Incorporating Numerical Relativity Waveforms into Gravitational Wave Data Analysis

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Connecting Gravitational Waves with Observational Astrophysics
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Outline

NR and GWDA to meet each other!

Numerical Relativity

- Enormous progress in the last few years - **2005 Breaktrough**
- Several NR groups worldwide with stable, accurate codes - already producing results and **waveforms** for BBH coalescence
- NR is able to simulate some of the most promising **sources** of gravitational radiation

Gravitational Wave Data Analysis

- LIGO has completed 5 science runs - producing large amounts of **data** and more to come in the future (advLIGO, Virgo, GEO600)
- **Detection** in the future seems plausible - no direct observations yet
- Clever **data analysis strategies** play a fundamental role

Multipole expansion of the wave (I)

- Output of NR $\longrightarrow$ full spacetime of a binary black hole system $\rightarrow \Psi_4$
- Required by GWDA $\longrightarrow$ strain $h(t)$ as measured by a detector far away

Different methods to extract h_{ij} from a numerical evolution:

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$$\Psi_4 = \ddot{h}_+ - i\ddot{h}_\times$$

Zerilli function: spacetime as perturbation of Schwarzschild

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- Data calculated in a numerical simulation contains **complex** quantities over the whole sphere

$$\boxed{\Psi_4 = \Psi_4(\theta, \phi)}$$

- In the detectors, the signal from a binary produces a **real** strain

$$\boxed{h(t) = F_+ h_+(t) + F_\times h_\times(t)}$$

$F_+, F_\times$ are the *antenna pattern* functions of the detector

Multipole expansion of the wave (II)

- A suitable way to make interchange of data manageable is to decompose the data over a sphere into **modes**

- $h_+ - ih_\times$ can be decomposed into modes using **spin weighted spherical harmonics** ${}^{-s}Y_{\ell m}$ of weight -2

  - $h_+ - ih_\times = \frac{M}{r} \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} H_{\ell m}(t) \; {}^{-2}Y_{\ell m}(\iota, \phi)\,.$

where $MH_{\ell m} = \oint \; {}^{-2}Y^{\star}_{\ell m}(\iota, \phi)(rh_+ - irh_\times)\, d\Omega.$

- Define $h_+^{(\ell m)}$ and $h_\times^{(\ell m)} \Longrightarrow rh_+^{(\ell m)}(t) - irh_\times^{(\ell m)}(t) \equiv MH_{\ell m}(t)\,.$

(Waveform reconstruction $\longrightarrow h_+ - ih_\times = \sum_{\ell m} {}^{-2}Y_{\ell m}(\iota, \phi) \left[h_+^{(\ell m)} - ih_\times^{(\ell m)}\right].$

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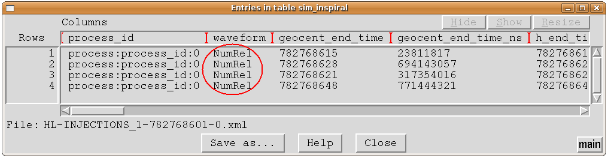
$\longrightarrow$ and incorporated to LAL with data analysis purposes in order to test the quality of the LSC searches

Modifications in LAL to allow for NR injections



Entries in table sim_inspiral

Rows	process_id	waveform	geocent_end_time	geocent_end_time_ns	h_end_ti
1	process:process_id:0	NumRel	782768615	23811817	78276861
2	process:process_id:0	NumRel	782768628	694143057	78276862
3	process:process_id:0	NumRel	782768621	317354016	78276862
4	process:process_id:0	NumRel	782768648	771444321	78276864

File: HL-INJECTIONS_1-782768601-0.xml

Save as... Help Close main

Specifying `--waveform NumRel` in the sim inspiral table turns on NR data injections

- A **metadata file** containing info about the parameters of the numerical simulations (mass ratio and spins) is parsed
- Suitable NR data ($h_{+,\times}^{(\ell m)}$ modes) according to the sim inspiral table is read
- $h_{+,\times}$ are reconstructed by multiplying $h_{+,\times}^{(\ell m)}$ by the corresponding $^{-s}Y_{\ell m}$ and performing the sum over the modes
- $h(t)$ is computed as $F_+ h_+(t) + F_\times h_\times(t)$
- $h(t)$ is **injected** into the data stream at the time(s) given in the sim inspiral table (coalescence time is computed as the maximum of the NR wave)

Sanity check

In order to test validity of the NR injection method, a sanity check has been performed by numerically injecting PN data into the detector stream.

→ 2048 s of **white gaussian simulated noise**
→ 1 **TaylorT1 3.5PN** waveform injected with two different (but equivalent) methods

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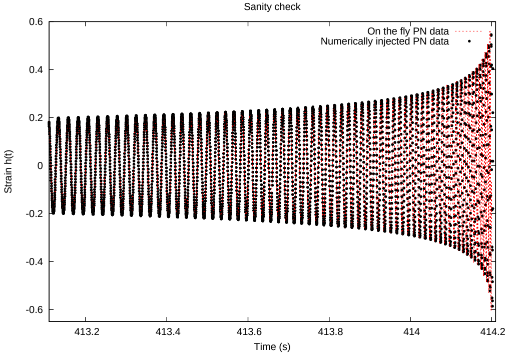
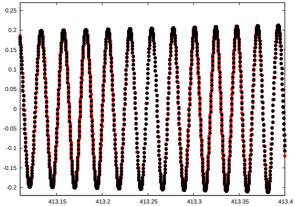
`--waveform TaylorT1threePointFivePN`
Generate PN waveform *on the fly* with the current built-in functions within the inspiral pipeline

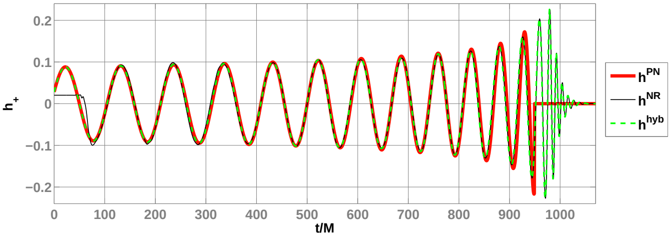
`--waveform NumRel`
Numerically read and inject a TaylorT1 3.5PN waveform (generated with LAL Inspiral package)



Sanity Check (Zoom)



Hybrid PN-NR Waveforms



Phenomenological template family for black-hole coalescence waveforms.
P. Ajith *et al.* Class.Quant.Grav.24:S689-S700,2007

Matched post-Newtonian and numerical relativity waveforms

$$h^{\mathrm{hyb}}_{+,\times}(t,\boldsymbol{\nu}) \equiv \begin{cases} h^{\mathrm{PN}}_{+,\times}(t,\boldsymbol{\mu}_0) & \text{if } t < t_1 \\ a_0\,\tau\,h^{\mathrm{NR}}_{+,\times}(t,\boldsymbol{\nu}) + (1-\tau)\,h^{\mathrm{PN}}_{+,\times}(t,\boldsymbol{\mu}_0) & \text{if } t_1 \le t < t_2 \\ a_0\,h^{\mathrm{NR}}_{+,\times}(t,\boldsymbol{\nu}) & \text{if } t_2 \le t \end{cases}$$

Best matched TaylorT1 3.5PN with AEI-CCT equal-mass ($\eta = 0.25$)* NR waves

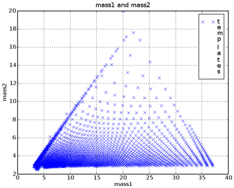
$^{*}\eta \equiv \frac{m_1 m_2}{(m_1+m_2)^2}$

Injecting and recovering hybrid waveforms (preliminary)

All simulated signals injected & recovered using **LAL/LALApps inspiral codes**.

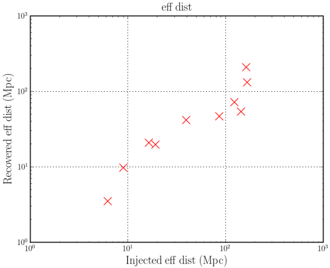
10 Hybrid injections:

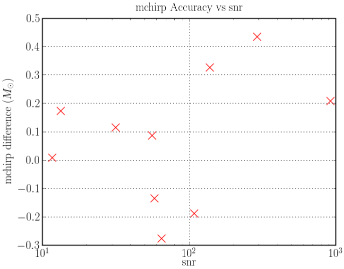
- $3M_\odot$ < $\text{mass}_{1,2}$ < $35M_\odot$
- $6\ M_\odot$ < totalMass < $40M_\odot$
- 1Mpc < dist < 100Mpc
- Over 2048 s of **white gaussian simulated noise** in order to test parameter recovery.
- The injections have been performed forcing `--inject-overhead` and $\iota, \phi = 0$



Taylor T1 2PN template bank (∼ 2100 templates)
$3M_\odot < mass < 35M_\odot$; $totalMass < 40M_\odot$;
$minMatch = 0.97$

Effective distance vs injected distance and m_{chirp} diff vs snr





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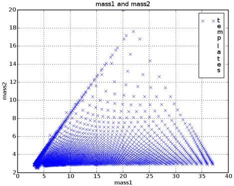

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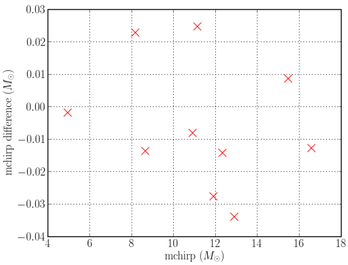
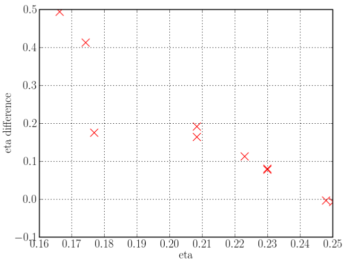
m_{chirp} and eta recovery (FRACTIONAL DIFF)





Summary and Outlook

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- NR and GWDA in an excellent moment to start fruitful collaboration
- Code to allow for injections of numerical data basically written
- Sanity check performed and validity of the method tested
- Some format details still under development (to allow NR data to be stored and read from frame files)

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Outlook

- Systematic analysis of results
- Study of different NR data and PN approximants
- Work in progress!
  $\longrightarrow$ "Data formats for numerical relativity waves". ArXiv:0709.0093 [gr-qc]