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Displacement-noise-free resonant speed meter for gravitational-wave detection

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Outline

Idea of displacement-noise-free measurement

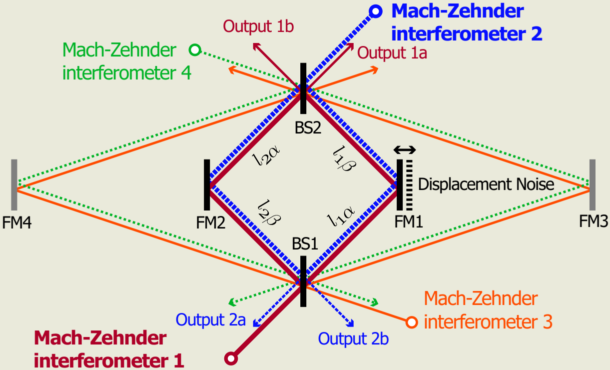
Idea

In 2004 S. Kawamura and Y. Chen put forward an idea of so called displacement-noise-free interferometer (DFI) which is free from displacement noise of the test masses as well as from optical laser noise[a]. They consider several variants.

[a] S. Kawamura, Y. Chen, PRL, **93**, 211103 (2004),
Y. Chen, S. Kawamura, PRL, **96**, 231102 (2006),
Y. Chen, A. Pai, K. Somiya, S. Kawamura, S. Sato, K. Kokeyama, R. Ward, K. Goda and E. Mikhailov, PRL, **97**, 151103 (2007),
S. Sato, K. Kokeyama, R. Ward, S. Kawamura, Y. Chen, A. Pai, K. Somiya, PRL, **98**, 141101 (2007)

2D Mach-Zehnder scheme

Kawamura, Chen *et al* [PRL,**97**, 151103(2006)]:

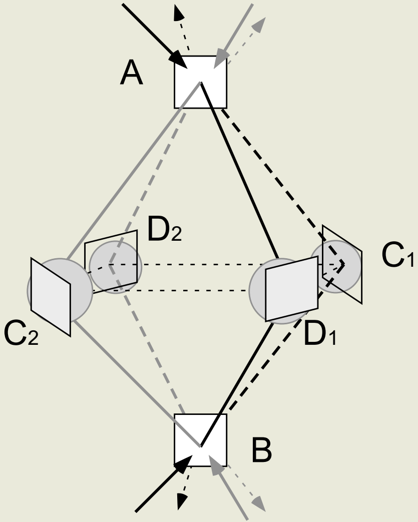


Manipulating by outputs one **can exclude information on displacement of each 6 mirrors** and keep information on GW signal.

The "price": in long wave approximation ($L \ll \lambda_{GW}$) the displacement-noise-free response signal decreases as $(\Omega_{\rm GW} L/c)^3$.

3D Mach-Zehnder scheme

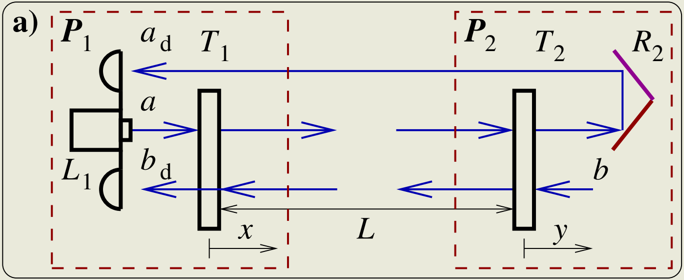
Kawamura, Chen *et al* [PRL,**97**, 151103(2006)]:



The "price": in long wave approximation ($L \ll \lambda_{GW}$) the displacement-noise-free response signal decreases as $(\Omega_{\rm GW} L/c)^2$.

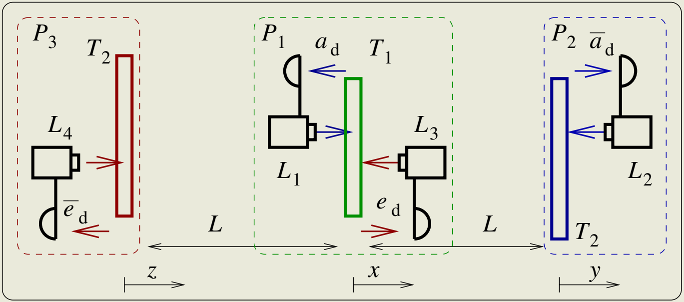
Double-pumped Fabry-Perot cavity

Tarabrin, Rakhubovsky and Vyatchanin (arXiv: 0804.395[qr-qc]; 0807.3824 [gr-qc]) proposed double-pumped Fabry-Perot cavity



Constructing linear combination of the reflection-output and transmission-output signals experimenter can exclude displacement noise of **one** (P_2) platform.

Two double-pumped Fabry-Perot cavities



Constructing linear combination of the reflection-output and transmission-output signals

experimenter can exclude displacement noise of **all** platforms.

The "price": in long wave approximation ($L \ll \lambda_{GW}$) the displacement-noise-free response signal decreases as $(\Omega_{\rm GW} L/c)^2$.

The aim of this presentation

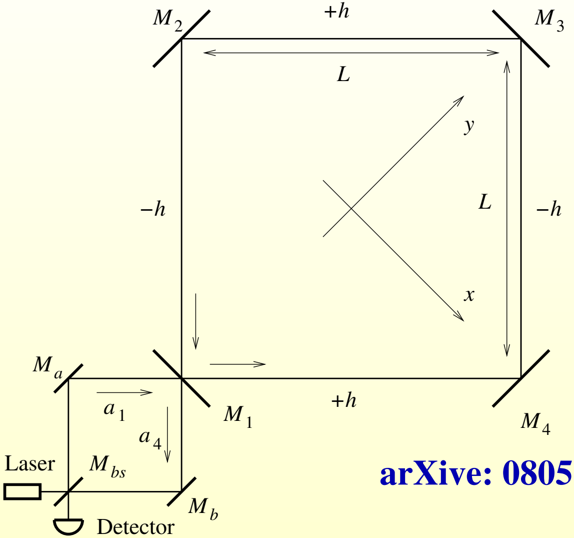
We propose displacement- and laser- noise free speed meter.

The "price" is modest: in long wave approximation ($L \ll \lambda_{GW}$) the displacement-noise-free response (gravitational) signal decreases only by factor

$$\sim (\Omega_{\rm GW} L/c)^{\mathbf{1}}$$

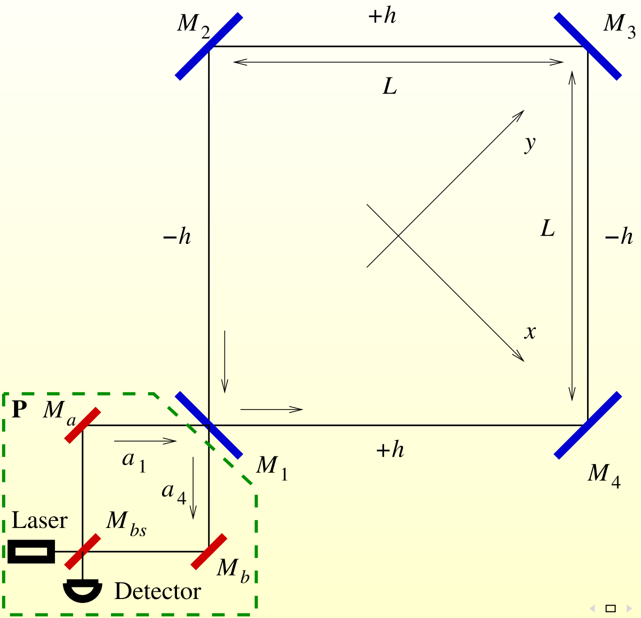
as compared with conventional Fabry-Perot (or LIGO) interferometer .

Speed meter proposed by Nishizawa, Kawamura, Sakagami

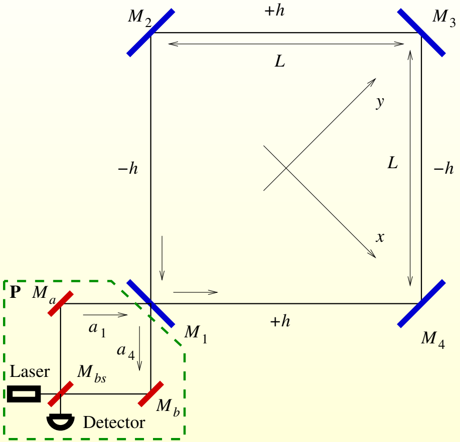


arXive: 0805.3750 [gr–qc]

Simplified configuration of speed meter (Not DFI yet!)



Assumptions of simplified configuration



No optical losses. Laser, detector, beam splitter M_{bs} with 50% transmissivity and completely reflective mirrors M_a, M_b are rigidly mounted on platform P which, in turn, can move as a free mass along axis y

Further assumptions

Mirrors $M_1,\ M_2,\ M_3.\ M_4$ can move as free masses. We consider only displacements $y_1,\ y_3$ of mirrors $M_1,\ M_3$, displacement $x_2,\ x_4$ of mirrors $M_2,\ M_4$ and displacement y_P of platform P.

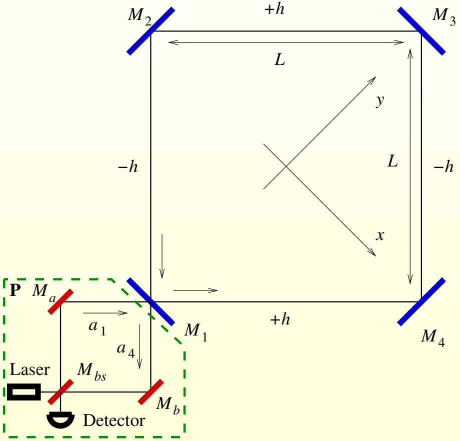
The interferometer lies in the plane perpendicular to direction of GW and directions of light propagations coincide with the GW principal axes.

Due to small size of platform P the phase advance of light propagating between mirrors of platform does not depend on frequency. We neglect GW effect on light propagating between mirrors of platform P and take into account only GW influence on wave propagating inside the ring cavity.

The detector is mounted on the same platform as the laser and, hence, we can work in inertial laboratory frame [a]. Moreover, we can use transverse-traceless (TT) gauge considering GW action as effective modulation of refractive index $(1 + h(t)/2)$ by weak GW perturbation metric $h(t)$.

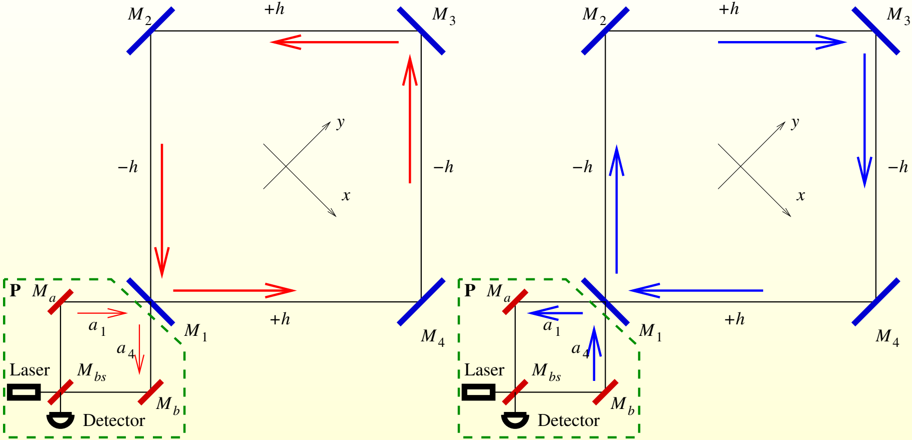
[a] S.P. Tarabrin, arXiv: gr-qc/0804.4292.

Cancellation of laser noise: dark port tuning



If mirrors and platform are at rest and GW is absent the light from the laser after circulating inside the interferometer returns to the laser port as well as the vacuum fluctuations wave from detector return to detector port.

Cancellation of displacements $y_1,\ y_3,\ y_P$



The symmetry is a crucial feature of this interferometer. Indeed, due to symmetry the clockwise and counterclockwise waves contain *identical* information on displacements $y_P,\ y_1,\ y_3$ which cancel after recombination on the beam splitter M_{bs} (actually after subtraction).

Speed meter

This interferometer is a kind of speed meter

Though displacements $y_P,\ y_1,\ y_3$ cancel in output signal d_p, the information on $x_2,\ x_4$ presents:

$$d_p \sim \big(\mathbf{x_4(t-\tau)} - \mathbf{x_4(t-3\tau)} + \mathbf{x_2(t-\tau)} - \mathbf{x_2(t-3\tau)}\big) \simeq$$
$$\simeq \tau\big(v_2(t-2\tau) + v_4(t-2\tau)\big), \quad \boxed{\tau = L/c}$$

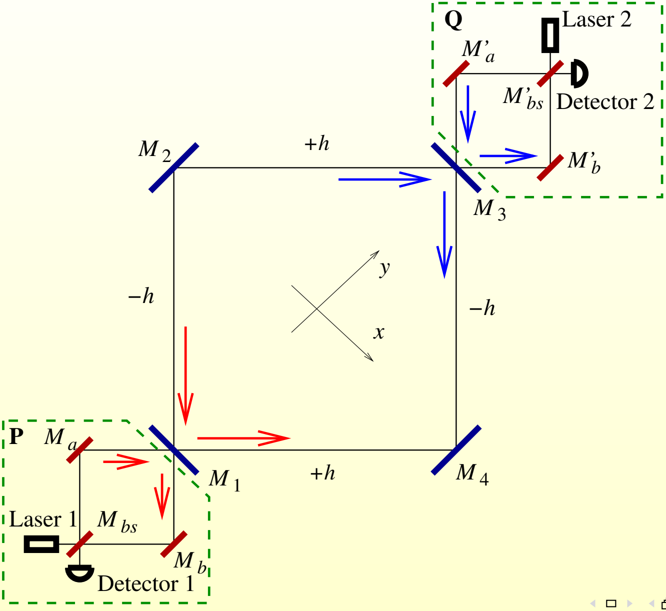
GW (signal) response is smaller by factor $\Omega\tau$

GW response of speed meter is also smaller if compared with GW response of conventional Fabry-Perot cavity by a factor (in spectral domain)

$$\frac{4\sin^2\frac{\Omega\tau}{2}\,\cos\Omega\tau}{\Omega\tau} \simeq \Omega\tau\,.$$

In time domain GW signal is proportional to $\tau(dh(t))/dt$ instead of $h(t)$ in conventional GW detector.

Displacement noise free speed meter



Assumptions

Laser 2, detector 2 with reflective mirrors M'_a, M'_b and beam splitter M'_{bs} are rigidly mounted on platform Q, which can move as a free mass along y axis.

Power and frequency of laser 2 are the same as those of laser 1.

The beams of laser 1 and 2 have orthogonal polarization.

The mirror M_1 has transmissivity T for waves emitted by laser 1 but it is completely refractive for waves emitted by laser 2.

The mirror M_3 has transmissivity T for waves emitted by laser 2 but is completely refractive for waves emitted by laser 1.

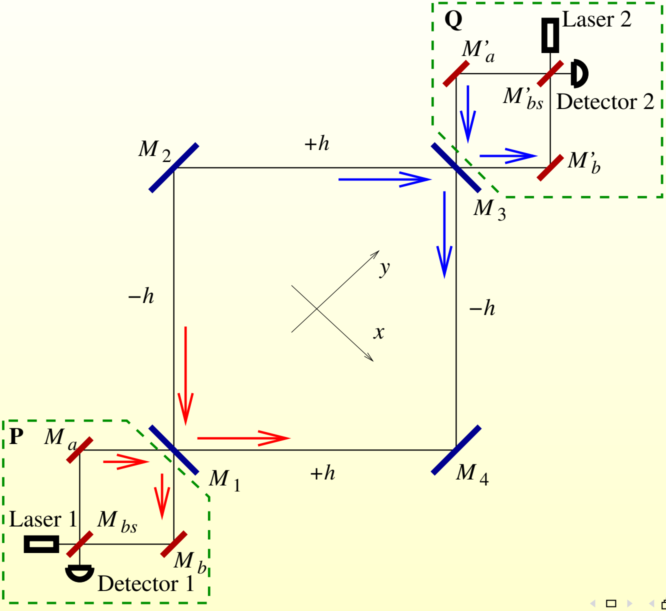
So light emitted by laser 1 completely returns through mirror M_1 to platform P as well as light emitted by laser 2 completely returns through mirror M_3 to platform Q.

Symmetry

The symmetry of interferometer relatively axis x plays a key role. Indeed, the wave from laser 1 circulating **counterclockwise** in ring cavity and wave from laser 2 circulating **clockwise** contain the *same* information on positions of mirrors M_1 — M_4. The same is valid for clockwise wave from laser 1 and counterclockwise wave from laser 2.

Hence, after recombination on beam splitters the wave in detector 1 port and wave in detector 2 port have to contain *identical* information on displacement x_2 and x_4 (information on positions y_1, y_3, as well as on platform positions y_P (y_Q), cancels after recombination on beamsplitters

Symmetry (cont.)



Symmetry (cont.)

However, the contribution of GW into phase advances of counterclockwise wave from laser 1 and of clockwise wave from laser 2 have **opposite** signs.
Indeed, for counterclockwise wave from laser 1 GW dimensionless metric h has first positive sign (between mirrors M_1 and M_4) then negative sign and so on, but for clockwise wave from laser 2 GW metric h has first negative sign (between mirrors M_3 and M_4) then positive sign and so on.

The same is valid for clockwise wave from laser 1 and counterclockwise wave from laser 2.

As a result: the output waves on platforms P and Q contain *identical* terms proportional to displacements x_2, x_4.
But tems proportional to GW action have *opposite* signs.

Complete exclusion of displacement noise

Now we just subtract the responces of detector 1 and detector 2:

$$C = \frac{d_P(\Omega) - d_Q(\Omega)}{\sqrt{2}} \simeq$$

$$\simeq \frac{a_{\text{vac}}(\Omega) - b_{\text{vac}}(\Omega)}{i\sqrt{2}}\,\frac{\gamma + i(\delta+\Omega)}{\gamma - i(\delta+\Omega)} +$$

$$+ 2\sqrt{2}\, i\, A\, kLh(\Omega)\left(\frac{\gamma\Omega}{\big(\gamma - i(\delta+\Omega)\big)\big(\gamma - i\delta\big)}\right), \quad \gamma = \frac{T^2}{8},$$

$$\left|C_{\Omega\tau\ll 1}\right| < \left|\frac{a_{\text{vac}}(\Omega) - b_{\text{vac}}(\Omega)}{i\sqrt{2}}\right| + 2\sqrt{2}\, A\, kLh(\Omega).$$

Note that resonance gain is presented both in our DFI speed meter and in conventional Fabry-Perot GW detector. However, the resonance gain is almost compensated by small factor $\Omega\tau$.

Conclusion

Modest decrease of sensitivity

At low frequencies GW response in our DFI turns out to be better (sensitivity decreases by factor $\Omega\tau$ only) than that in Mach-Zehnder-based DFIs or two double pumped Fabry-Perot cavities.

The symmetry

The symmetry plays a key role in analyzed DFI. First, due to the symmetry relatively clockwise and counterclockwise waves we have the opportunity of excluding both laser noise and noise of displacements y_P (y_Q), y_1, y_3 in the output signal in each detector port. Second, the symmetry relatively axis x allows excluding information on displacements x_2, x_4 and converting resonant speed meter into DFI.

Conclusion (cont.)

Vulnurable assumption

Our analysis is based on the statement that we can work in laboratory frame using TT gauge if laser and detector are mounted on the same platform. However, this statement was proved for a single round trip configuration and, strictly speaking, it should be checked independently for analyzed configuration.

Gedanken device

The proposed configuration of DFI is a gedanken (mental) device, however, it may be a promising base candidate for the future configurations of GW detectors with displacement and laser noise exclusion which, in turn, will allow overcoming the Standard Quantum Limit.

Acknowledgments

Many thanks to
V.B. Braginsky,
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A. Nishizawa,
S.P. Tarabrin
for fruitful discussions and critical remarks.

5 Some additional formulas

Output amplitude I

$$d_P(\Omega) = -ia_{\text{vac}}(\Omega)\,\frac{\theta^4 - R}{1 - R\theta^4} + \frac{T^2\,A\,ik\,\theta_0\theta\,(\theta_0^2 - \theta^2)\big(x_4(\Omega) + x_2(\Omega)\big)}{\sqrt{2}(1 - R\theta^4)(1 - R\theta_0^4)} + \tag{1}$$

$$+\frac{4i\,T^2\,A\,\theta_0^2\theta^2\,g(\Omega)}{(1 - R\theta^4)(1 - R\theta_0^4)},$$

$$g(\Omega) \equiv h(\Omega)\,kL\,\frac{\sin^2\frac{\Omega\tau}{2}\,\cos\Omega\tau}{\Omega\tau}, \quad \theta_0 = e^{i\delta\tau}, \quad \theta = e^{i(\delta+\Omega)\tau}, \quad \tau = \frac{L}{c}\,.$$

Here $R = \sqrt{1 - T^2}$ is reflectivity of mirror M_1. We assume that laser frequency ω is detuned by δ from resonance frequency $\omega_0 = \omega - \delta$ of ring interferometer. It is worth repeating that fluctuational amplitude $a(\Omega)$ is absent in $d_P(\Omega)$ — it means laser noise exclusion.