



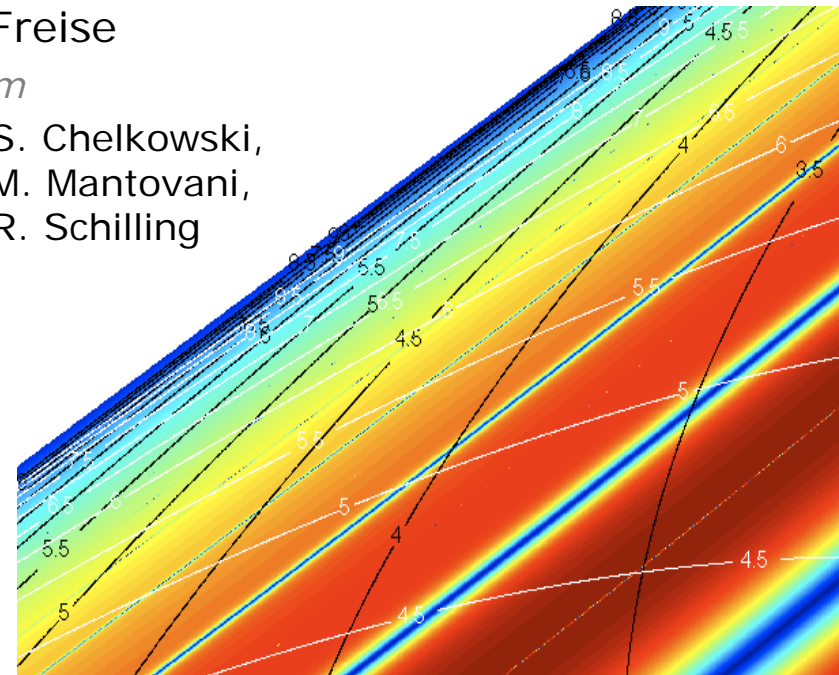
# On Aspects of the Advanced Virgo Arm Cavity Design

Stefan Hild and Andreas Freise

*University of Birmingham*

with input from F. Bondu, A. Brillet, S. Chelkowski,  
J. Degallaix, G. Losurdo, C.N. Man, M. Mantovani,  
J. Marque, G. Mueller, L. Pinard, R. Schilling  
and others ...

LSC-Virgo meeting Amsterdam,  
September 2008



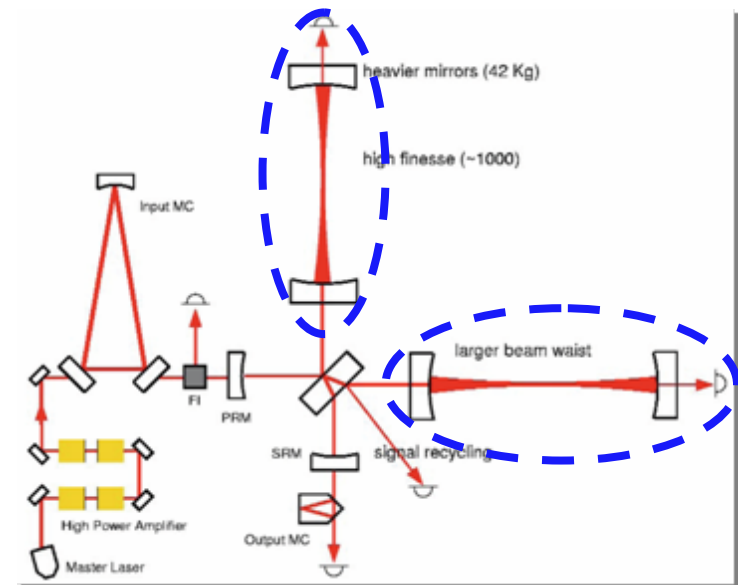


## The Context

- Advanced Virgo design is organized in several subsystems.
- I work on the subsystem: “Optical simulation and Design” (OSD) *subsystem-manager: A. Freise*
- One of the primary tasks of the OSD-subsystem is the **Advanced Virgo Arm Cavity Design.**

# Arm Cavities: The Core of GW Detectors

- In principle arm cavities are rather simple objects, consisting of just **two mirrors** and **a space** between them.
- In reality one has to carefully choose the characteristics of the arm cavities:
  - ➡ Detector sensitivity and bandwidth.
  - ➡ Actual arm cavity design sets constraints for other subsystems.
  - ➡ Design of other subsystems sets constraints for the arm cavity design.





## Characteristics of the Arm Cavity to be chosen

- Beam geometry (waist position)
- Beam size at the test masses
- Radius of curvature of the test masses
- Finesse of the arm cavity

Brief overview  
of the principle  
considerations

- Wedges or Etalon

... going a bit more into detail ...  
(Discussion of various  
requirements and constraints)



## Beam Geometry

- Where to put the waist inside the arm cavity?
  - ➡ Initial detectors have the waist close/at the input mirrors
- Advanced detectors: Move waist towards the cavity center.
  - ➡ Larger beam at input mirror
  - ➡ Lower overall coating Brownian noise
  - ➡ BUT: much larger beams in the central interferometer
    - may need larger BS
    - much larger optics for input and output telescope
    - Non-degenerate recycling cavities might help

# Beam Geometry

- Intuitively one would think the lowest coating noise is achieved when beam waist is at the center of the cavity ( $\Rightarrow$  equal beam size at ITM and ETM),

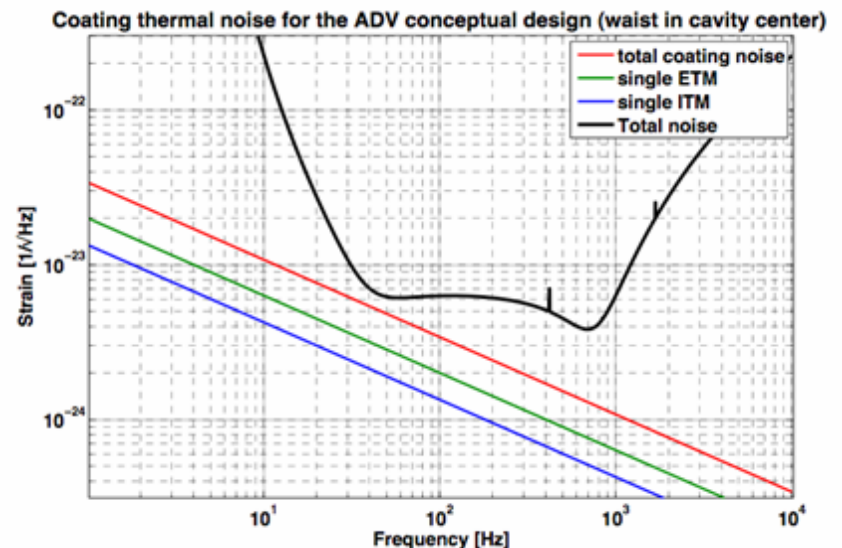
**BUT:**

- Coating noise for ITM and ETM are different, due to their different number of coating layer:

$$\bar{v} = C(S_T + \gamma^{-1}S_S),$$

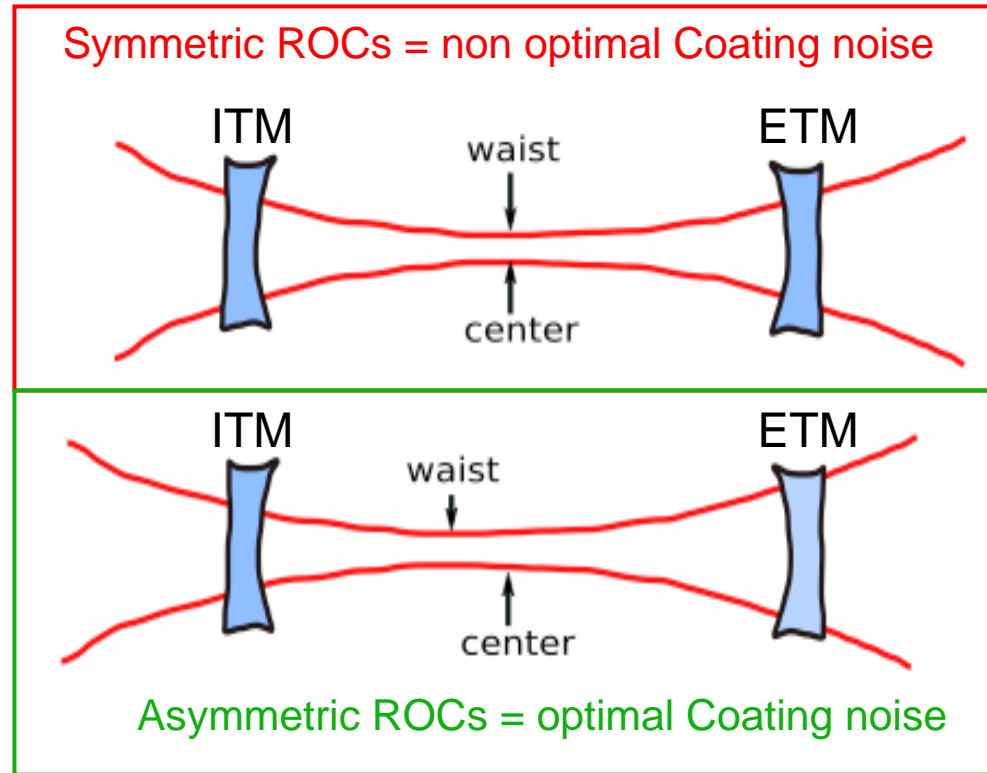
J. Agresti et al (LIGO-P060027-00-Z)

- For equal beam size ETM has higher noise.



## Optimal Waist Position

- In order to minimize the thermal noise we have to make the beam larger on ETM and smaller on ITM.
- Equivalent to moving the waist closer to ITM.
- Nice side effect, the beam in the central central area would be slightly smaller.





## Beam Size

### ➤ Principle Rule:

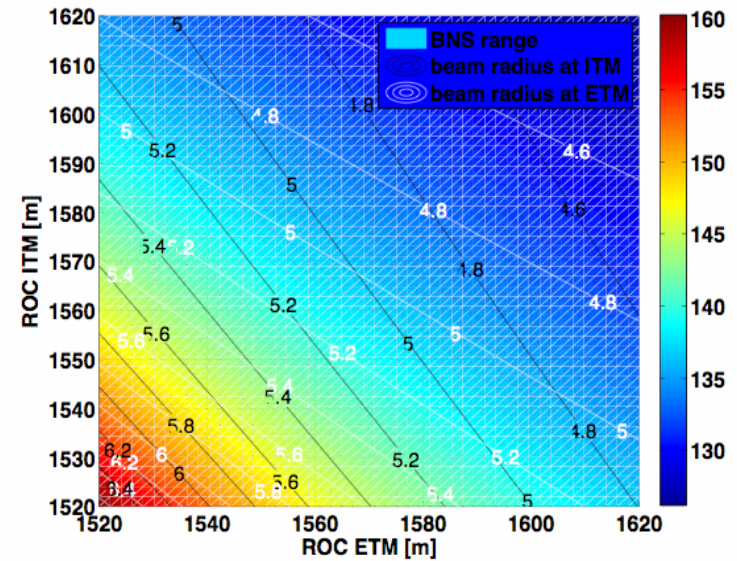
- ➡ The larger the beam the better the detector sensitivity
- ➡ Larger beams make nearly everything else more complicated / more expensive.

### ➤ Advantages of large beams:

- ➡ Reduced thermal noise of test masses (especially coating Brownian)
- ➡ Slightly reduced contribution from residual gas pressure
- ➡ Reduced thermal lensing

### ➤ Disadvantages of large beams:

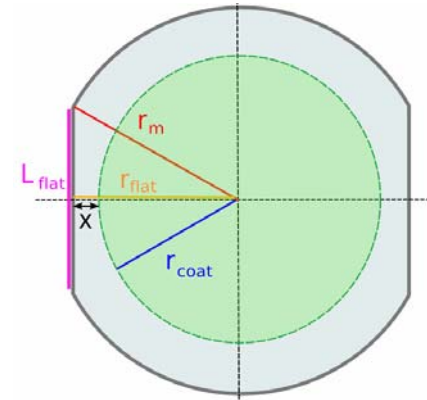
- ➡ Higher clipping losses
- ➡ Larger test masses (especially BS, because of 45deg angle)
- ➡ Larger apertures are required (vacuum system, actuators, etc)
- ➡ Large telescopes (input, output, pick-off beams)
- ➡ More sensitive to ROC deviations





## How to decide on Beam Size ?

- Order of constraints:
  1. Mirror weight (from suspension)
  2. Aspect ratio of mirror
  3. Coating size
  4. Choose affordable losses
  
- Final decision needs to trade off:
  - Detector sensitivity
  - Clipping losses inside the arm cavity (mirror/coating size)
  - Clipping losses inside recycling cavities (actuator geometry, BS size)
  - Scattered light noise contribution of the clipped light
  - Cavity stability (see following slides)
  
- In the end we will probably choose a beam radius ( $1/e^2$  in power) of about 5.5 to 6.5cm.



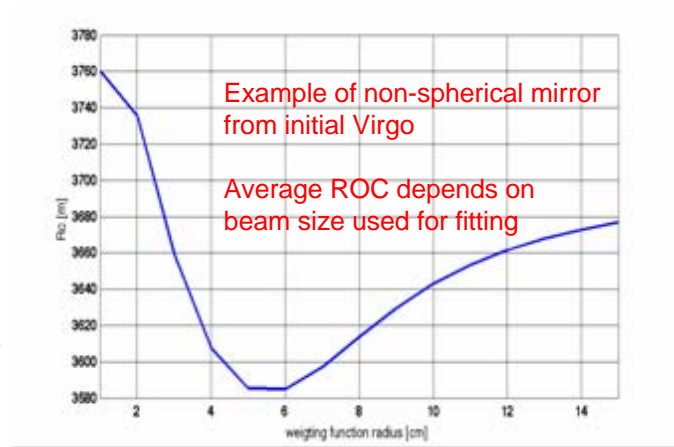
More detail in  
[Hild et al: VIR-038B-08](#)



# Cavity Stability and Choice of ROCs

- ROCs and beam size are connected.
- We want ROCs that give stable cavity:
  - ➔ Account for potential manufacturing accuracy

- AdVirgo example:  $L = 3000\text{m}$ , beam radius at ITM and ETM =  $6\text{cm}$  => ROCs of  $1531\text{m}$  are required.
- Deviation of only a few ten meters can make cavity unstable.
- Additional problem: polished spheres are not spherical.



- ➔ Avoid resonance of higher order optical modes
  - Use mode-non-degeneracy as figure of merit

# Cavity Stability and Choice of ROCs

➤ Definition of mode-non-degeneracy:

- Gouy-phase shift of mode of order  $l+m$ :

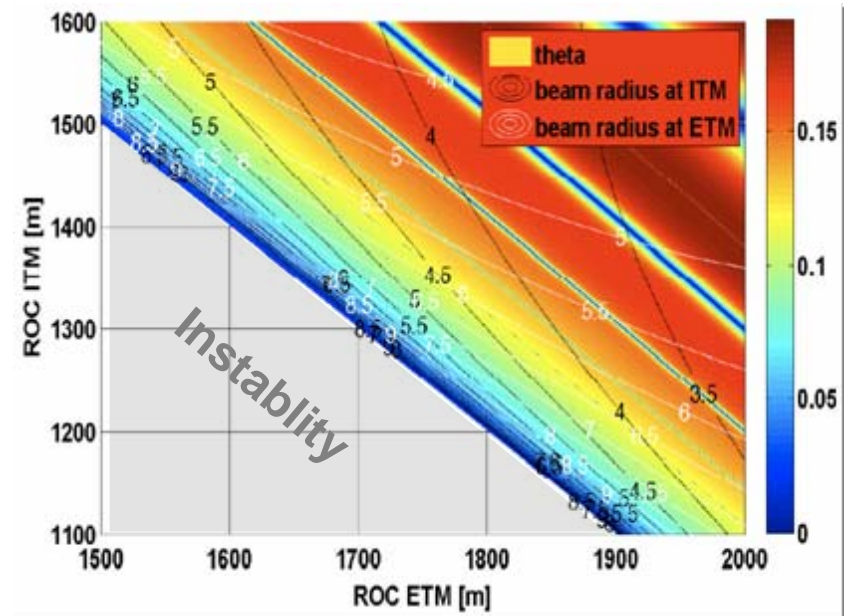
$$\phi_{l+m} = (l+m) \frac{1}{\pi} \arccos \sqrt{\left(1 - \frac{L}{R_{c,i}}\right) \left(1 - \frac{L}{R_{c,e}}\right)}$$

- Mode-non-degeneracy for a single mode is:

$$\Psi_{l+m}(L, R_{c,i}, R_{c,e}) = |\phi_{l+m} - \text{round}(\phi_{l+m})|$$

- Figure of merit for combining all modes up to the order  $N$ :

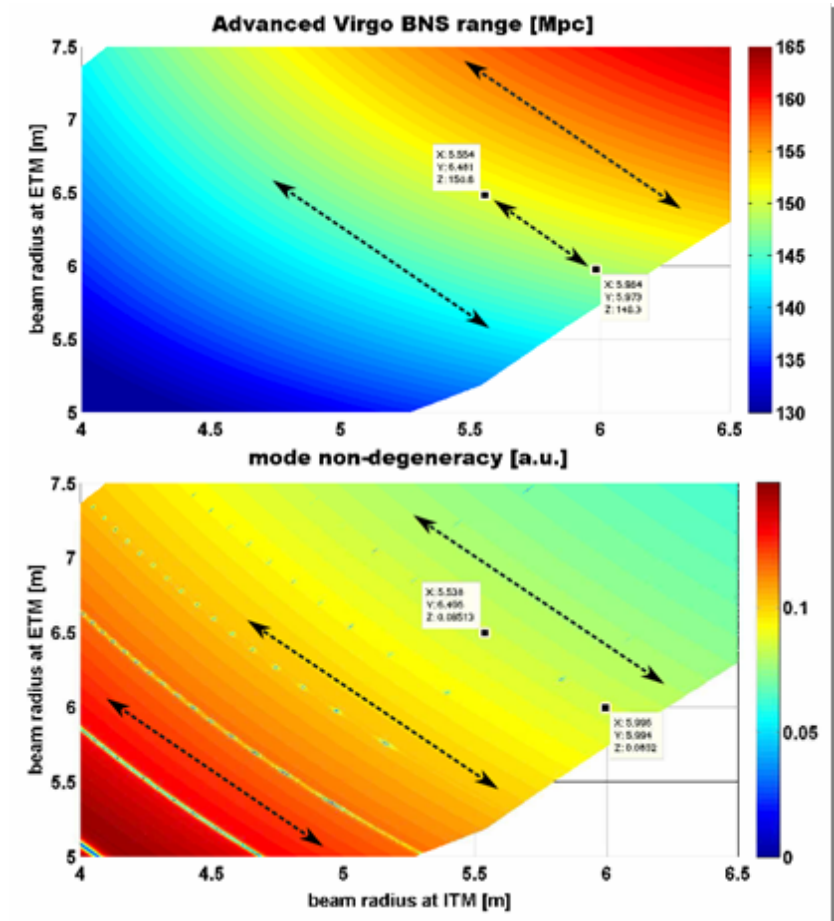
$$\Theta_N(L, R_{c,i}, R_{c,e}) = \frac{1}{\sqrt{\sum_{k=1}^N \frac{1}{\Psi_k^2} \frac{1}{k!}}}$$





## Choice of ROCs/beam size: Sensitivity vs Mode-non-degeneracy

- In general mode-non-degeneracy and sensitivity go opposite.
- Asymmetric ROCs are beneficial:
  - ➡ For identical mode-non-degeneracy (parallel to arrows in lower plot) we can increase sensitivity (parallel to arrow in upper plot) by going towards the upper left corner.
  - ➡ This means making beam larger on ETM and smaller on ITM.





## Arm Cavity Finesse

- Advantages of higher finesse:
  - ➡ Reduced noise coupling from MICH to DARM
  - ➡ Less thermal load in central interferometer
  
- Disadvantages of higher finesse:
  - ➡ More sensitive to losses inside the arm cavities
  - ➡ Increased coating Brownian noise of the ITM (due to more required coating layers)
  - ➡ Power problems (parametric instabilities)?
  
- In the end we will probably go for a finesse between 400 and 700.



## Characteristics of the Arm Cavity to be chosen

- Beam geometry (waist position)
- Beam size at the test masses
- Radius of curvature of the test masses
- Finesse of the arm cavity

Brief overview  
of the principle  
considerations

- Wedges or Etalon

... going a bit more into detail ...  
(Discussion of various  
requirements and constraints)

# Wedges vs Etalon

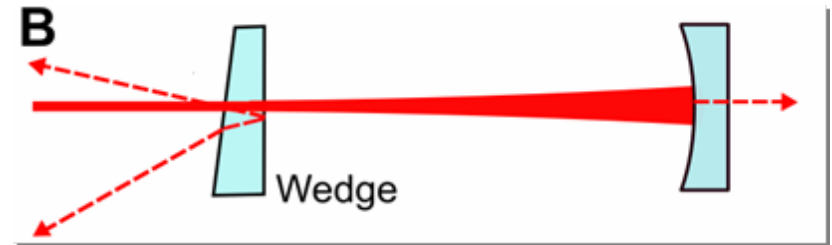
## Input mirror etalon:

- Initial Virgo has no wedges in the input mirrors
- The etalon effect could be used for adjusting the cavity finesse (compensating for differential losses)
- If etalon effect is not controlled it might cause problems



## Input mirror with wedge:

- Used by initial LIGO
- Reflected beams from AR coating can be separated from main beam => pick-off beams provide additional ports for generation of control signals.
- No etalon effect available.





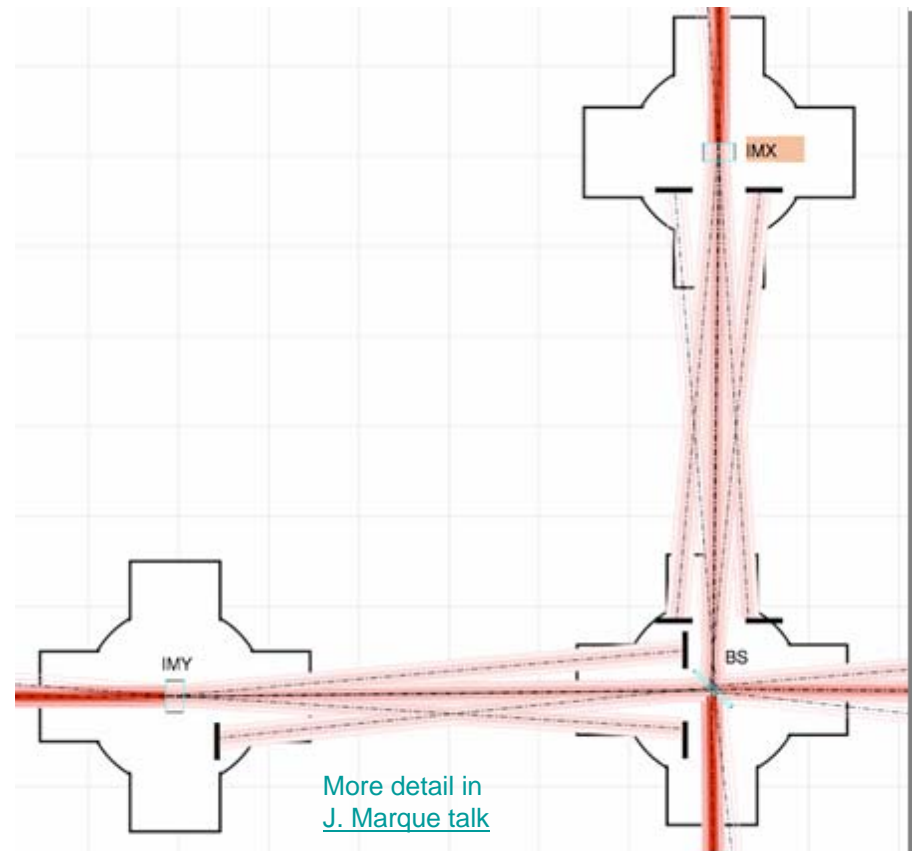
## Possible design option: Wedges at input mirrors and etalon effect at end mirrors



- Wedge at input mirrors:
  - ➡ Allows for additional pick-off beams
- Use etalon effect at end test mass
  - ➡ Tune etalon to balance arms => reduce noise couplings => might speed up commissioning
  - ➡ Tune etalon to change readout quadrature in DC-readout.
  - ➡ Replace AR-coating by a coating of about 10% reflectivity.
  - ➡ Ideally use a curved back surface (same curvature as front).

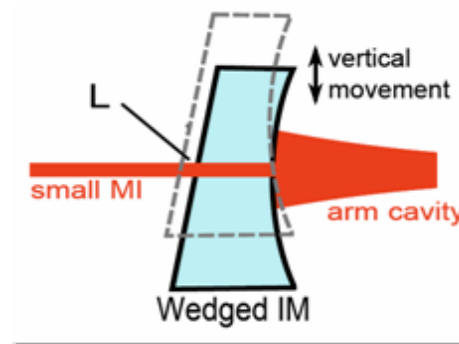
## Wedges at Input Mirrors

- Need a wedge large enough to separate beams within about 5 meter (distance ITM to BS).
- For 6cm beam radius a wedge of about 1.5 deg is required.
- High hardware impact (larger vacuum tube in central IFO, more optical elements)



# Differential Arm Length Noise from vertical Movement of wedged Input Mirrors

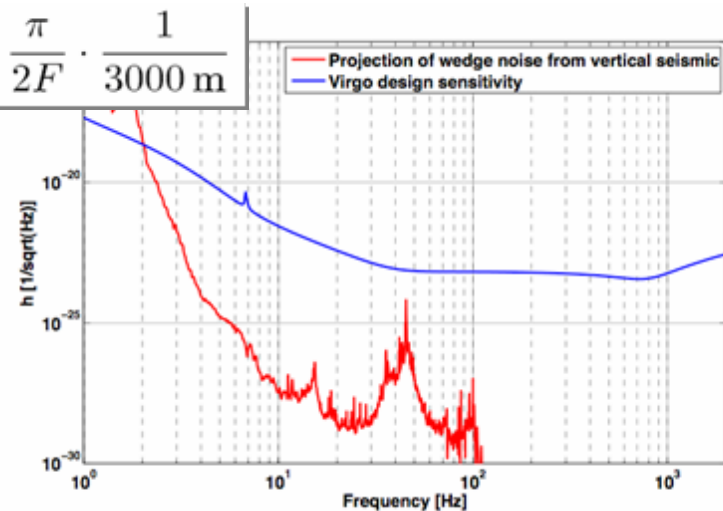
- Lateral movement of a wedged mirror cause length sensing noise.
- Need to do a projection of seismic noise to DARM:



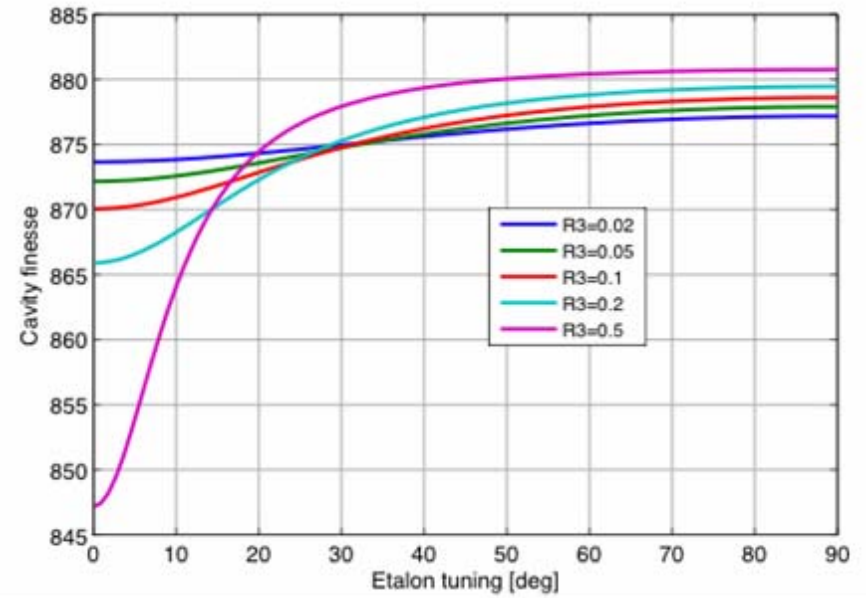
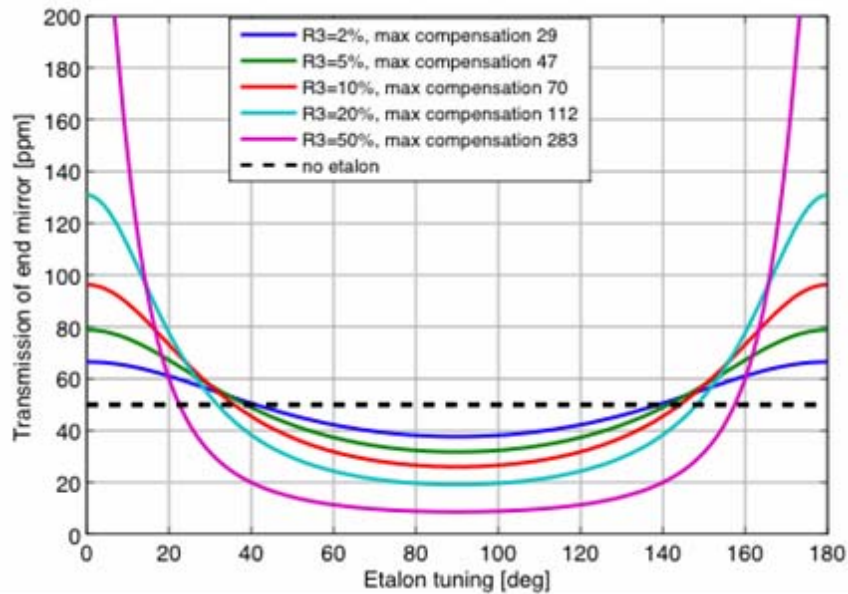
More detail in  
[Hild et al: VIR-037A-08](#)

$$h_{\text{wedge}} = 2 \cdot \tilde{V}_{\text{sei}} \cdot TF_{\text{SA}} \cdot \tan(\alpha) \cdot (n_{\text{sub}} - 1) \cdot \frac{\pi}{2F} \cdot \frac{1}{3000 \text{ m}}$$

- Not limiting within the detection band.
- Please note: No actuation noise considered.



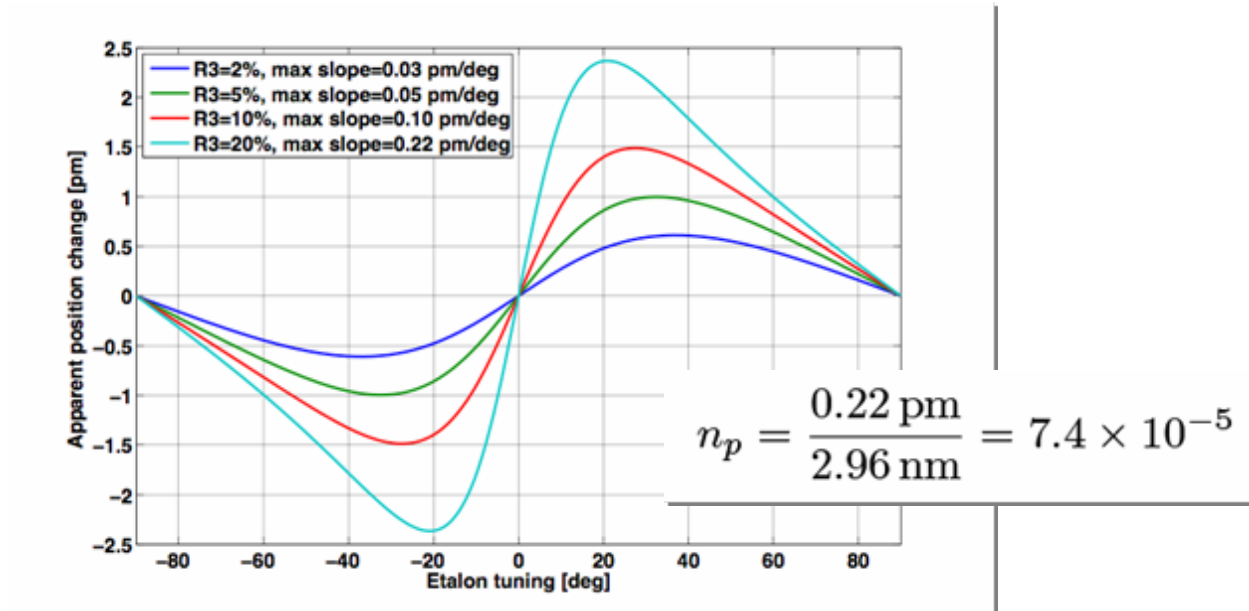
# Balancing Range due to Etalon Effekt



- Examples of figures of merit:
  - ➡ Transmittance of end mirror (etalon)
  - ➡ Finesse of arm cavity



## Etalon changes Optical Phase



- When changing the etalon tuning the optical-phase changes as well. (noise!)
- The two etalon surfaces build a compound mirror, whose apparent position depends on the etalon tuning.



# Requirement for Temperature Stability of Etalon Substrate

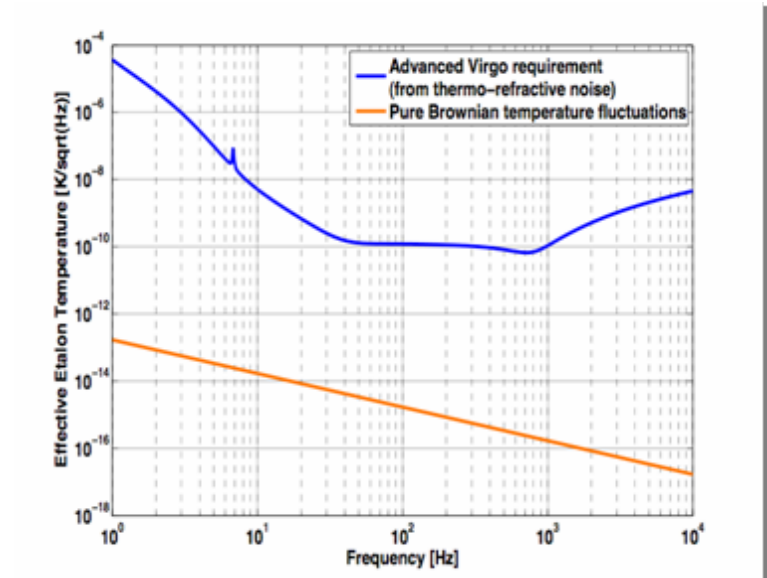
- Certain temperature stability of Etalon substrate required to not spoil Adv sensitivity

$$\tilde{h}_{\text{adv}}(f) = \tilde{T}_{\text{req}}(f) \cdot \frac{dn}{dT} \cdot l_{\text{eta}} \cdot n_p \cdot \frac{1}{L},$$

- Can compare this requirement to substrate thermal noise

$$\tilde{T}_{\text{mirror}}(f) = \sqrt{\frac{4k_b T^2 \kappa}{(\rho C)^2 l_{\text{eta}}} \frac{1}{\pi R_b^4 (2\pi f)^2}},$$

- Not limiting.
- Please note: Did not consider technically driven temperature fluctuations.



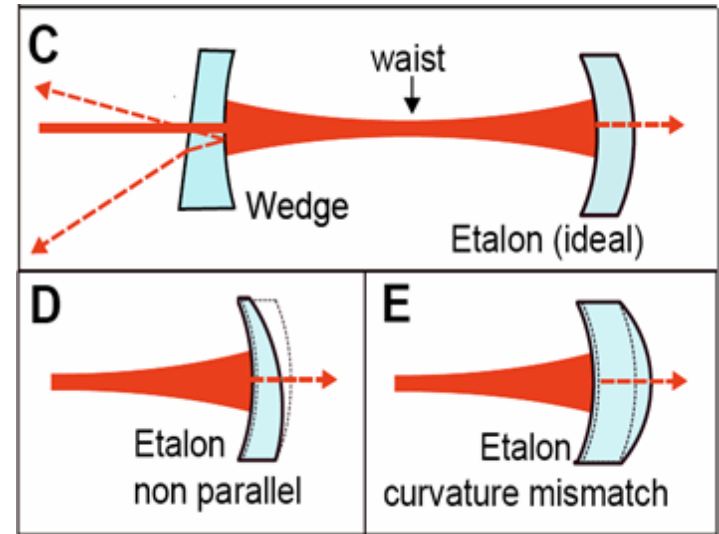
More detail in  
[Hild et al: VIR-058A-08](#)

# Optical Design: Check System Integrity for Deviations from Specs

- A deviation in the **relative misalignment (parallelism)** and **relative curvature** of the two etalon surfaces:

- Imperfect wave front overlap...
- Reduces tuning range ...
- Beam shape distortions ...

- Two methods for analysis:
  - FFT based code (Waveprop)
  - Coupling coefficients







# FFT-simulation of a Non-Perfect Etalon

➤ Using R. Schilling's WaveProp, <http://www.rzg.mpg.de/~ros/WaveProp/>

➤ Cross checking with DarkF. [DarkFstatus\\_08\\_03\\_2006.ppt](#)

➤ Parameters:

- ➡ Field: 256x256
- ➡ Computing 3000 roundtrips
- ➡ End mirror front:
  - 50ppm transmission
- ➡ End mirror back:
  - Varying three parameters
  - **Reflectance**
  - **Misalignment** (parallelism)
  - **Curvature mismatch**

```

23 | Set parameters:
24 | sog = -400 | size of the grid in one dimension [m]
25 | lambda = 1.064e-6 | light wavelength [m]
26 | dist = 3000 | mirror distance [m]
27 | w1 = .03517543 | radius of initial beam [m]
28 | m1Kdx = 0.175 | radius of cavity input mirror M1 [1/m]
29 | m1Kc = -1./1910 | curvature of cavity input mirror [1/m]
30 | m1R = sqrt(0.9929) | amplitude reflectance of cavity input mirror
31 | m2Fr = m1
32 | m2FrKr = sqrt(0.99995)
33 | m2FrKn2 = 1.44963 | refractive index of end mirror substrate
34 | m2Ft = m2Fr
35 | m2bKdx = 0.175 | radius of rear surface of M2 [1/m]
36 | m2bKc = 0. | curvature of rear surface of M2 [1/m]
37 | m2bKr = sqrt(0.20) | amplitude reflectance of rear surface of M2
38 | m2bKn1 = m2FrKn2 | refractive index of end mirror substrate
39
40 | call wp_init(nx,sog,lambda,flw='m') | initialize WaveProp
41
42 | call wp_setup(m1) | set up cavity input mirror
43 | call wp_setup(m2Fr) | set up cavity end mirror (front surf.)
44 | call wp_setup(m2Ft,'tF') | set up cavity end mirror (back surf.)
45 | call wp_setup(m2b) | set up cavity end mirror (back surf.)
46 | call wp_hgmodes(psi_in,w1,-m1Kc,pw=1-0.99295) | generate initial field
47
48 | phi=0
49 | do (iph=0:nph) | do nph phase values for substrate
50 | phi=phi+(360./nph)*deg | next phase for substrate cavity
51 | xp(iph)=iph*(360./nph) | build array representing an X axis
52 | call wp_setup(psi_co) | set up and clear cavity field
53 | call wp_setup(psi_su,m2FrKn2) | set up and clear substrate field
54 | do (irt=0:nrt) | do nrt round trips
55 | X(irt)=exp(i*irt) | build array representing an X axis
56 | call wp_reflect(psi_co,m1) | reflect at rear mirror
57 | call wp_interfere(psi_in,psi_co,psi_co,'c',pw_c=pw(irt)) | constr. interf.
58 | call wp_propagate(psi_co,dist) | propagate to far mirror
59 | call wp_transmit(psi_co,m2Ft,psi_out=psi_ou) | emit through M2
60 | call wp_reflect(psi_su,m2Fr,'b') | reflect @ rear of front surf.
61 | call wp_interfere(psi_ou,psi_su,psi_su) | interference for su cavity
62 | call wp_reflect(psi_su,m2b,phi-phi) | reflect at back surface of M2
63 | call wp_transmit(psi_su,m2Ft,'b',psi_out=psi_ou) | emit through M2
64 | call wp_reflect(psi_co,m2Fr) | reflect at front surface of M2
65 | call wp_interfere(psi_ou,psi_co,psi_co) | interf. of co with su field
66 | call wp_propagate(psi_co,dist) | propagate to rear mirror
67 | and do
68 | pwc(iph)=pw(nrt) | store final cavity power
69 | end do
70
71 | call wp_plot3d(psi_in,4,'Input field',zoom=2.) | plot input field
72

```

## Analytic Approximations using Higher-Order Modes

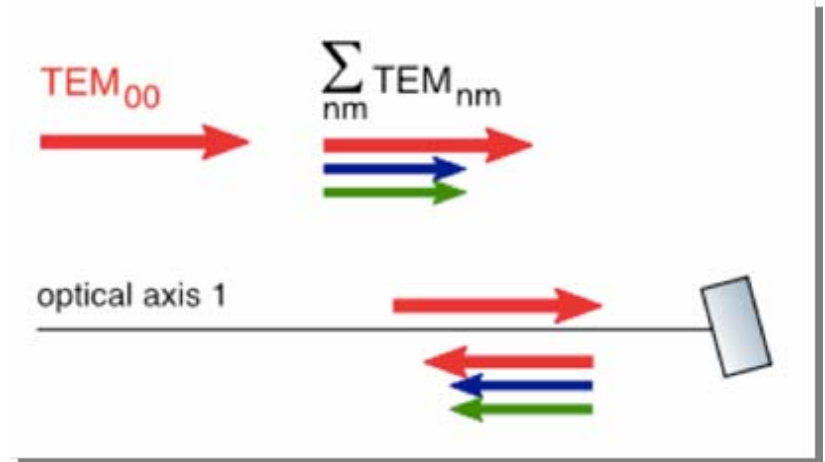
- Reflection at a (slightly) misaligned component can be characterised by scattering into higher order TEM modes
- This model is valid for misalignments below half the diffraction angle (paraxial approximation)
- The amplitude in the outgoing fields is given by **coupling coefficients**  $k_{nmnm}$

$$a_{nm} = k_{00nm} a_{00}$$

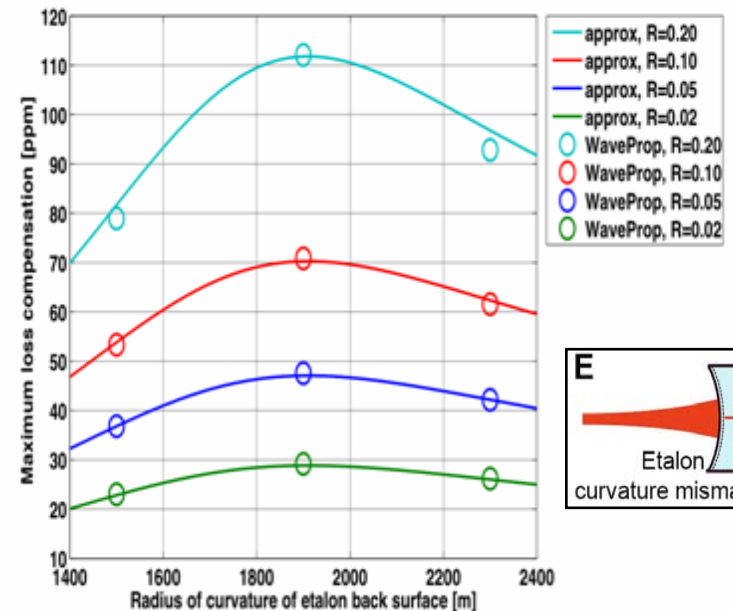
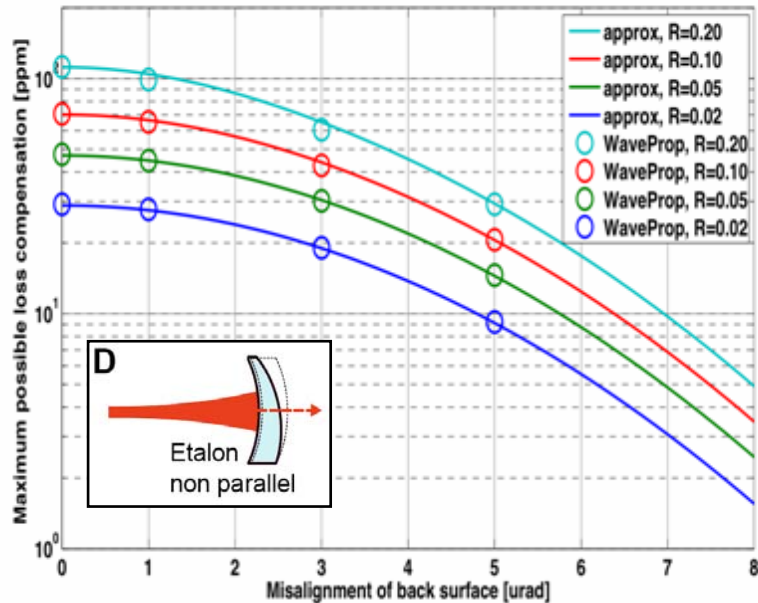
- For small misalignments the **coupling coefficients**  $k_{nmnm}$  can be approximated. The amount of light which remains in a  $TEM_{00}$  mode is given by:

$$k_{0000}(q, \gamma) = \exp\left(-\frac{\pi|q|^2 \sin^2(2\gamma)}{2\lambda \Im(q)}\right)$$

( $q$  is the Gaussian beam parameter of the light at the mirror)



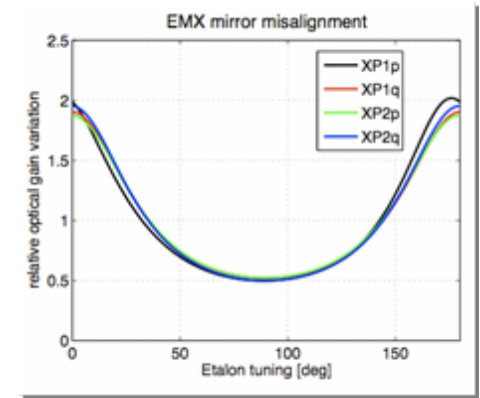
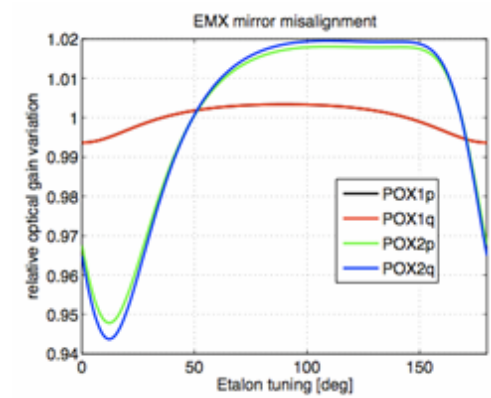
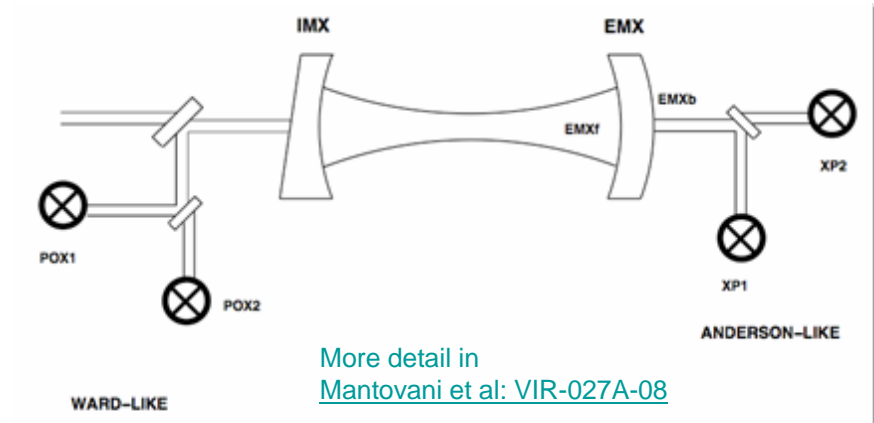
# Tuning Range of imperfect Etalon



- Requirements for Etalon manufacturing accuracy:
  - Parallelism better than a few urad.
  - ROC deviation: uncritical

# Influence of Etalon Tuning to other Subsystems: Example Alignment

- Evaluation of global alignment sensing and control.
- Simulated Ward-technique and Anderson-technique.
- For perfect etalon: No surprises.
- For non perfect etalon:
  - ➡ Coupling of etalon rear surface misalignment is 4 to 5 orders below etalon front surface misalignment.
  - ➡ Amount of first order optical modes inside the arm cavity origination from etalon imperfections is found to be negligible.





## Summary

- Presented overview of how to choose the main characteristics of the Advanced Virgo arm cavity.
- More detailed analysis for wedges vs etalon:
  - ➡ Presented potential design (wedged ITM, etalon at ETM)
  - ➡ Presented requirements for:
    - Seismic isolation (wedge)
    - Temperature stability of etalon (optical phase noise)
    - Manufacturing accuracy of the etalon
  - ➡ Checked for negative implications of other subsystems:
    - Alignment sensing and control
- Publication on the arxiv:  
[Hild et al: "Using the etalon effect for in-situ balancing of the Advanced Virgo arm cavities"](#)  
[arXiv:0807.2045](#)

*Thanks for the helpful  
input from the LSC review !*