Coherent network searches for gravitational-wave bursts

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Outline

- Gravitational Waves and Detectors
- Gravitational-Wave Bursts (GWBs)
- Standard Formulation of Coherent Analysis for GWBs
  - waveform estimation
  - detection
  - consistency / veto test
  - source location
- History of Coherent Techniques for GWBs
- Recent Advances:
  - "Constrained" likelihoods
  - Improved consistency tests
  - Improved source location
  - Maximum Entropy waveform estimation
- Status of Coherent Searches in LIGO

The Global Network

Several km-scale detectors, bars now in operation

Network gives:

- Detection confidence
- Direction by triangulation
- Waveform extraction

Coherent Analysis for GWBs

“Standard Likelihood” Formulation

The Basic Problem & Solution

- Output of D detectors with noise amplitudes σ_i:
  - Waveforms $h_+(t)$, $h_x(t)$, source direction Ω all unknown.
  - How do we find them?

data | antenna responses (Ω unknown) | GWB (unknown) | noise (unknown)

$$\begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_D \end{bmatrix} = \begin{bmatrix} F_1^+(\Omega)/\sigma_1 & F_1^\times(\Omega)/\sigma_1 \\ F_2^+(\Omega)/\sigma_2 & F_2^\times(\Omega)/\sigma_2 \\ \vdots & \vdots \\ F_D^+(\Omega)/\sigma_D & F_D^\times(\Omega)/\sigma_D \end{bmatrix} \begin{bmatrix} h^+ \\ h^\times \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_D \end{bmatrix}$$

- Approach: Treat Ω and h_+, h_x at each instant of time as independent parameters to be fit by the data.
  - Scan over the sky (Ω).
  - At each sky position construct the least-squares fit to h_+, h_x from the data ("noisy templates").
  - Amplitude of the template and the quality of fit determine if a GWB is detected.

The Modern View

Follow formulation by Rakhmanov, gr-qc/0604005.
Adopt matrix notation:

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_D \end{bmatrix} \qquad \mathbf{F} = \begin{bmatrix} F_1^+(\Omega)/\sigma_1 & F_1^\times(\Omega)/\sigma_1 \\ F_2^+(\Omega)/\sigma_2 & F_2^\times(\Omega)/\sigma_2 \\ \vdots & \vdots \\ F_D^+(\Omega)/\sigma_D & F_D^\times(\Omega)/\sigma_D \end{bmatrix} \qquad \mathbf{h} = \begin{bmatrix} h^+ \\ h^\times \end{bmatrix} \qquad \mathbf{n} = \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_D \end{bmatrix}$$

Dx1 Dx2 2x1 Dx1

Vector of network data values:

$$\mathbf{d} = \mathbf{F}\mathbf{h} + \mathbf{n}$$

Wavefrom Estimation by Least-Squares

For trial sky position Ω, compute $F(\Omega)$ and find best-fit waveform h that minimizes residual $(d\text{-}Fh)^2$. Simple linear problem!

$$\mathbf{0} = \left.\frac{\partial(\mathbf{d}-\mathbf{F}\mathbf{h})^*(\mathbf{d}-\mathbf{F}\mathbf{h})}{\partial \mathbf{h}^*}\right|_{\mathbf{h}=\hat{\mathbf{h}}} = \mathbf{F}^*(\mathbf{d}-\mathbf{F}\hat{\mathbf{h}})$$

* = conjugate transpose

$$\hat{\mathbf{h}} = (\mathbf{F}^*\mathbf{F})^{-1}\mathbf{F}^*\mathbf{d}$$

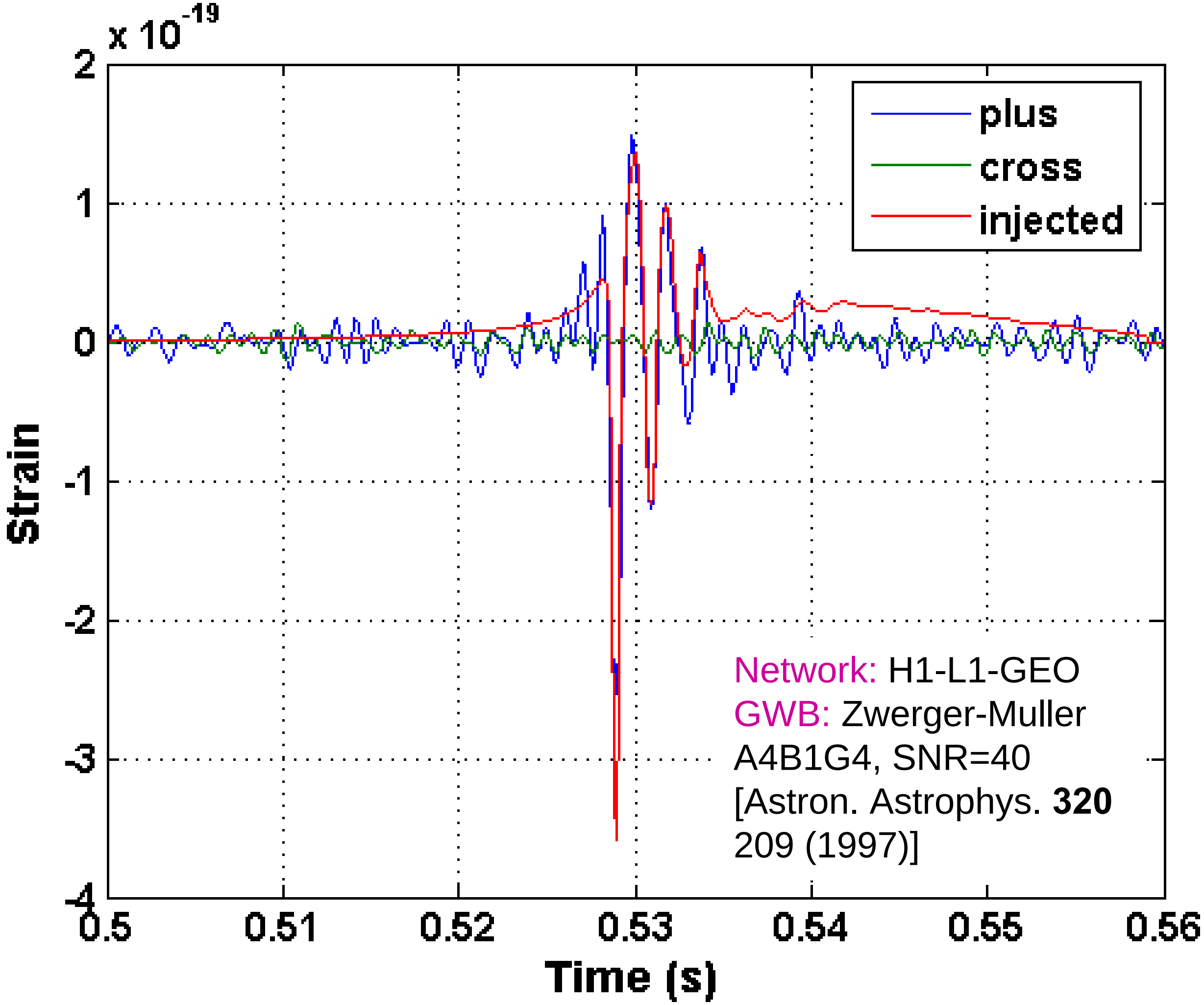
linear best-fit solution for h_+, h_x

"Moore-Penrose inverse" (2xD matrix) :

$$\mathbf{F}_{\mathbf{MP}}^{-1} := (\mathbf{F}^*\mathbf{F})^{-1}\mathbf{F}^* \qquad \mathbf{F}_{\mathbf{MP}}^{-1}\mathbf{F} = \mathbf{I}$$

$$\hat{\mathbf{h}} = \mathbf{F}_{\mathbf{MP}}^{-1}(\Omega)\,\mathbf{d}$$

Example: Supernova GWB Recovery



Recovered signal (blue) is a noisy, band-passed version of injected GWB signal (red)

Injected GWB signal has $h_x = 0$.

Recovered h_x (green) is just noise.

Detection from Likelihood Ratio

Is **d** due to a GWB (**h**) or Gaussian noise (**h**=0)?

Detection statistic: threshold on maximum of the *likelihood ratio*

$$\mathbf{L} \equiv \log \frac{\mathbf{P}(\mathbf{d}\,|\,\hat{\mathbf{h}})}{\mathbf{P}(\mathbf{d}\,|\,\mathbf{0})} = -\frac{1}{2}\underbrace{(\mathbf{d}-\mathbf{F}\hat{\mathbf{h}})^*(\mathbf{d}-\mathbf{F}\hat{\mathbf{h}})} + \frac{1}{2}\underbrace{\mathbf{d}^*\mathbf{d}}$$

"null energy" after subtracting $\hat{\mathbf{h}}$

"total energy" in original data

Maximum value of likelihood is attained for $\mathbf{h} = \hat{\mathbf{h}}$

$$\mathbf{L}_{\mathbf{max}} = \mathbf{L}(\hat{\mathbf{h}}) = -\frac{\mathbf{1}}{\mathbf{2}}\mathbf{d}^*\mathbf{F}\mathbf{F}_{\mathbf{MP}}^{\mathbf{-1}}\mathbf{d} = \frac{\mathbf{1}}{\mathbf{2}}\left(\mathbf{E}_{\mathbf{total}} - \mathbf{E}_{\mathbf{null}}\right)$$

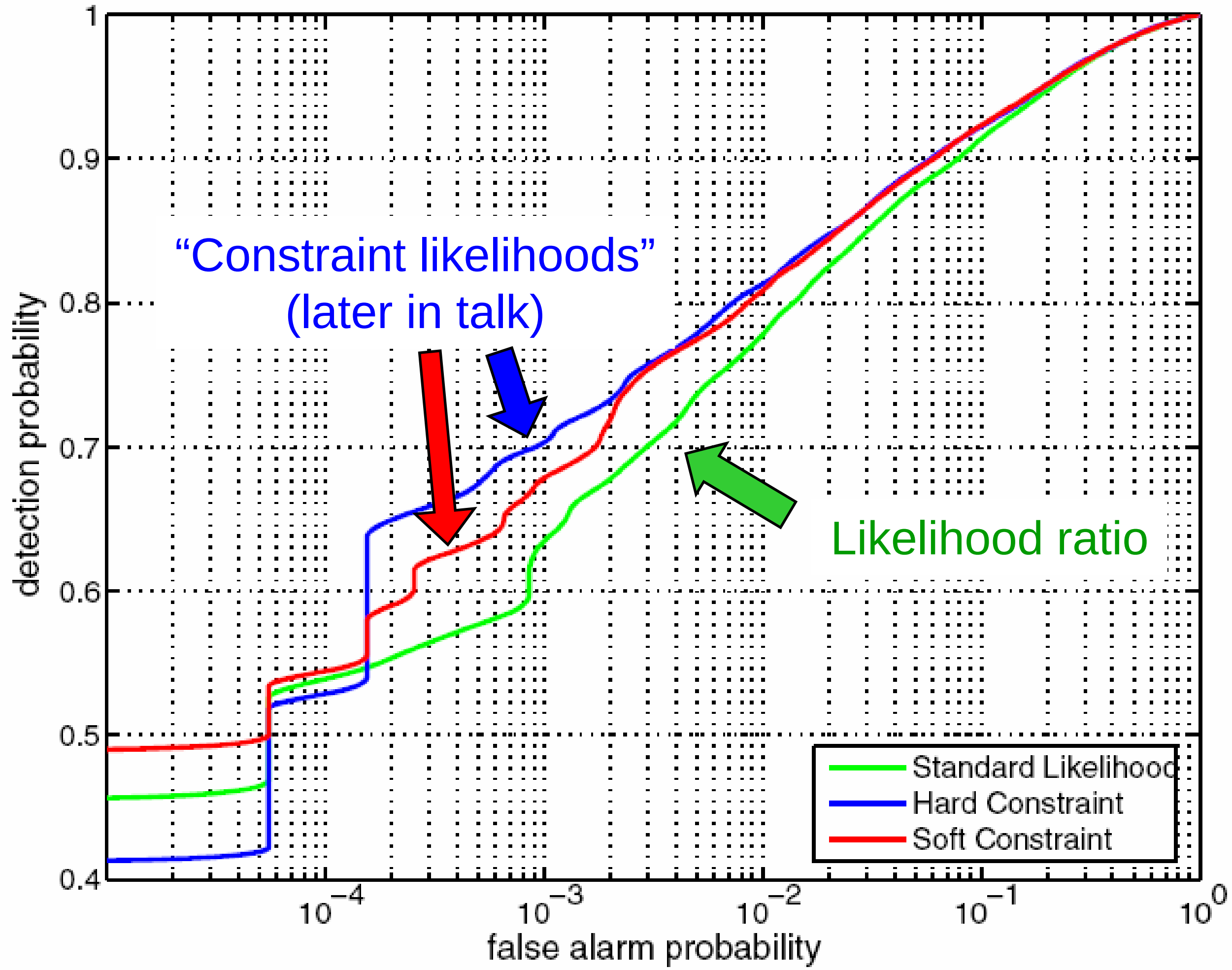
← detection if $\mathbf{L}_{\mathbf{max}}$ > threshold

Example: ROC for Detecting Black-Hole Mergers

From Klimenko et al., PRD **72** 122002 (2005)

Injected signal:
"Lazarus" black-hole merger, SNR=6.9 [Baker et al., PRD **65** 124012 (2002)]

Network:
H1-L1-GEO (white noise approximation)



Consistency Test

Consistency: Is the transient a true GWB or a noise "glitch"? If a GWB, then residual data should be χ^2 distributed:

$$\mathbf{E}_{\mathbf{null}} \equiv \left(\mathbf{d}-\mathbf{F}\hat{\mathbf{h}}\right)^{*}\left(\mathbf{d}-\mathbf{F}\hat{\mathbf{h}}\right) \sim \chi^2\left(\left[\mathbf{D}-\mathbf{2}\right]\mathbf{N}\right)$$

true GWB

$$\mathbf{E}_{\mathbf{null}} \approx \left[\mathbf{D}-\mathbf{2}\right]\mathbf{N}\left[\mathbf{1} \pm O\left(\frac{\mathbf{1}}{\sqrt{\left[\mathbf{D}-\mathbf{2}\right]\mathbf{N}}}\right)\right]$$

If E_{null} >> [D-2] N then reject event as noise "glitch".

Source Location

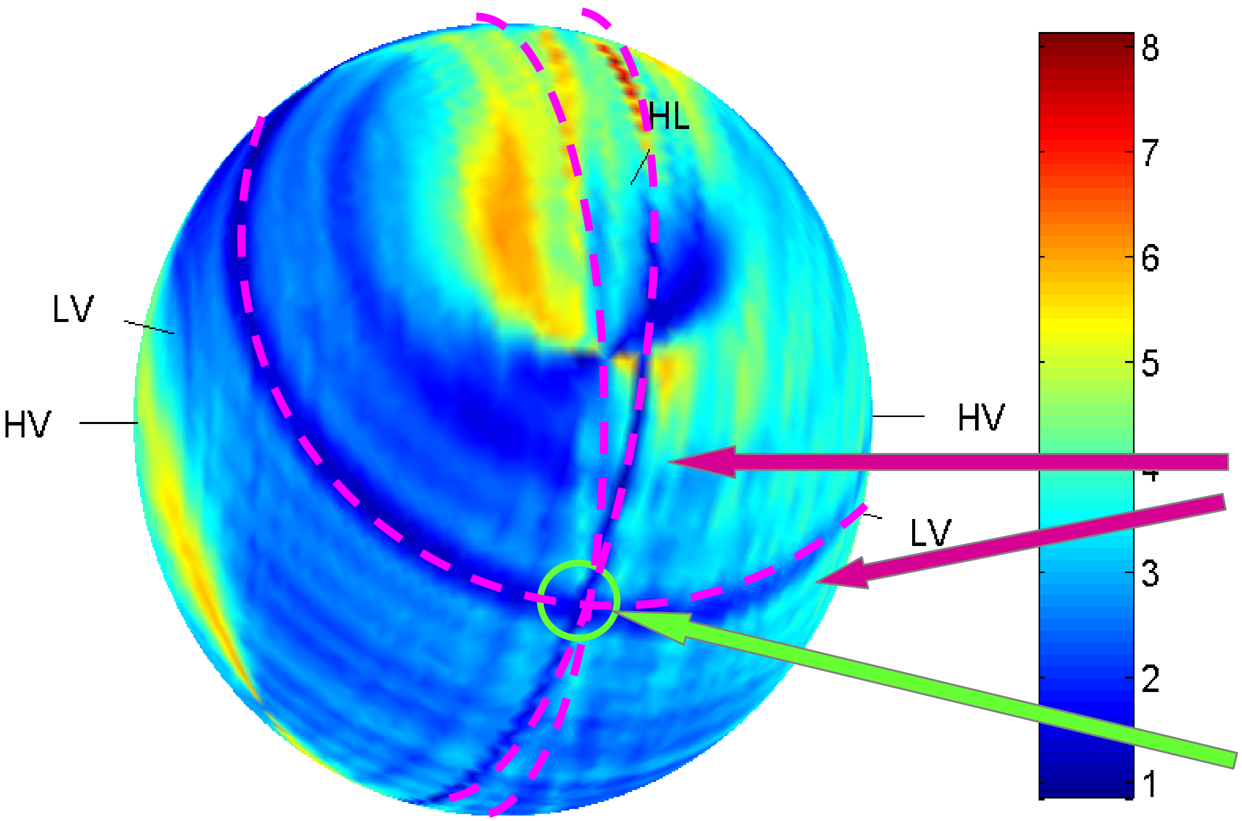
Source location: Usually we do not know the direction of the GWB source a priori (with exceptions: supernova, GRB, …)

Simplest solution: Moore-Penrose inverse depends on sky position:

$$\mathbf{F}_{\mathbf{MP}}^{-1} = \mathbf{F}_{\mathbf{MP}}^{-1}(\mathbf{\Omega})$$

Test over grid of sky positions. Estimate Ω as sky position with lowest χ^2.

Example: Supernova GWB



χ^2 / DOF consistency with a GWB as a function of direction for a simulated supernova (~1 kpc)

Interference fringes from combining signal in two detectors.

True source location:
– intersection of fringes
– χ^2 / DOF ~ 1

GWB: Dimmelmeier et al. A1B3G3 waveform, Astron. Astrophys. 393 523 (2002) , SNR = 20
Network: H1-L1-Virgo, design sensitivity

(Brief) History of Coherent Techniques for GWBs

Ancient History

- Y. Gursel & M. Tinto PRD **40** 3884 (1989)
  - *“Near Optimal Solution to the Inverse Problem for GWBs”*.

- First solution of inverse problem for GWBs.
  - Source location, waveform extraction.
  - For detectors at 3 sites.

- Procedure:
  - Use 2 detectors to estimate GWB waveforms at each point on the sky.
  - Check estimated waveform for consistency with data from 3rd detector (χ^2 test).
  - Symmetrize χ^2 expression over the 3 detectors.
  - Used timing estimates to restrict region of sky to be scanned, find minimum of $\chi^2(\Omega)$.

Medieval Times

- E.E. Flanagan & S.A. Hughes, PRD 57 4566 (1998)
  - *“Measuring gravitational waves from binary black hole coalescences: II. the waves' information and its extraction, with and without templates”*
  - Appendix A (!)

- Discovered maximum-likelihood formulation of detection & inverse problems.
  - Generalized to 3+ detectors, colored noise.
  - Equivalent to Gursel-Tinto for 3 detectors.

Pros and Cons

- This standard approach is known as the "maximum likelihood" or "null stream" formalism.

- Very powerful:
  - Can detect, distinguish from noise, locate, and extract GWB waveform with no *a priori* knowledge of the waveform!

- Standard approach also has significant weaknesses:
  1. Need 3 detector sites at a minimum to fit out 2 waveforms!
  2. Very expensive on data (squanders statistics). Use up 2 detectors just fitting h_+, h_x. *(More on next slide.)*
  3. Can break down at some sky positions & frequencies (**F** becomes singular, so $\mathbf{F}_{\mathbf{MP}}^{\mathbf{-1}}$ does not exist).
  4. Very slow: quadratic in data, must be evaluated separately for each sky position (both unlike linear matched filter).

Cost in Statistical Power compared to Templated Searches

Consistency: Is the transient a true GWB or a noise "glitch"? If a GWB, then residual data should be χ^2 distributed:

$$E_{null} \equiv \frac{1}{2}\left(d - F\hat{h}\right)^* \left(d - F\hat{h}\right) \sim \chi^2\left(\underbrace{[D-2]N}\right)$$

D: number of detectors ~ 3-5

N: number of data samples per detector ~ 100

[D-2]N, *not* DN: Lose 2 data streams to make best-fit h_+, h_x. Very expensive loss of data and loss of statistical power for the consistency test!

Compare to matched filter (h_+, h_x templates known *a priori*); e.g., inspiral search:

- Templates have only a few parameters to be fit with the data.
- E.g.: binary neutron star signal has 2 (mass of each star)
  Consistency test: $\chi^2(DN - 2)$ instead of $\chi^2(DN - 2N)$.

Not a replacement for templated searches (you have a good template)!

Post-Modernism

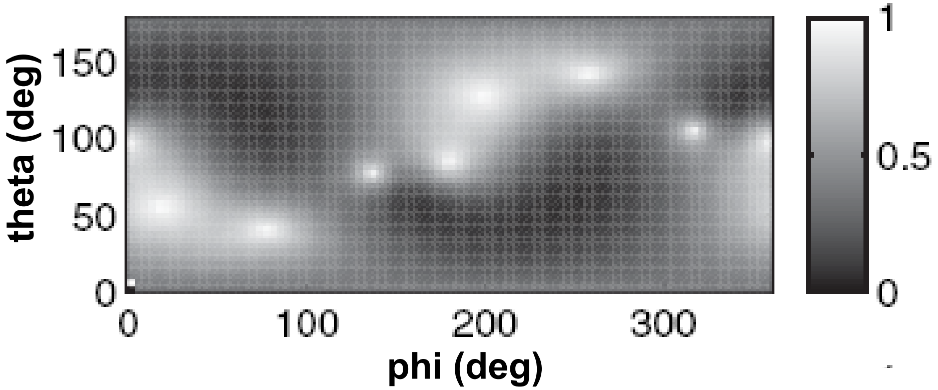
- Over the past year, several LIGO collaboration groups have rediscovered the maximum likelihood formalism and have extended and improved it.
- Advances on all fronts of coherent analyses:
  - detection
  - consistency / veto
  - source location
  - waveform extraction
  - first application to real data
- Also some amelioration of weaknesses on previous slide.
- Rest of talk: walk through examples from each area.

Breakdown of standard approach

Moore-Penrose inverse can be singular or near singular (ill-conditioned) for some sky positions.

- Sky regions where network has poor sensitivity to one or both GW polarizations.

Alignment factor ε for LIGO-GEO-Virgo network



Klimenko et al., PRD **72** 122002 (2005): can choose polarization gauge ("dominant polarization frame") such that

$$\mathbf{F}_{\mathbf{MP}}^{-1} = \frac{1}{g}\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\varepsilon} \end{pmatrix} \mathbf{F}^{*}$$

For some Ω, ε(Ω) << 1. Estimated waveform for that polarization becomes noise dominated:

$$\hat{\mathbf{h}} \equiv \mathbf{F}_{\mathbf{MP}}^{-1}\mathbf{d} = \mathbf{h} + \mathbf{F}_{\mathbf{MP}}^{-1}\mathbf{n}$$

~n/ε for one polarization

Regularization Schemes

- Breakdown of Moore-Penrose inverse explored in several recent papers:
  - Klimenko, Mohanty, Rakhmanov, & Mitselmakher: PRD **72** 122002 (2005), J. Phys. Conf. Ser. **32** 12 (2006), gr-qc/0601076
  - Rakhmanov gr-qc/0604005
  - General problem: for some sky positions $F(\Omega)$ is singular and one or both reconstructed waveforms dominated by noise ("blows up").

- Key advance: Regularization of Moore-Penrose inverse.
  - Effectively impose penalty factor for large values of h_+, h_x.
  - Important side benefit: allows application to 2-detector networks.

One Example: Constraint Likelihood

- Klimenko, Mohanty, Rakhmanov, & Mitselmakher: PRD **72** 122002 (2005), J. Phys. Conf. Ser. **32** 12 (2006).
- In dominant polarization frame:

$$L_{max} = L_{max,1} + L_{max,2} \qquad F_{MP}^{-1} = \frac{1}{g}\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\varepsilon} \end{pmatrix} F^*$$

$$L_{max,1} = \frac{1}{2g}\left(\left[F_1/\sigma\right]^* d\right)^2 \approx \frac{1}{2}\left(g\,h_1^2 + n^2\right)$$

$$L_{max,2} = \frac{1}{\varepsilon}\frac{1}{2g}\left(\left[F_2/\sigma\right]^* d\right)^2 \approx \frac{1}{2}\left(\varepsilon\, g\,h_2^2 + n^2\right)$$

small GWB contribution

full noise contribution

Constraint Likelihood

- Constraint likelihood: Lower weighting of less sensitive polarization “by hand”.

$$L_{max} \Rightarrow L_{max,1}$$ “hard constraint”

$$L_{max} \Rightarrow L_{max,1} + \varepsilon L_{max,2}$$ “soft constraint”

$$L_{max,hard} \approx \frac{1}{2}(g h_1^2 + n^2)$$

zero signal and noise contribution from second polarization

$$L_{max,soft} \approx \frac{1}{2}(g h_1^2 + \varepsilon^2 g h_2^2 + [1+\varepsilon] n^2)$$

very small GWB contribution from second polarization

reduced noise contribution from second polarization

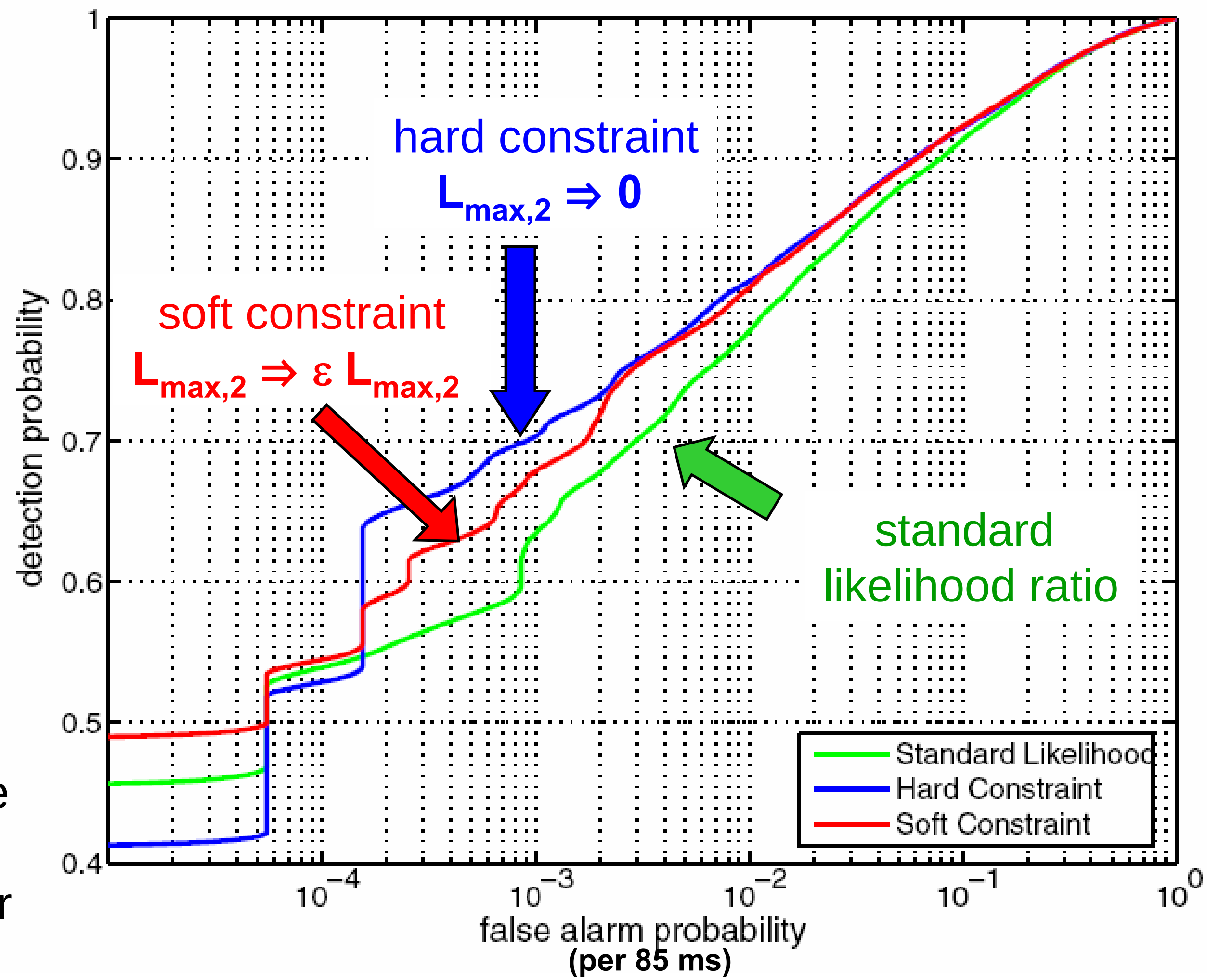
Example: ROC for Detecting Black-Hole Mergers (again)

From Klimenko et al., PRD **72** 122002 (2005)

Injected signal:
"Lazarus" black-hole merger, SNR=6.9 [Baker et al., PRD **65** 124012 (2002)]

Network:
H1-L1-GEO (white noise approximation)

Constraint likelihoods have better detection efficiency than standard likelihood for some false alarm rates.



Improved Consistency / Veto Test

- Real interferometers have populations of glitches, bursts of excess power not due to gravitational waves
  - Can fool null-stream analysis.

- A χ^2 test can be fooled by, e.g., calibration errors (GWB not exactly subtracted out, so $\chi^2 > 1$), or weak glitches (so $\chi^2 \sim 1$).

- Chatterji, Lazzarini, Stein, Sutton, Searle, & Tinto, grqc/0605002 proposed a robust consistency test.
  - Compare energy in residual (the χ^2) to that expected for *uncorrelated* glitches.

How much cancellation is enough?

- Look at energy in residual data:

projection is function of $F^{+,\times}(\Omega)$, $\sigma(f)$

$$\mathbf{E}_{\mathbf{null}} = \left(\mathbf{d} - \mathbf{F}\hat{\mathbf{h}}\right)^2 = \begin{bmatrix} d_1^* & d_2^* & \cdots & d_D^* \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} & & Q_{1D} \\ Q_{21} & Q_{22} & & Q_{2D} \\ & & \ddots & \vdots \\ Q_{D1} & Q_{D2} & \cdots & Q_{DD} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_D \end{bmatrix}$$

cross-correlation terms
“correlated energy”
glitch: ~ 0
GWB: = -1 x incoherent energy

auto-correlation terms
“incoherent energy”

$$\text{cancellation measure} \equiv \frac{\text{null energy } \mathbf{E}_{\mathbf{null}}}{\text{incoherent energy } \mathbf{E}_{\mathbf{inc}}} = \begin{cases} \ll \mathbf{1} & \text{GWB} \\ \sim \mathbf{1} & \text{glitch} \end{cases}$$

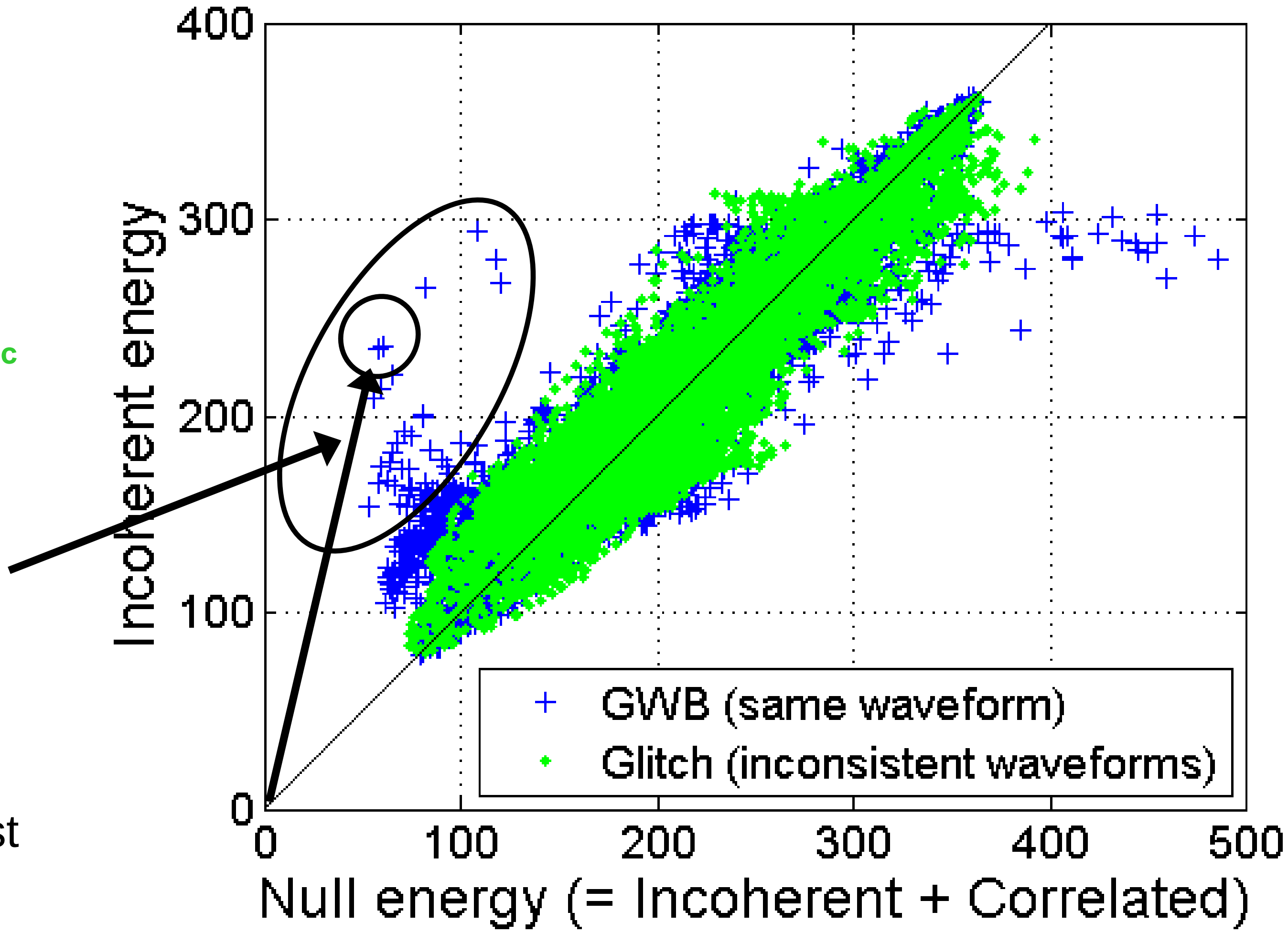
Example: 1 GWB vs. 1 Glitch

One point for each sky position tested (10^4 total).

Glitch lies on diagonal $\mathbf{E_{null} \sim E_{inc}}$ (low correlation)

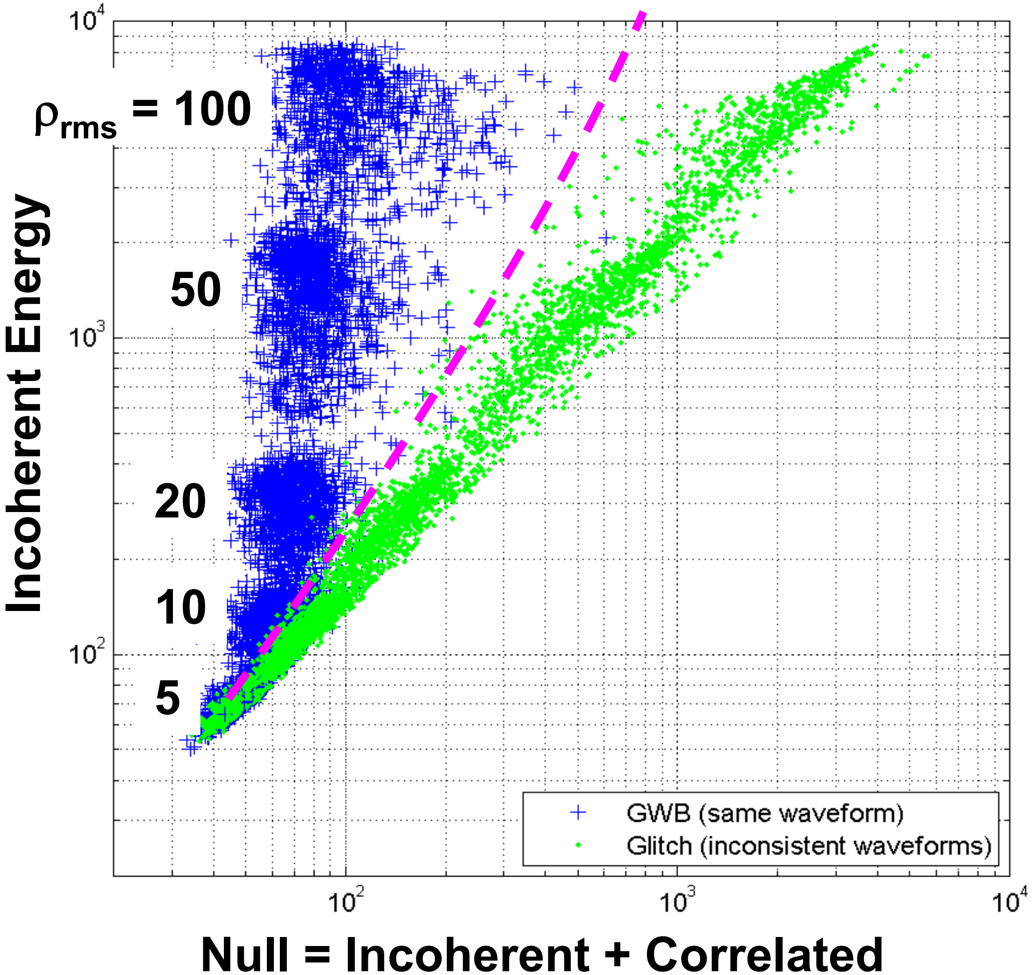
GWB has off-diagonal points $\mathbf{E_{null} \sim E_{inc}}$ (high correlation)

Strongest cancellation (lowest $\mathbf{E_{null}/E_{inc}}$)



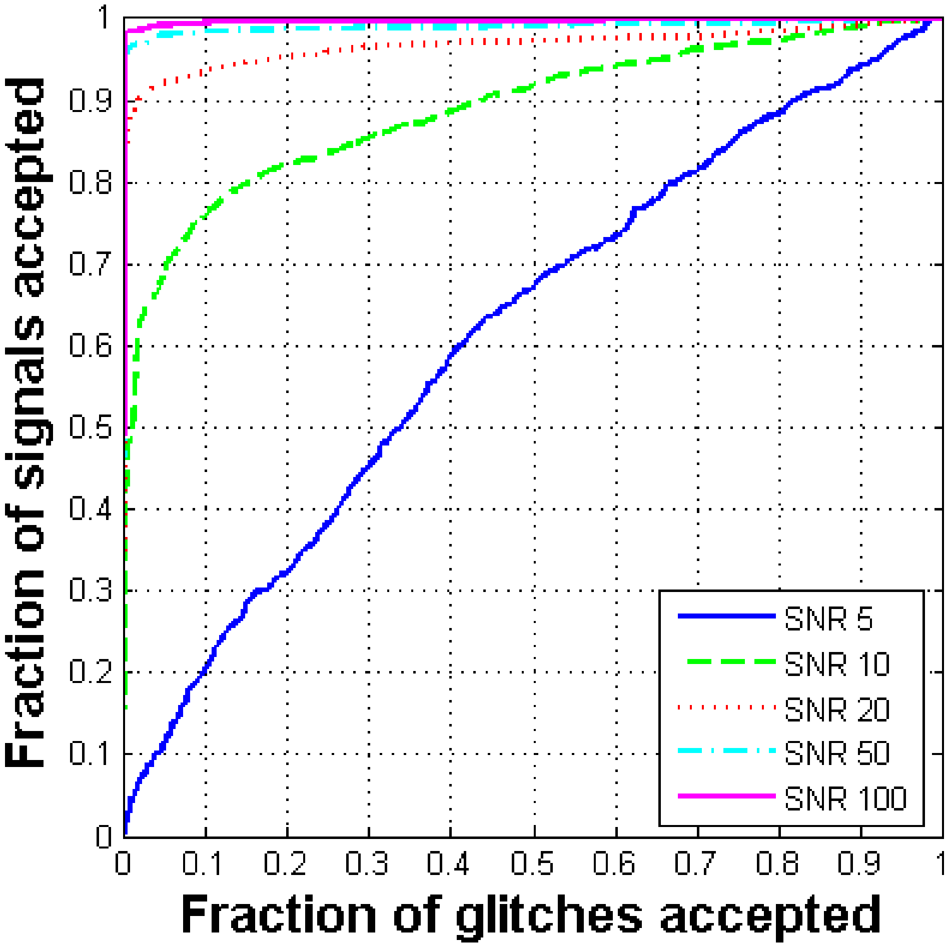
Example: 5000 GWBs vs. 5000 Glitches

- One point from each simulation.
  - sky position giving strongest cancellation
- GWB and glitch populations clearly distinguished for SNR > 10-20.
  - Similar to detection threshold in LIGO.



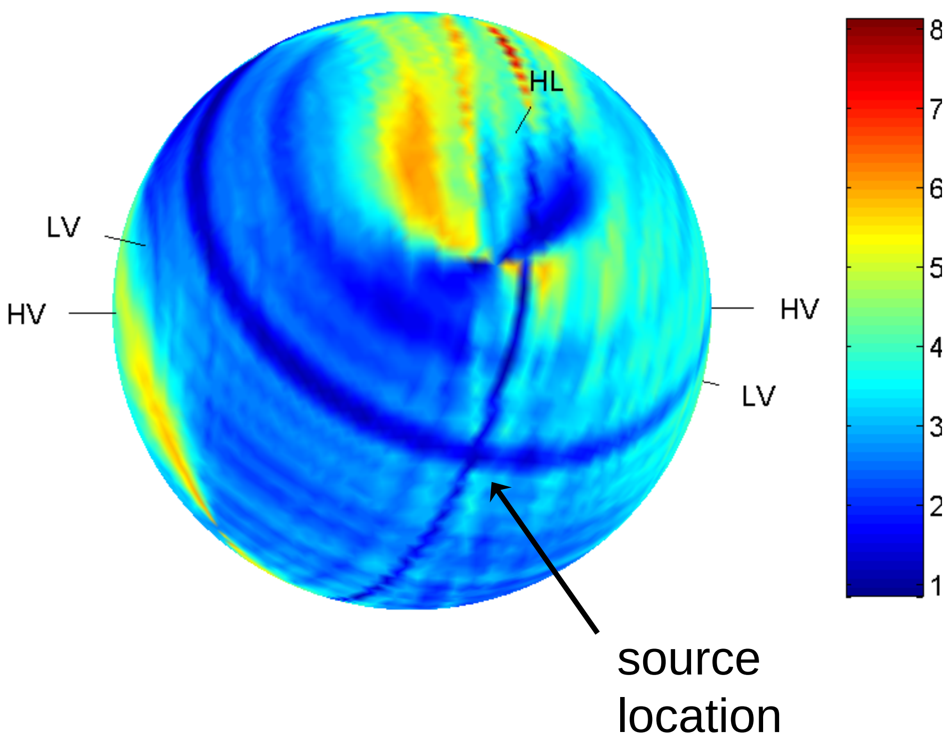
ROC: Distinguishing GWBs from Glitches

- Good discrimination for SNR > 10-20.
  - Similar to detection threshold in LIGO.



Source Localization: Not Good!

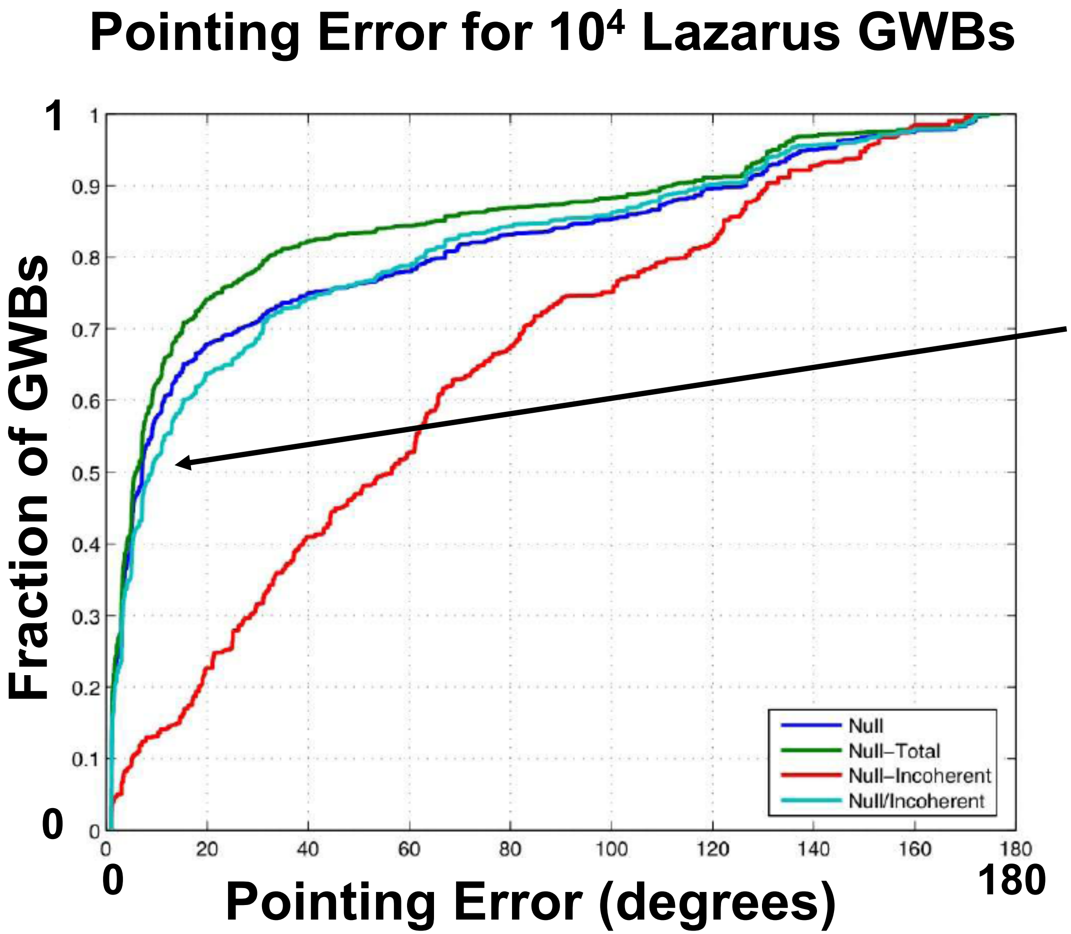
$\mathbf{E_{null}}$ across the sky for 1 GWB
(Hanford-Livingston-Virgo network)



Null energy E_{null} varies slowly along rings of constant time delay with respect to any detector pair.

Noise fluctuations often move minimum away from source location (pointing error)

Example: Lazarus Black Hole Mergers, SNR = 100 (!) (H-L-V network)



L. Stein B.Sc. Thesis, Caltech, 2006.

Median pointing error O(10) degrees.

Must use additional information for accurate source localization!

- E.g.: timing or global ring structure rather than local energy
- More research required!

Maximum Entropy Waveform Estimation

- *This section: work by Summerscales, Ph.D. Thesis, Pennsylvania State University (2006).*

- Another way to regularize waveform reconstruction and minimize fitting to noise.
- Add entropy prior P(h) to maximum-likelihood formulation:

I: model

$$P(\mathbf{h} \,|\, \mathbf{d}, I) \propto P(\mathbf{d} \,|\, \mathbf{h}, I) P(\mathbf{h} \,|\, I)$$

standard likelihood

prior on GWB waveform

Maximum Entropy Cont.

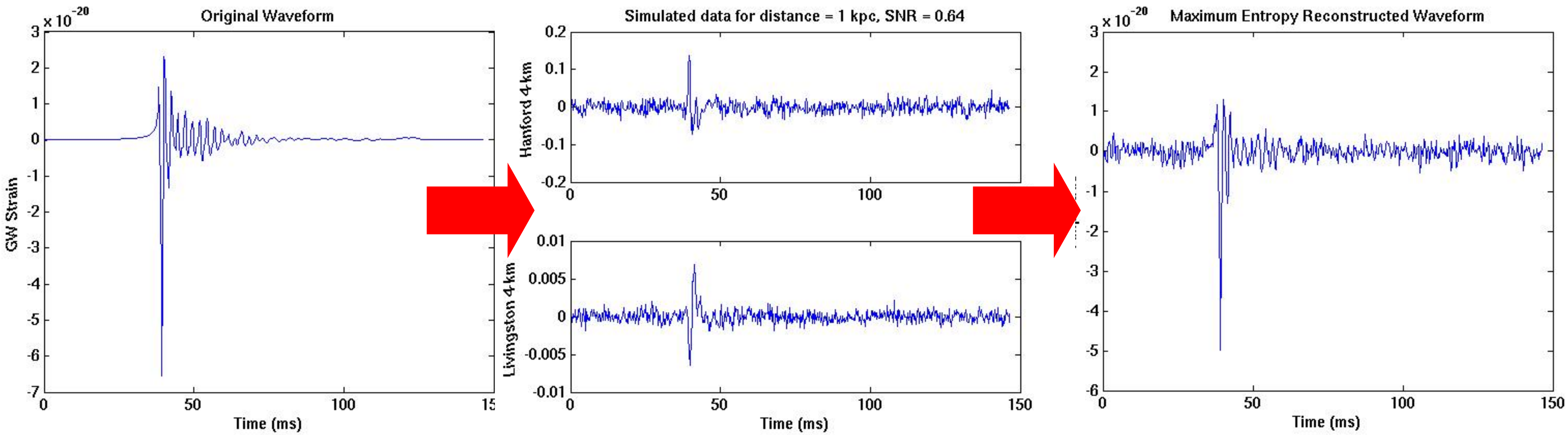
- Choice of prior:

$$P(\mathbf{h} \mid I) = \exp[\alpha S(\mathbf{h}, \mathbf{m})]$$

$$S(\mathbf{h}, \mathbf{m}) = \sum_{time\ i} \left(4m_i^{\ 2} + h_i^{\ 2}\right)^{1/2} - 2m_i - h_i \log \frac{\left(4m_i^{\ 2} + h_i^{\ 2}\right)^{1/2} + h_i}{2m_i}$$

  - S: Related to Shannon Information Entropy (or number of ways quanta of energy can be distributed in time to form the waveform).
    - Not quite usual $\rho \ln \rho$ form of entropy because h can be negative.
    - Hobson and Lasenby MNRAS **298** 905 (1998).
  - Model m_i: Mean number of “positive” or “negative” quanta per time bin *i*.
    - Determined from data **d** using Bayesian analysis.
  - α is a Lagrange parameter that balances being faithful to the signal (minimizing χ^2) and avoiding overfitting (maximizing entropy)

Maximum Entropy Performance, Weak Signal



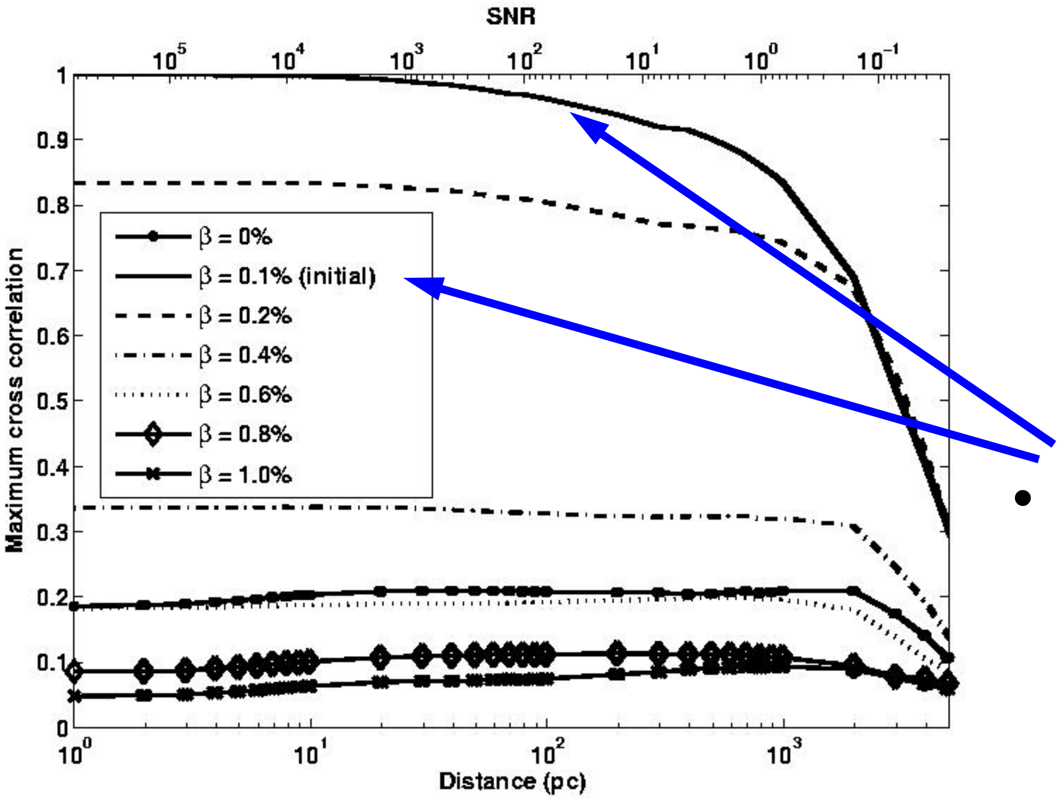
input GWB signal

noisy data from two detectors

recovered GWB signal

Forthcoming paper by Summerscales, Finn, Ott, & Burrows: study ability to recover supernova waveform parameters (rotational kinetic energy, degree of differential rotation, equation of state polytropic index).

Extracting Rotational Information



- Cross correlations between reconstructed signal and waveforms from models that differ only by rotation parameter β (rotational kinetic energy).
- Reconstructed signal most closely resembles waveforms from models with the same rotational parameters

Summary

- Gravitational-wave bursts are an interesting class of GW signals
  - Probes of physics of supernovae, black-hole mergers, gamma-ray burst engines, …

- Coherent data analysis for GWBs using the global network of GW detectors is a potentially powerful tool for
  - detection
  - source localization
  - waveform extraction
  - consistency testing

Summary

- The past year has seen rapid advances in coherent analysis techniques:
  - Regularization of data inversion
    - Improved detection efficiency, can apply to 2-detector networks
  - Exploration of priors on GWB waveforms (e.g. entropy)
    - Tests of ability extract physics from GWBs (supernovae)
  - Improved tests for discriminating GWBs from background noise
  - Much more work remains to be done (e.g., source localization)!

- The first application of fully coherent techniques to real data is in progress
  - Constraint likelihood applied to LIGO S4 data from 2005.

- The future of GWB astronomy looks bright!