Parameter estimation of spinning binary black-hole inspirals using MCMC

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Outline

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Goals of this project

Intermediate goals

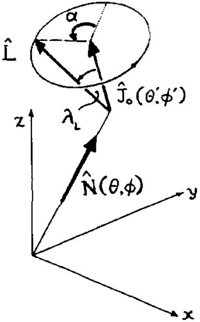
- Show that Markov-Chain Monte Carlo (MCMC) with a large number of parameters (> 10) on LIGO data can be done
- Test MCMC code on software and hardware injections

Final goals

- Do parameter estimation on LIGO/Virgo detection of inspiral
- Use as a follow-up for template-based search to:
  - Confirm spinning inspiral nature of signal
  - Determine physical parameters (masses, spin, position, . . .)
- Provide final stage in automated CBC pipeline
- Learn about compact binaries and their evolution

Spinning BH binaries: Simple waveform

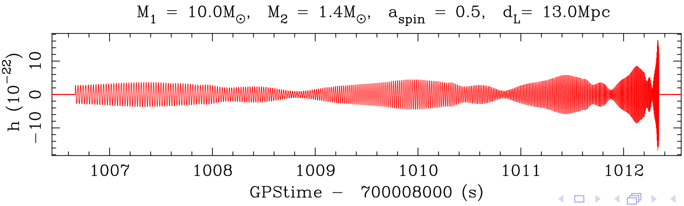
- Röver non-spinning code
- Waveform template:
  - Analytic waveform
  - Restricted 1.5 PN
  - Simple precession
  - 12D parameter set: $\vec{\lambda}$



Apostolatos et al., 1994

Typical data stretch (f_{low} – coalescence):

5.5s, 400 gravitational-wave cycles, 5 precession cycles



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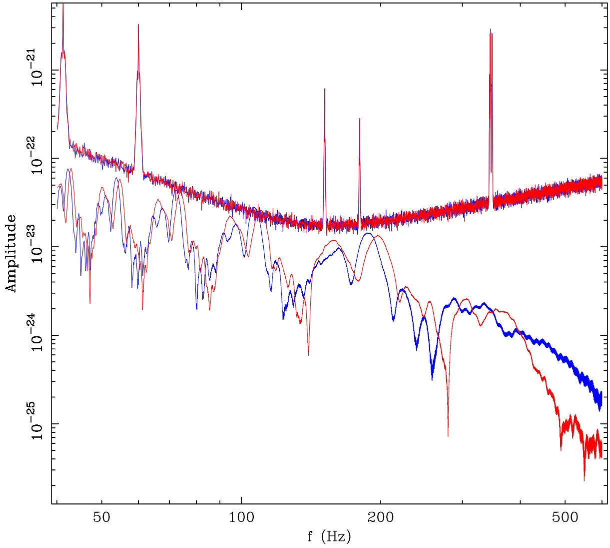
Signal injection into detector noise



- Using 1–2 4-km detectors H1, L1
- Gaussian, stationary noise
- Do 1.5PN software injections
- Retrieve physical parameters with 1.5PN template

Here, ΣSNR = 17

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Compute posterior distribution

- Find posterior density of the model parameters
- Bayesian approach
- The likelihood for each detector i is:

$$L_i(d|\vec{\lambda}) \propto \exp\left(-2\int_0^{\infty} \frac{\left|\tilde{d}(f) - \tilde{m}(\vec{\lambda}, f)\right|^2}{S_n(f)}\, df\right) \propto \exp\left(\frac{\mathrm{SNR}^2}{2}\right)$$

- Coherent network of detectors:
  - $\mathrm{PDF}(\vec{\lambda}) \propto \mathrm{prior}(\vec{\lambda}) \times \prod_i L_i(d|\vec{\lambda})$
- Use Markov-Chain Monte Carlo to sample the posterior

Generating a Markov chain

Basic MCMC scheme

at any point j in the chain with state $\vec{\lambda}_j$, prior $p_j \equiv p(\vec{\lambda}_j)$ and likelihood $L_j \equiv L(d|\vec{\lambda}_j)$:

- propose random jump to new state $\vec{\lambda}_{j+1}$ with p_{j+1} and L_{j+1}
- if $\left(\frac{p_{j+1}}{p_j}\frac{L_{j+1}}{L_j} > \texttt{ran\_uniform}[0,1]\right)$ then
  - **accept** new state $\vec{\lambda}_{j+1}$
- else
  - **reject** new state; $\vec{\lambda}_{j+1} = \vec{\lambda}_j$
- save state $\vec{\lambda}_{j+1}$

MCMC runs

MCMC parameters

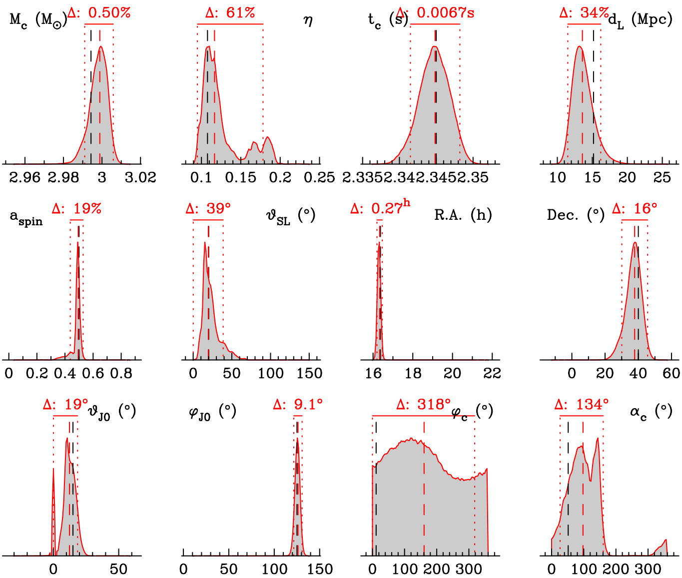
masses: M_c & η, distance: $\log d_L$, time, phase and precession phase at coalescence: t_c, φ_c & α_c, position: R.A. & sin Dec, spin magnitude: a_{spin}, angle between $\vec{S}$ and $\vec{L}$: $\cos\theta_{SL}$, orientation of $\vec{J}_0$: $\sin\theta_{J_0}$ & φ_{J_0}

MCMC set-up

- 5 serial chains per run, starting from the true parameter values
- Chain length: 5×10^6 states, burn-in: 5×10^5 states
- Run time: 10 days on a 2.8 GHz CPU

- Signals injected in simulated noise for H1L1 @ SNR $\approx$17.0
- Fiducial binary: $M_{1,2} = 10 + 1.4\,M_\odot$, $d_L = 16$–21 Mpc
- Spin: $a_{spin} = 0.0, 0.1, 0.5, 0.8$, $\theta_{SL} = 20^\circ, 55^\circ$

Results



Parameters:

- H1 & L1
- $M = 10, 1.4\,M_\odot$
- $d_L = 18.7$ Mpc
- $a_{spin} = 0.5$, $\theta_{SL} = 20°$
- ΣSNR $\approx$ 17.0
- Black dashed line: true value
- Red dashed line: median
- Δ's: 90% probability

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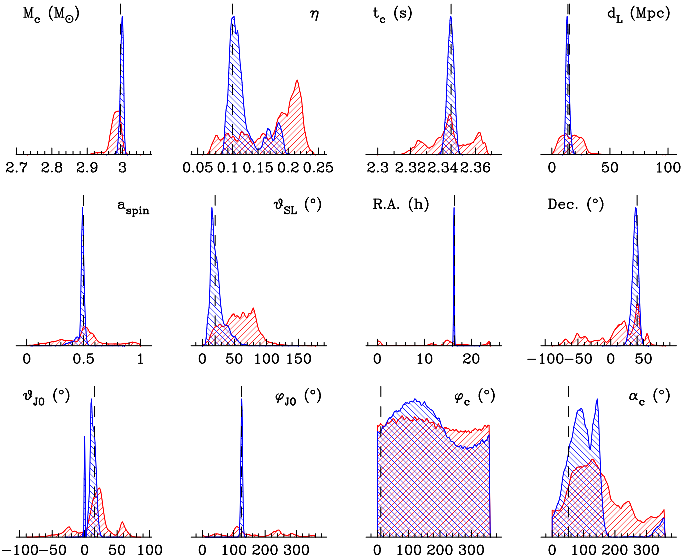
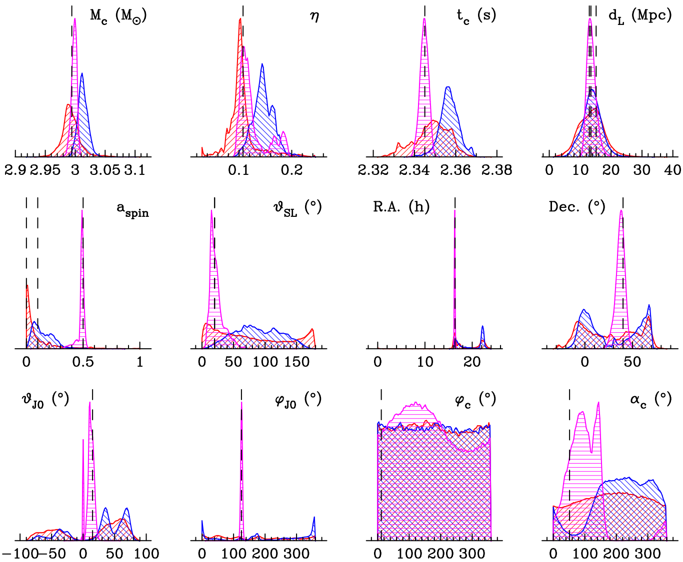
Dependence on the number of detectors



Parameters:

- H1, H1 & L1
- $M = 10, 1.4\, M_\odot$
- $d_L = 18.7\,\mathrm{Mpc}$
- $a_{\mathrm{spin}} = 0.5$
- $\theta_{\mathrm{SL}} = 20^\circ$
- $\Sigma\mathrm{SNR} \approx 12.7, 17.0$
- Dashed lines show true values
- PDFs scaled to surface area

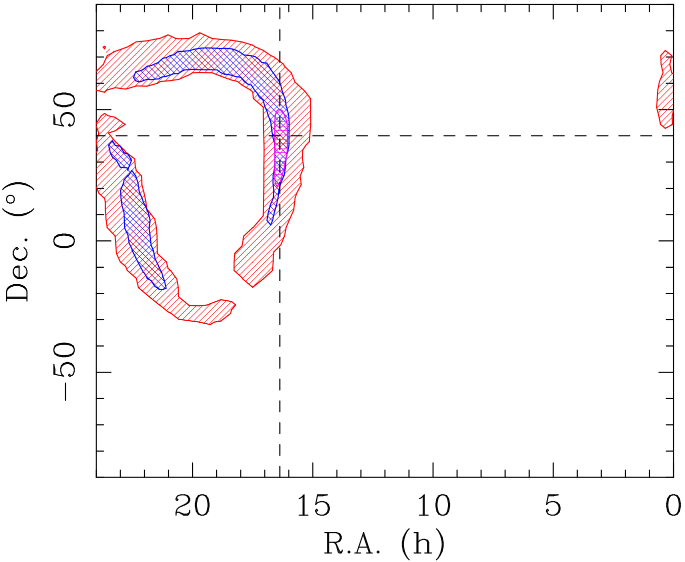
Dependence on spin



Parameters:

- H1 & L1
- $M = 10, 1.4\, M_\odot$
- $d_L \approx 16 - 21$ Mpc
- $a_{spin} = 0.0, 0.1, 0.5$
- $\theta_{SL} = 20°$
- ΣSNR ≈ 17.0
- Dashed lines show true values
- PDFs scaled to surface area

Position in the sky



Parameters:

- H1 & L1
- $M = 10, 1.4\,M_\odot$
- $d_L \approx 16 - 21\,\mathrm{Mpc}$
- $a_{\mathrm{spin}} = 0.0, 0.1, 0.5$
- $\theta_{\mathrm{SL}} = 20^\circ$
- $\Sigma\mathrm{SNR} \approx 17.0$
- Dashed lines show true position

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Conclusions

MCMC code:

We have developed an MCMC code that can recover the 12 parameters of a binary inspiral, including the spin

Accuracies:

- Detection with only 2 detectors can produce astronomically relevant information when spin is present, with typical accuracies for low/higher spin:
  - individual masses: $\sim 32\%/39\%$
  - dimensionless spin: $0.17 - 0.18$
  - distance: $\sim 55\%/45\%$
  - sky position: $\sim 25^\circ/7^\circ$
  - binary orientation: $\sim 55^\circ/15^\circ$
  - time of coalescence: 11ms / 6ms
- Combination of the above can lead to association with an electromagnetic detection (*e.g.* gamma-ray burst)