

Displacement- and laser-noise free gravitational-wave detection with Fabry-Perot cavities

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Displacement-noise-free GW detection.

General idea

A proper linear combination of the interferometer response signals cancels “displacement noise”, “tidal GW force” and “time delays” terms but does not cancel the “direct coupling of the GW-light” term which has the order of $O\left[h(L/\lambda_{\text{GW}})^2\right]$

□ □ □ □ □ □ □ □ □ □ localized effect □ □ □ □ □ □ □ □ □ □

response □ displacement noise + tidal GW force +

□ □ □ □ □ □ □ □ □ □

$$a_0 h(L/\lambda_{\text{GW}})^0$$

□ □ □ □ □ distributed effect □ □ □ □ □ □

+ time delays +

□ □ □ □ □ □ □ □ □ □

$$a_1 h(L/\lambda_{\text{GW}})^1$$

direct coupling GW-light

□ □

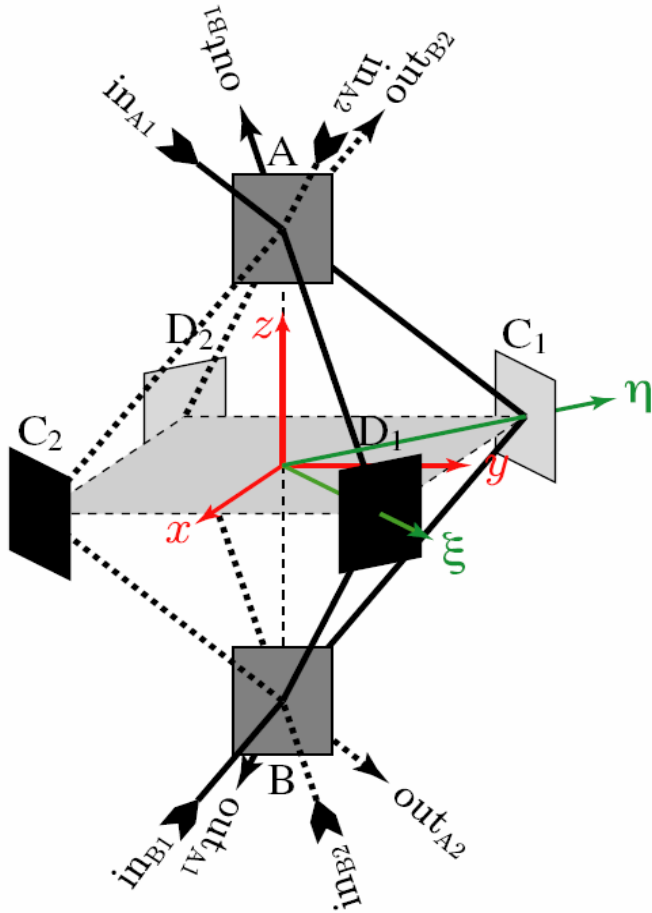
$$a_2 h(L/\lambda_{\text{GW}})^2 + a_3 h(L/\lambda_{\text{GW}})^3 + \dots$$

Interferometers for displacement-noise-free GW detection. 3D configuration

3D configuration (Octahedron)

Y. Chen *et al.*, PRL **97**, 151103 (2006).

Two Mach-Zehnder interferometers has 4 output ports. Their proper linear combination cancels displacement noise of 2 beamsplitters (A and B) and 4 mirrors (C₁, C₂, D₁ and D₂).

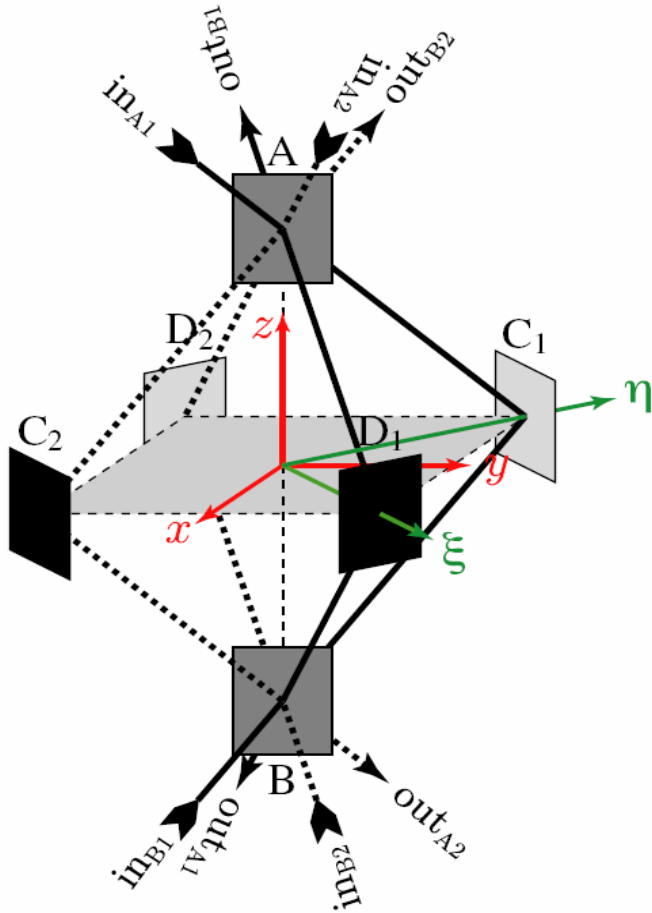


Interferometers for displacement-noise-free GW detection. 3D configuration

3D configuration (Octahedron)

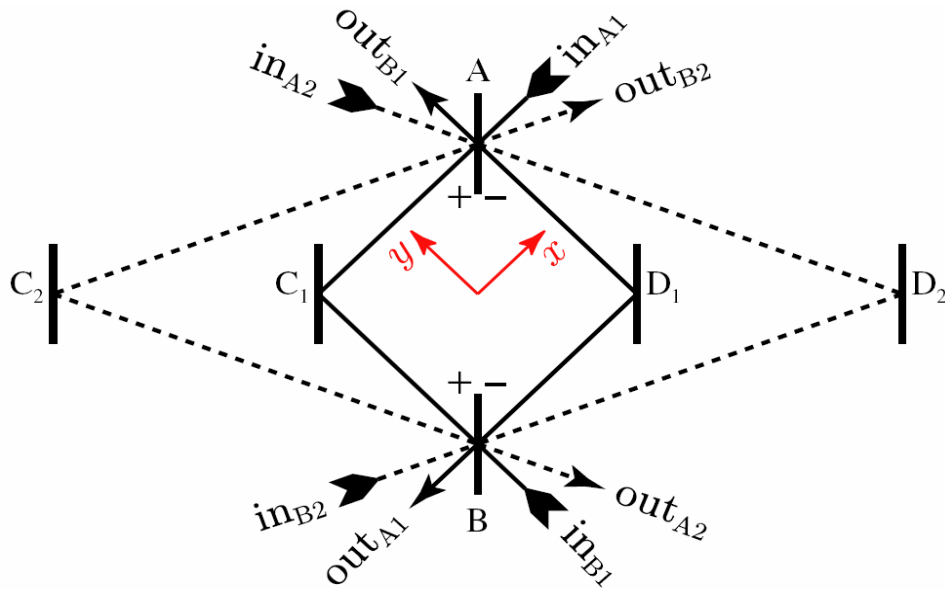
Y. Chen *et al.*, PRL **97**, 151103 (2006).

However, the displacement-noise-free interferometer (DFI) response signal is proportional to $h(L / \lambda_{\text{GW}})^2$ in low frequencies.



$$\text{DFI} \sim \text{noise} + a_0 h(L/\lambda_{\text{GW}})^0 + a_1 h(L/\lambda_{\text{GW}})^1 + a_2 h(L/\lambda_{\text{GW}})^2 + a_3 h(L/\lambda_{\text{GW}})^3 + \dots$$

Interferometers for displacement-noise-free GW detection. 3D configuration

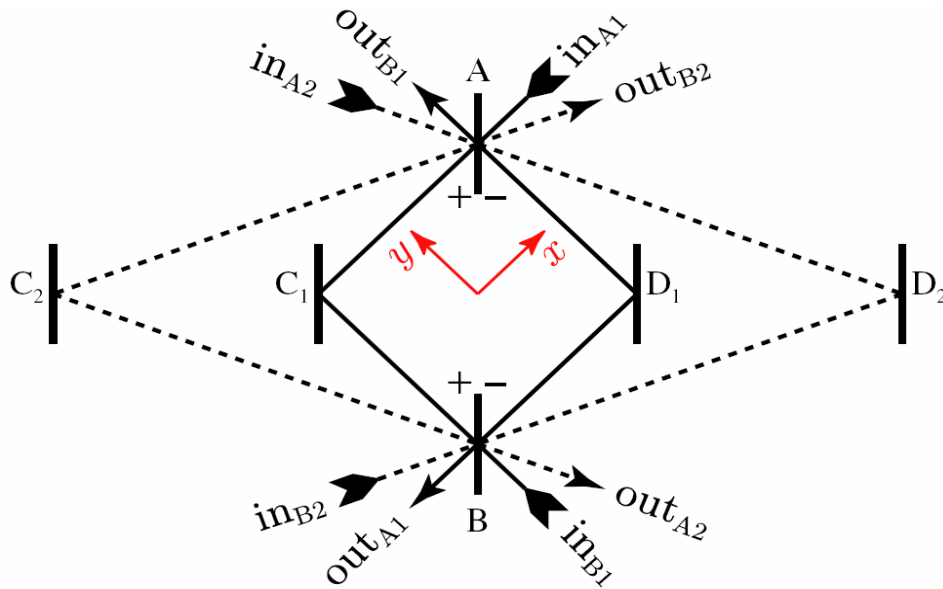


2D configuration

Y. Chen *et al.*, PRL **97**, 151103 (2006).

Proper linear combination of 4 output ports cancels displacement noise of A, B, C_1 , C_2 , D_1 and D_2 .

Interferometers for displacement-noise-free GW detection. 3D configuration



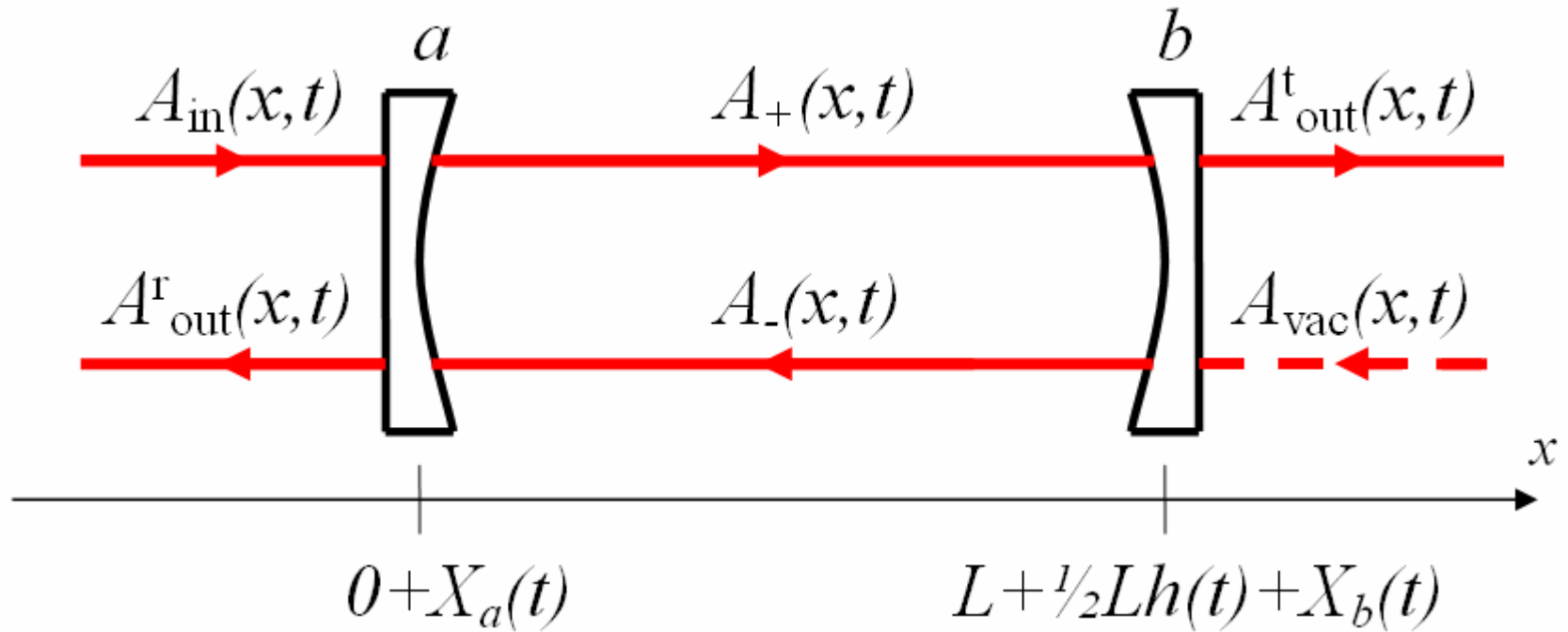
2D configuration

Y. Chen *et al.*, PRL **97**, 151103 (2006).

The displacement-noise-free response signal of the interferometer is proportional to $h(L / \lambda_{\text{GW}})^3$ in low frequencies.

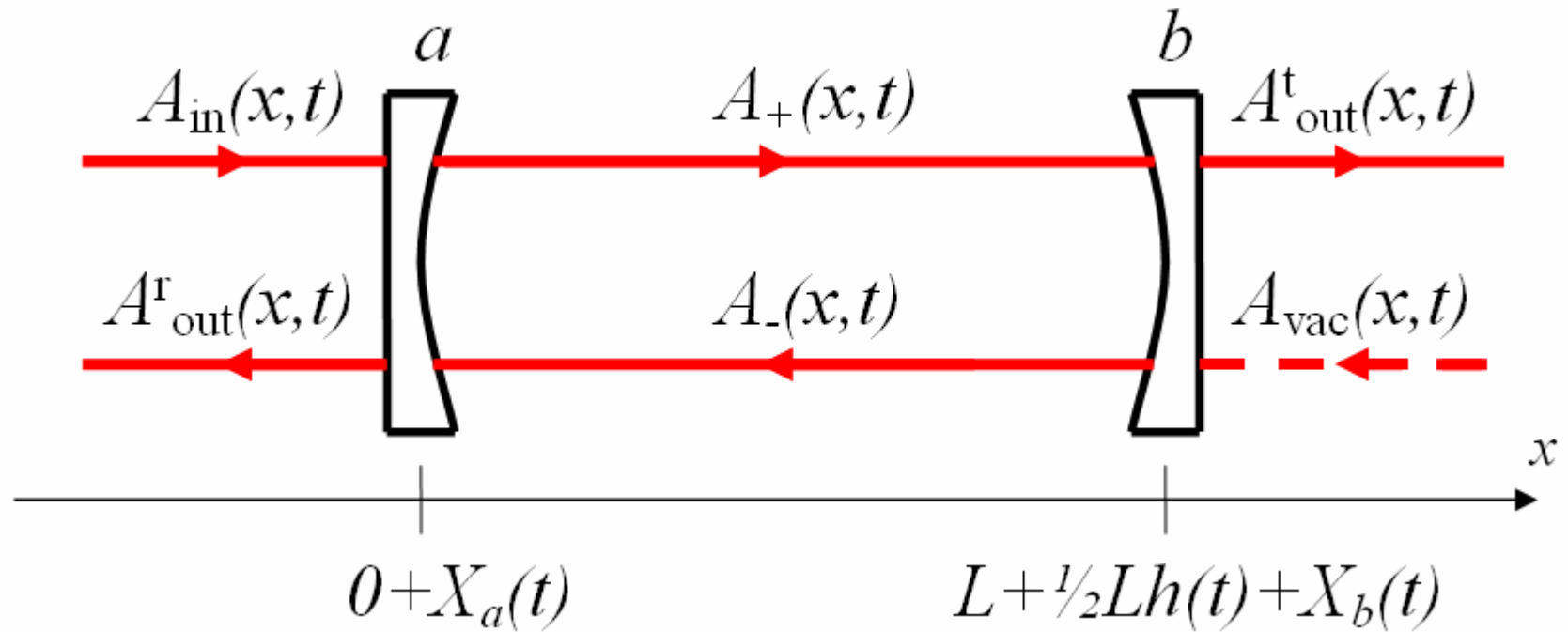
$$\text{DFI} \sim \text{noise} + a_0 h(L/\lambda_{\text{GW}})^0 + a_1 h(L/\lambda_{\text{GW}})^1 + a_2 h(L/\lambda_{\text{GW}})^2 + a_3 h(L/\lambda_{\text{GW}})^3 + \dots$$

Fabry-Perot cavity with movable mirrors



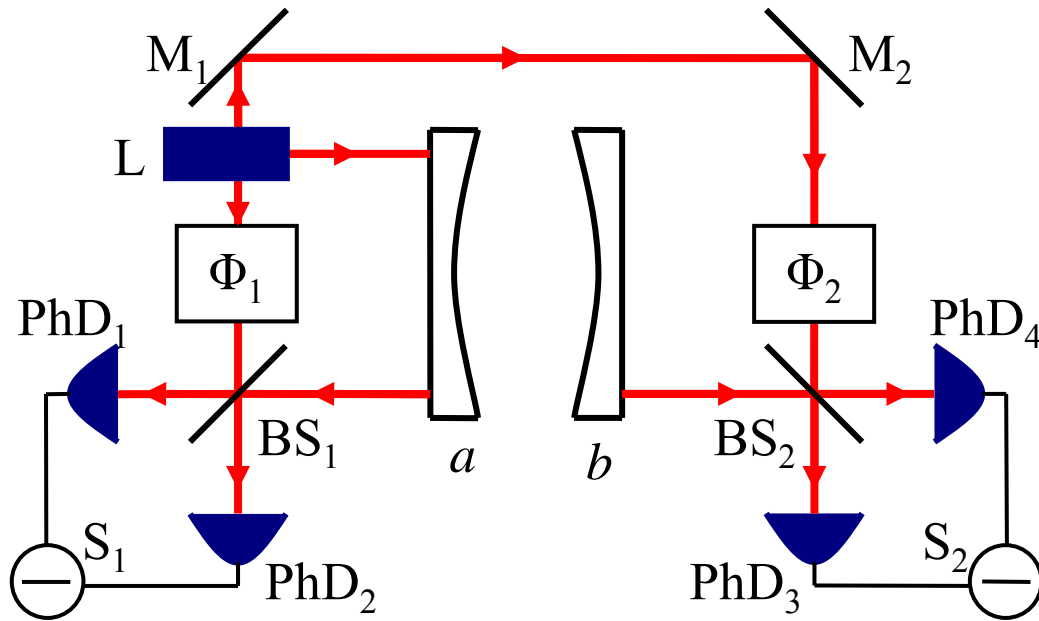
We consider another class of DFI detectors which do not utilize the effect of direct coupling of GW to light and thus are not limited with the $(L / \lambda_{\text{GW}})^n$ factor.

Fabry-Perot cavity with movable mirrors



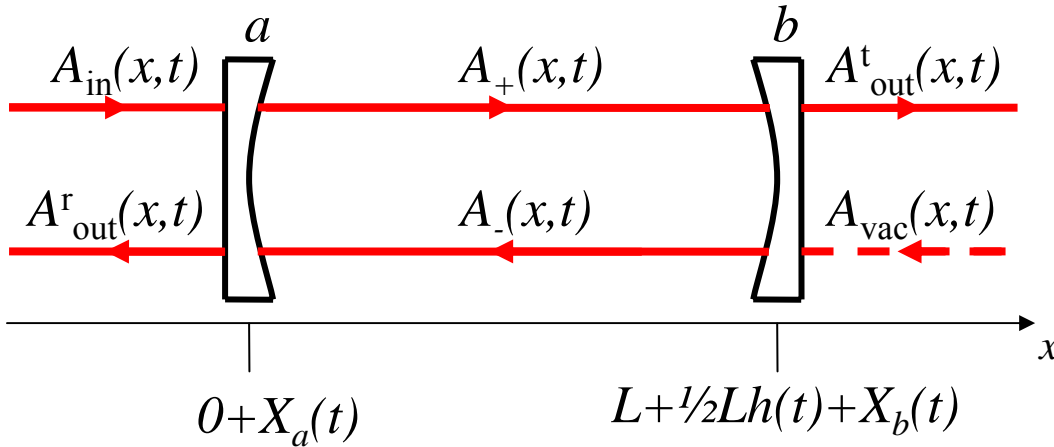
We analyze a Fabry-Perot (FP) cavity with movable mirrors having equal amplitude reflectivities R and transmittances T . Experimentalists can detect and manipulate with the output signals $A_{\text{out}}^r(x, t)$ and $A_{\text{out}}^t(x, t)$.

Ideal model. Laser and homodyne detectors are displacement-noise-free



Consider the simplest model. Laser and homodyne detectors are displacement-noise-free devices.

Response signals of a Fabry-Perot cavity



A_0 – input amplitude,
 R – amplitude reflectivity,
 δ_1 – detuning,
 $\tau = L / c$,
 X_a, X_b – fluctuating
 coordinates of mirrors
 “a” and “b”,
 X_{GW} – GW signal

$$a_{out}^r = \frac{R - R e^{2i(\delta_1 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}} a_{in} - \frac{RT^2 A_0 e^{2i\delta_1\tau}}{1 - R^2 e^{2i\delta_1\tau}} 2ik_0 \frac{(X_b + X_{GW}) e^{i\Omega\tau} - (1 + \Delta\sigma_1) X_a}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}},$$

$$a_{out}^t = \frac{T^2 e^{i(\delta_1 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}} a_{in} + \frac{R^2 T^2 A_0 e^{3i\delta_1\tau}}{1 - R^2 e^{2i\delta_1\tau}} 2ik_0 \frac{(X_b + X_{GW}) e^{i\Omega\tau} - X_a e^{i\Omega\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}},$$

$$\Delta\sigma_1 = \left(1 - e^{2i\delta_1\tau}\right) \frac{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}}{1 - R^2} e^{-2i\delta_1\tau} \approx -2i\delta_1\tau, \quad X_{GW} = \frac{1}{2} Lh \frac{\sin \Omega\tau}{\Omega\tau} \approx \frac{1}{2} Lh.$$

Double-pumped Fabry-Perot (DPFP) cavity



The pump wave A_{in} through mirror “a” has detuning δ_1 and polarization in the plane of incidence, and pump wave B_{in} through mirror “b” has detuning δ_2 and polarization normal to the plane of incidence.

Double-pumped Fabry-Perot (DPFP) cavity



The experimentalist can now separately detect and manipulate with 4 output signals (their quadrature components): reflected waves A^r_{out} and B^r_{out} , and transmitted waves A^t_{out} and B^t_{out} .

Response signals of a DPFP cavity



$$a_{out}^r = \frac{R - Re^{2i(\delta_1 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}} a_{in} - \frac{RT^2 A_0 e^{2i\delta_1\tau}}{1 - R^2 e^{2i\delta_1\tau}} 2ik_0 \frac{(X_b + X_{GW}) e^{i\Omega\tau} - (1 + \Delta\sigma_1) X_a}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}},$$

$$a_{out}^t = \frac{T^2 e^{i(\delta_1 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}} a_{in} + \frac{R^2 T^2 A_0 e^{3i\delta_1\tau}}{1 - R^2 e^{2i\delta_1\tau}} 2ik_0 \frac{(X_b + X_{GW}) e^{i\Omega\tau} - X_a e^{i\Omega\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}},$$

$$b_{out}^r = \frac{R - Re^{2i(\delta_2 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_2 + \Omega)\tau}} b_{in} + \frac{RT^2 B_0 e^{2i\delta_2\tau}}{1 - R^2 e^{2i\delta_2\tau}} 2ik_0 \frac{(-X_a + X_{GW}) e^{i\Omega\tau} + (1 + \Delta\sigma_2) X_b}{1 - R^2 e^{2i(\delta_2 + \Omega)\tau}},$$

$$b_{out}^t = \frac{T^2 e^{i(\delta_2 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_2 + \Omega)\tau}} b_{in} + \frac{R^2 T^2 B_0 e^{3i\delta_2\tau}}{1 - R^2 e^{2i\delta_2\tau}} 2ik_0 \frac{(-X_a + X_{GW}) e^{i\Omega\tau} + X_b e^{i\Omega\tau}}{1 - R^2 e^{2i(\delta_2 + \Omega)\tau}}.$$

Response signals of a DPFP cavity



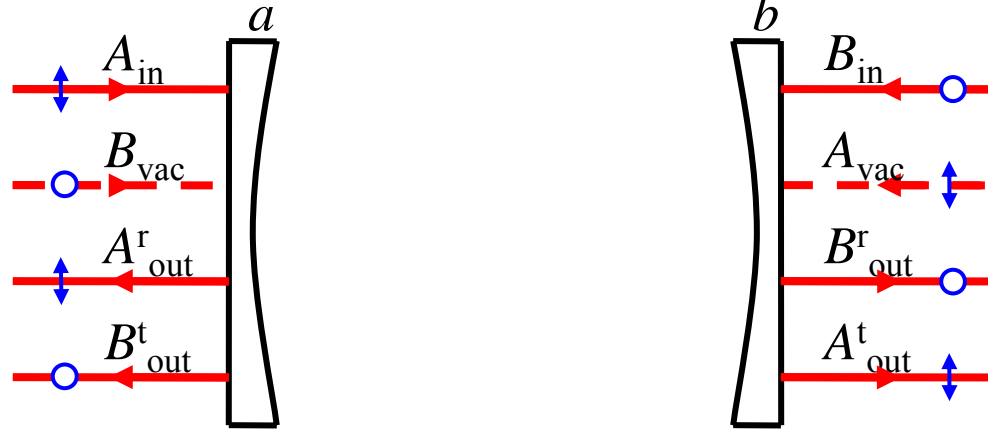
$$a_{\text{out}}^r = \frac{R - Re^{2i(\delta_1 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}} a_{\text{in}} - \frac{RT^2 A_0 e^{2i\delta_1\tau}}{1 - R^2 e^{2i\delta_1\tau}} 2ik_0 \frac{(X_b + X_{\text{GW}}) e^{i\Omega\tau} - (1 + \Delta\sigma_1) X_a}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}},$$

$$a_{\text{out}}^t = \frac{T^2 e^{i(\delta_1 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}} a_{\text{in}} + \frac{R^2 T^2 A_0 e^{3i\delta_1\tau}}{1 - R^2 e^{2i\delta_1\tau}} 2ik_0 \frac{(X_b + X_{\text{GW}}) e^{i\Omega\tau} - X_a e^{i\Omega\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}},$$

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Cancelation of X_a



$$a_{out}^r = \frac{R - Re^{2i(\delta_1 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}} a_{in} - \frac{RT^2 A_0 e^{2i\delta_1\tau}}{1 - R^2 e^{2i\delta_1\tau}} 2ik_0 \frac{(X_b + X_{GW}) e^{i\Omega\tau} - (1 + \Delta\sigma_1) X_a}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}},$$

$$a_{out}^t = \frac{T^2 e^{i(\delta_1 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}} a_{in} + \frac{R^2 T^2 A_0 e^{3i\delta_1\tau}}{1 - R^2 e^{2i\delta_1\tau}} 2ik_0 \frac{(X_b + X_{GW}) e^{i\Omega\tau} - X_a e^{i\Omega\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}},$$

$$s_1 = Ra_{out}^r e^{i(\delta_1 + \Omega)\tau} + (1 + \Delta\sigma_1) a_{out}^t =$$

$$= \text{laser noise}_1 + \frac{R^2 T^2 A_0 e^{3i\delta_1\tau}}{1 - R^2 e^{2i\delta_1\tau}} 2ik_0 \frac{\Delta\sigma_1 (X_b + X_{GW}) e^{2i\Omega\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}}. \quad (\text{no } X_a)$$

Response signals of a DPFP cavity



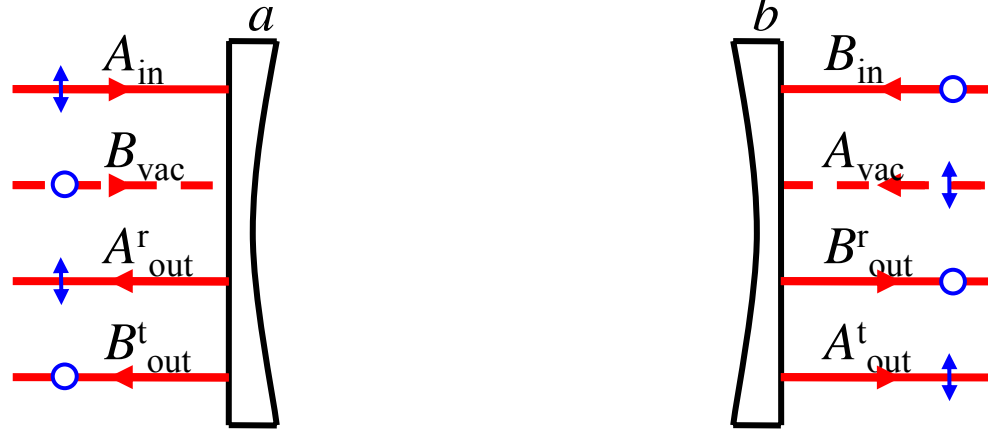
$$a_{out}^r = \frac{R - Re^{2i(\delta_1 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}} a_{in} - \frac{RT^2 A_0 e^{2i\delta_1\tau}}{1 - R^2 e^{2i\delta_1\tau}} 2ik_0 \frac{(X_b + X_{GW}) e^{i\Omega\tau} - (1 + \Delta\sigma_1) X_a}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}},$$

$$a_{out}^t = \frac{T^2 e^{i(\delta_1 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}} a_{in} + \frac{R^2 T^2 A_0 e^{3i\delta_1\tau}}{1 - R^2 e^{2i\delta_1\tau}} 2ik_0 \frac{(X_b + X_{GW}) e^{i\Omega\tau} - X_a e^{i\Omega\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}},$$

$$b_{out}^r = \frac{R - Re^{2i(\delta_2 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_2 + \Omega)\tau}} b_{in} + \frac{RT^2 B_0 e^{2i\delta_2\tau}}{1 - R^2 e^{2i\delta_2\tau}} 2ik_0 \frac{(-X_a + X_{GW}) e^{i\Omega\tau} + (1 + \Delta\sigma_2) X_b}{1 - R^2 e^{2i(\delta_2 + \Omega)\tau}},$$

$$b_{out}^t = \frac{T^2 e^{i(\delta_2 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_2 + \Omega)\tau}} b_{in} + \frac{R^2 T^2 B_0 e^{3i\delta_2\tau}}{1 - R^2 e^{2i\delta_2\tau}} 2ik_0 \frac{(-X_a + X_{GW}) e^{i\Omega\tau} + X_b e^{i\Omega\tau}}{1 - R^2 e^{2i(\delta_2 + \Omega)\tau}}.$$

Cancelation of $-X_a + X_{GW}$



$$b_{\text{out}}^r = \frac{R - R e^{2i(\delta_2 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_2 + \Omega)\tau}} b_{\text{in}} + \frac{RT^2 B_0 e^{2i\delta_2\tau}}{1 - R^2 e^{2i\delta_2\tau}} 2ik_0 \frac{(-X_a + X_{GW}) e^{i\Omega\tau} + (1 + \Delta\sigma_2) X_b}{1 - R^2 e^{2i(\delta_2 + \Omega)\tau}},$$

$$b_{\text{out}}^t = \frac{T^2 e^{i(\delta_2 + \Omega)\tau}}{1 - R^2 e^{2i(\delta_2 + \Omega)\tau}} b_{\text{in}} + \frac{R^2 T^2 B_0 e^{3i\delta_2\tau}}{1 - R^2 e^{2i\delta_2\tau}} 2ik_0 \frac{(-X_a + X_{GW}) e^{i\Omega\tau} + X_b}{1 - R^2 e^{2i(\delta_2 + \Omega)\tau}} e^{i\Omega\tau},$$

$$s_2 = R b_{\text{out}}^r e^{i(\delta_1 + \Omega)\tau} + b_{\text{out}}^t =$$

$$= \text{laser noise}_2 - \frac{R^2 T^2 B_0 e^{3i\delta_2\tau}}{1 - R^2 e^{2i\delta_2\tau}} 2ik_0 \frac{\Delta\sigma_2 X_b e^{i\Omega\tau}}{1 - R^2 e^{2i(\delta_2 + \Omega)\tau}}. \quad (\text{no } -X_a + X_{GW})$$

Cancelation of X_b . Displacement-noise-free response signal of a DPFP cavity

$$s_1 = \text{laser noise}_1 + \frac{R^2 T^2 A_0 e^{3i\delta_1\tau}}{1 - R^2 e^{2i\delta_1\tau}} 2ik_0 \frac{\Delta\sigma_1 (X_b + X_{\text{GW}}) e^{2i\Omega\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}},$$

$$s_2 = \text{laser noise}_2 - \frac{R^2 T^2 B_0 e^{3i\delta_2\tau}}{1 - R^2 e^{2i\delta_2\tau}} 2ik_0 \frac{\Delta\sigma_2 X_b e^{i\Omega\tau}}{1 - R^2 e^{2i(\delta_2 + \Omega)\tau}},$$

$$s_{\text{DFI}} = s_1 + \frac{e^{3i\delta_1\tau} \Delta\sigma_1 / (1 - R^2 e^{2i(\delta_1 + \Omega)\tau})}{e^{3i\delta_2\tau} \Delta\sigma_1 / (1 - R^2 e^{2i(\delta_2 + \Omega)\tau})} s_2 e^{i\Omega\tau} = \left\{ \frac{A_0}{1 - R^2 e^{2i\delta_1\tau}} = \frac{B_0}{1 - R^2 e^{2i\delta_2\tau}} \right\} =$$

$$= \text{laser noise} + \frac{R^2 T^2 A_0 e^{3i\delta_1\tau}}{1 - R^2 e^{2i\delta_1\tau}} 2ik_0 \frac{\Delta\sigma_1 X_{\text{GW}} e^{2i\Omega\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}}.$$

Cancelation of X_b . Displacement-noise-free response signal of a DPFP cavity

$$s_1 = \text{laser noise}_1 + \frac{R^2 T^2 A_0 e^{3i\delta_1\tau}}{1 - R^2 e^{2i\delta_1\tau}} 2ik_0 \frac{\Delta\sigma_1 (X_b + X_{\text{GW}}) e^{2i\Omega\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}},$$

$$s_2 = \text{laser noise}_2 - \frac{R^2 T^2 B_0 e^{3i\delta_2\tau}}{1 - R^2 e^{2i\delta_2\tau}} 2ik_0 \frac{\Delta\sigma_2 X_b e^{i\Omega\tau}}{1 - R^2 e^{2i(\delta_2 + \Omega)\tau}},$$

$$s_{\text{DFI}} = s_1 + \frac{e^{3i\delta_1\tau} \Delta\sigma_1 / (1 - R^2 e^{2i(\delta_1 + \Omega)\tau})}{e^{3i\delta_2\tau} \Delta\sigma_1 / (1 - R^2 e^{2i(\delta_2 + \Omega)\tau})} s_2 e^{i\Omega\tau} = \left\{ \frac{A_0}{1 - R^2 e^{2i\delta_1\tau}} = \frac{B_0}{1 - R^2 e^{2i\delta_2\tau}} \right\} =$$

$$= \text{laser noise} + \frac{R^2 T^2 A_0 e^{3i\delta_1\tau}}{1 - R^2 e^{2i\delta_1\tau}} 2ik_0 \frac{\Delta\sigma_1 X_{\text{GW}} e^{2i\Omega\tau}}{1 - R^2 e^{2i(\delta_1 + \Omega)\tau}} \approx$$

$$\approx a_{\text{in}} + b_{\text{in}} - \frac{i\delta_1}{\gamma - i\delta_1} A_0 2ik_0 Lh, \quad \text{if } \delta_1 = \delta_2.$$

Difference between conventional and DPFP displacement-noise-free GW detectors

Conventional displacement-noise-free interferometers utilize the asymmetry between localized nature of displacement noise and distributed nature of GWs.

Noise and GW tidal force-field are canceled while direct coupling of the GW to light is kept. The measure of asymmetry is $(L / \lambda_{\text{GW}})^3$ for 2D scheme.

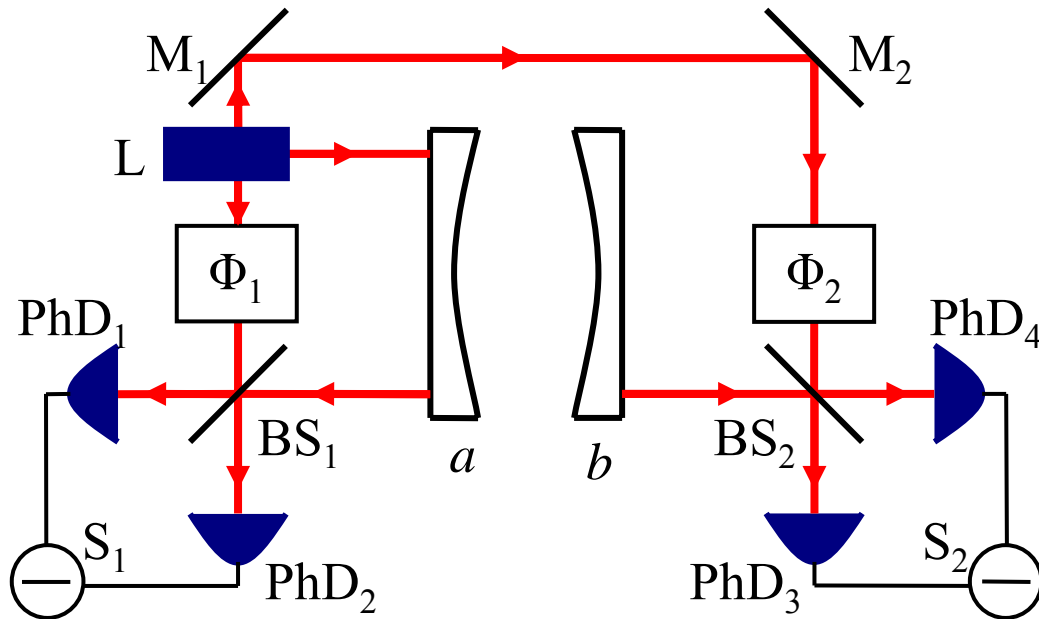
Double-pumped Fabry-Perot cavity utilizes the asymmetry between reflected and transmitted response signals of the cavity and does not rely on direct coupling of the GW to light. The measure of the asymmetry is $\delta\tau$.

Disadvantages of the DPFP displacement-noise-free GW detector

The resonant gain of the Fabry-Perot cavity is lost in the algorithm of displacement noise cancelation. The proposed scheme has a comparable shot-noise-limited sensitivity to a one-round-trip GW detector.

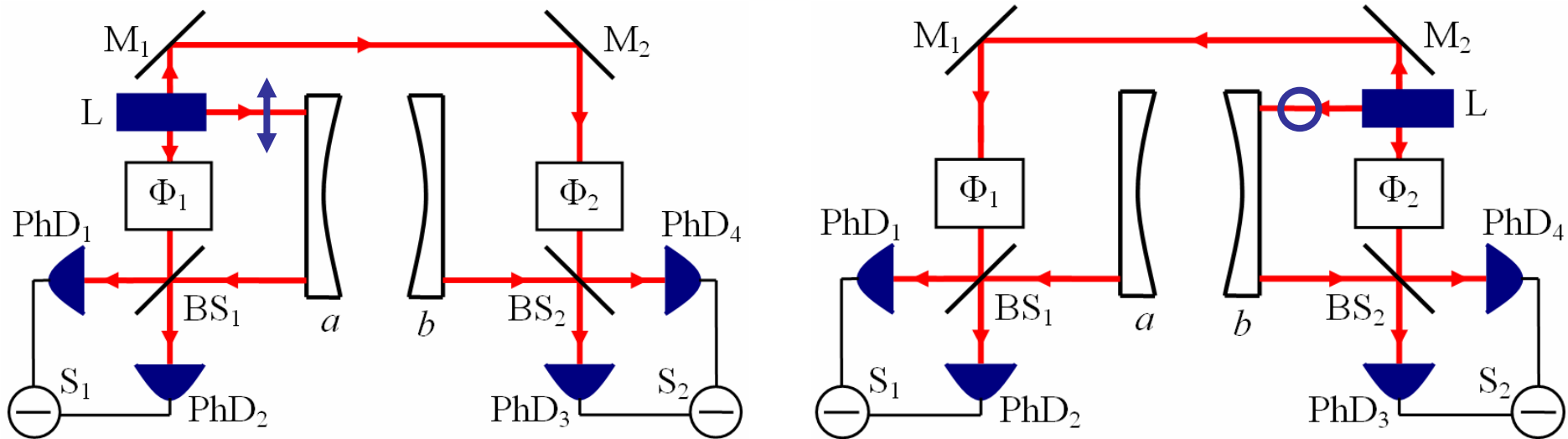
Obviously, displacement noise of the emission-detection scheme will be crucial due to the loss of the optical resonant gain in the GW signal. Experimentalists should carefully cool, isolate, etc the emission-detection scheme.

Realistic model. Noisy lasers and homodyne detectors



Consider a single pump situation. Reflected wave is detected with the homodyne detector in the vicinity of mirror “ a ”. Transmitted wave is detected in the vicinity of mirror “ b ”. Local oscillator is laser L .

Realistic model. Noisy lasers and homodyne detectors



Both the lasers generating two pumps and all the homodyne detectors undergo fluctuative motions.

“Pseudo”-displacement-noise-free response signal of a DPFP cavity

The DFI response signal changes if motions of lasers and detectors are taken into account:

$$S_{\text{DFI}} \approx a_{\text{in}} + b_{\text{in}} - \frac{i\delta_1}{\gamma - i\delta_1} A_0 2ik_0 Lh +$$

+ lasers and detectors displacement noise.

Though optical laser noises $a_{\text{in}} + b_{\text{in}}$ can be canceled in the laser-noise cancelation scheme, the displacement noise of the lasers and homodyne detectors cannot.

“Pseudo”-displacement-noise-free response signal of a DPFP cavity

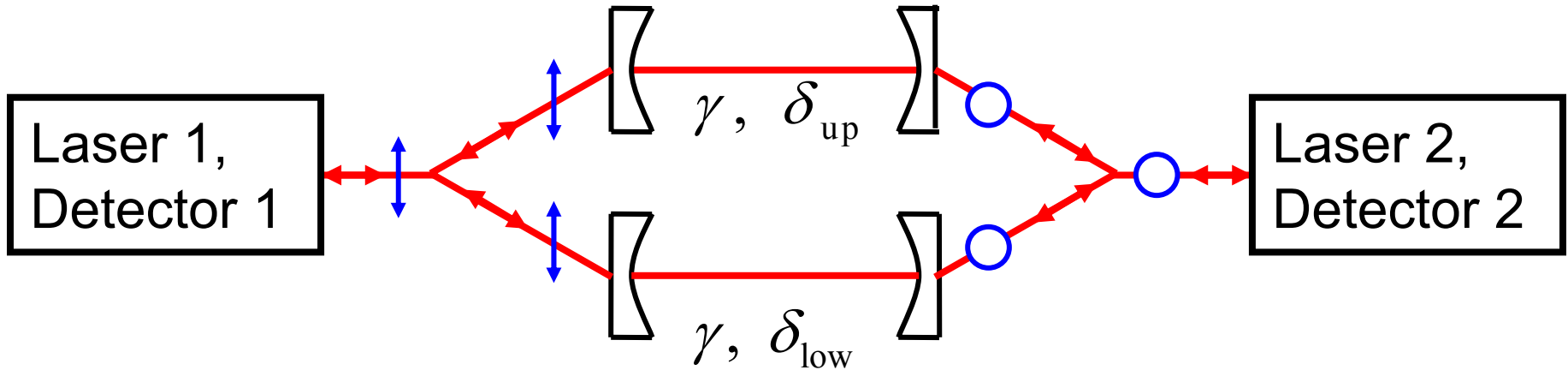
The DFI response signal changes if motions of lasers and detectors are taken into account:

$$S_{\text{DFI}} \approx a_{\text{in}} + b_{\text{in}} - \frac{i\delta_1}{\gamma - i\delta_1} A_0 2ik_0 Lh +$$

+ lasers and detectors displacement noise.

Optical noise terms do not depend on cavity bandwidth and detuning and thus can be removed from the combined DFI responses of the pair of DPFP cavities.

Cancelation of laser noise. General idea

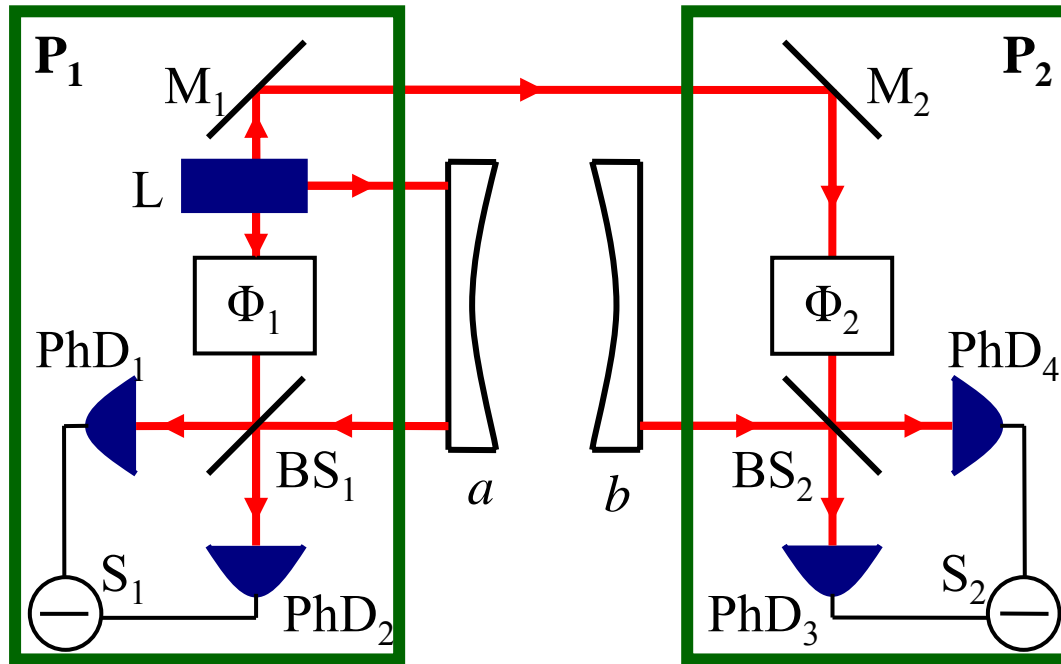


$$S_{DFI}^{low} \approx a_{in} + b_{in} + \frac{i\delta_{low}}{\gamma - i\delta_1} A_0 2ik_0 Lh + (\text{lasers and HDs disp. noise})_1,$$

$$S_{DFI}^{up} \approx a_{in} + b_{in} + \frac{i\delta_{up}}{\gamma - i\delta_u} A_0 2ik_0 Lh + (\text{lasers and HDs disp. noise})_2,$$

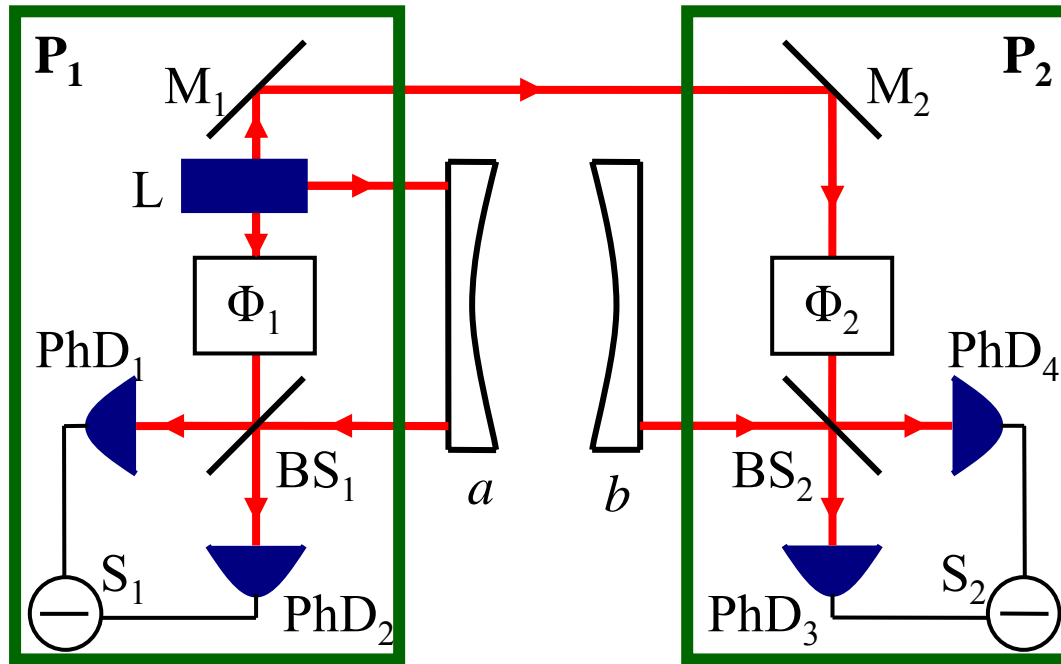
$$S_{DFI}^{low} - S_{DFI}^{up} = \frac{i\gamma(\delta_{low} - \delta_{up})}{(\gamma - i\delta_{low})(\gamma - i\delta_{up})} A_0 2ik_0 Lh + (\text{lasers and HDs disp. noise}).$$

Drawbacks of the scheme



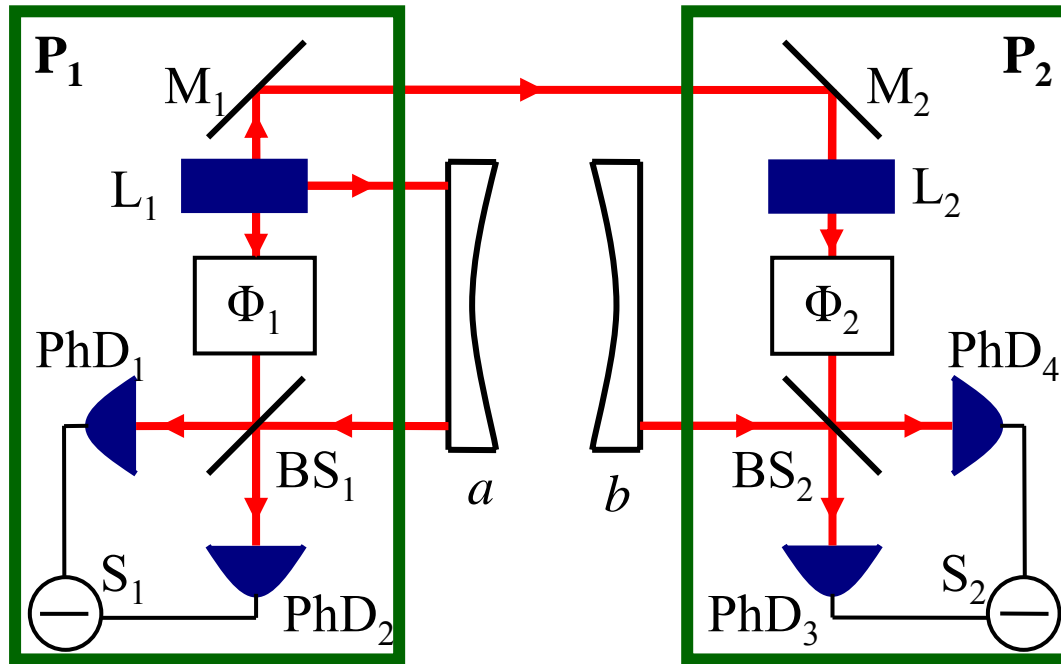
Possible solution: install laser and the homodyne detectors on very heavy platforms P_1 and P_2 which will undergo little fluctuative motions then. Cool down all the optical elements on the platforms to cryogenic temperature.

Drawbacks of the scheme



Strong optical field from the laser needs to be transmitted to the location of the second homodyne detector for the detection of transmitted wave. This is a kilometer-scale distance!

Drawbacks of the scheme



Possible solution: use the second laser (which is already on the second platform) as the local oscillator for the transmitted wave. Possible problem: laser synchronization procedure.

Conclusion

- We proposed a double-pumped FP cavity operating as the GW detector free from the displacement noise of the mirrors.
- This may be considered as the back-action evasion scheme without the implementation of QND schemes which are very vulnerable to losses.
- The isolation of the GW signal from mirrors displacement noise is possible due to the asymmetry between the reflected and the transmitted output signals of the cavity. The sensitivity of our detector is limited by the (δ / γ) factor instead of the $(L / \lambda_{\text{GW}})^n$ factor.
- Due to the loss of the resonant factor our scheme becomes highly susceptible to the displacement noises of the emission-detection scheme. This problem can be overcome in principle by installing the emission-detection scheme on heavy platforms and cooling it down to cryogenic temperatures.

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**Thank you for your
attention!**