Semi-analythic optimization of future interferometers

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October 26, 2007

LIGO-G070803-00-Z

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Methods of enhancing AdvLIGO sensitivity

1. Squeezed light
2. Phase filter cavity(ies)
3. Amplitude filter cavity(ies)
4. Two-pass schemes
5. . . .   + any combinations

Methods of enhancing AdvLIGO sensitivity

1. Squeezed light
2. Phase filter cavity(ies)
3. Amplitude filter cavity(ies)
4. Two-pass schemes
5. … + any combinations

$$\bigotimes$$

1. Low-frequency (inspirals)
2. Broad-band (bursts)
3. Narrow-band (periodic)
4. …

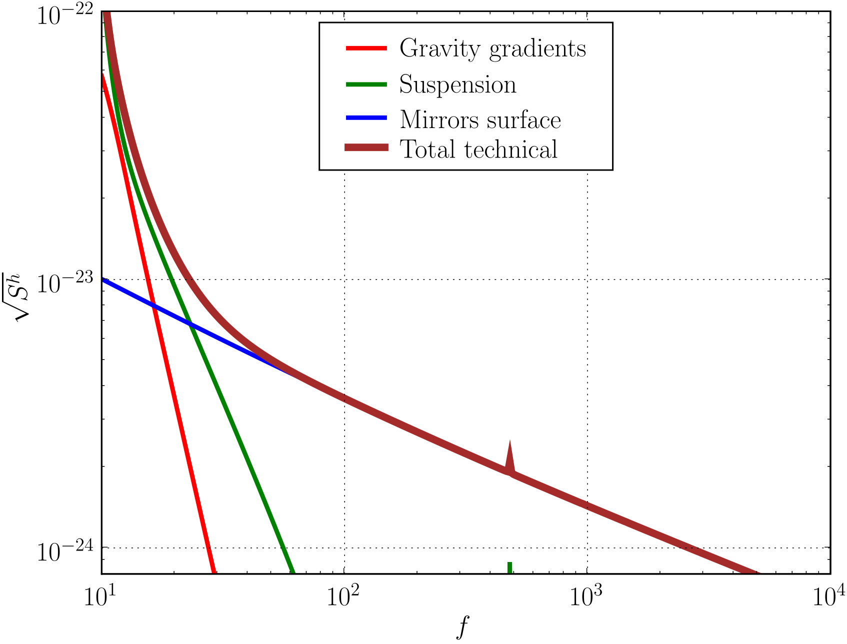
Parameters space

Universal constants

$$\hbar \quad \omega_p \quad L \quad M$$

Technical noises (tend to drift up with time)

Technical noises (bench 6.1)



Parameters space

Universal constants

$$\hbar \quad \omega_p \quad L \quad M$$

Technical noises (tend to drift up with time)

Variables

Arm cavities:	γ_{arm}	δ_{arm}
SR cavity:	ρ_{SRC}	ϕ_{SRC}
Pumping:	W_{arm}	ϕ
Squeezing:	r	θ

Filter cavity(ies) parameters

$\ldots$

The motivation

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However, only (semi)analythical optimization is able to provide heuristics in new configurations search.

Reduction of the parameters space, stage 1

$$J = \frac{8\omega_p W_{\rm arm}}{McL}\,; \quad \text{AdvLIGO: } J = (2\pi\times 100\,{\rm s}^{-1})^3\,.$$

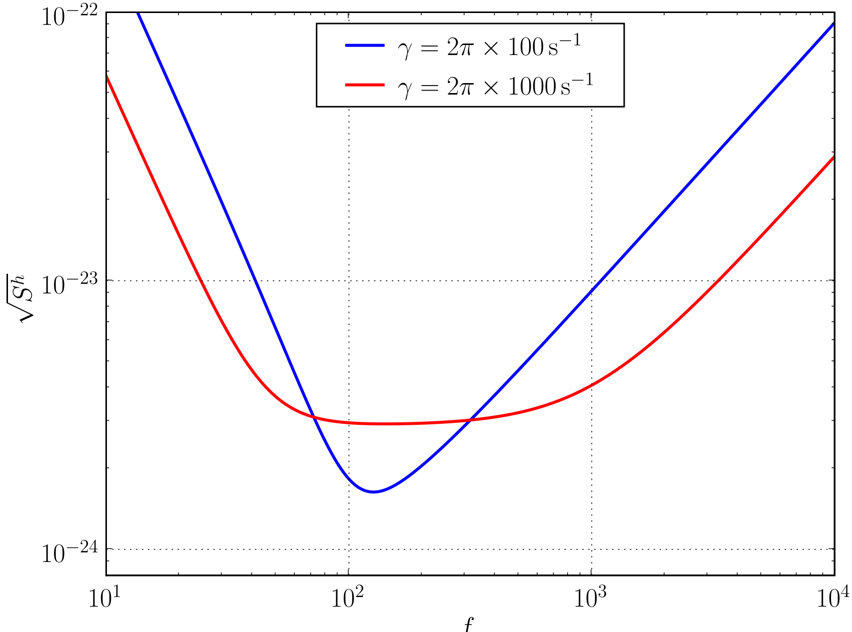
A.Buonanno, Y.Chen, PRD **67**, 062002 (2003):

$$\gamma = \gamma_{\rm arm}\,{\rm Re}\,\frac{1+\rho_{\rm SRC}e^{2i\phi_{\rm SRC}}}{1-\rho_{\rm SRC}e^{2i\phi_{\rm SRC}}}\,,$$

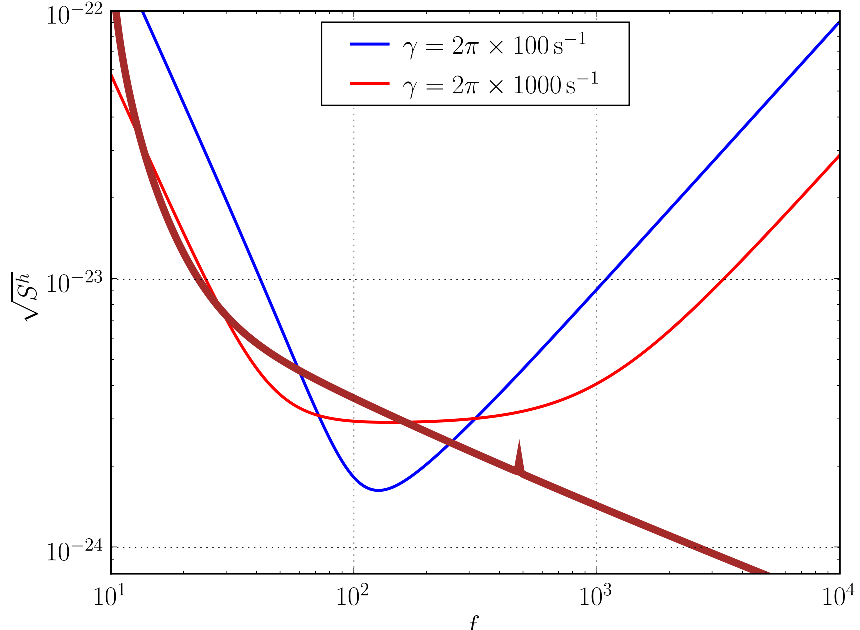
$$\delta = \delta_{\rm arm} + \gamma_{\rm arm}\,{\rm Im}\,\frac{1+\rho_{\rm SRC}e^{2i\phi_{\rm SRC}}}{1-\rho_{\rm SRC}e^{2i\phi_{\rm SRC}}}$$

$$\Leftrightarrow\ \Gamma = \sqrt{\gamma^2+\delta^2}\,, \quad \beta = \arctan\frac{\delta}{\gamma}\,.$$

Conventional vs. broadened



Conventional vs. broadened



Conventional vs. broadened

Due to the technical noises (mainly, mirrors surface fluctuations), the optimal γ should be large:

$$\gamma \gg J^{1/3} = 2\pi \times 100\,\mathrm{s}^{-1}\,.$$

Reduction of the parameters space, stage 2

$$S^h(\Omega) = \frac{8}{m^2L^2\Omega^4}\left\{[K_{\rm eff}(\Omega) - m\Omega^2]^2 S_x(\Omega) + \frac{\hbar^2}{4S_x(\Omega)}\right\} + S^h_{\rm tech}(\Omega)\,.$$

Here

$S_x(\Omega)$: shot noise.

$S_{xF}(\Omega)$: cross-correlation of shot noise and radiation-pressure noise; depends on ϕ.

$K_{\rm eff}(\Omega) = K(\Omega) + \dfrac{S_{xF}(\Omega)}{S_x(\Omega)}$: effective rigidity.

By some cryptic reason, without optical losses $K_{\rm eff}$ is always real.

Reduction of the parameters space, stage 2

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If $\Omega \ll \Gamma$, then

$$S^h(\Omega) \approx \frac{8}{m^2L^2\Omega^4}\left\{[K_{\rm eff}(0) - m\Omega^2]^2 S_x(0) + \frac{\hbar^2}{4S_x(0)}\right\} + S^h_{\rm tech}(\Omega)\,,$$

i.e., all optical parameters are reduced to

$$S_x(0) \quad \text{and} \quad K_{\rm eff}(0)\,.$$

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If $\Omega \gtrsim \Gamma \gg J^{1/3}$, then

$$S^h(\Omega) \approx \frac{8}{L^2}\,S_x(\Omega\to\infty) + S^h_{\rm tech}(\Omega) = \frac{dS_x(\Omega\to\infty)}{d\Omega^2}\times\Omega^2 + S^h_{\rm tech}(\Omega)\,.$$

$i.e.$, all optical parameters are reduced to

$$\frac{dS_x(\Omega\to\infty)}{d\Omega^2}\,.$$

Reduction of the parameters space, stage 2

$$S^h(\Omega) = \frac{8}{m^2L^2\Omega^4}\left\{[K_{\text{eff}}(\Omega) - m\Omega^2]^2 S_x(\Omega) + \frac{\hbar^2}{4S_x(\Omega)}\right\} + S^h_{\text{tech}}(\Omega)\,.$$

Therefore, if $\Gamma \gg J^{1/3}$, then optimization for low-frequency sources gives

$$S_x(0) \quad \text{and} \quad K_{\text{eff}}(0).$$

and optimization for high-frequency ones gives

$$\frac{dS_x(\Omega \to \infty)}{d\Omega^2}\,.$$

Low-frequency

Numerical optimization of

$$\mathrm{SNR}_{\mathrm{NSNS}} = \int_0^{f_{\mathrm{ISCO}}} \frac{f^{-7/3} df}{S^h(2\pi f)}$$

gives

$$\frac{M S_x(0)}{\hbar} = \frac{\Gamma}{4J\cos\beta\cos^2(\phi+\beta)} = (2\pi \times 48\,\mathrm{s}^{-1})^{-2}\,,$$
$$\frac{K_{\mathrm{eff}}(0)}{M} = \frac{J[\cos\beta\sin 2(\phi+\beta)+\sin\beta]}{\Gamma} = (2\pi\times 51\,\mathrm{s}^{-1})^2\,.$$

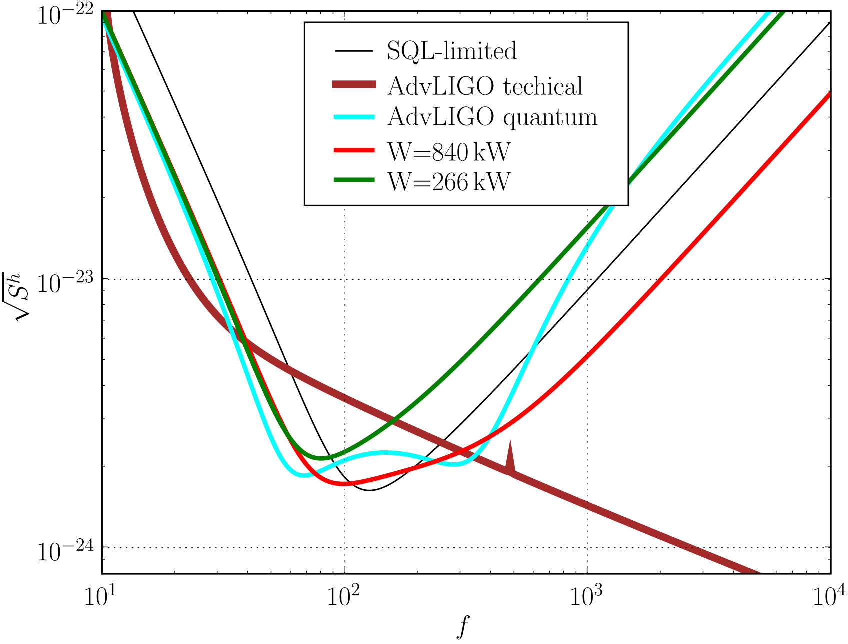
High-frequency

$$\frac{M}{\hbar}\frac{dS_x(\Omega \to \infty)}{d\Omega^2} = \frac{1}{4J\varGamma\cos\beta\cos^2\phi} = \min\ .$$

A minimization problem with 2 additional conditions. The solution is the following:

$$\frac{\varGamma}{J} \approx \frac{2300}{(2\pi \times 100)^3}\,\mathrm{s}^{-2}\,, \quad \beta \approx 0.29\,, \quad \phi \approx 0.08\,.$$

AdvLIGO, coherent pumping



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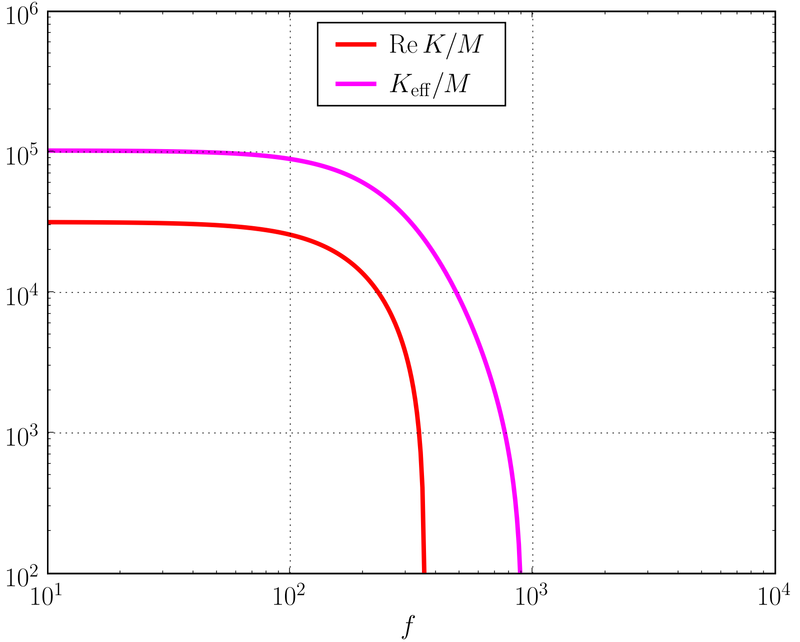


AdvLIGO, coherent pumping

Pseudo-resonance (the effective rigidity K_{eff}) is created mostly by the cross-correlation term S_{xF}; the real rigidity K is much smaller.

Estimates of h_{SQL}/h

Low-frequency (NSNS):

$$\frac{h_{\mathrm{SQL}}}{h} = \sqrt{\frac{\displaystyle\int_{f_c}^{f_{\mathrm{ISCO}}} \frac{f^{-7/3}}{S^h(2\pi f) + S^h_{\mathrm{tech}}(2\pi f)}\,df}{\displaystyle\int_{f_c}^{f_{\mathrm{ISCO}}} \frac{f^{-7/3}}{S^h_{\mathrm{conv}}(2\pi f) + S^h_{\mathrm{tech}}(2\pi f)}\,df}}$$

High-frequency:

$$\frac{h_{\mathrm{SQL}}}{h} = \sqrt{\frac{S^h_{\mathrm{conv}}(2\pi f) + S^h_{\mathrm{tech}}(2\pi f)}{S^h(2\pi f) + S^h_{\mathrm{tech}}(2\pi f)}}\,\Bigg|_{2\pi f \gg \gamma}$$

Estimates of h_{SQL}/h

	W	NSNS	$\gtrsim 1\,\mathrm{kHz}$
SQL-limited	840 kW	1	1
AdvLIGO	840 kW	1.18	0.50
Broadened	266 kW	1.07	0.58
	840 kW	1.17	1.85

Low-frequency

The same numerical optimization of low-frequency sensitivity can be used:

$$\frac{M S_x(0)}{\hbar} = \frac{\Gamma[\cosh 2r + \sinh 2r \cos(\phi + \theta + 2\beta)]}{4J \cos\beta \cos^2(\phi + \beta)}$$
$$= (2\pi \times 48\,\mathrm{s}^{-1})^{-2}\,,$$

$$\frac{K_{\mathrm{eff}}(0)}{M} = \text{a long formula which does not fit here}$$
$$= (2\pi \times 51\,\mathrm{s}^{-1})^{2}\,.$$

High-frequency

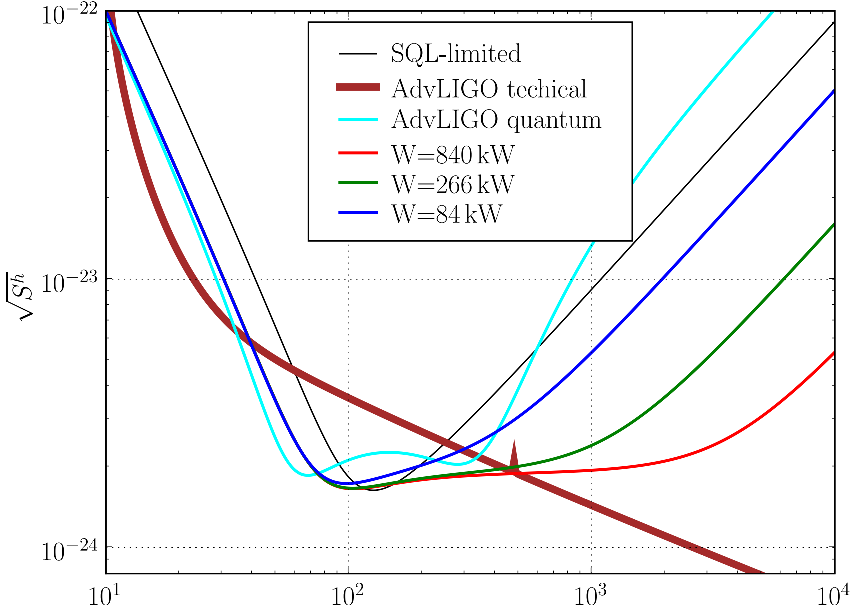
$$\frac{M}{\hbar}\frac{dS_x(\Omega \to \infty)}{d\Omega^2} = \frac{\cosh 2r + \sinh 2r \cos(\phi + \theta)}{4J\mathit{\Gamma} \cos\beta \cos^2\phi} = \min\ .$$

Again, a minimization problem with 2 additional conditions. The solution is the following:

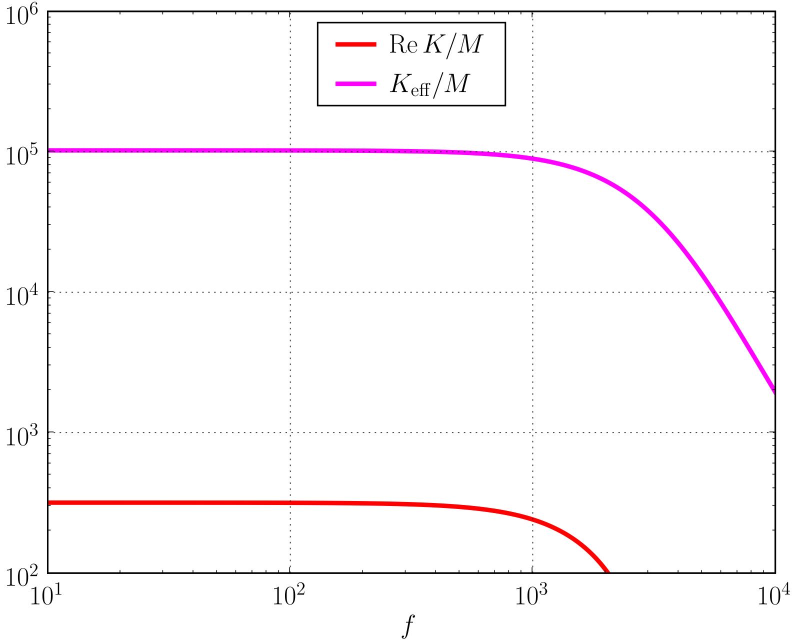
$$\frac{\mathit{\Gamma}}{J} \approx \frac{2.1 \times 10^4}{(2\pi \times 100)^3}\,\mathrm{s}^{-2}\,, \qquad \beta \approx 0.03\,,$$

$$\phi \approx -0.05\,, \qquad \theta \approx -1.52\,.$$

AdvLIGO, squeezed pumping



AdvLIGO, squeezed pumping



AdvLIGO, squeezed pumping

Pseudo-resonance (the effective rigidity $K_{\rm eff}$) is created almost completely by the cross-correlation term S_{xF}; the real rigidity K is negligibly small.

Estimates of h_{SQL}/h

	W	NSNS	$\gtrsim 1\,\mathrm{kHz}$
SQL-limited	840 kW	1	1
AdvLIGO	840 kW	1.18	0.50
Broadened	266 kW	1.07	0.60
	840 kW	1.17	1.89
Sqz.+Broad.	84 kW	1.17	1.80
	266 kW	1.21	5.67
	840 kW	1.22	16.9

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- Squeezed light in combination with the broadened interferometer configuration allows to obtain sensitivity, comparable with the AdvLIGO sensitivity at low frequencies, and significantly better — at high frequencies, using mush smaller optical power.
- In order to obtain the necessary interferometer bandwidth $\gamma \sim 10^4\,\mathrm{s}^{-1}$ with narrow-band arm cavities:
  $$T_{\mathrm{ITM}}^2 = 5 \times 10^{-4} \;\Rightarrow\; \gamma_{\mathrm{arm}} = 93.75\,\mathrm{s}^{-1}\,,$$
  the SR mirror reflectivity have to be increased up to $R_{\mathrm{SR}} \gtrsim 1 - 10^{-2}$.

To do:

Configurations with filter cavity(ies).