, Mechanical loss: A serious problem from the LIGO pr*
- *Crack propagation and hydrolytic weakening in SiO<sub>2</sub>*
- *Chemo-mechanical processes*

# Quantum modeling for materials properties

Density functional theory with general gradient approximations:  
A state-of-the-art method for first-principles material simulation with chemical accuracy

The Kohn-Sham equation for a system with periodic boundary condition is written as follows,

$$\left[ -\frac{1}{2} \nabla^2 + V(\rho(r)) \right] \psi_\alpha = \varepsilon_\alpha \psi_\alpha; \alpha = (n, \mathbf{k})$$

$$V(\rho) = V_{e-nuc} + V_H(\rho) + V_{XC}(\rho)$$

*~10<sup>3</sup> electrons*

$$\psi_\alpha(r) = \psi_{n,k}(r) = \sum_{\substack{\mathbf{r} \\ G, \frac{1}{2}|G+k|^2 \leq E_{cut}}} c_{n,G+k}^{\mathbf{r}} e^{i(G+k)\cdot\mathbf{r}} \quad \text{or} \quad \psi_\alpha(r) = \psi_{n,k}(\mathbf{r}) = \sum_{\mu} c_{\mu n}^k \phi_{\mu}^k(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}$$

# Molecular dynamics (MD) method

$$m_i \frac{d^2 \mathbf{R}_i}{dt^2} = -\nabla_{\mathbf{R}} U(\{\mathbf{R}\}, \rho(\mathbf{r}))$$

$\mathbf{R}$ : Nuclei position,  $\rho$  electron density, in general, one solve the whole wave function to get energy and forces. In classical MD, we replace  $U$  by empirical functions

Van Beest, Kramer, van Santen (BKS) Potential for  $\text{SiO}_2$

PRL 64, 1955 (1990)

$$U_{MD}(\{\mathbf{r}_{ij}\}) = \sum_{i < j} V_{ij}(\mathbf{r}_{ij})$$

$$V_{ij}(\mathbf{r}_{ij}) = \frac{q_i q_j}{r_{ij}} + A_{ij} e^{-b_{ij} r_{ij}} - \frac{C_{ij}}{r_{ij}^6}$$

Classical MD

$\sim 10^6 - 10^8$  atoms

$\sim$  nano-seconds

Good for obtaining statistics, but accuracy is limited by energy functions

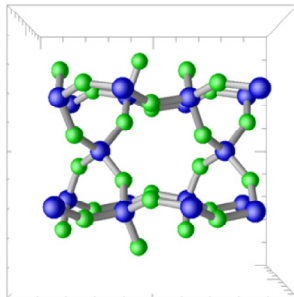
$A, b, C, q$ : potential parameters for Si-Si, O-O, & Si-O

# Modeling and Simulation -- What we can do and what to expect

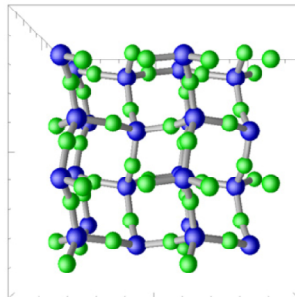
- Quantum Modeling -- based on density function theory  
Electronic Properties: **Energy barriers, dielectric functions, Young's modulus, Poisson ratio**, effects of dielectric doping, parameters for classical simulations,
- Classical molecular dynamics  
Mechanical and thermodynamical properties, structure: **Young's modulus, Poisson ratio, thermal expansion coefficient, thermal conductivity**
- New development: Hybrid Quantum-classical simulation  
Embedding a quantum model cluster in a classical environment for better description of energy barriers.
- Challenges: Accuracy in energy barrier, quality of classical potential

# Quantum calculation of crystal properties

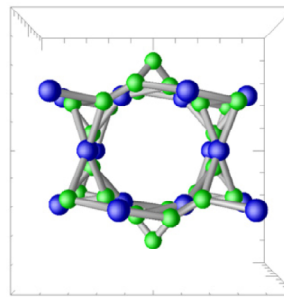
	$\alpha$ -quartz	$\alpha$ -cristobalite	$\beta$ -quartz	$\beta$ -cristobalite
$a$ (•) EXPTL	4.92	4.96	5.00	
$a$ (•) SIESTA	5.02	4.93	5.18	
$a$ (Å) PWSCF	5.06	5.13	5.13	
$c/a$ EXPTL	1.10	1.39	1.09	
$c/a$ SIESTA		1.41	1.09	
	1.10			
$c/a$ PWSCF	1.11	1.40	1.09	
$E_c$ (eV/SiO <sub>2</sub> )				
EXPTL	19.23	19.20	19.18	
SIESTA	21.34	21.30	21.29	21.13
VASP				
PWSCF				



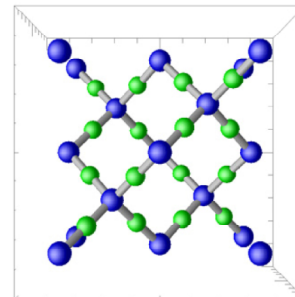
(a)



(b)



(c)



(d)

(a)-(d)  
 $\alpha$ -quartz,  
 $\alpha$ -cristobalite,  
 $\beta$ -quartz  
 $\beta$ -  
cristobalite.

# Amorphous silica

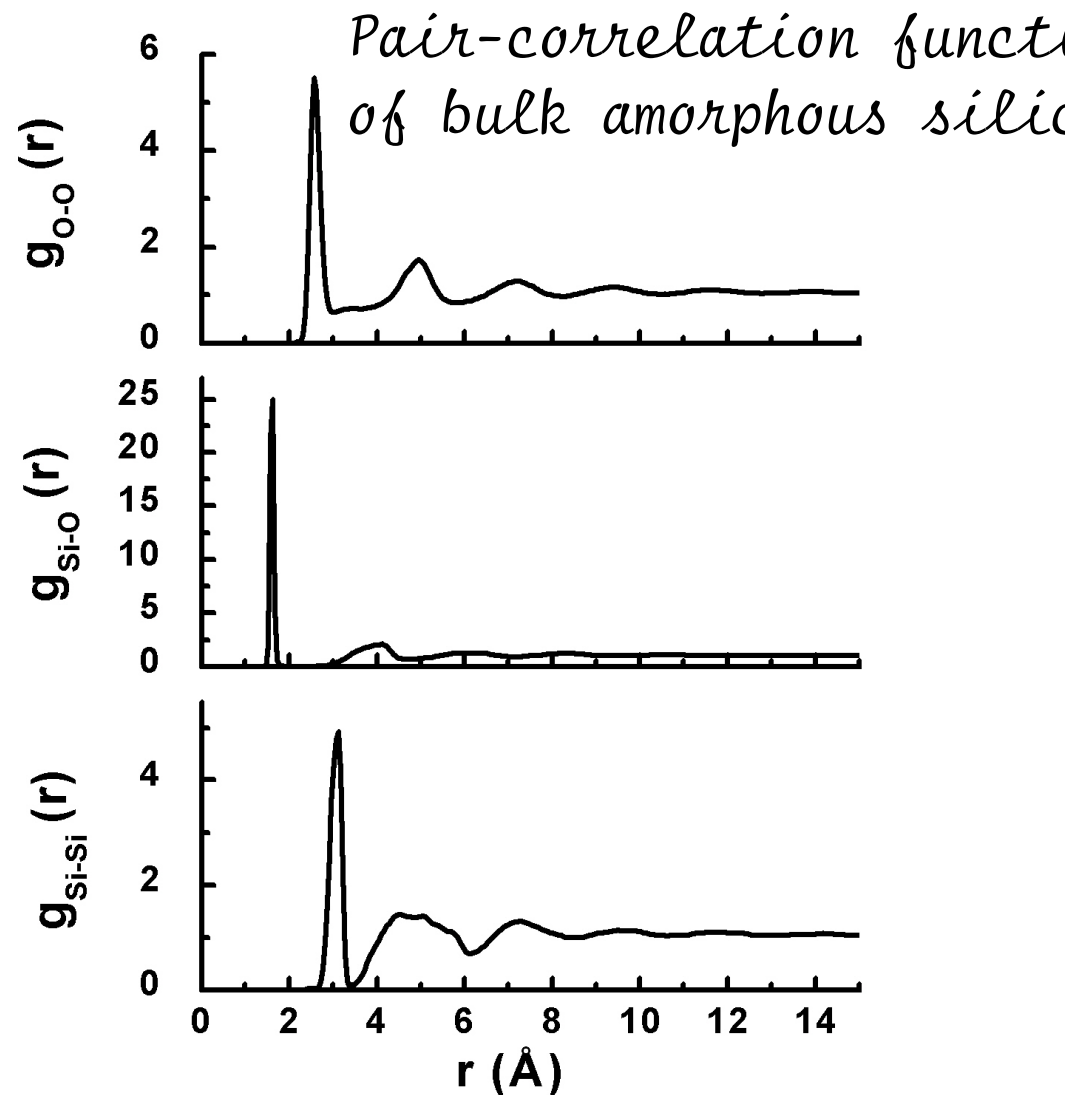
- The amorphous silica bulk is obtained by annealing of the liquid glass from 8000K to 300K.

Huff et al, *J. Non-Cryst. Solids* 253, 133 (1999)

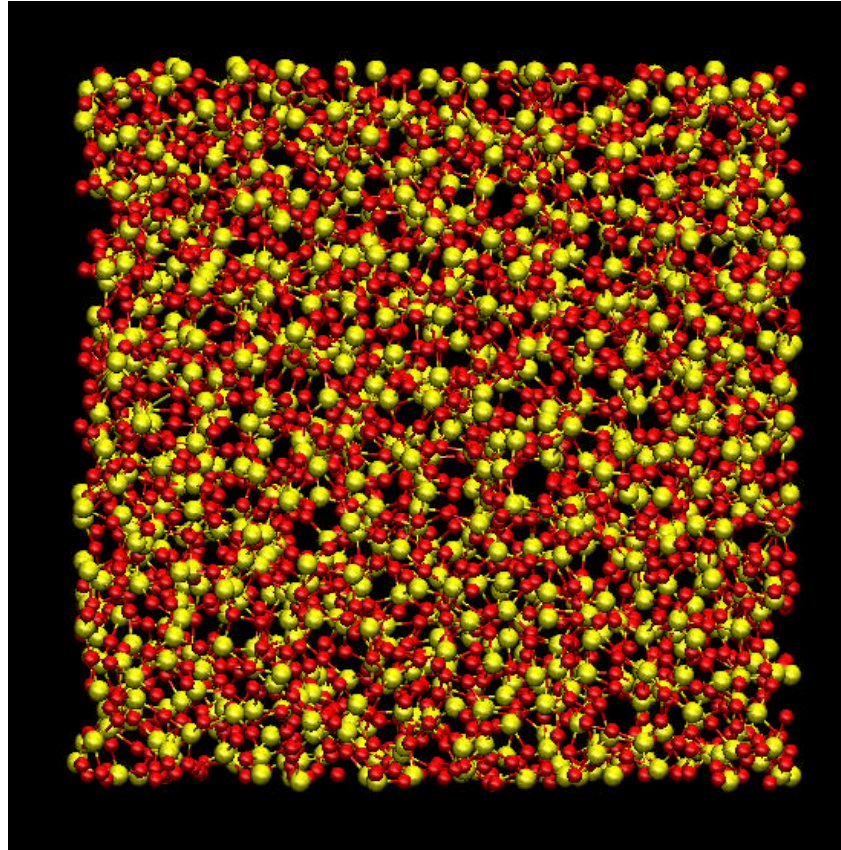
- A  $10^4$ -atom slab is used to simulate the surface.

- Density, pair-correlation functions are in agreement with experimental data

Wright *J. Non-Cryst. Solids*, 170, 61 (1994)



# Properties of amorphous silica surfaces



- In the absence of strain, the Si-O bonds are inert to H<sub>2</sub>O and NH<sub>3</sub>, etc.
- Strained Si-O bonds greatly increase the reactivity by creating acidic and basic adsorption sites on silicon and oxygen.
- Reactive sites (surface defects) play crucial roles in the surface corrosion
- Two-membered-ring (TMR) is a surface defect with high abundance

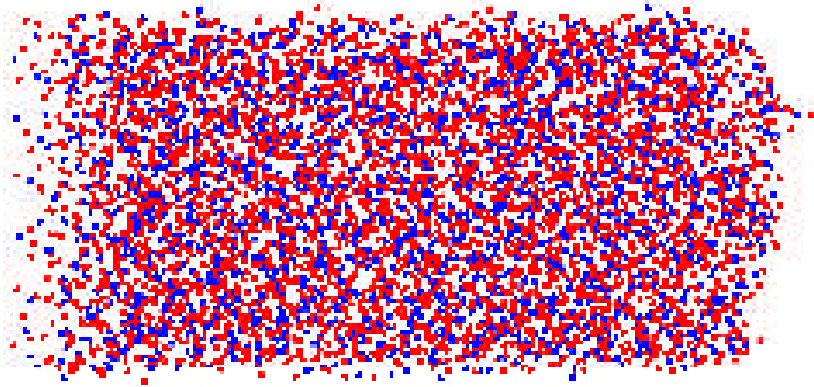
Water destroys TMR, heating above 500 °C restores the TMR, surface dehydroxylation

Bunker et al, Surf. Sci. 222, 95 (1989); Bunker et al, Surf. Sci. 210, 406 (1989)  
Walsh et al, JCP B13, 919 (1989)  
S. Iarori et al, JPC B105, 8007 (2001)  $\beta$ -cristobalite model

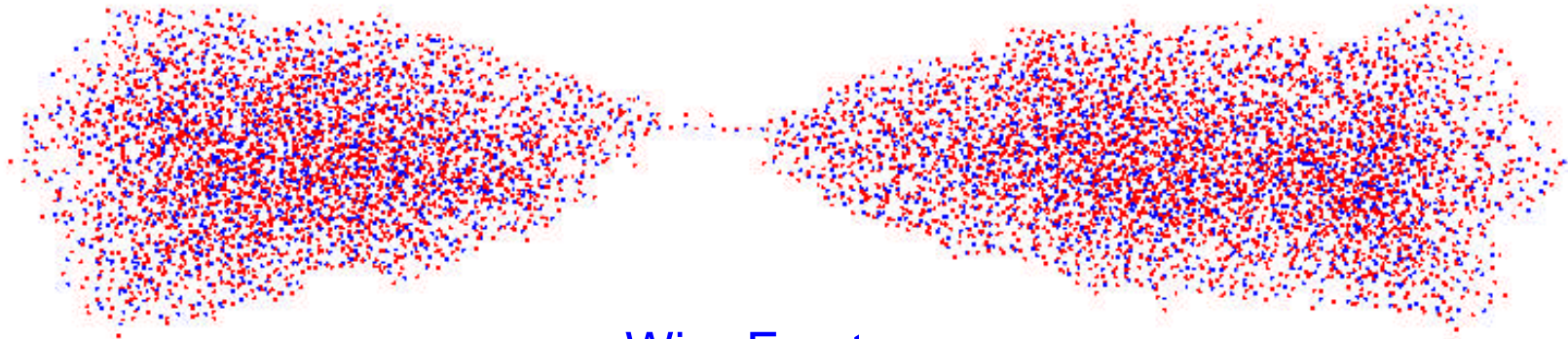
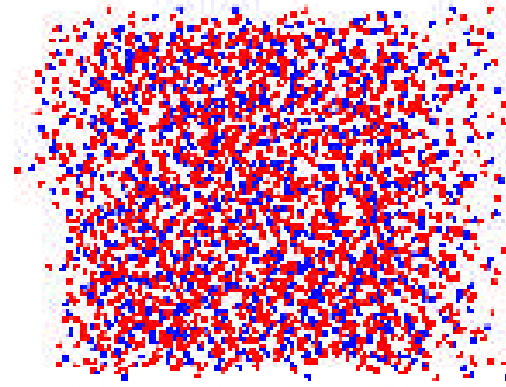


# Results: Fracture Point Snapshot

Comparison between amorphous systems



Bulk Fracture



Wire Fracture

# The Problem related to LIGO: Coating Thermal Noise

Relaxations of glasses affect:

- Neutron and light scattering
- Sound wave attenuation
- Dielectric loss

A direct relation between a microscopic quantity  $V$  and a macro-scopic measurement  $X''$  is (Wiedersich et al. PRL (2000) 2718

$$\chi''(\nu) \propto Q^{-1} \propto \int_0^{\infty} \frac{2\pi\nu\tau}{1 + (2\pi\nu\tau)^2} g(V) dV$$

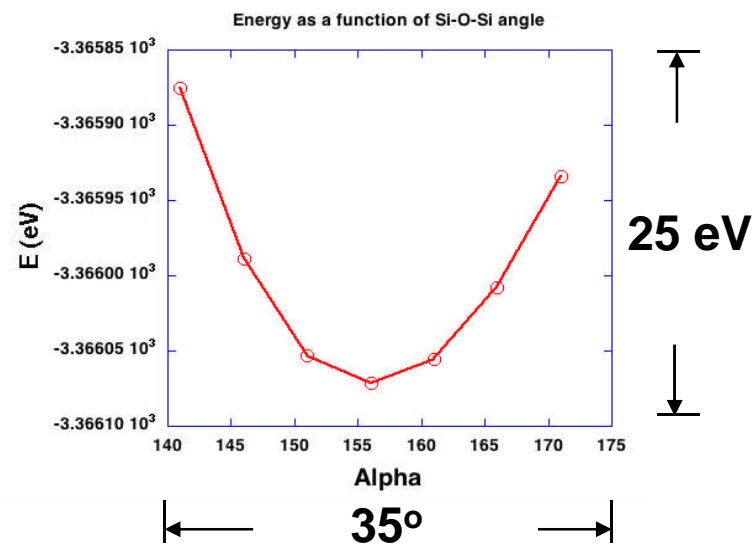
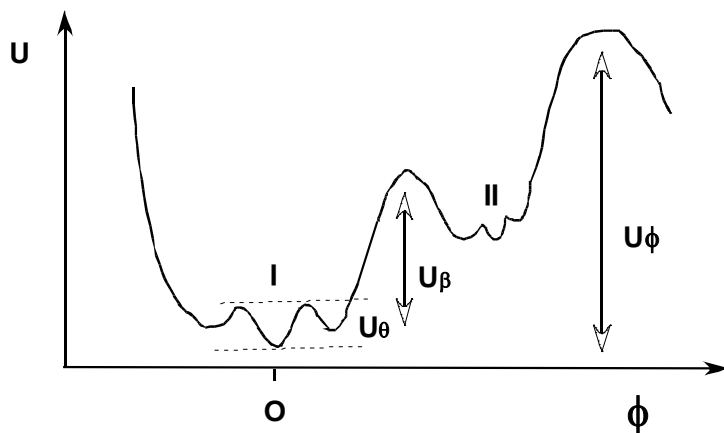
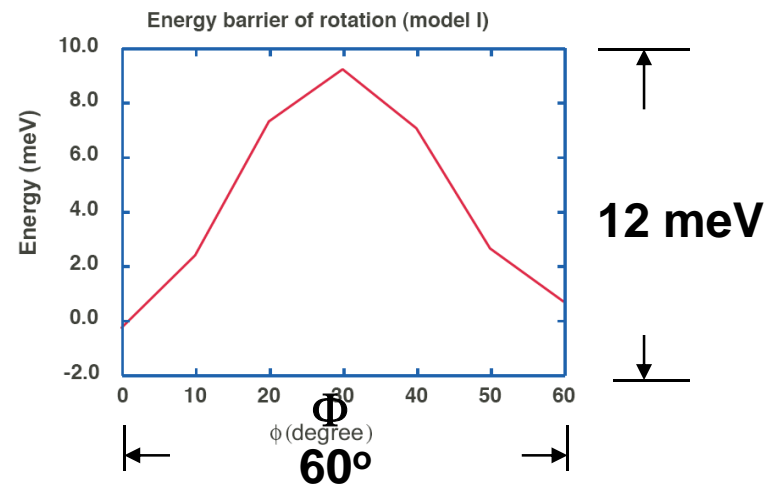
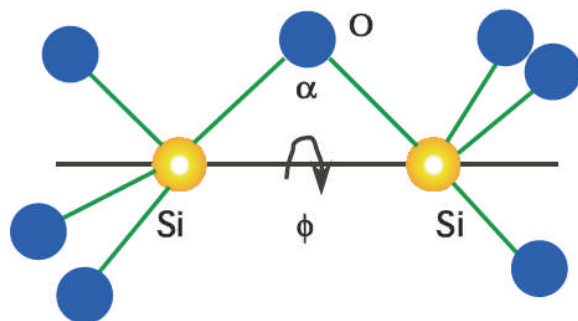
$\chi''(\nu)$  : light scattering scattering susceptibility,

$V$ : barrier,  $Q^{-1}$ : internal friction

$g(V)$ : barrier distribution,  $\tau$ : relaxation time

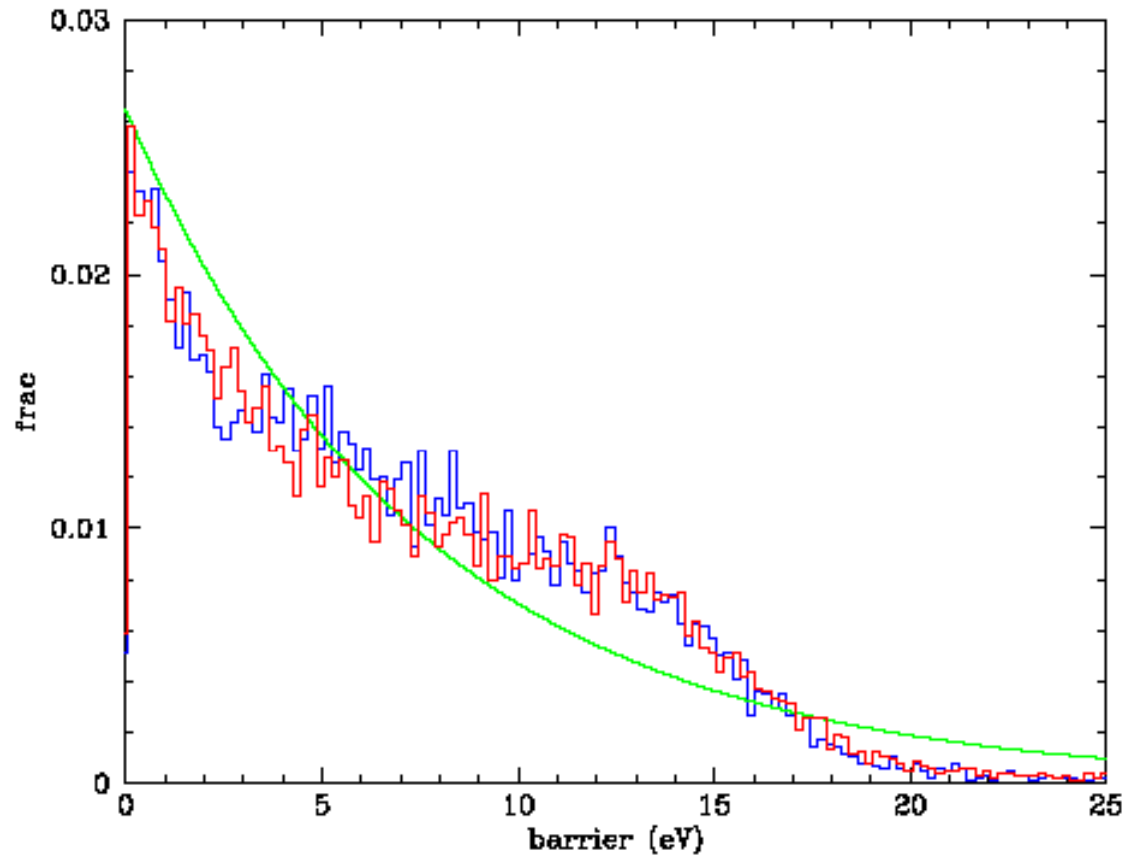
Thermal noise relates to  $Q$  via Young's modules, Poisson ratio,... G. Harry et al. Class Quantum. Grav. 19 (2002) 897-927 **Recent reference:** *G.Harry talk in LIGO/Virgo Thermal Noise Workshop October 2006*

# Quantum calculations of silica



M.R. Vukcevic, *J. Non-Crystl. Solids*  
 Vol. 11 (1972) pg.26-63  
 C.M. Bartenev, et al. *Inorganic Materials*  
 Vol.32, No.6 (1996) pg.671-682

## Barrier distribution from classical MD



Blue: Bulk

Red: Surface

Green:

$$g(V) = V_0^{-1} \exp(-V / V_0)$$

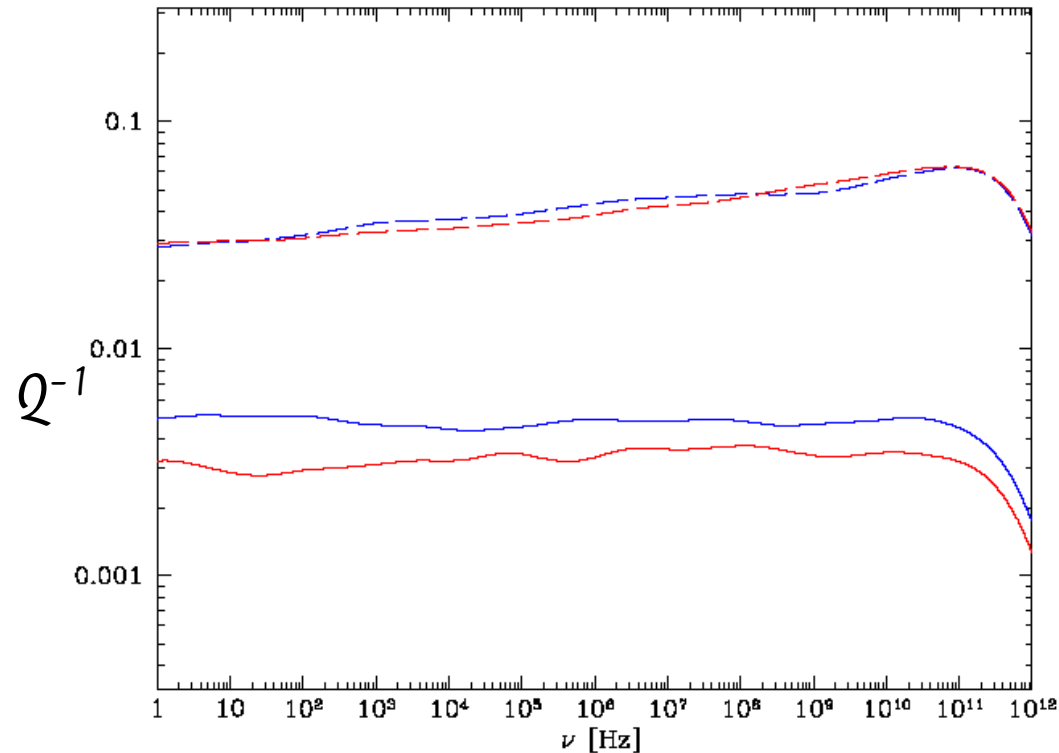
$$V_0 \approx 8eV$$

$$\sim 319 k_B T$$

Wiedersich et al.

84, 2718, PRL (2000)

# Calculated $Q^{-1}$ vs. frequency



Solid: 300 K  
Dashed: 32x300 K  
Blue: Bulk  
Red: Surface

Conclusion:  
Bad compared to  
experiments!

## Improvements

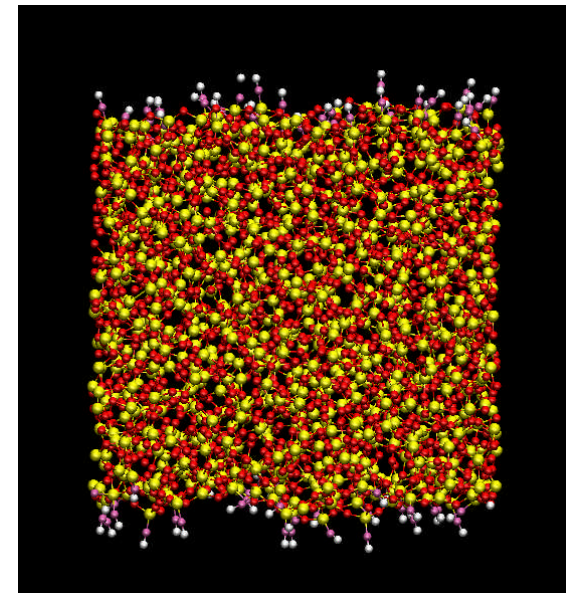
Relaxation during barrier calculation

Locating all possible low barriers

Improving potential energy function

Improving statistics

Also: Investigate hydroxylated surfaces 



# $Ta_2O_5$ : Structure

QuickTime™ and a  
TIFF (LZW) decompressor  
are needed to see this picture.

A high-temperature structure for  $Ta_2O_5$  with modulations by  $TiO_2$  substitution  
*Makovec et al.* Journal of Solid State Chemistry 179 (2006) 1782–1791

# PLAN: quantum calculations of $Ta_2O_5 + TiO_2$

QuickTime™ and a  
TIFF (LZW) decompressor  
are needed to see this picture.

Optimized  $Ta_2O_5$  high-temperature crystal structure (via DFT). High-T structure is closer to amorphous structure, a better model than the low-T one.  
Structure and mechanical properties of pure and doped  $Ta_2O_5$   
Effect of local chemistry on Young's moduli, Poisson ratio

$$S_x(f) = 2k_B T \phi_{\text{eff}} (1 - \sigma^2) / (\pi^{3/2} f w Y),$$

$$\begin{aligned} \phi_{\text{eff}} = \phi + d / (\sqrt{\pi} w Y_{\perp}) & \left( (Y / (1 - \sigma^2) - 2\sigma_{\perp}^2 Y Y_{\parallel} / (Y_{\perp} (1 - \sigma^2) (1 - \sigma_{\parallel}))) \phi_{\perp} \right. \\ & + Y_{\parallel} \sigma_{\perp} (1 - 2\sigma) / ((1 - \sigma_{\parallel}) (1 - \sigma)) (\phi_{\parallel} - \phi_{\perp}) \\ & \left. + Y_{\parallel} Y_{\perp} (1 + \sigma) (1 - 2\sigma)^2 / (Y (1 - \sigma_{\parallel}^2) (1 - \sigma)) \phi_{\parallel} \right), \end{aligned}$$

G. Harry et al. Class. Quantum Grav. 24 (2007) 405–415

# *Simulation Milestones*

- Examine properties of silica using the current model and compare with existing experimental measurement on  $\text{SiO}_2$  bulk and surface, extend our investigation to silicates. (Advance LIGO)
- Investigate the effects of coating and dopant materials used in the LIGO experiment, and understand the change of physical properties. ( $\text{Ta}_2\text{O}_5$ ,  $\text{TiO}_2$ ,  $\text{HfO}_2$ ,  $\text{Nb}_2\text{O}_5$ , ..titania, zirconia lutetium doping...) (Advance LIGO)
- In collaboration with experiments, seek for new coating materials that have optimal combinations of low thermal noise and optical absorption, computer-aided material design (beyond Advance LIGO).



# Collaborators

- **Group members**

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