TDI Network Simulation

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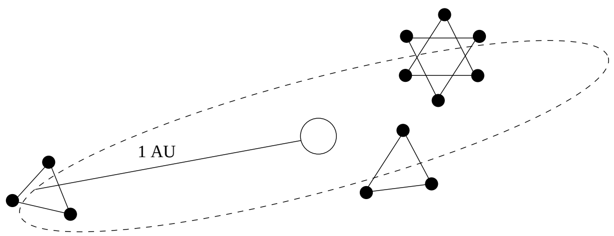
LSC meeting, Hannover

October 25, 2007

LIGO-G070757-00-Z

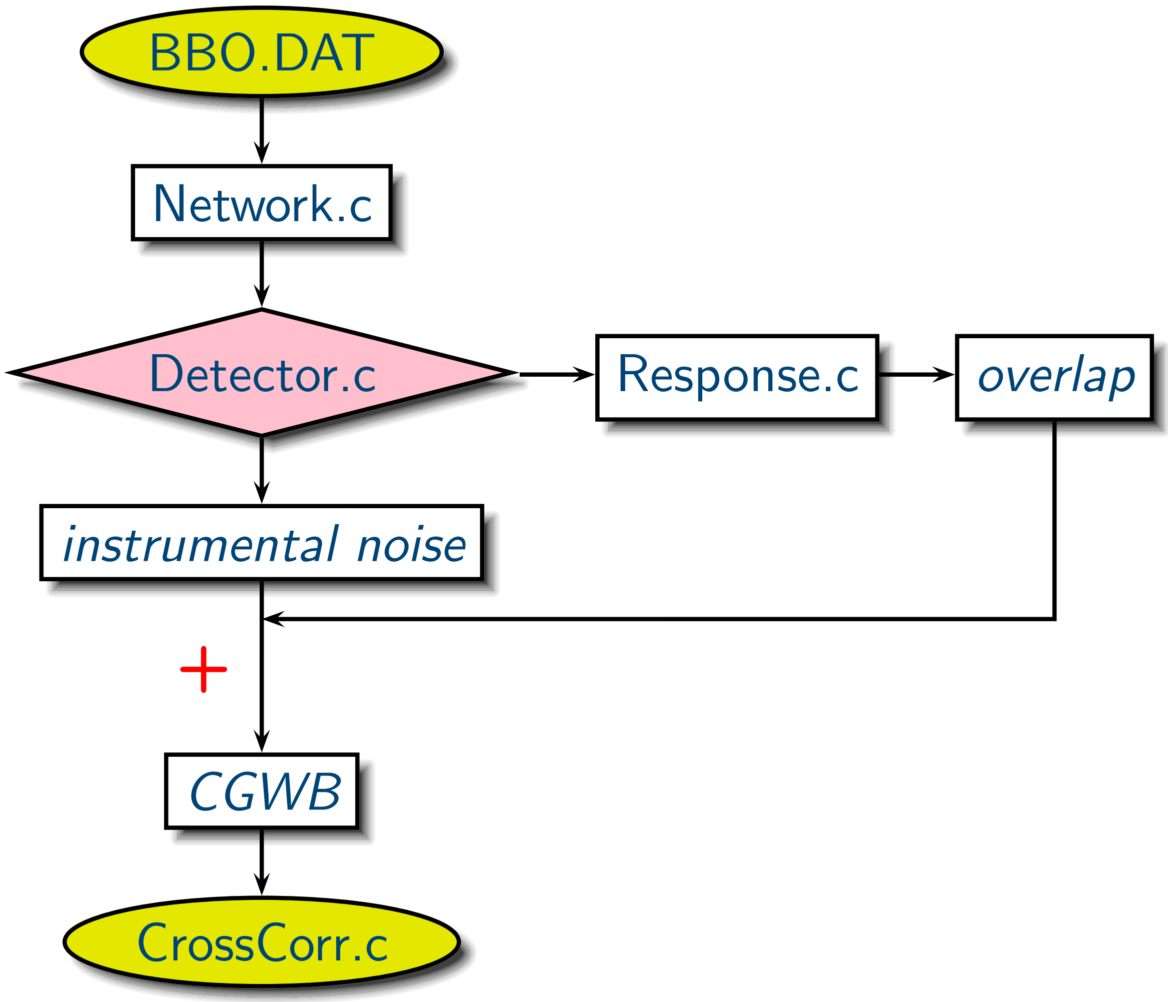
The Big Bang Observer



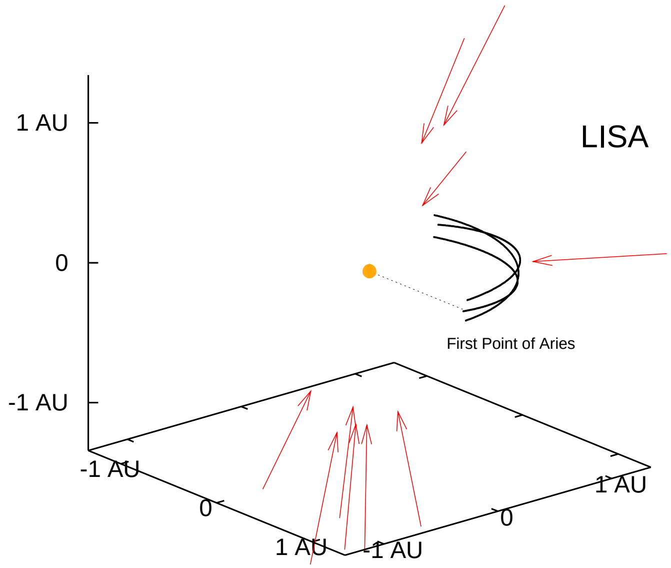
- ✔ 4 LISA type space detectors
- ✔ Orbiting the sun at 1 AU
- ✔ Two collocated detectors

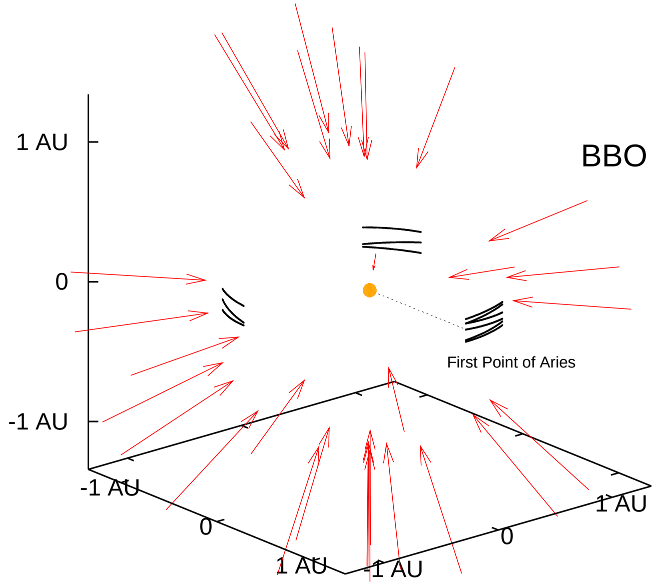
	Symbol	Value
Laser power	P_0	$300\,\mathrm{W}$
Received power	P_r	$\sim 9\,\mathrm{W}$
Mirror diameter	D	$2.5\,\mathrm{m}$
Arm length	L	$5 \cdot 10^7\ \mathrm{m}$
Wavelength of laser light	λ	$355\,\mathrm{nm}$
Acceleration noise	$\sqrt{S_\mathrm{acc}}$	$3 \cdot 10^{-17}\,\mathrm{m}/(\mathrm{s}^2\sqrt{\mathrm{Hz}})$

Simple Pipeline



Moving detectors



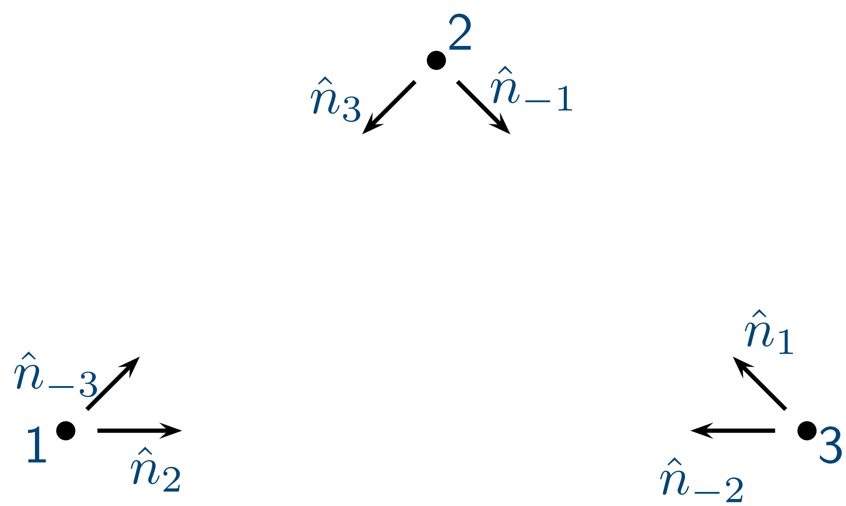


Time-dependent distance between spacecrafts:

$$L_l(t) = |\vec{p}_r(t) - \vec{p}_s(t - L_l(t)/c)|$$

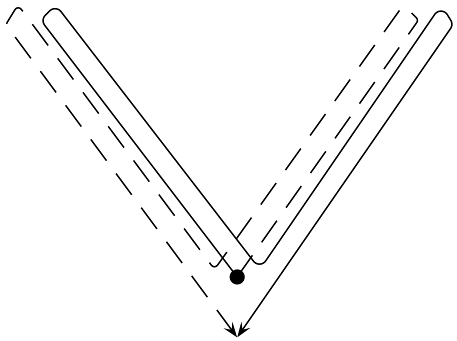
Time-dependent light propagation direction:

$$\hat{n}_l(t) = \frac{1}{L_l(t)} \left(\vec{p}_r(t) - \vec{p}_s(t - L_l(t)/c)\right)$$



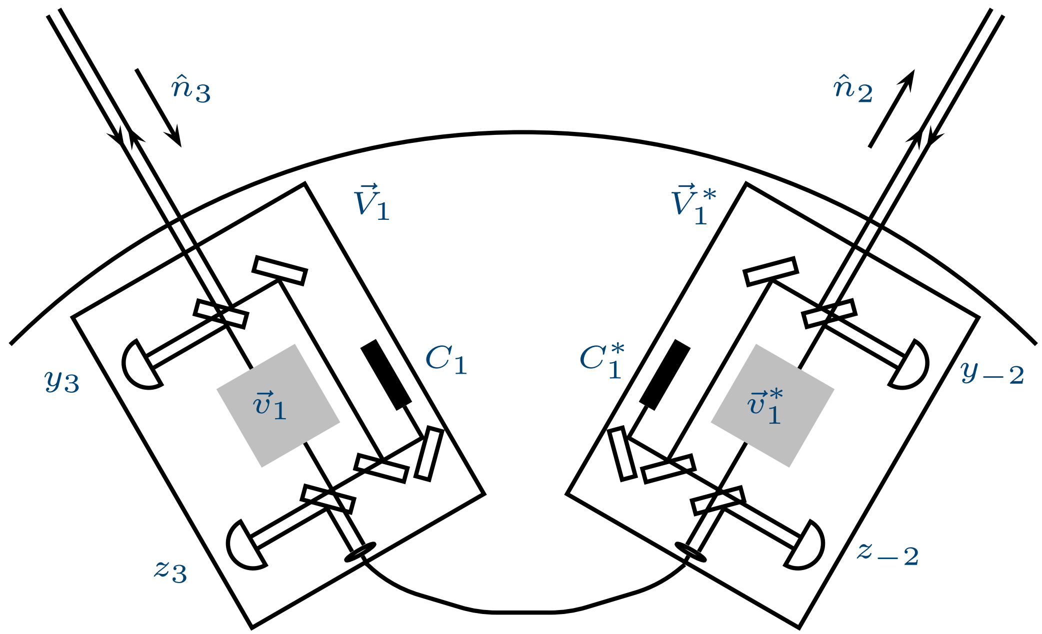
TDI

BBO
Network.c (1)
Orbits.c
Detector.c
TDI
GW Doppler signal
Response.c
Stochastic GW
Network.c (2)
Noise projection



Unequal-arm Michelson

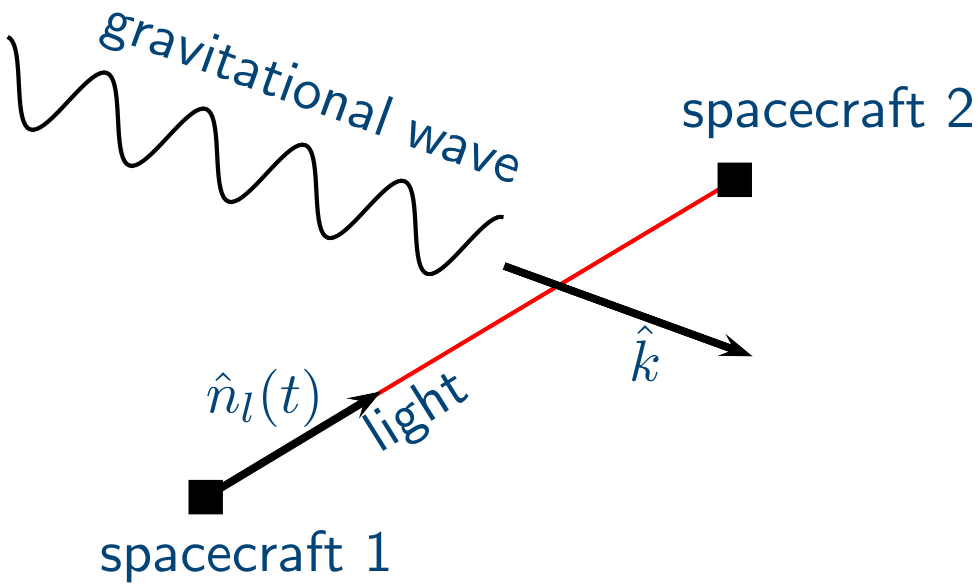
$$
\begin{aligned}
X(t) = & [y_{-3,32-2} + y_{3,2-2} + y_{2,-2} + y_{-2}] \\
& - [y_{2,-2-33} + y_{-2,-33} + y_{-3,3} + y_{3}] \\
& - 0.5 \cdot (z_{-2,2-2-33} - z_{3,-332-2}) - 0.5 \cdot (z_{-2} - z_{3}) \\
& + 0.5 \cdot (z_{-2,2-2} - z_{3,2-2}) + 0.5 \cdot (z_{-2,-33} - z_{3,-33})
\end{aligned}
$$



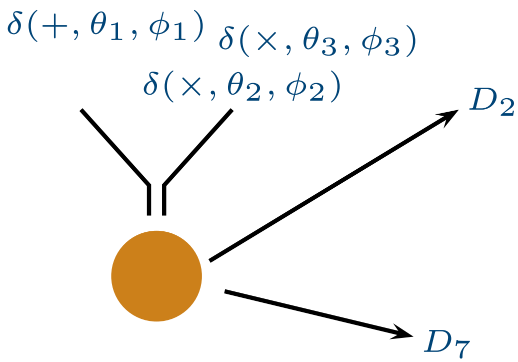
GW Doppler signal

Projected GW Doppler signal:

$$y^{\mathrm{GW}}(t) = \frac{1}{2(1-\hat{k}\cdot\hat{n}_l(t))}$$
$$\hat{n}_l(t)\cdot\Big(h(t_{\mathrm{s}}-\hat{k}\cdot\vec{p}_s(t_{\mathrm{s}})) - h(t-\hat{k}\cdot\vec{p}_r(t)\Big)\cdot\hat{n}_l(t)$$



FD properties from dynamical models

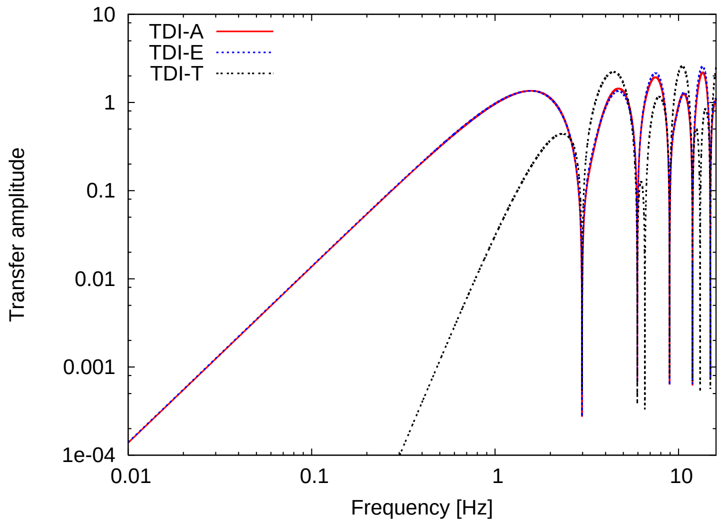


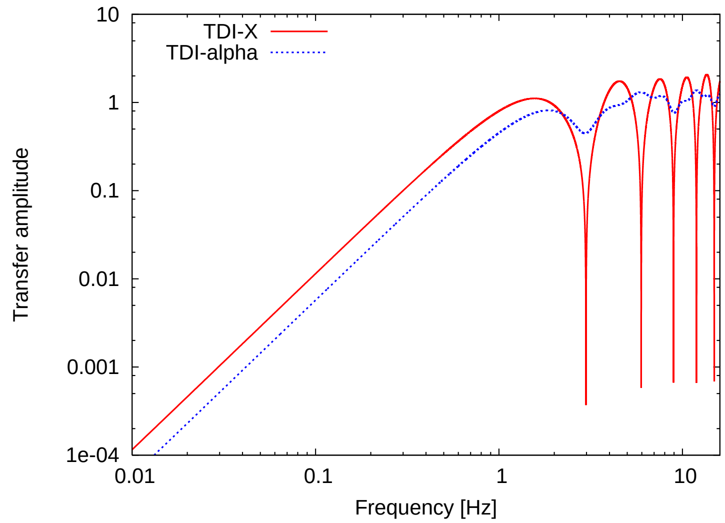
1. Inject δ-peaks at the origin of a master coordinate system
2. Use retarded time to calculate GW signal at each detector
3. Calculate a detector output
4. Obtain FD functions via FFT

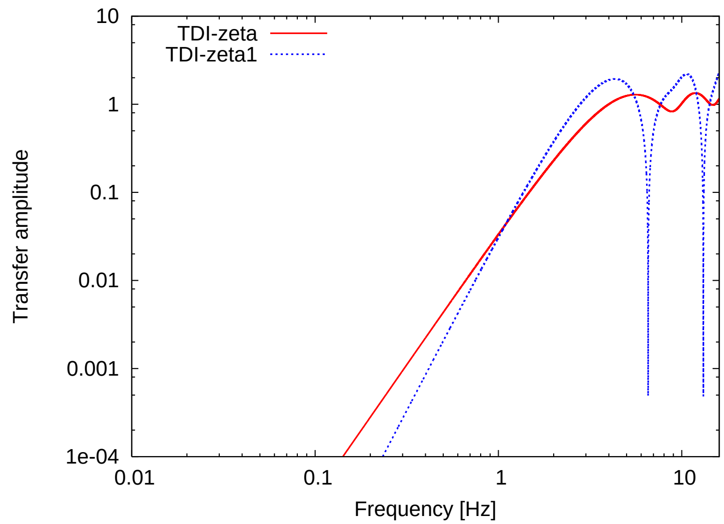
Example:

$$\gamma_{jk}(f_i) = \frac{1}{A} \sum_{\theta,\phi} \frac{\tilde{y}_j^{+}(f_i)[\tilde{y}_k^{+}(f_i)]^* + \tilde{y}_j^{\times}(f_i)[\tilde{y}_k^{\times}(f_i)]^*}{T}$$

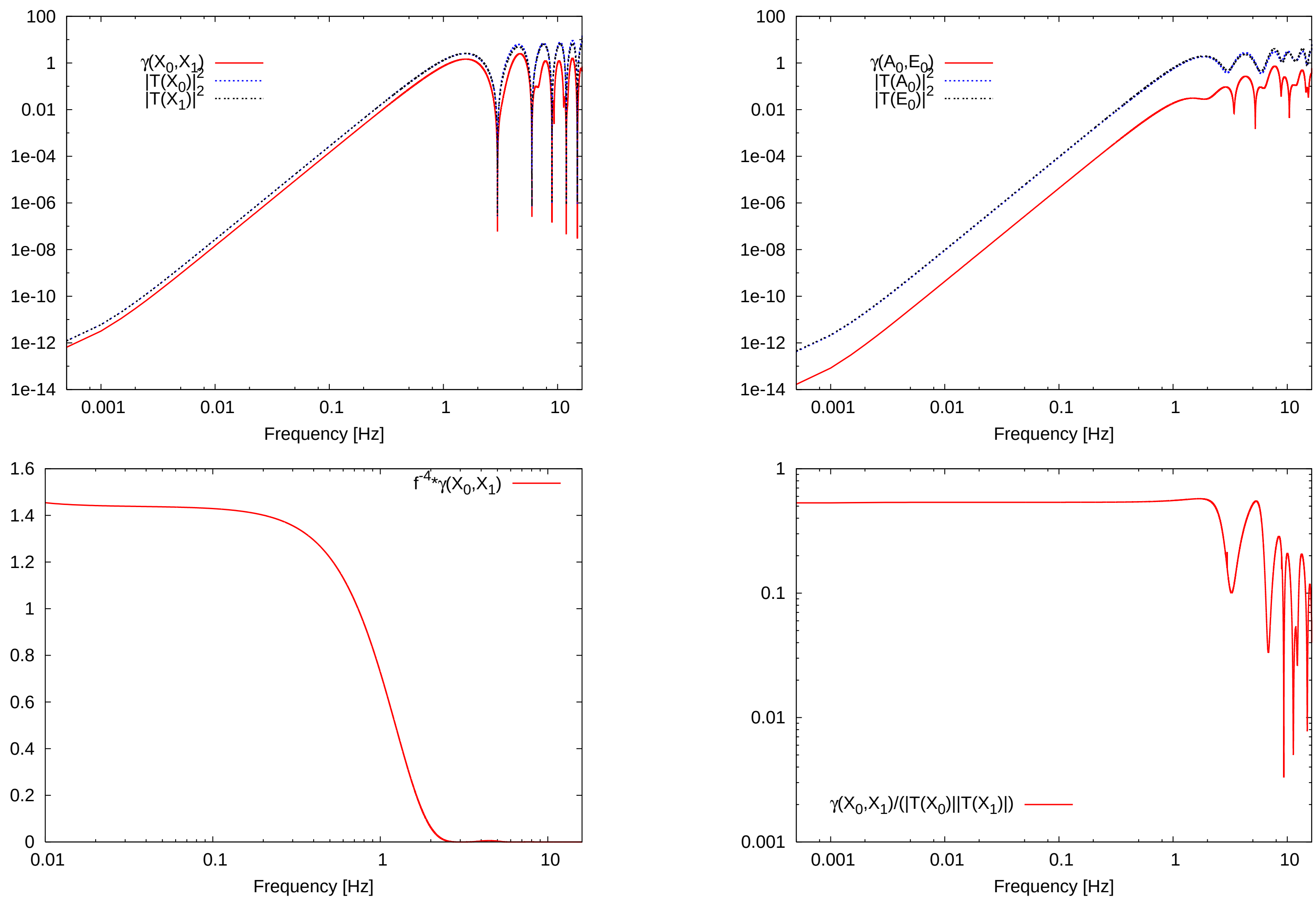
Transfer functions



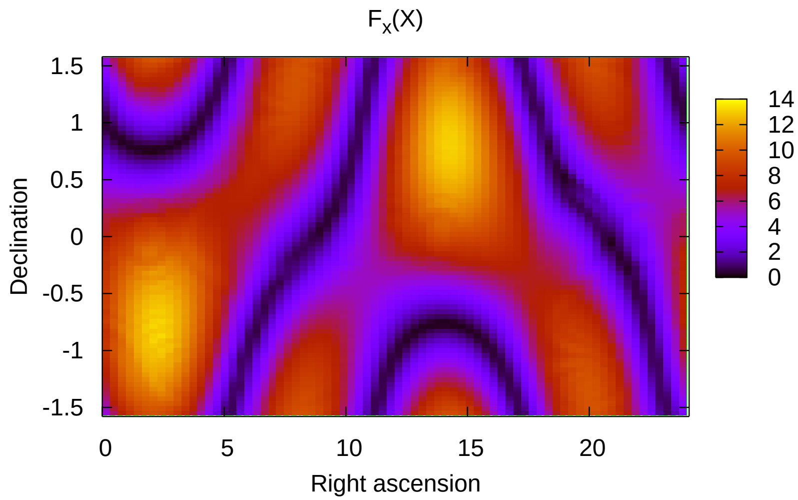


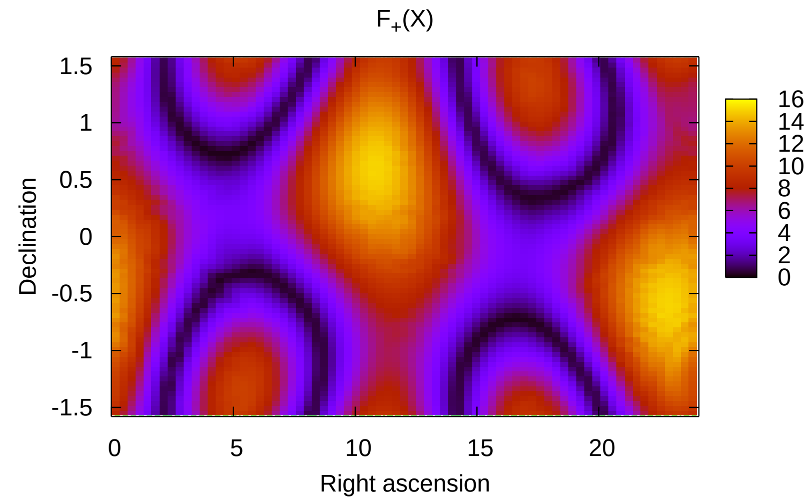


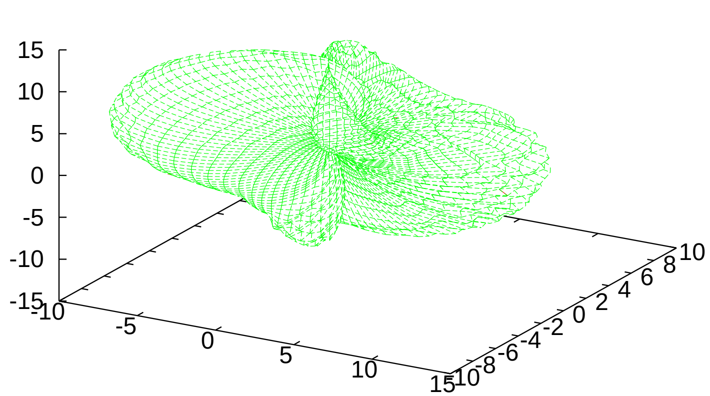
Overlap-reduction functions

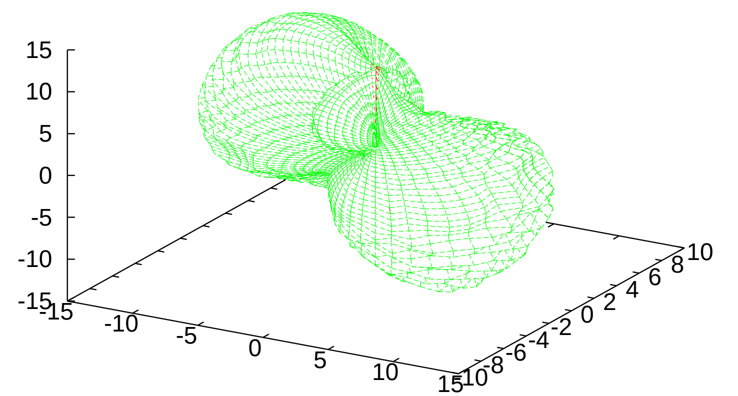


Response functions

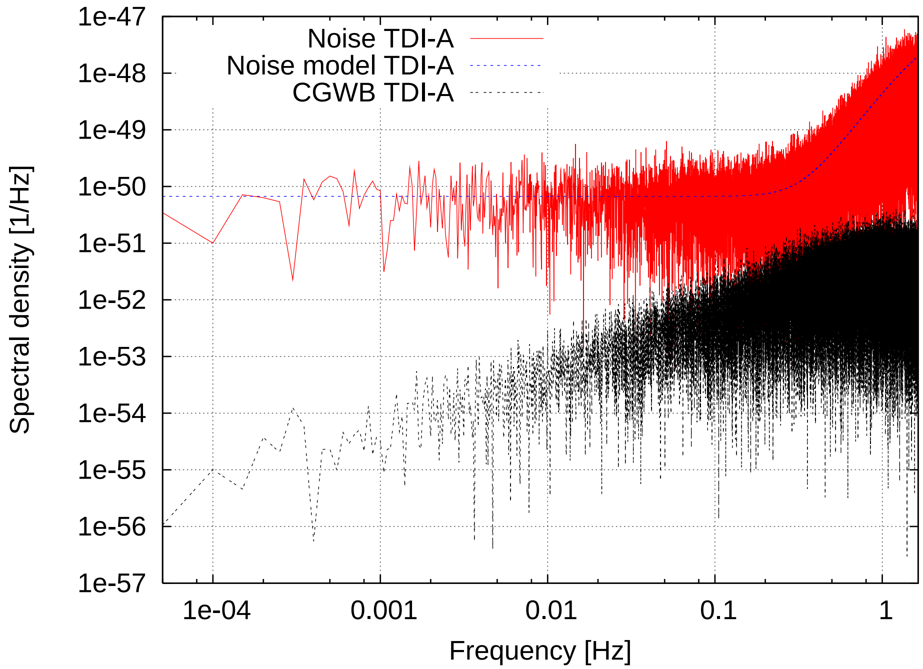








Primordial.c



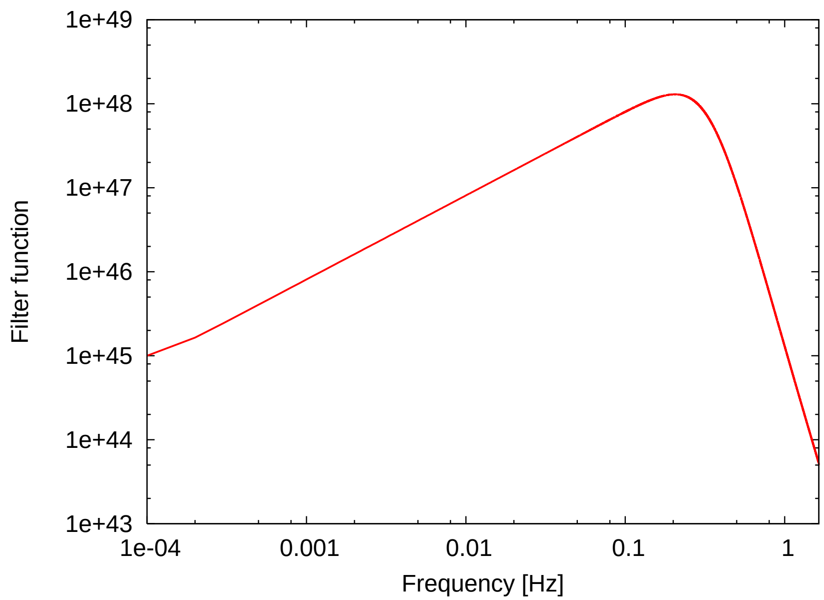
$$\tilde{y}_1^{\mathrm{h}} = \sqrt{\frac{T \cdot S_{\mathrm{gw}}}{2}\gamma_{11}}\,(n_1 + i n_2)$$

$$\tilde{y}_2^{\mathrm{h}} = \tilde{y}_1^{\mathrm{h}}\frac{\gamma_{12}}{\gamma_{11}} + \sqrt{\frac{T \cdot S_{\mathrm{gw}}}{2}}\sqrt{\gamma_{22} - \frac{|\gamma_{12}|^2}{\gamma_{11}}}\,(n_3 + i n_4)$$

CrossCorr.c

Cross correlation: $\langle S \rangle = \frac{1}{N} \left| \sum_{i=0}^{N} \frac{\tilde{y}_1(f_i)[\tilde{y}_2(f_i)]^*}{T} \tilde{Q}(f_i) \right|$

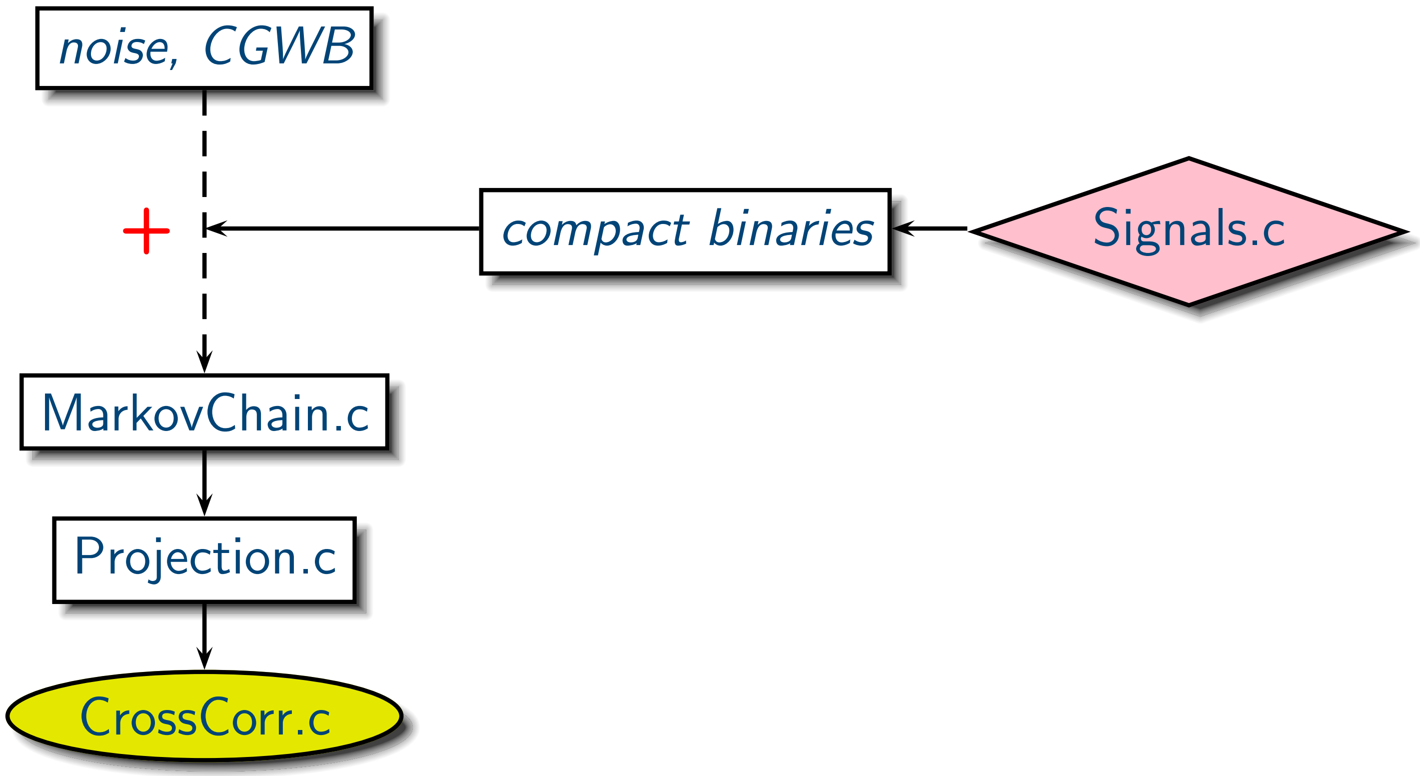
Optimal filter: $\tilde{Q}(f_i) = \frac{\gamma_{12}(f_i) S_{\rm gw}(f_i)}{S_1^{\rm n}(f_i) S_2^{\rm n}(f_i)}$



Sensitivities with respect to stochastic backgrounds:

- ✔ LISA: $\Omega \gtrsim 10^{-12}$ (needs to be checked)
- ✔ BBO: $\Omega \gtrsim 10^{-17}$

Complete Pipeline



FisherMatrix.c

- Fisher matrix depends on template derivatives:

$$\Gamma_{\alpha\beta}(\vec{\lambda}) = \langle \partial_\alpha h(\vec{\lambda}) | \partial_\beta h(\vec{\lambda}) \rangle$$

- Scalar products:

$$\langle g|h \rangle = \frac{1}{T} \sum_i^{N/2} \frac{\tilde{h}(f_i)\tilde{g}^*(f_i) + \tilde{g}(f_i)\tilde{h}^*(f_i)}{S^{\mathrm{n}}(f_i)}$$

- Numerical problems:

$$(\Gamma_{\alpha\beta}) = \begin{pmatrix} & D & \theta & \phi & \Psi & \cdots \\ D & 3.2 \cdot 10^{-37} & -5.8 \cdot 10^{-16} & -1.4 \cdot 10^{-15} & -1.2 \cdot 10^{-18} & \\ \theta & & 1.1 \cdot 10^{9} & 1.2 \cdot 10^{9} & 2.6 \cdot 10^{5} & \\ \phi & & & 1.8 \cdot 10^{9} & 4.0 \cdot 10^{5} & \\ \Psi & & & & 1.3 \cdot 10^{4} & \\ \vdots & & & & & \end{pmatrix}$$

- Brute-force solution: use GMP

Projection.c

1. Generate best fits: $\hat{\lambda}^\alpha = \Gamma_N^{\alpha\beta}\langle n|\partial_\beta h\rangle$
2. Subtract best fits from data: $\delta s = s - \hat{h}$, with $\hat{h} = h(\hat{\lambda})$
3. Project residual data: $\delta s_\perp = \delta s - \Gamma^{\alpha\beta}\langle \delta s|\partial_\alpha h\rangle \cdot \partial_\beta h$