Beams of the Future

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LIGO-G070767-00-Z

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JOBS WANTED!!

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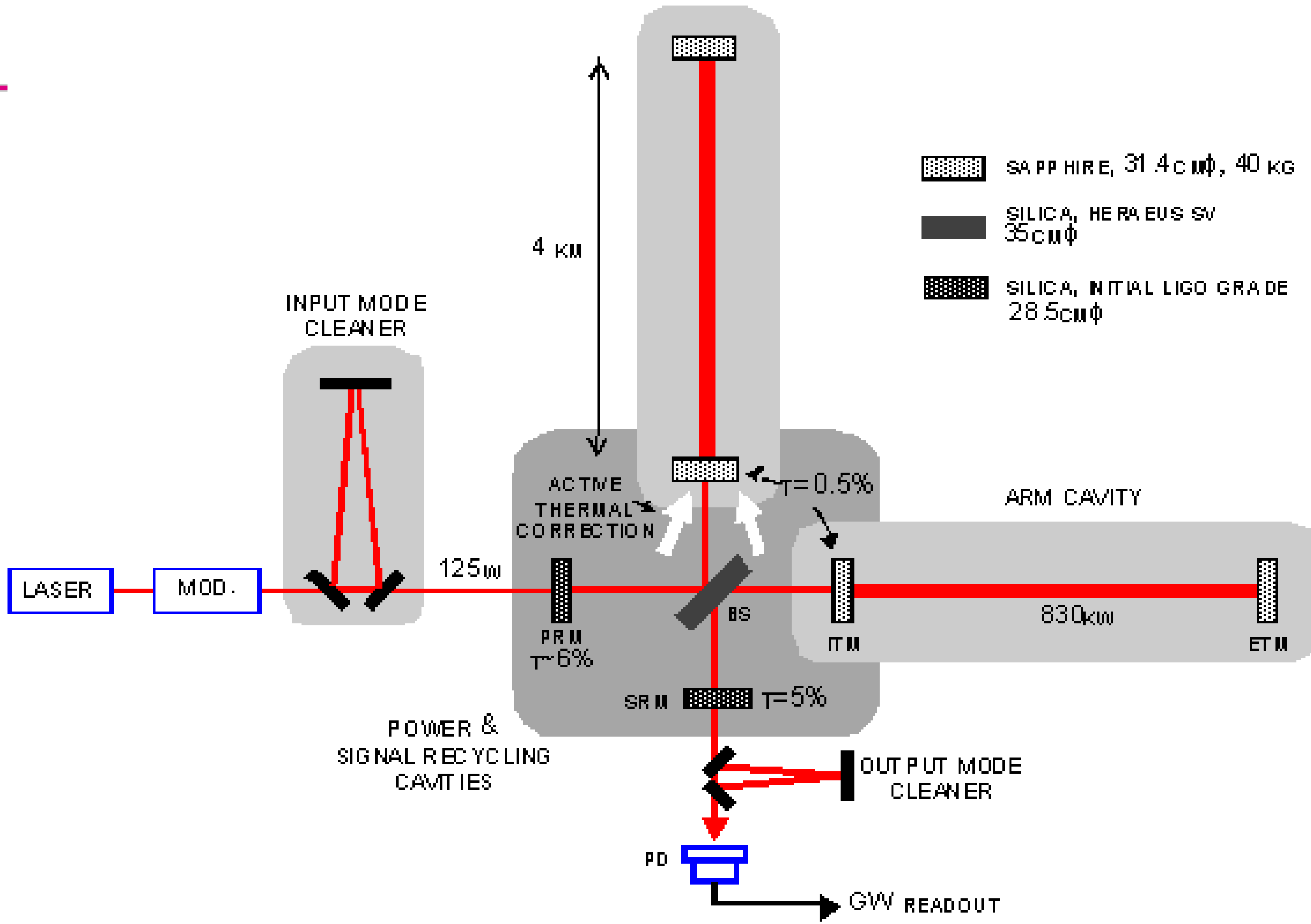
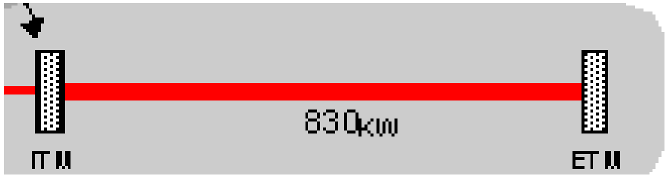
Advanced LIGO design



Arm Cavities - Current Status

- Circulating power over 830 kw
  - » Radiation pressure : ~$3*10^{-3}$ N
  - » Compare to 9-12 kw and ~ 3-4* 10^{-5} N in initial LIGO
- Gaussian Beams - Baseline Design
  - » High thermal noise
  - » Nearly Flat Spherical Mirrors (r= 53.7 km)
    - – To be changed to nearly concentric
- Hyperboloidal beams
  - » Mesa
  - » Finite Mirror Effects

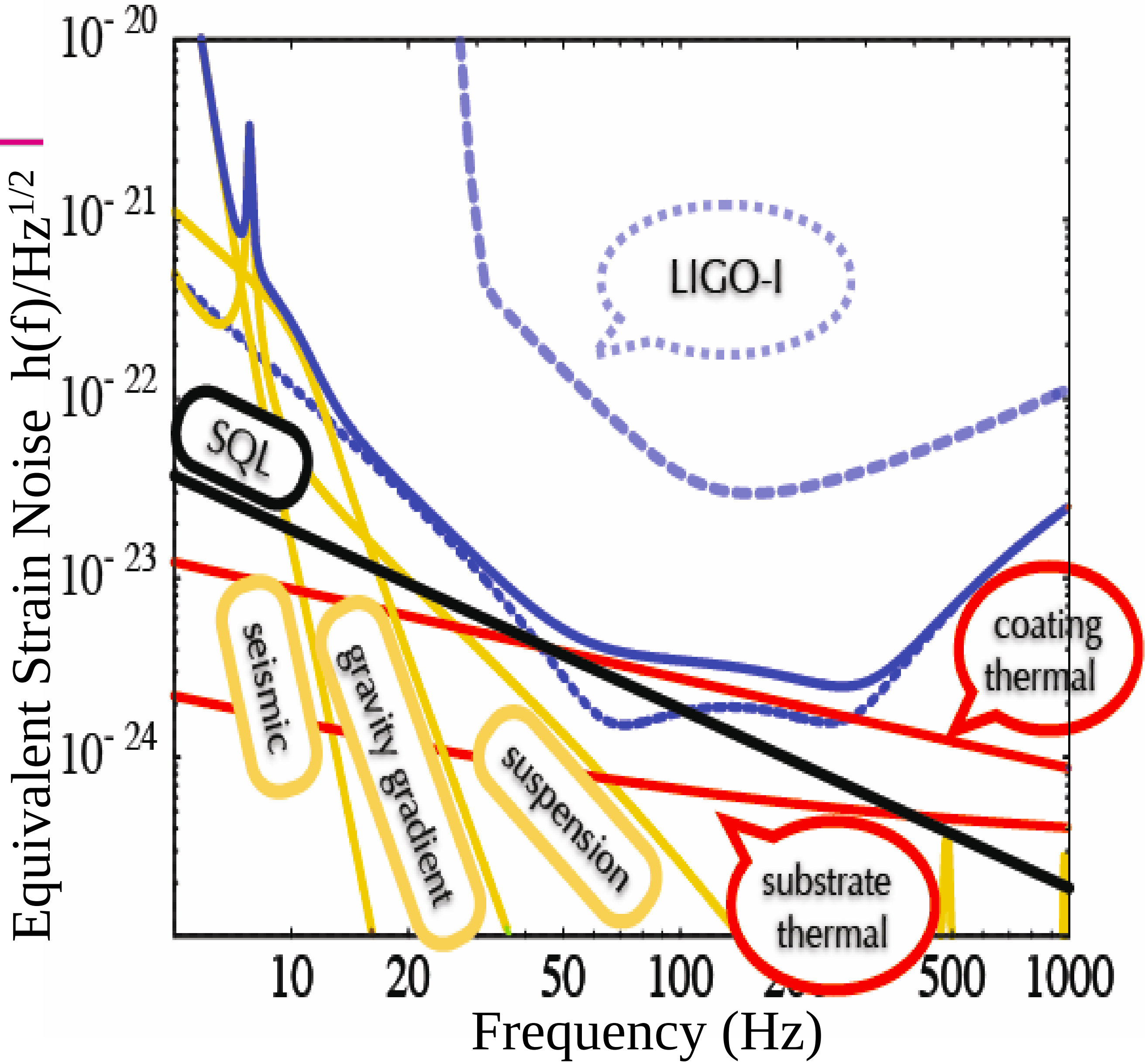


- Conical Beams
  - » Largest Noise Reduction to date

Noise in LIGO

Coating Thermal Noise is the leading noise source in Advanced LIGO at 100 Hz

It can be reduced.



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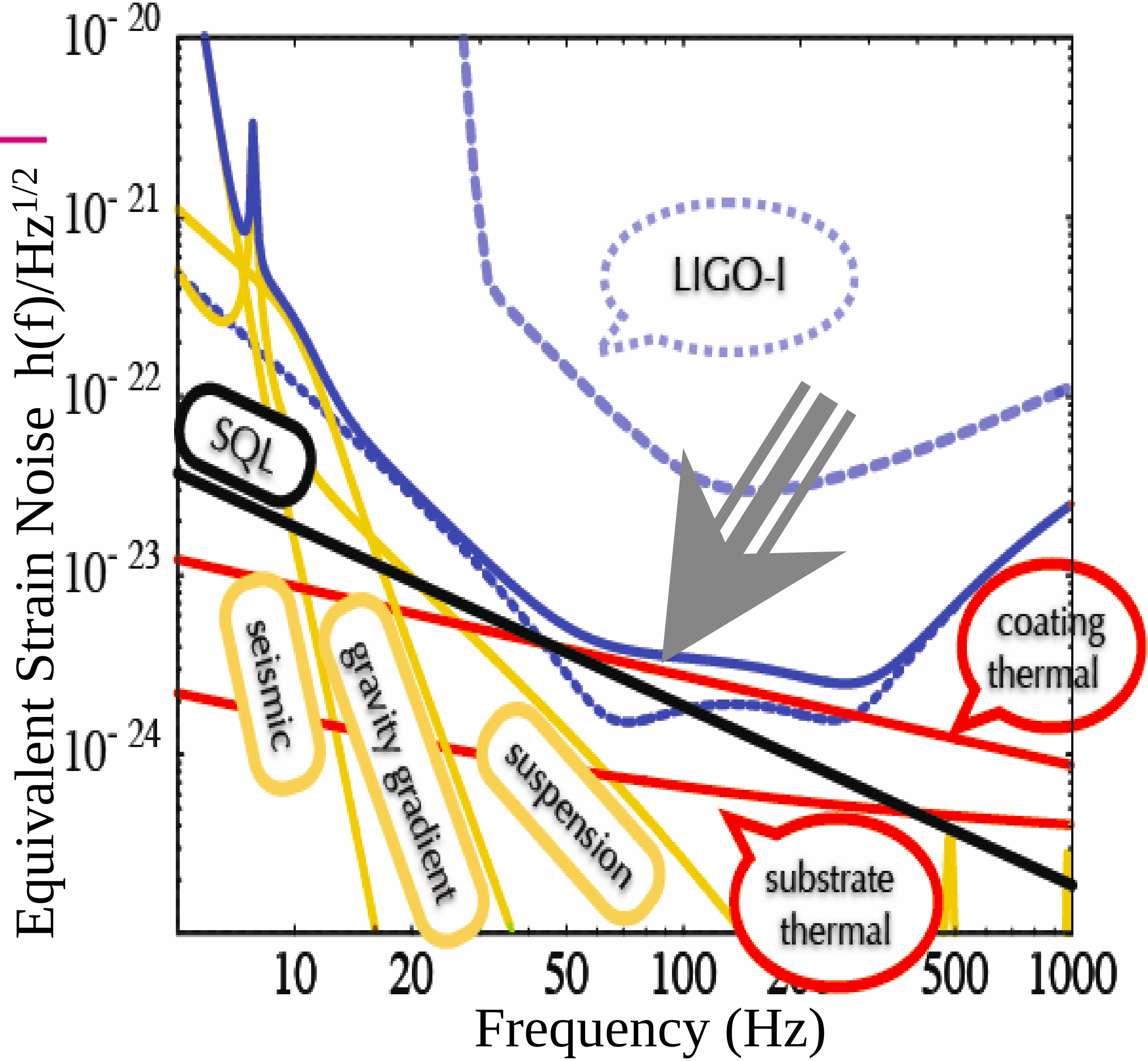
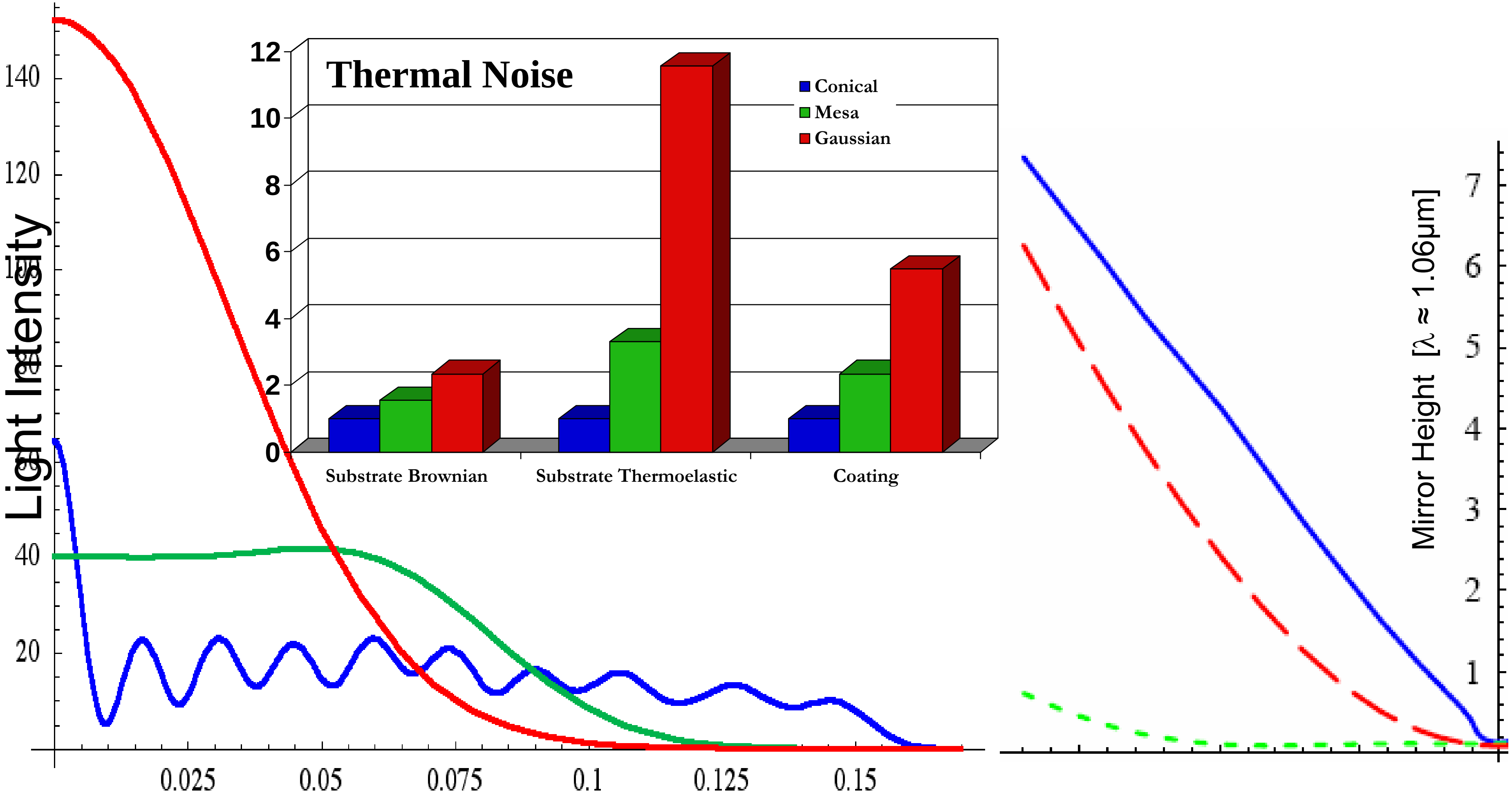
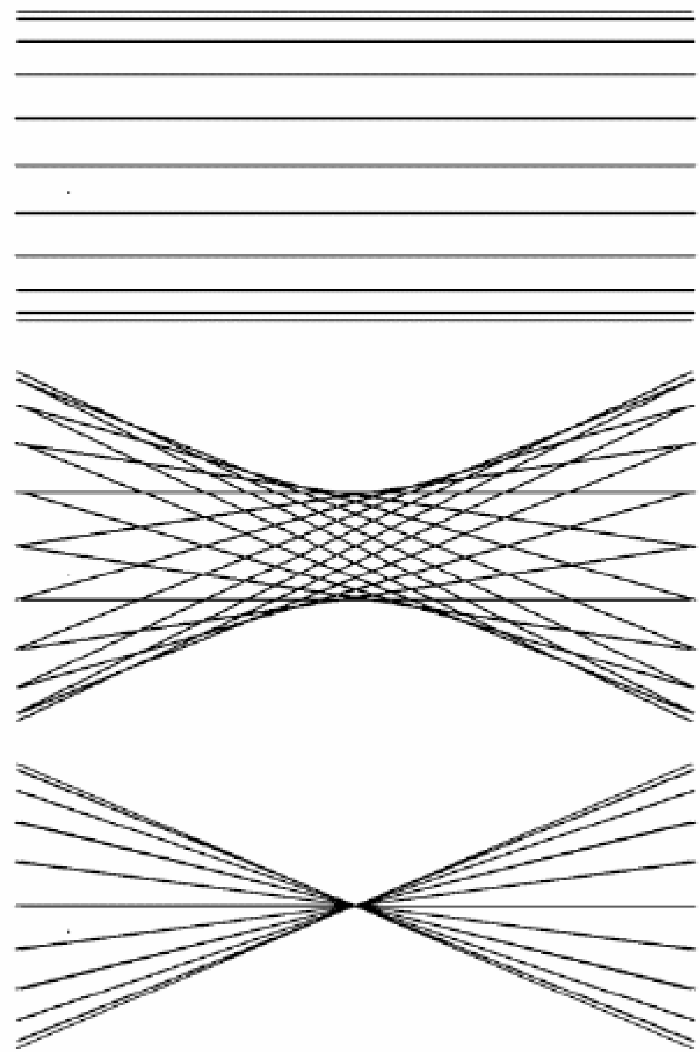


Conical, Mesa and Gaussian Beams



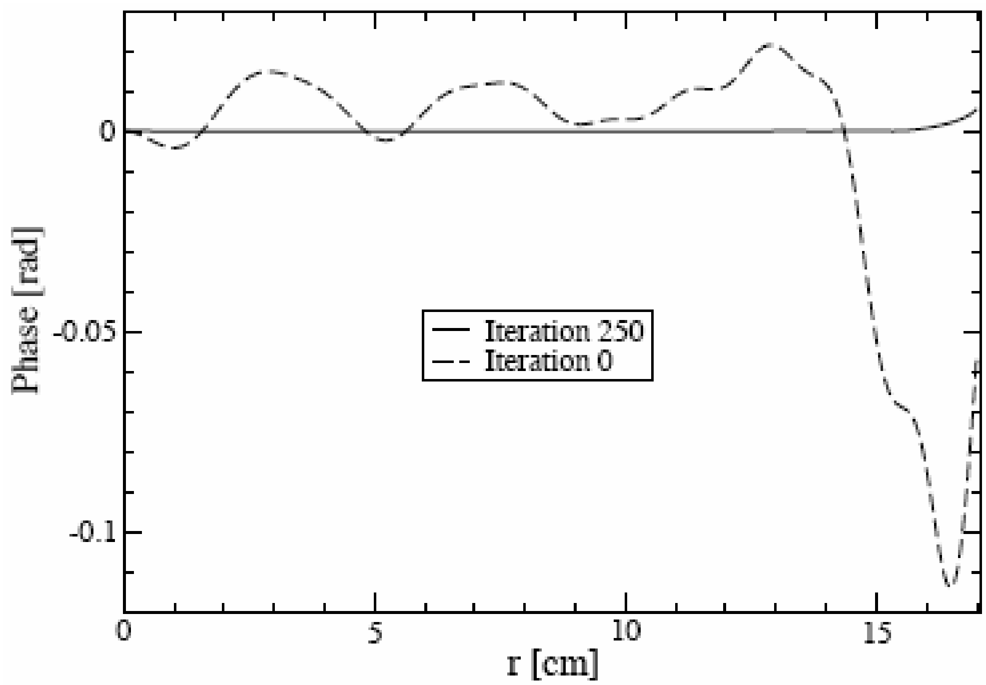
Hyperboloidal and Mesa Beams

- Composed of minimal Gaussians propagating on generators of coaxial hyperboloids parametrized by a twist angle α and falling on the mirror inside a disk of radius D.
  - » $\alpha=0$ Original Mesa
  - » $\alpha=\pi$ No Tilt Instability
  - » $\alpha=\pi/2$ Minimal Gaussian
  - » $\alpha=0.91\,\pi$ Has Coating Thermal Noise 12% Lower than Mesa when finite mirror effects are taken into account
  - » 28 % Coating Noise Reduction Possible by reshaping the mirror to conform to the finite cavity eigenbeam phasefront



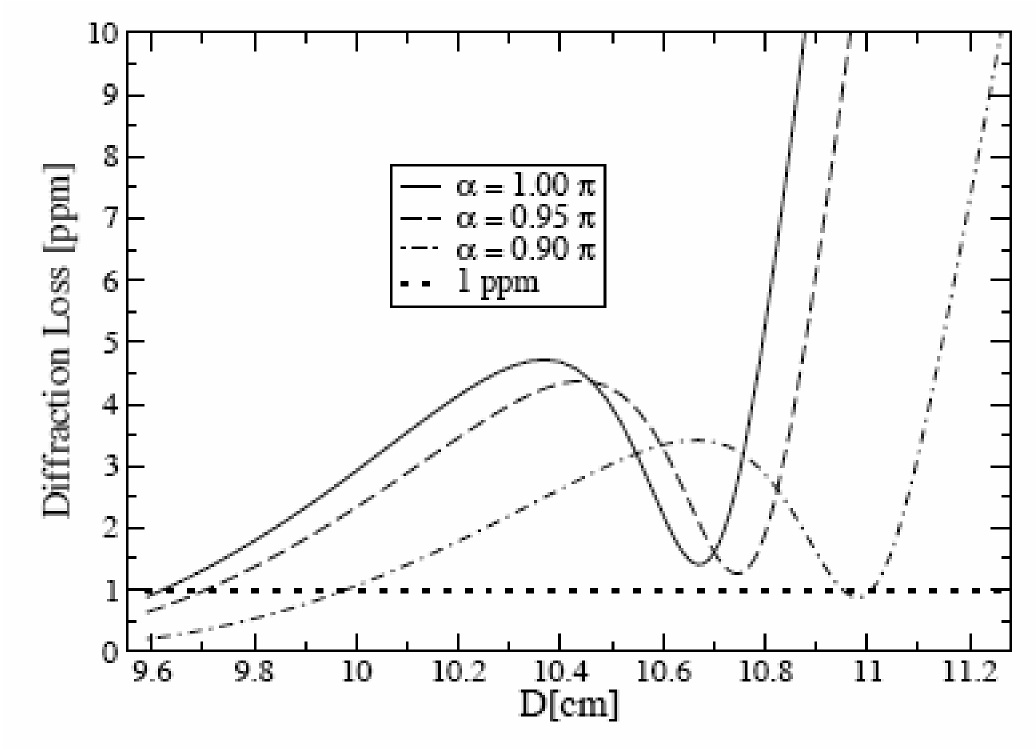
Mirror Construction

- Classically, a Mesa mirror is the innermost 17 cm of the phasefront of the *infinite* theoretical beam.
- The mirror is finite
- Phasefront of the *finite* beam fails to match the mirror surface.
- Shaping the mirror to match the phasefront of the finite beam dramatically decreases diffraction.



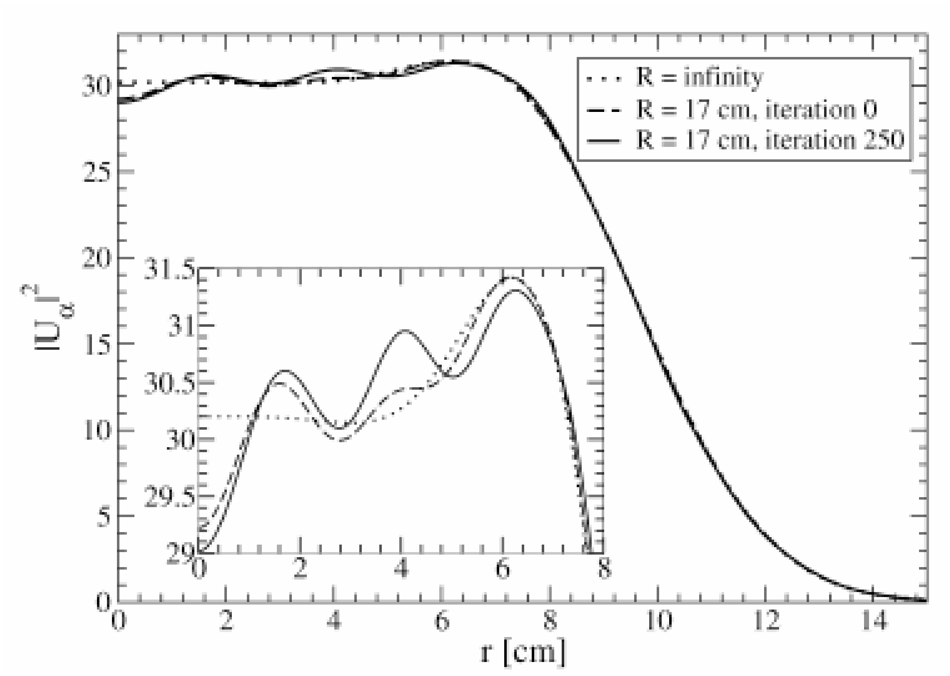
Finite Mirror Effects

- Mirror is finite. Normally, this leads to higher diffraction loss compared to clipping approximation.
- In a few cases, this can be used to our advantage to reduce coating thermal noise [compared to Mesa] by
- 12% - α=0.91 π hyperboloidal beam. No mirror reshaping
- 28% - by shaping the mirror to match the phasefront of the eigenbeam supported by *finite* mirrors.



Finite Mirror Effects

- 28% Coating Noise Reduction
  - » Power Distribution remains Mesa
  - » Mirror remains close to Mexican Hat
  - » A factor of 30 reduction in diffraction loss depends on the fine structure and correct positioning of the mirror.



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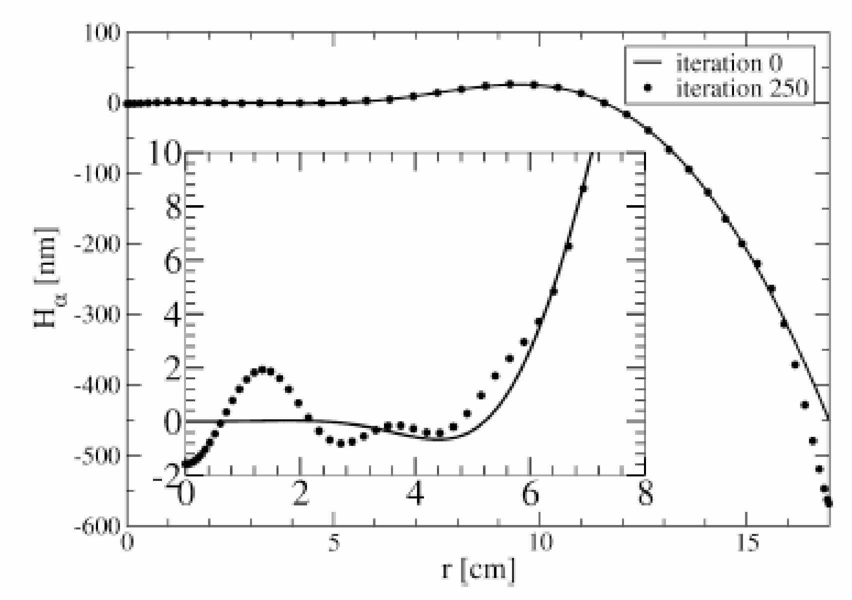

The Devil is in the details …

… **Details Follow.**

Thermal Noise

- Geoffrey Lovelace and others derived simple scaling laws.

- Valid under the assumptions:
  - » Infinite Mirrors
    - – No Mirror Edge effects
    - – No Finite Thickness effects
  - » Quasi-Static approximation
    - – GW frequency is far below the mirror resonant frequencies.

Noise	Brownian	Thermoelastic
Coating	$\sim \int_{k=0}^{\infty} dkk \, \lvert \tilde{p}^2(k) \rvert$	$\sim \int_{k=0}^{\infty} dkk \, \lvert \tilde{p}^2(k) \rvert$
Substrate	$\sim \int_{k=0}^{\infty} dk \, \lvert \tilde{p}^2(k) \rvert$	$\sim \int_{k=0}^{\infty} dkk^2 \, \lvert \tilde{p}^2(k) \rvert$

Thermal Noise

- Geoffrey Lovelace and others derived simple scaling laws.

- Valid under the assumptions:
  - » Infinite Mirrors
    - – No Mirror Edge effects
    - – No Finite Thickness effects
  - » Quasi-Static approximation
    - – Assumes the mirror surface does not change with time

Noise	Brownian	Thermoelastic
Coating $\sim \int_{k=0}^{\infty} drrp^2(r)$	$\sim \int_{k=0}^{\infty} dkk \mid \tilde{p}^2(k) \mid$	$\sim \int_{k=0}^{\infty} dkk \mid \tilde{p}^2(k) \mid$
Substrate	$\sim \int_{k=0}^{\infty} dk \mid \tilde{p}^2(k) \mid$	$\sim \int_{k=0}^{\infty} dkk^2 \mid \tilde{p}^2(k) \mid$

Coating noise is the dominant one is Fused Silica mirrors.
It is the best candidate for the minimization process.
Bonus: involves no Fourier Transforms.

Thermal Noise

- Geoffrey Lovelace and others derived simple scaling laws.
- Valid under the assumptions:
  - » Infinite Mirrors
    - – No Mirror Edge effects
    - – No Finite Thickness effects
  - » Quasi-Static approximation
    - – Assumes the mirror surface does not change with time

$\frac{\text{Mesa Noise}}{\text{Cone Noise}}$	Brownian	Thermoelastic
Coating	2.339	2.339
Substrate	1.534	3.302

Gauss-Laguerre

- For minimization to be possible, we need a coordinate system in the space of LIGO beams
- Gauss-Laguerre basis
  - » Orthonormal
  - » Complete
- Used to analytically analyze hyperboloidal beams in gr-qc 0602074 (Galdi, Castaldi, Pierro, Pinto, Agresti, D'Ambrosio, De Salvo)

In the center of the cavity

$$\psi_m(\xi) = \sqrt{2}\exp\left(-\frac{\xi^2}{2}\right) L_m(\xi^2)$$

$L_n(\zeta)$ denotes an nth-order Laguerre polynomial

$$\int_0^\infty \psi_p(\xi)\psi_q(\xi)\xi d\xi = \delta_{pq}$$

$$U\ \ (r,0) = \sum_{m=0}^{\infty} A_m\ \ \psi_m\left(\frac{\sqrt{2}r}{w_0}\right)$$

For all real U, A's can be real

Gauss-Laguerre

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$$\Psi_m(r,z) = \frac{w_0}{w(z)}\psi_m\left[\frac{\sqrt{2}r}{w(z)}\right]\exp\left[i\frac{k_0 r^2}{2R(z)}\right] \times \exp\left\{i\left[k_0 z - (2m+1)\Phi(z)\right]\right\},$$

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2}, \quad R(z) = z + \frac{z_R^2}{z},$$

$$\Phi(z) = \arctan\left(\frac{z}{z_R}\right),$$

$$U(r,z) = \sum_{m=0}^{\infty} A_m \; \Psi_m(r,z)$$

Coating Thermal Noise Minimization Process

- Thermal Noise

$$N \sim \int_0^R p(r)^2 r dr \sim \sum_{i,j,k,l=0}^{\infty} A_i A_j A_k A_l \int_0^R \Psi_i(r,z_0)\Psi^*{}_j(r,z_0)\Psi_k(r,z_0)\Psi^*{}_l(r,z_0) r dr$$

$$p = |U|^2$$

- Constraints
  - » Normalization

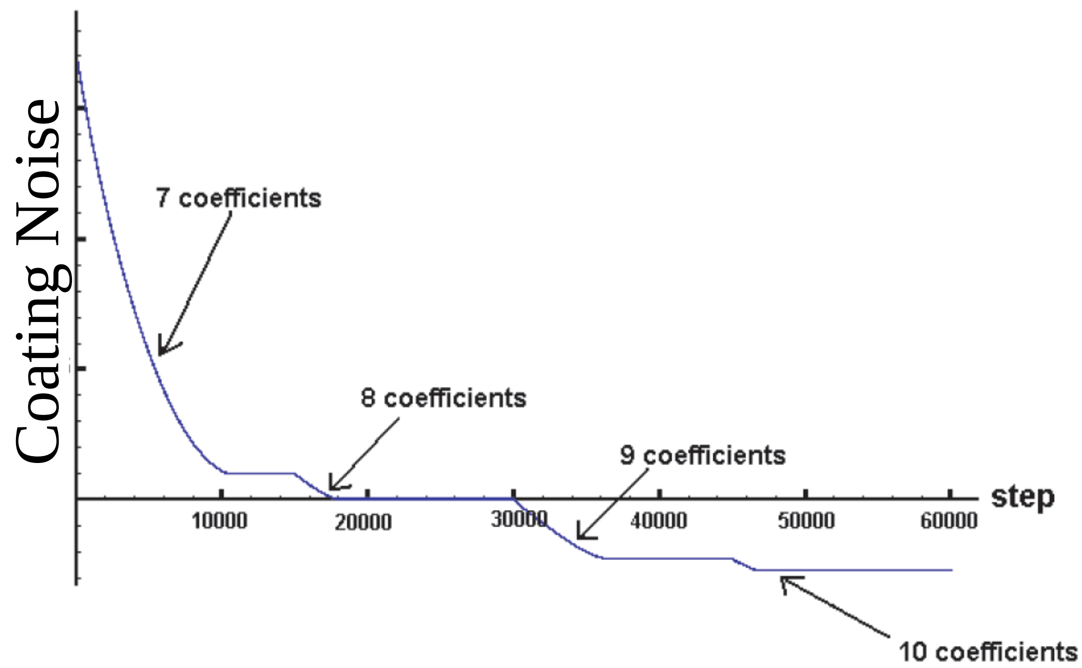
$$1 = \int_0^{\infty} p(r) r dr = \sum_{i,j=0}^{\infty} A_i A_j \int_0^{\infty} \Psi_i(r,z_0)\Psi^*{}_j(r,z_0) r dr$$

  - » Constant Diffraction Loss

$$10^{-6} = \int_R^{\infty} p(r) r dr = \sum_{i,j=0}^{\infty} A_i A_j \int_R^{\infty} \Psi_i(r,z_0)\Psi^*{}_j(r,z_0) r dr$$

Coating Thermal Noise Minimization Process

- Simple Gradient Flow
  - » Variable step size
- Subject to constraints
  - » Diffraction Loss = 1 ppm
  - » Power normalization
- Local Minima exist
  - » Increase dimension one by one to avoid.



Coating Thermal Noise Minimization Process

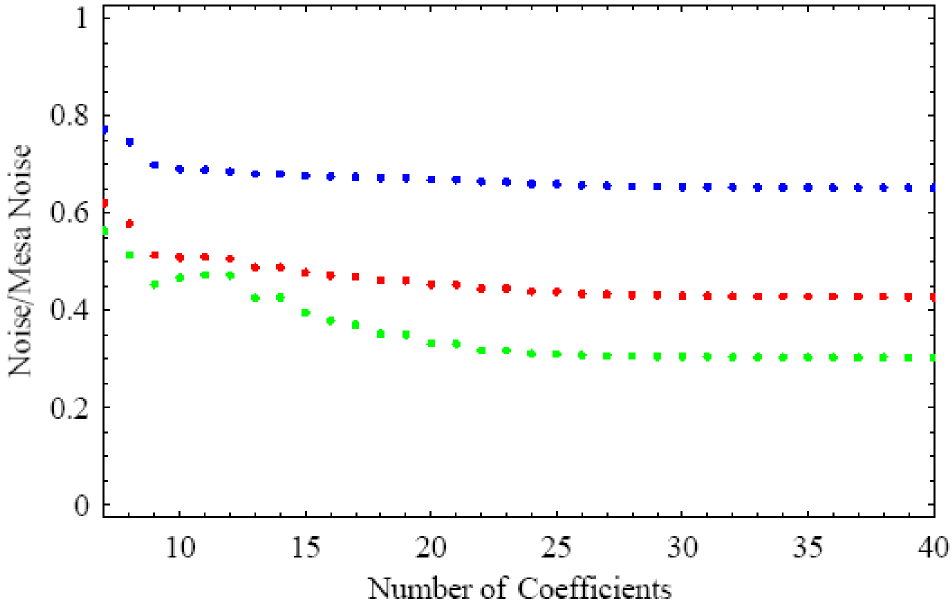
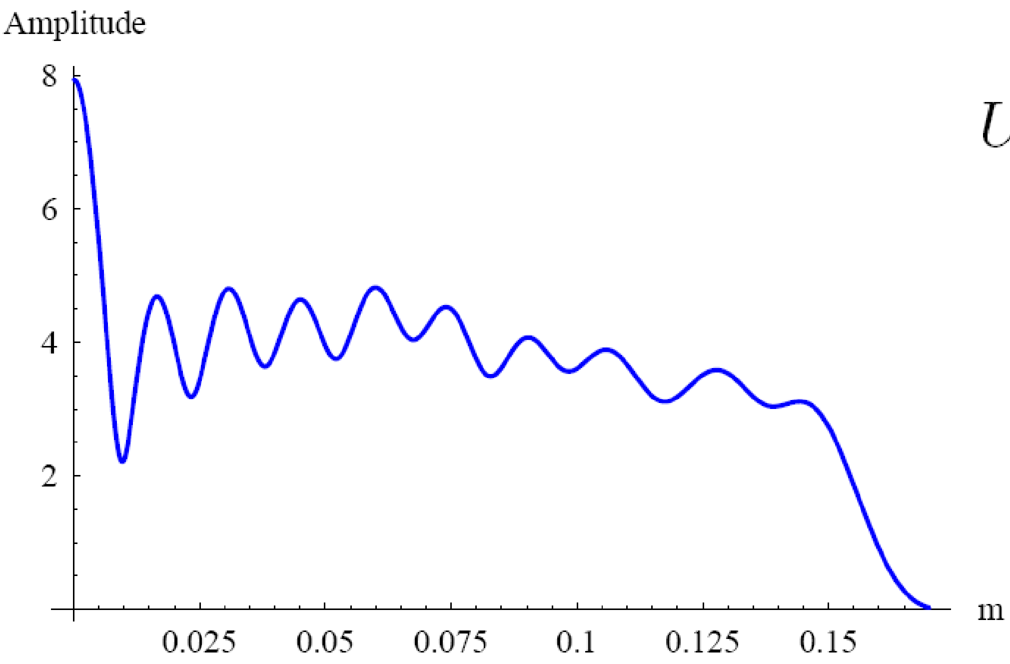


Figure 3.10: Thermal noise as a function of the number of Gauss-Laguerre coefficients employed in the minimization code. The blue dots represent substrate Brownian noise, the coating noise in in red and the substrate thermoelastic noise is in green. Although we minimized for the coating noise alone, the substrate thermoelastic noise decreases more than the coating noise while the substrate brownian noise decreases less. Both coating brownian noise and coating thermoelastic noise obey the same scaling law. The coating thermal noise is expected to be the largest in the Fused Silica mirrors considered for LIGO design while thermoleastic noise would have the leading contribution if Sapphire mirrors where to be used. In this figure is normalized so that the Mesa noise is 1 for each of the three noises.

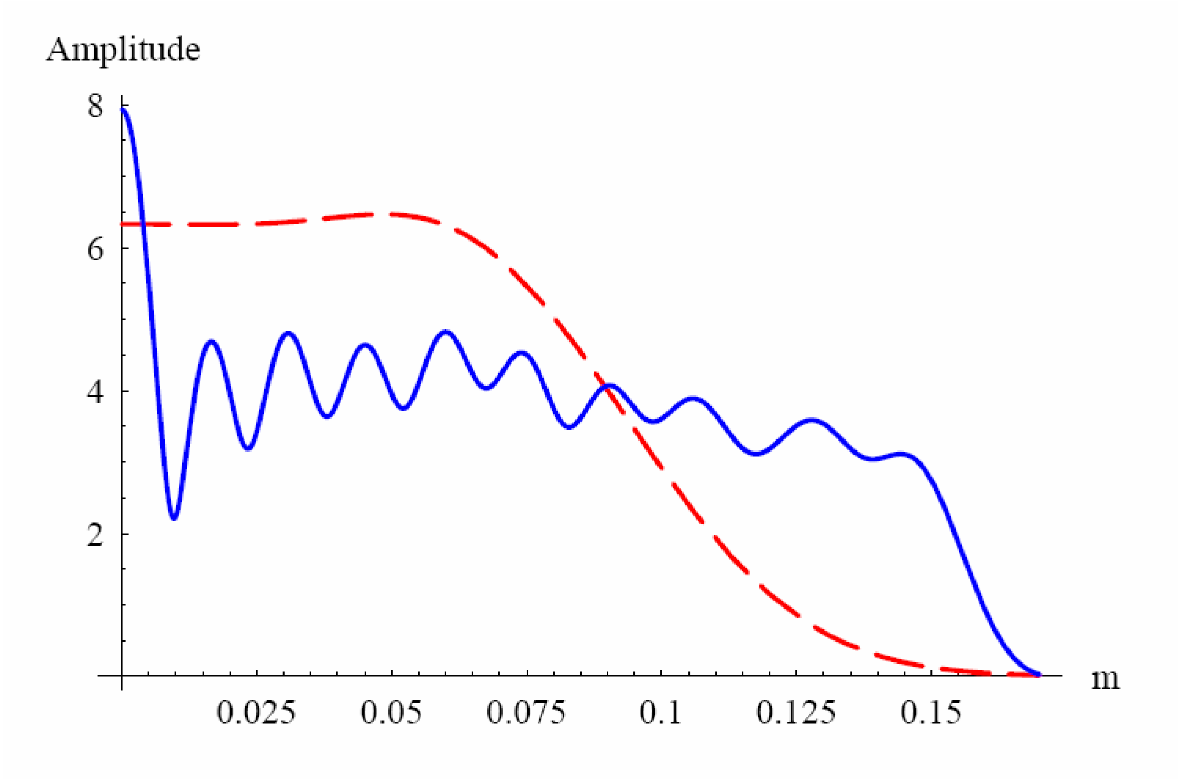
Amplitude Profile



$$U(r,z) = \sum_{m=0}^{\infty} A_m \Psi_m(r,z)$$

35 Coefficients

Amplitude Profile



35 Coefficients conical beam vs. Mesa

Amplitude Profile

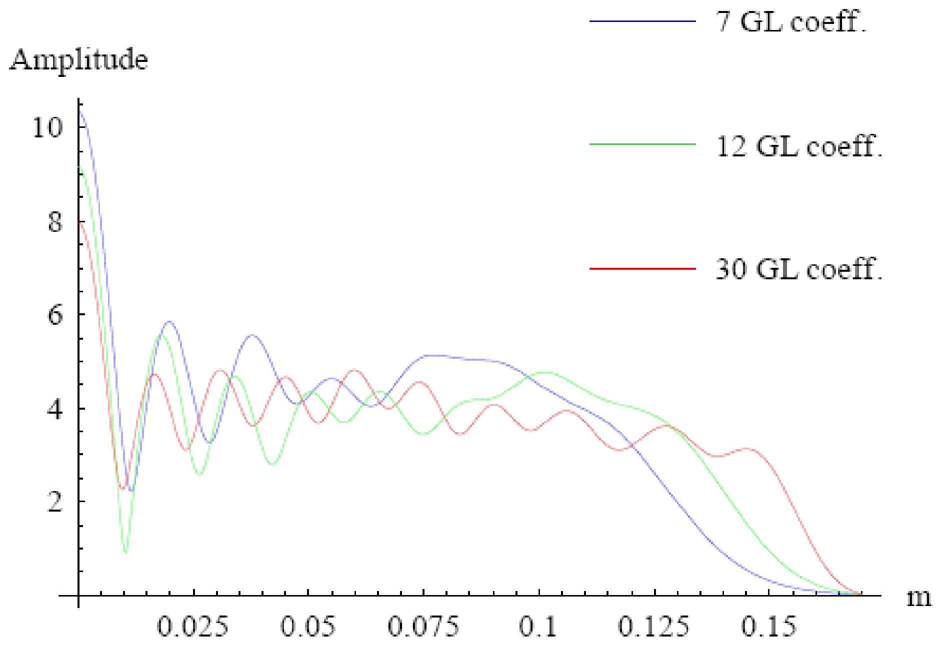
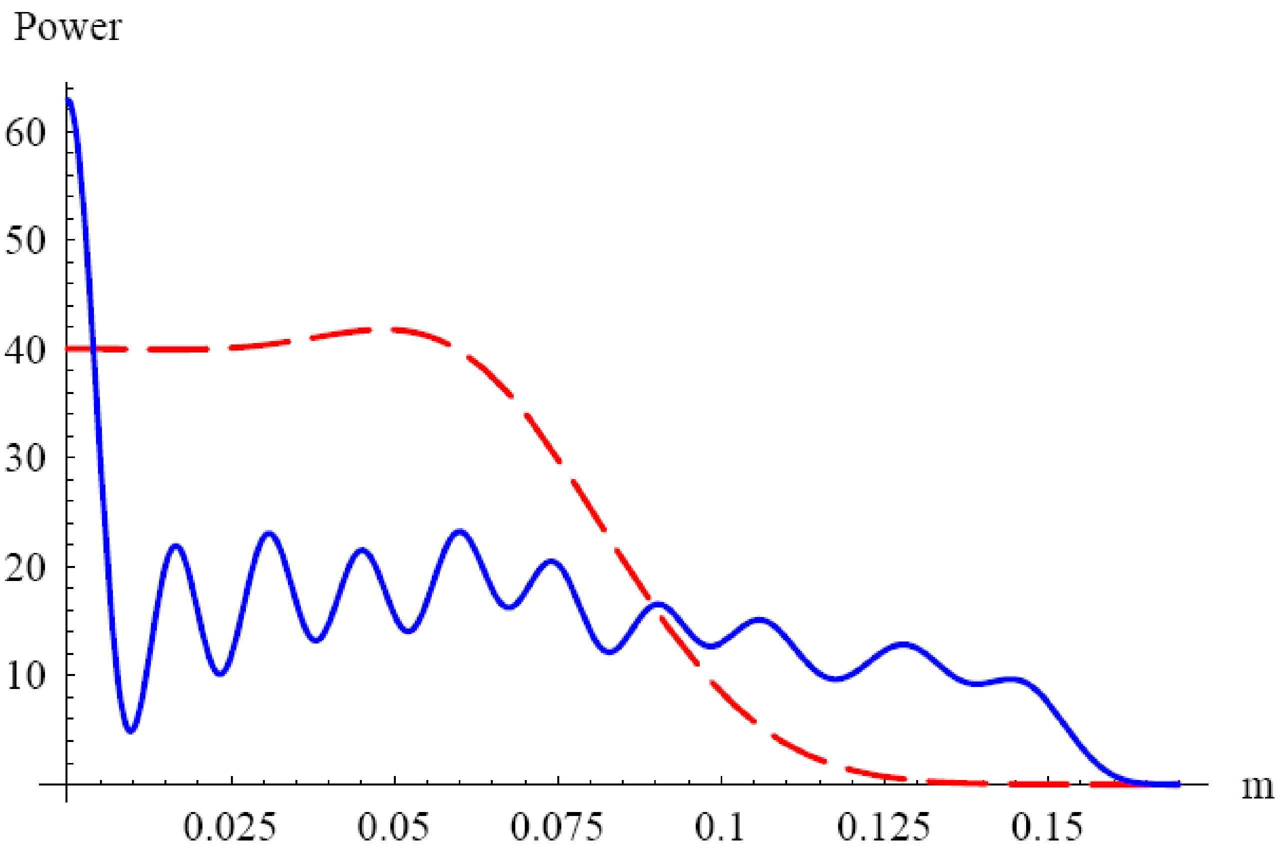


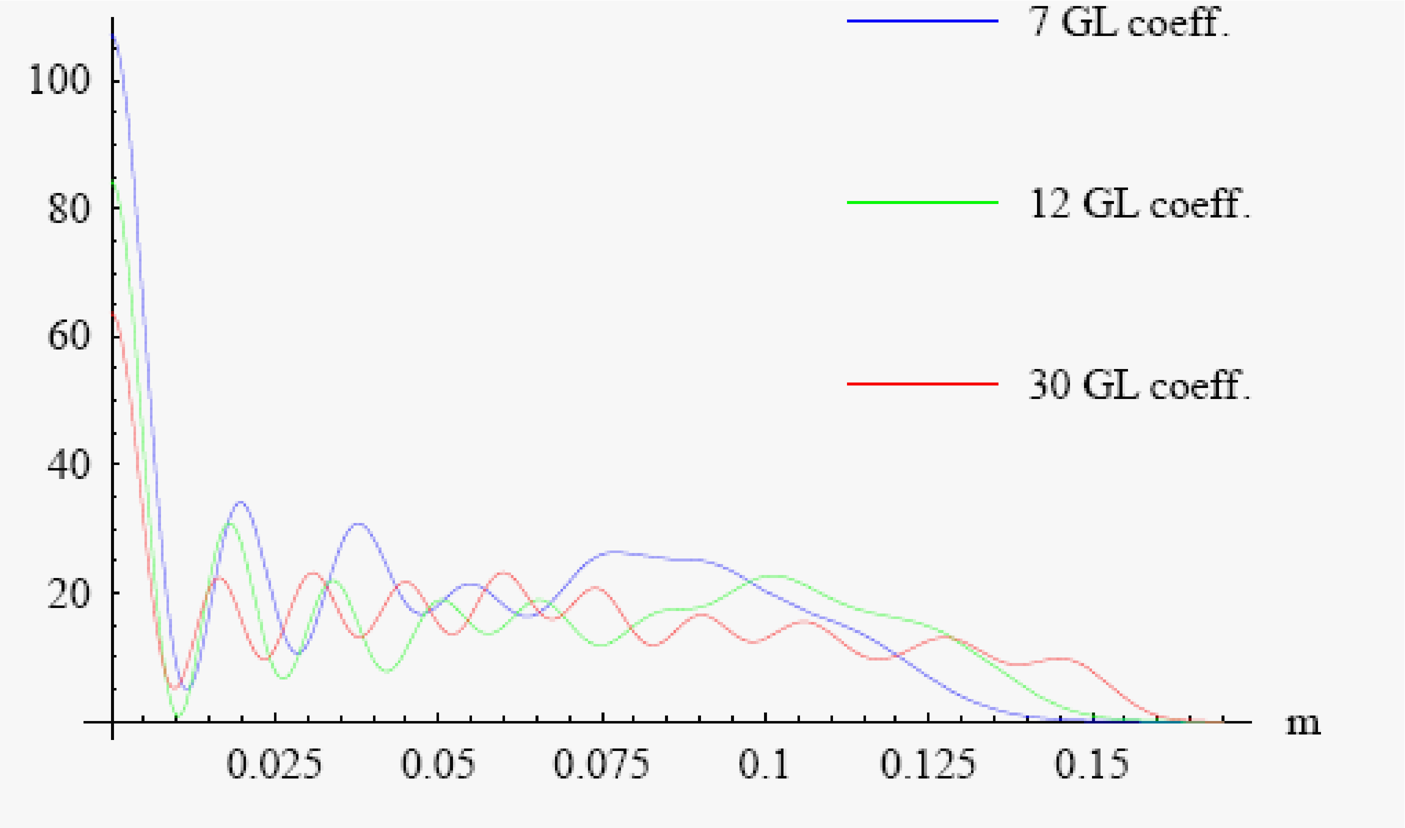
Figure 3.11: Amplitude of the electric field at the mirror as a function of the number of Gauss-Laguerre coefficients employed in the minimization code. When fewer coefficients are used, the beam is spread over a smaller area of the mirror.

Power Distribution



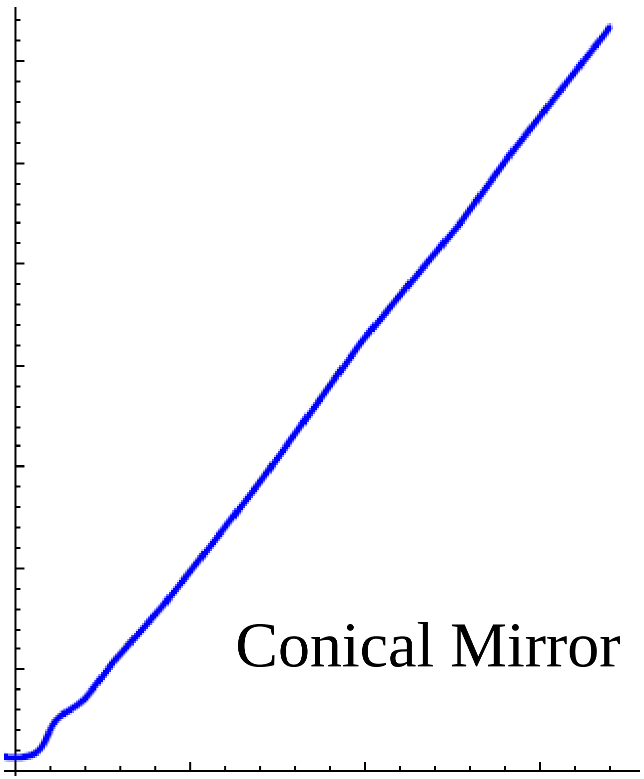
35 Coefficients conical beam vs. Mesa

Power Distribution



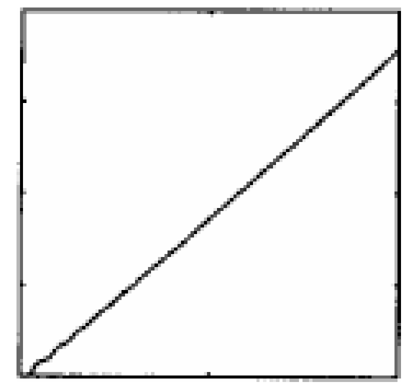
Mirror

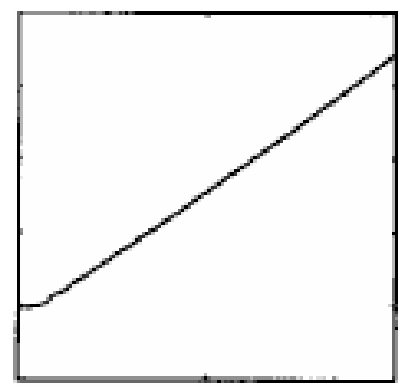
Our conical mirror is similar to mirrors supporting Bessel-Gauss Beams

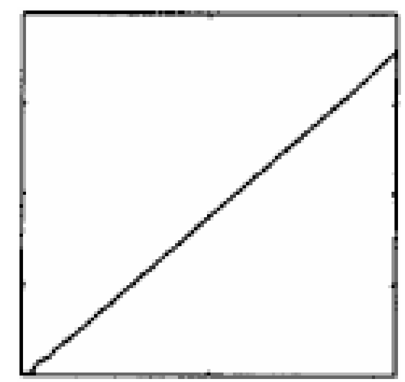


Mirror = Phasefront (Arg[U]=constant)

Mirrors for
Bessel-Gauss beams
[Durnin et al.]







Mirror

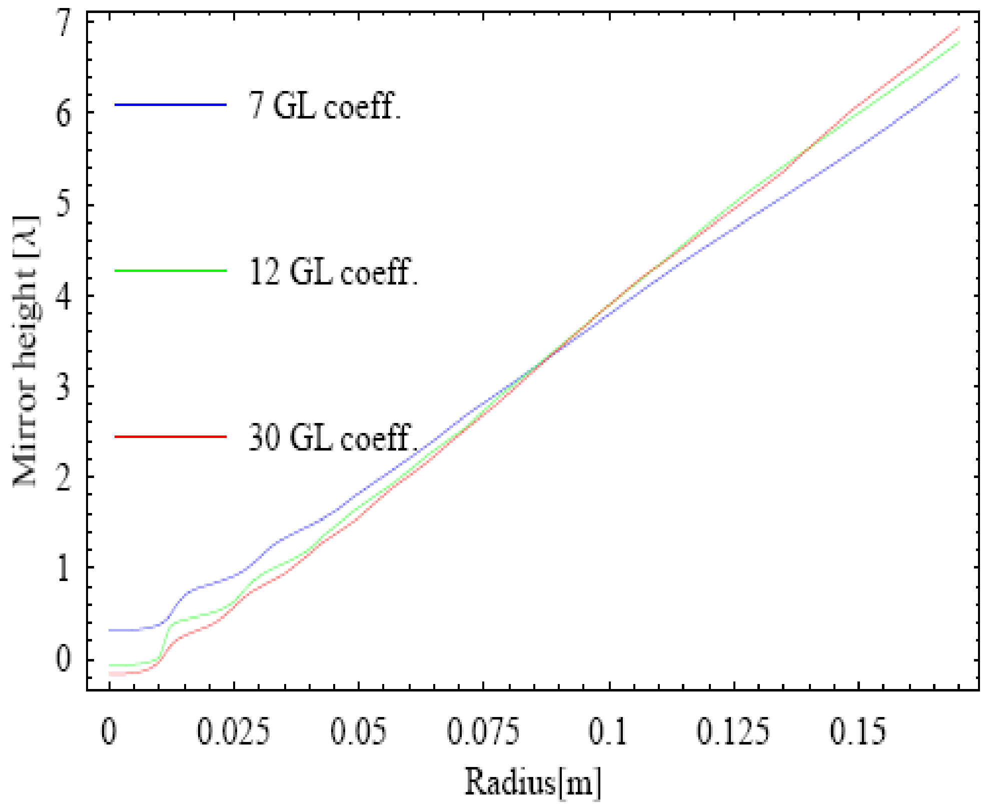


Figure 3.13: Mirrors supporting the 7, 12 and 30 coefficients modes. The mirror height is measured in units of λ, where $\lambda = 1.06\mu m$ is the wavelength of the light used in the interferometer.

Bessel and Bessel-Gauss Beams

- Bessel Beams
  - » Diffraction-Free
  - » Not physically realizable (infinite energy)
  - » Conical phasefronts
  - » Intensity independent of z (direction of propagation)

$$E(r,z) = E_0 e^{i(\beta z - \omega t)} J_0(\alpha r)$$

$$E_n(r,z) = E_0 e^{i(\beta z - \omega t - n\phi)} J_n(\alpha r)$$

Filed distribution everywhere
Bessel Beams

- Bessel-Gauss beams
  - » Finite energy derivative of Bessel Beams
  - » Physically realizable
  - » Intensity distribution and phasefronts shape depend on z
  - » "Nearly" diffraction-free in a finite region
  - » Nearly conical mirrors in some regime

$$E(r,z) = E_0 e^{i(\beta z - \omega t)} J_0(\alpha r) e^{-\left(\frac{r}{w_o}\right)^2}$$

$$E_n(r,z) = E_0 e^{i(\beta z - \omega t - n\phi)} J_n(\alpha r) e^{-\left(\frac{r}{w_o}\right)^2}$$

Bessel-Gauss Beams field distribution
near z=0

LIGO

- Bessel Beams
  - » Diffraction-Free
  - » Not physically realizable (infinite energy)
  - » Conical phasefronts
  - » Intensity independent of z (direction of propagation)

- Bessel-Gauss beams
  - » Finite energy derivative of Bessel Beams
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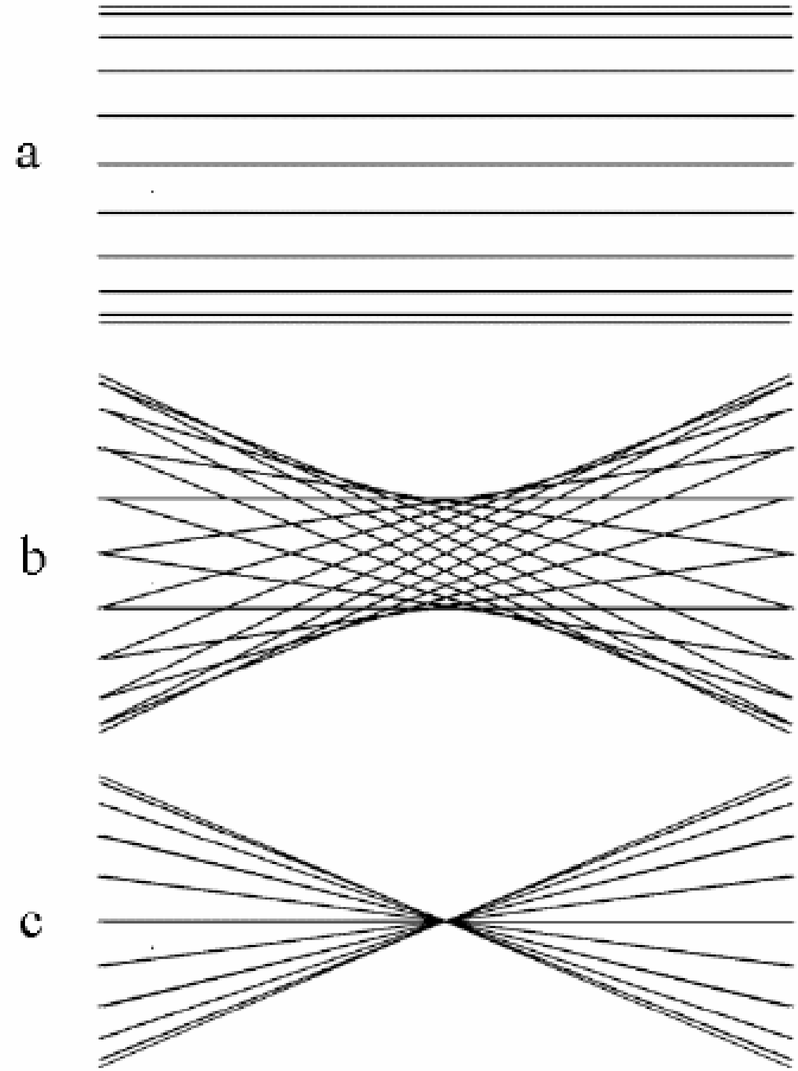
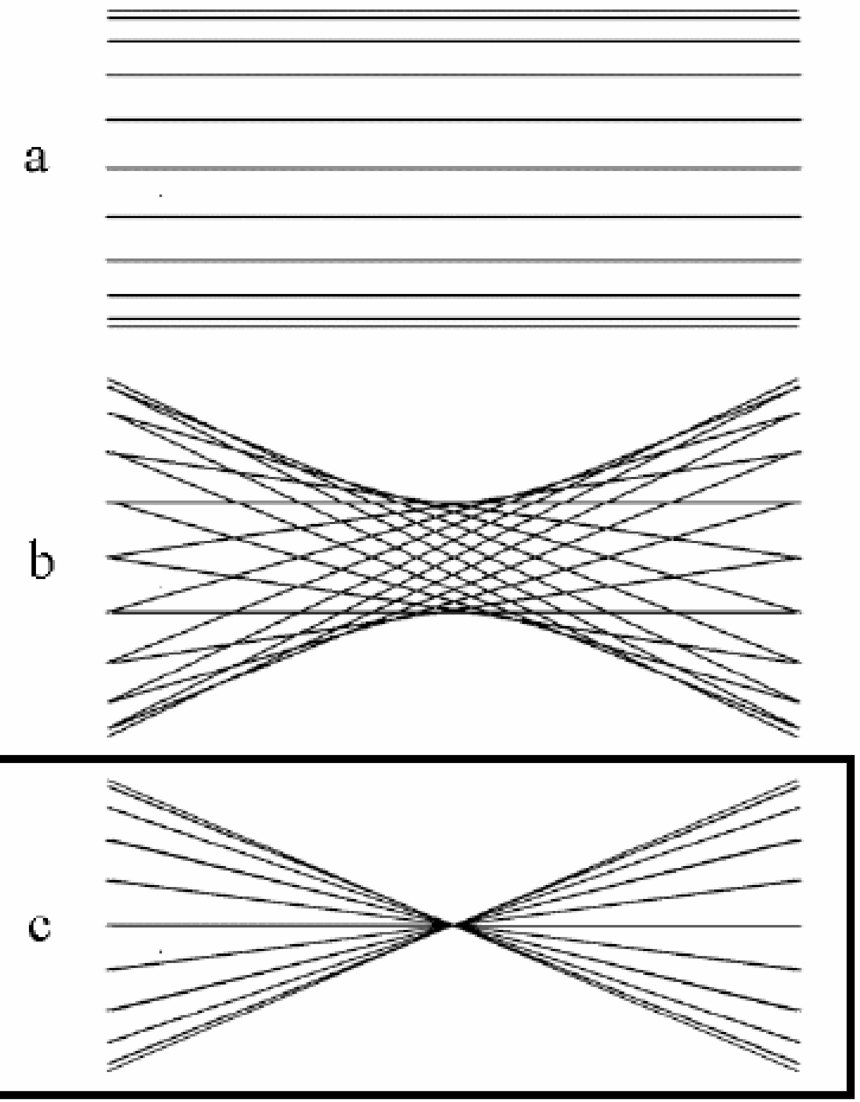


FIG. 1: Optical axes of the families of minimal Gaussians beams used to construct: (a) an FM mesa beam [6], denoted in this paper $\alpha = 0$; (c) our new CM mesa beam, denoted $\alpha = \pi$; (b) our new family of hyperboloidal beams, which deform, as α varies from 0 to π, from a FM beam (a) into a CM beam (c).

- Bessel Beams
  - » Diffraction-Free
  - » Not physically realizable (infinite energy)
  - » Conical phase fronts
  - » Intensity independent of z (direction of propagation)
  - » Can be thought as a set of plane waves propagating along the generators of a cone
- Bessel-Gauss beams
  - » Finite energy derivative of Bessel Beams
  - » Physically realizable
  - » Intensity distribution and phase fronts shape depend on z
  - » "Nearly" diffraction-free in a finite region
  - » Nearly conical mirrors in some regime
  - » Can be thought of as a set of Gaussian beams centered on the generators of a cone interfering in a region close to the vertex.



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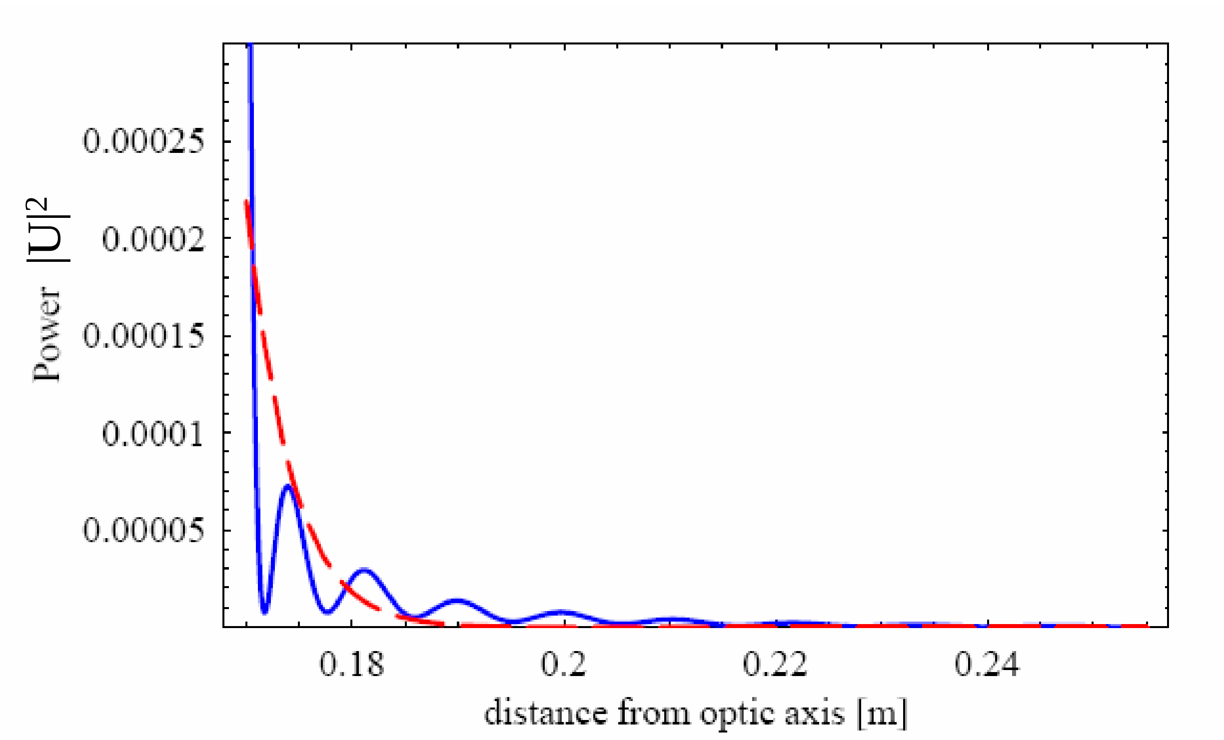
Diffraction Losses

Clipping Approximation

$$D = \frac{\int_{R_{mirror}}^{\infty} 2\pi U^2 r dr}{\int_0^{\infty} 2\pi U^2 r dr}$$



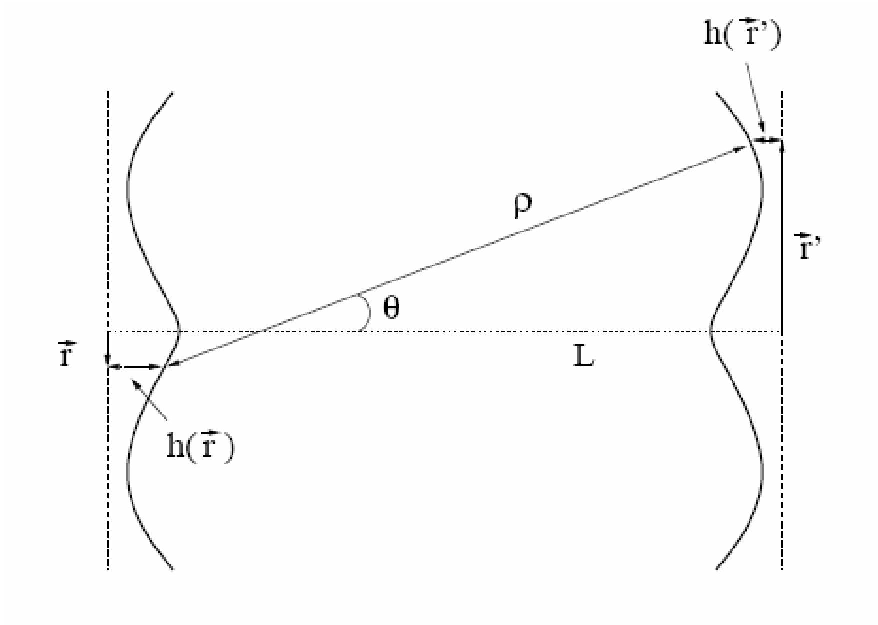
Diffraction Losses

We followed gr-qc/0511062 (Agresti, Chen, D'Ambrosio, Savov).

$$v_2(\vec{r}) = \int \mathrm{d}^2\vec{r}\,' \; \mathcal{K}(\vec{r}, \vec{r}\,')\, v_1(\vec{r}\,')$$

$$\mathcal{K}(\vec{r}, \vec{r}\,') = \frac{ik}{4\pi\rho}(1 + \cos\theta)e^{-ik\rho}$$

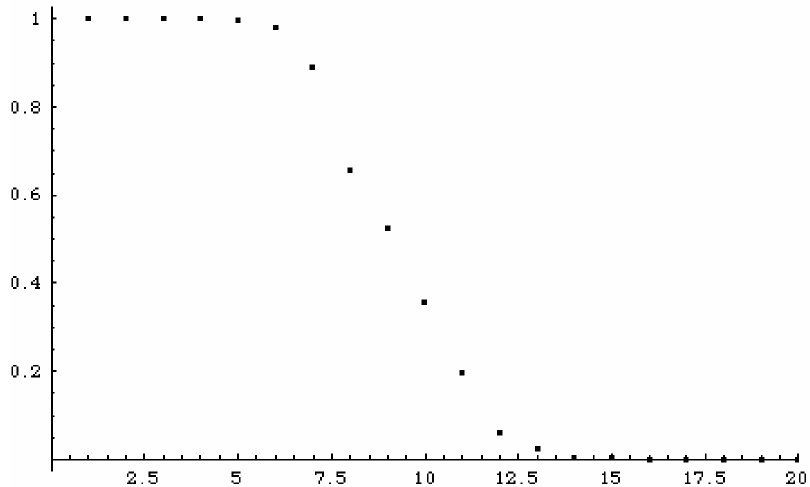
$$k = \frac{2\pi}{\lambda}$$

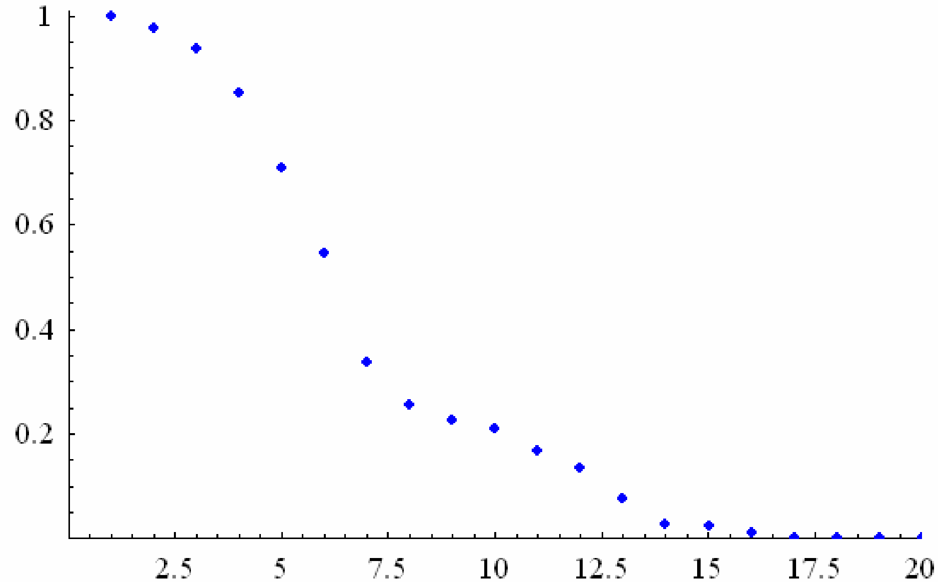


Solve the Fresnel-Kirchoff eigenequation

$$\gamma\, v(\vec{r}) = \int \mathrm{d}^2\vec{r}\,' \; \mathcal{K}(\vec{r}, \vec{r}\,')\, v(\vec{r}\,')$$

Eigenvalues of axisymmetric propagator





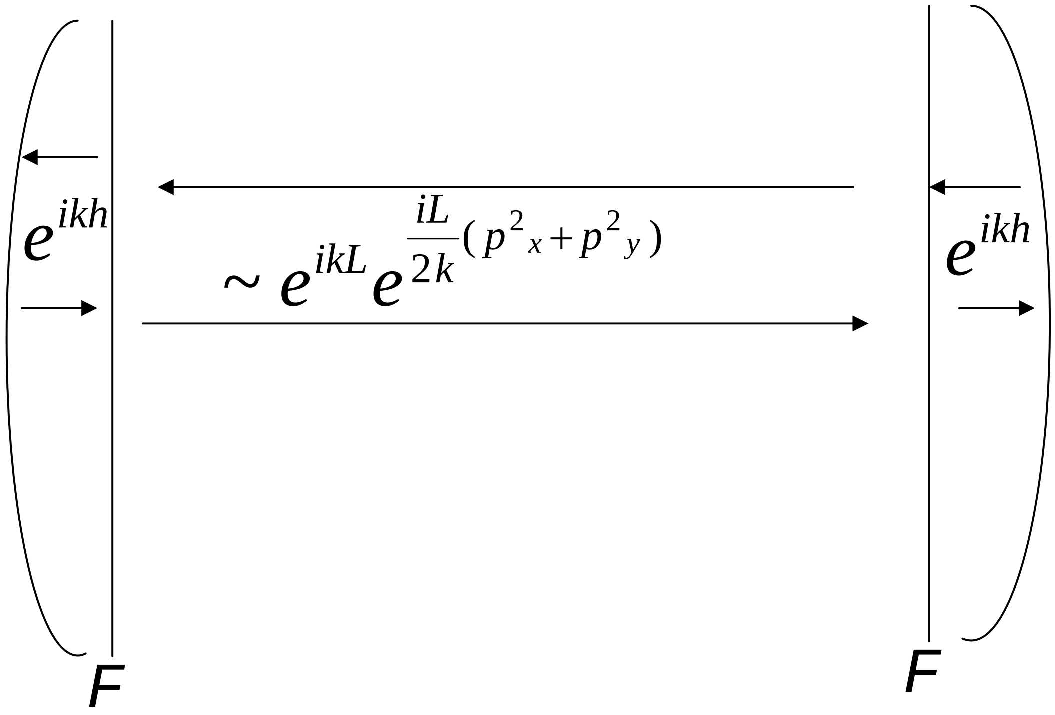
Mesa – many viable parasitic modes

Conical cavity – parasitic modes have high losses and die away

$$K^h_{\mathrm{f}(m)}(r,r') = \frac{i^{m+1}k}{L} J_m\left(\frac{krr'}{L}\right) e^{ik\left[-L+h(r)+h(r')-\frac{r^2+r'^2}{2L}\right]}$$

Sensitivity to errors

Full 3 D “FFT” code



Mirror Tilt

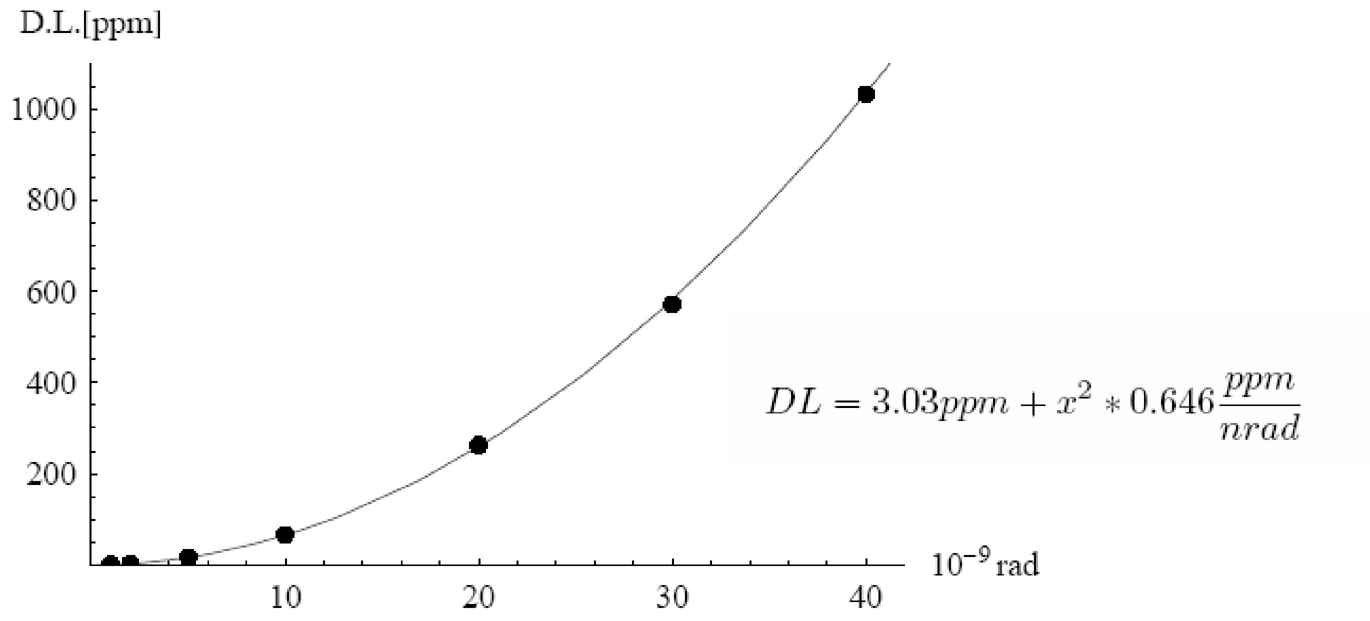
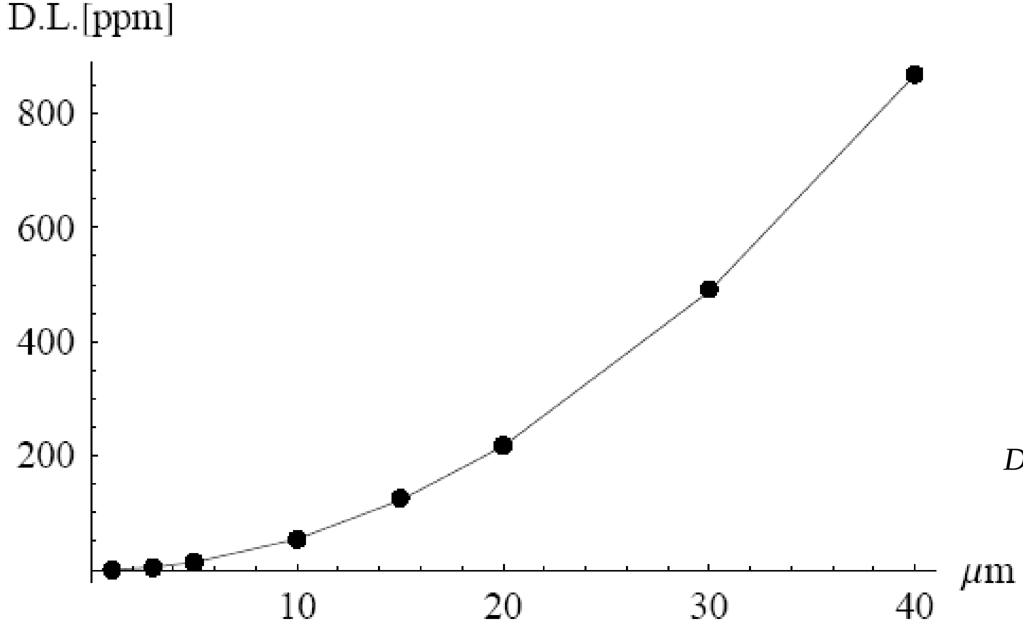


Figure 3.21: Diffraction Loss in a conical cavity when its mirrors are symmetrically tilted as a function of the tilt

TILT TOLERANCE ~ 3 *10^{-9} rad

Mirror Displacement



$$DL = 3.203\,ppm + 0.542x^2\,\frac{ppm}{\mu m^2}$$

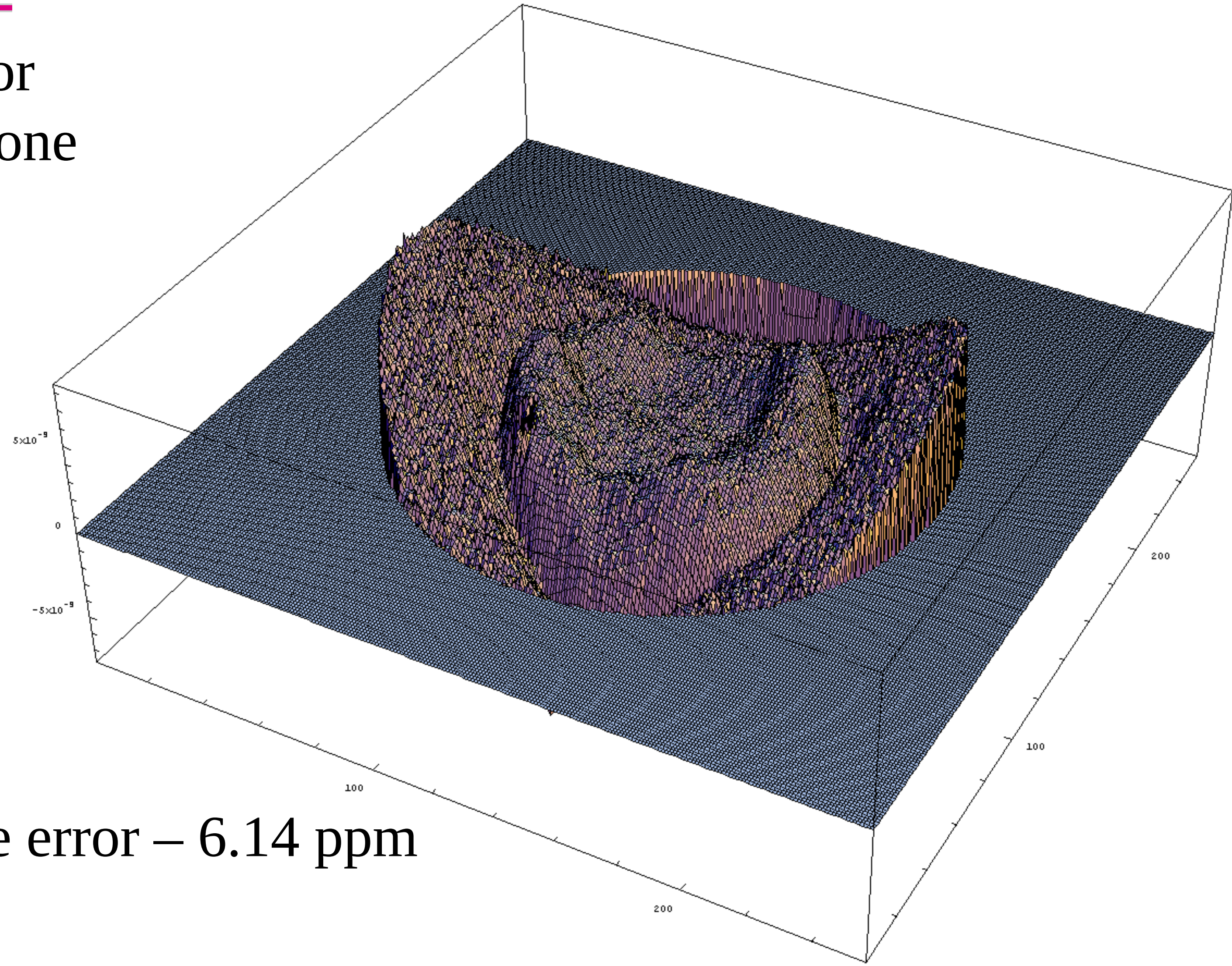
Figure 3.25: Dots represent the diffraction loss in ppm for a Conical Cavity perturbed by moving one mirror away from the optic axis as a function of this displacement. The continuous line is a quadratic function fit to the data.

TRANSLATION TOLERANCE ~ 4 $*10^{-6}$ m

This can be traded off if the tilt can be controlled within 1 nanoradian.

Mirror Figure Error

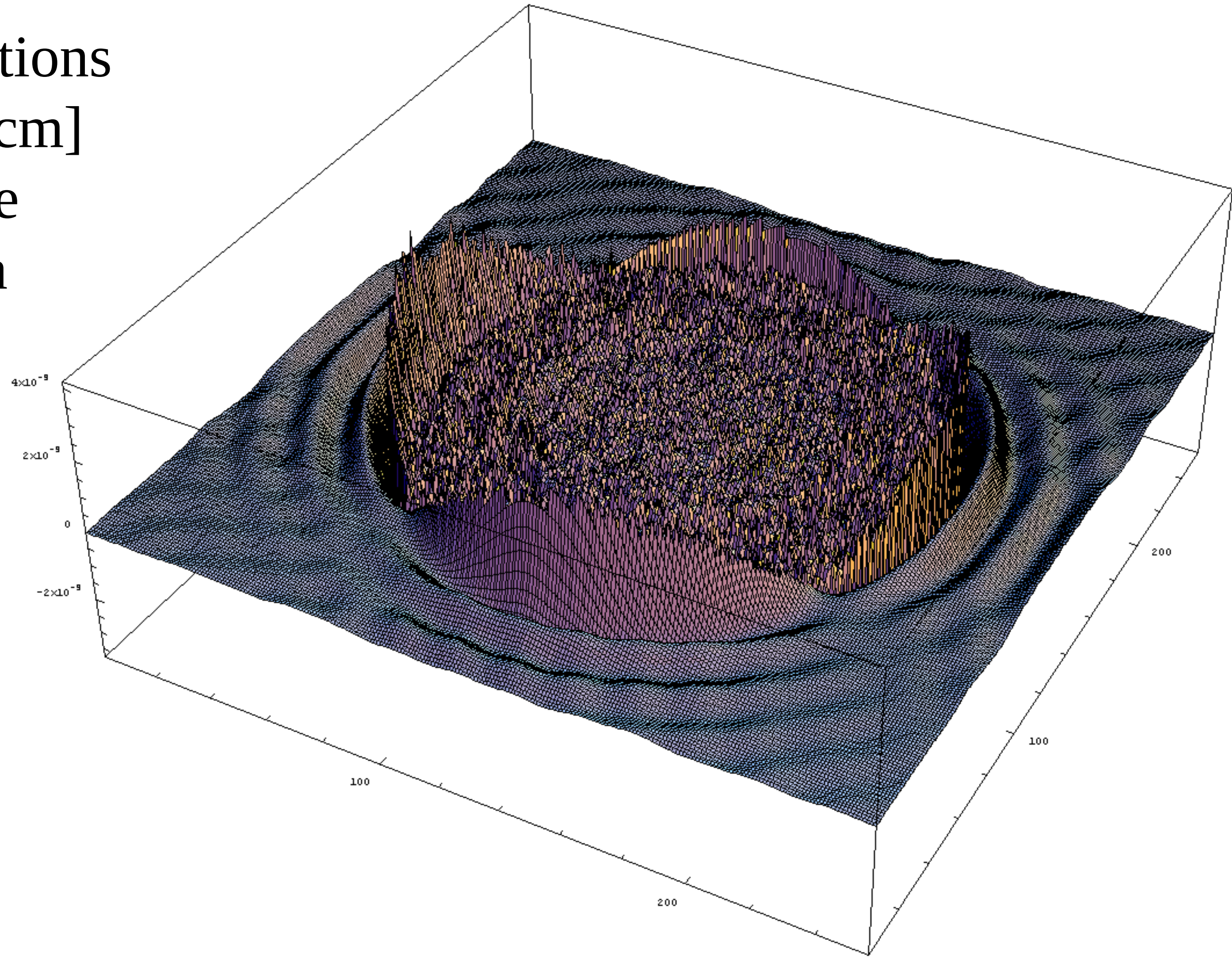
LIGO I figure error
404.83 ppm for Cone



1/10 LIGO I figure error – 6.14 ppm

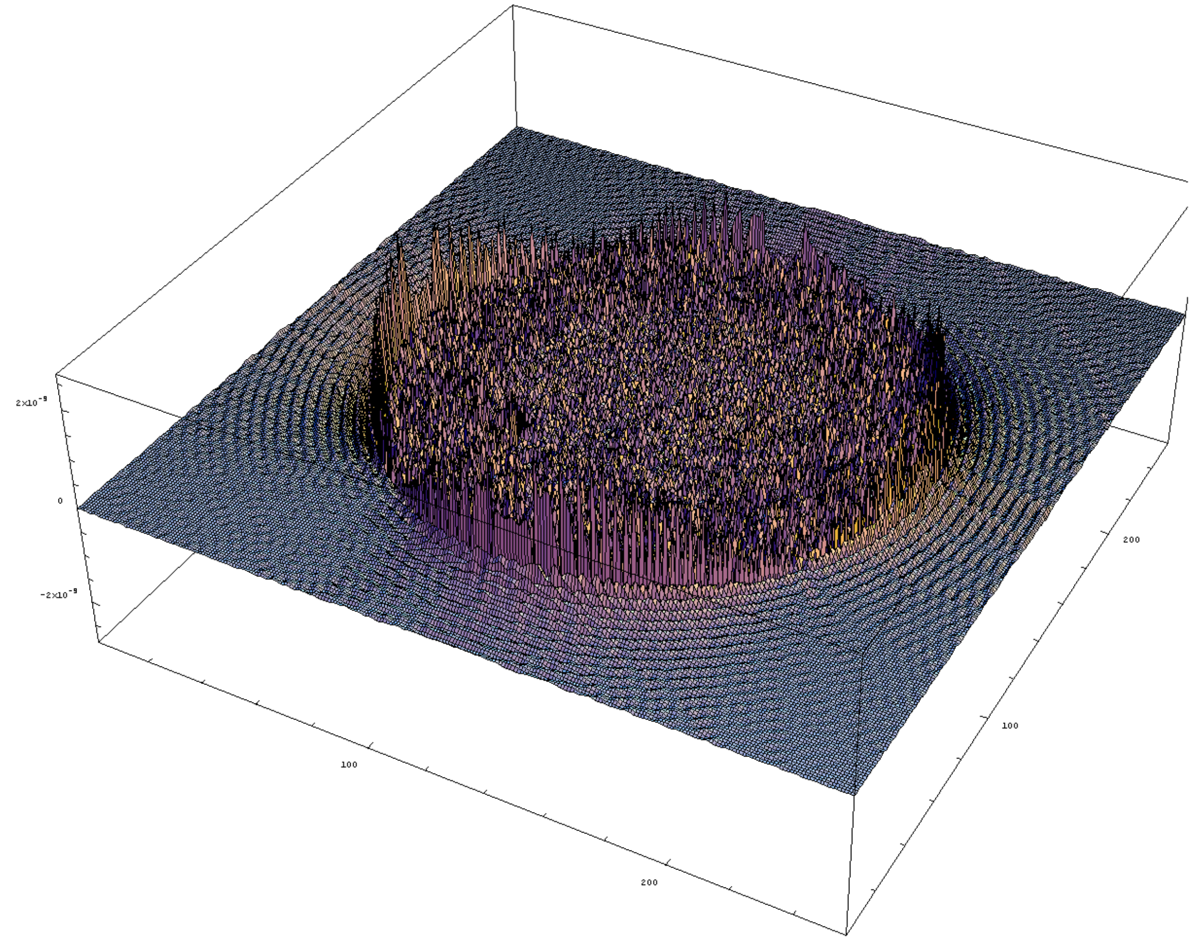
Mirror Figure Error

Removed perturbations larger than R/4 [4 cm]
6.34 ppm for Cone
5.24 ppm for Mesa



Mirror Figure Error

Removed perturbations larger than R/16 [1 cm]
3.16 ppm for Cone
2.68 ppm for Mesa



Experimental Requirements Summary

- Requirements :

Need to satisfy at least one limit in each of the following categories:

- Tilt
  - » $3*10^{-9}$ radians
- Displacement
  - » $4*10^{-6}$ m
  - » Equivalent to tilt 10^{-9} radians
- Figure Error
  - » Eliminate perturbations > 4 cm
  - » Decrease overall figure error 10 times

Noise reduction factors

$\frac{\text{Mesa Noise}}{\text{Cone Noise}}$	Brownian	Thermoelastic
Coating	2.339	2.339
Substrate	1.534	3.302

GR18 Poster

Using Conical Mirrors to See Further With LIGO

JOBS WANTED!!

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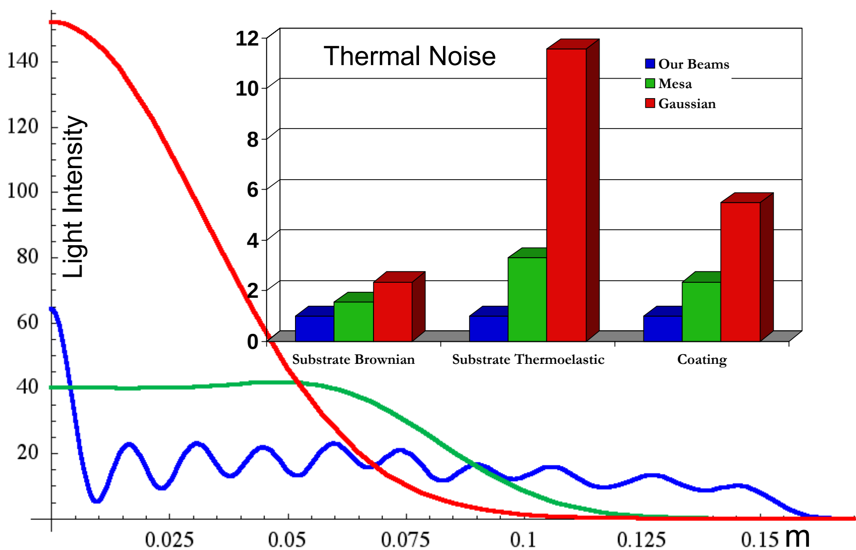
Mirror Internal Thermal Noise is the dominant noise source in Advanced LIGO's maximum frequency range. Coating Thermal Noise dominates over Substrate Thermal Noise.

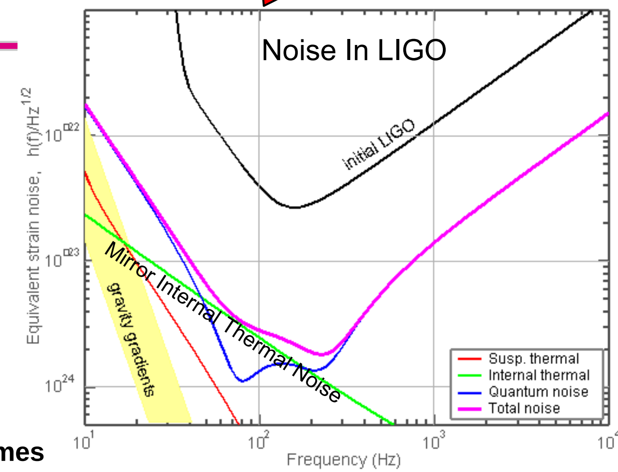
Thorne, O'Shaughnessy et al proposed changing the laser power distribution from Gaussian to flat-topped Mesa beams to reduce thermal noise by a factor of 2.5.

By systematically optimizing the laser intensity profile, we decreased thermal noise by a factor of 2.5 compared to Mesa and by a factor of 6 compared to baseline Gaussian. The resulting beams are supported by nearly conical mirrors and closely resemble Bessel-Gauss beams, previously known in the literature for their low diffraction loss.



Light Intensity and Different types of Internal Thermal Noise for our Conical beams compared with Baseline Gaussian

and Mesa, the leading non-Gaussian proposal.



Advantages:

Internal Thermal Noise 2.5 times lower than Mesa and 6 times lower than Baseline Gaussian.

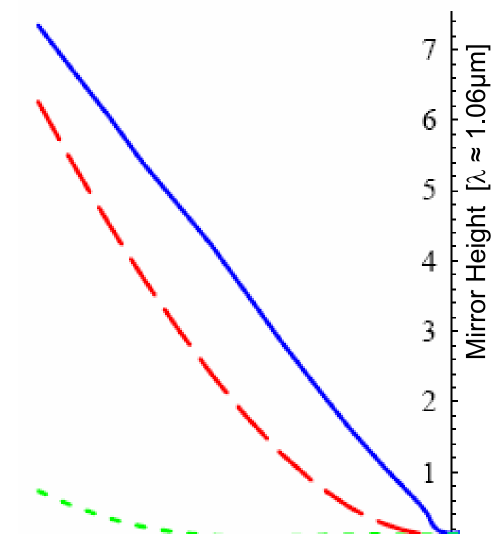
Event Rate in LIGO's maximum sensitivity frequency range ***roughly*** **3 times higher** than Mesa and 9 times higher than baseline Gaussian.

Only one nearly Lossless mode and thus easier to control light intensity distribution.

Challenges:

Arm Cavities need to be excited with non-Gaussian light.

More sensitive than Mesa to
- large-scale mirror figure error
- mirror positioning



Mirrors supporting our Conical beams compared to nearly-concentric and

nearly-flat Mesa beams