LIGO-G060321-00-Z

Laser Interferometer Space Antenna (LISA)

Simulating the WD-WD galactic background in the LISA data

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JPL - CIT

http://science.jpl.nasa.gov/Astrophysics/index.cfm

J. Edlund, M. Tinto, A. Krolak, and G. Nelemans, *Phys. Rev. D.* **71**, 122003 (2005)

GSFC·JPL

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Motivations

- Need for a numerical description of the WD-WD background as it will be observed in the LISA data.
  - Assess its magnitude in the various TDI combinations
  - Quantify the effects of the LISA motion around the Sun.
  - Test the effectiveness of various data analysis techniques for removing it from the LISA data.

D. Hils & P. Bender, R.F. Webbink, *Ap. J.* **360**, 75 (1990)
D. Hils & P. Bender, *CQG*, **14**, 1439 (1997)

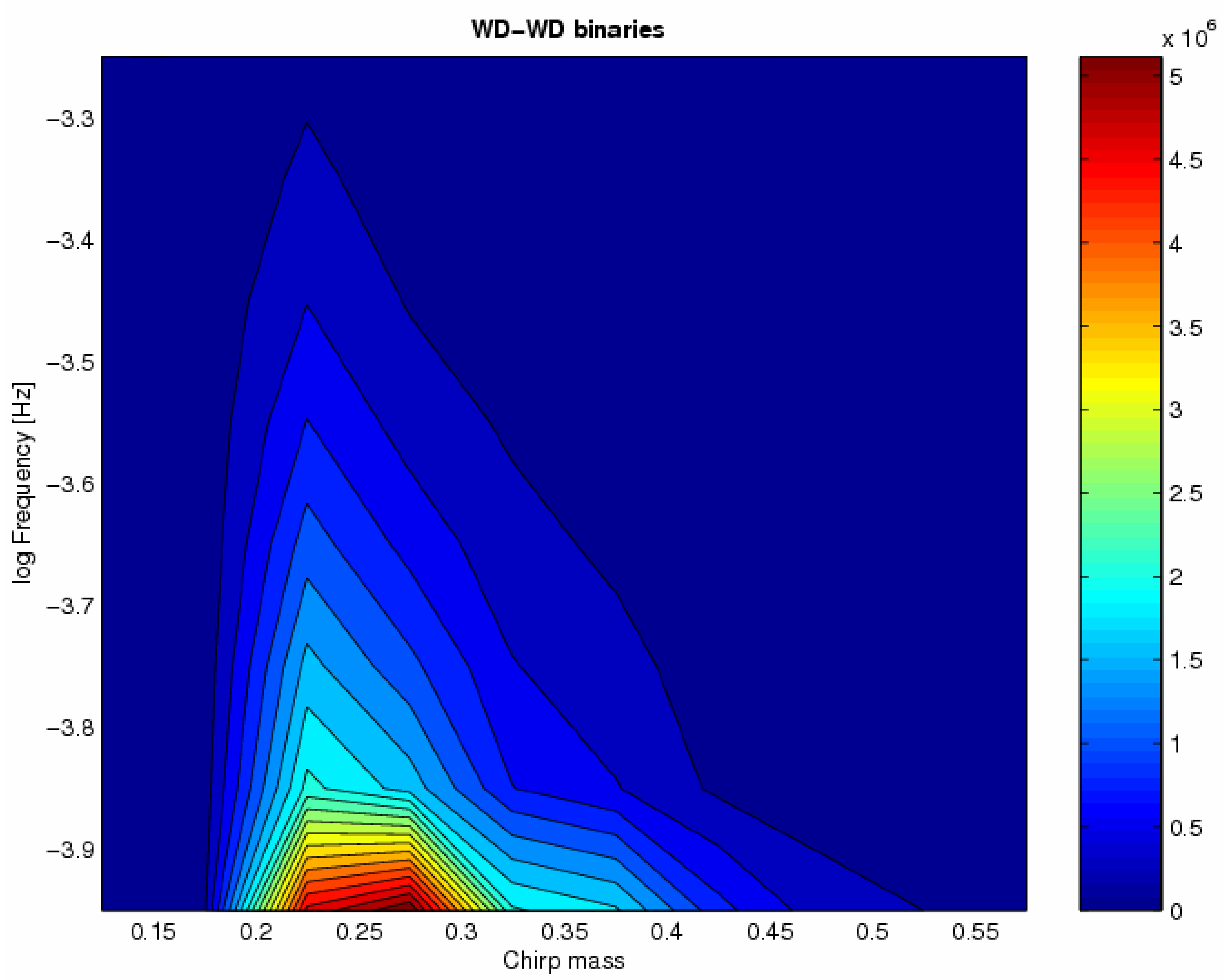
Parameters Distribution

Each GW signal depends on 8 parameters:
$(M_c, \omega, \lambda, \beta, \iota, \psi, \phi_0, D)$

- The overall P.D.F can be assumed to have the following form:

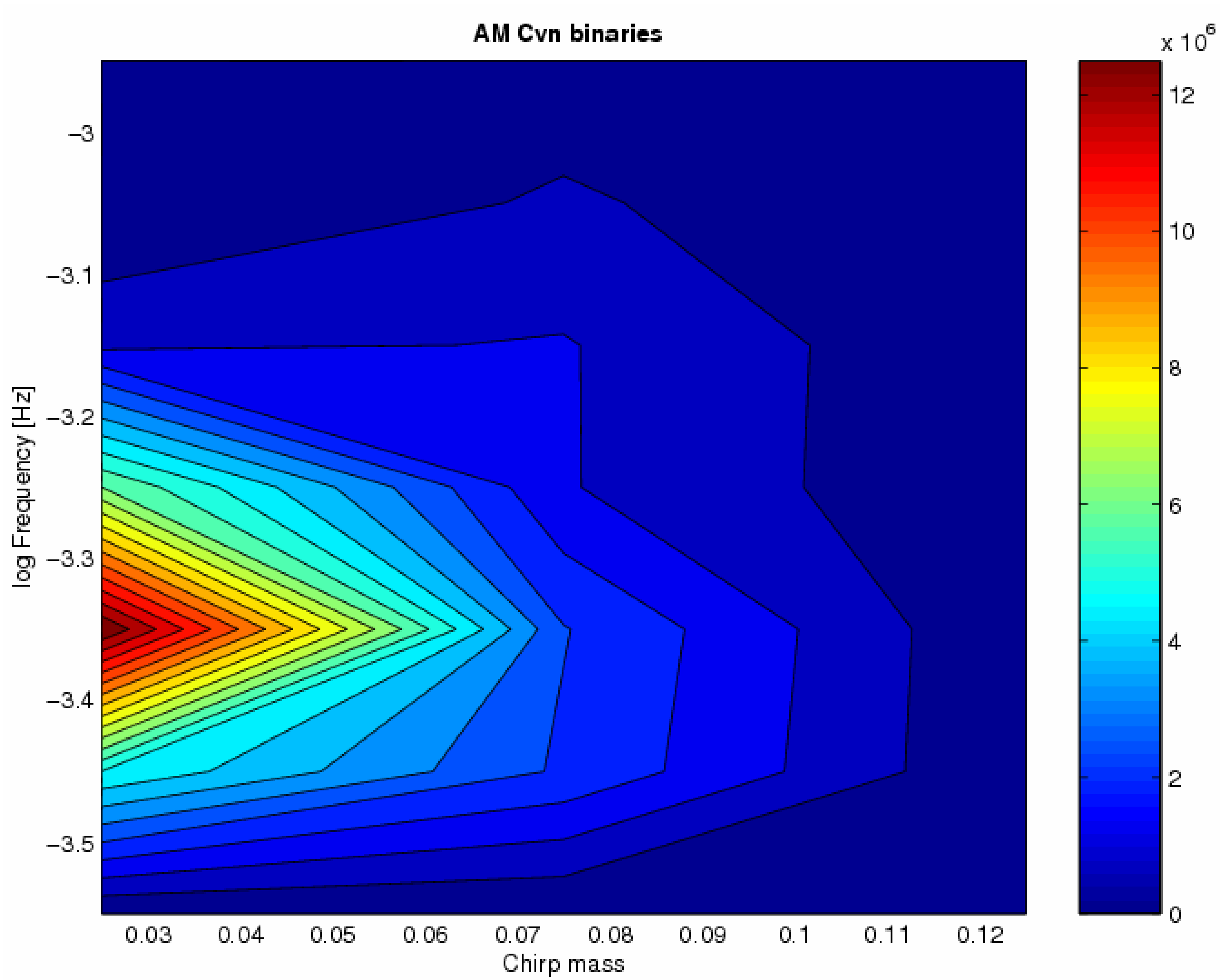
$$P(M_c, \omega, \lambda, \beta, \iota, \psi, \phi_0, D) = P_1(M_c, \omega)\, P_2(\psi)\, P_3(\iota)\, P_4(\lambda, \beta, D)\, P_5(\phi_0)$$

WD-WD Binaries Distribution



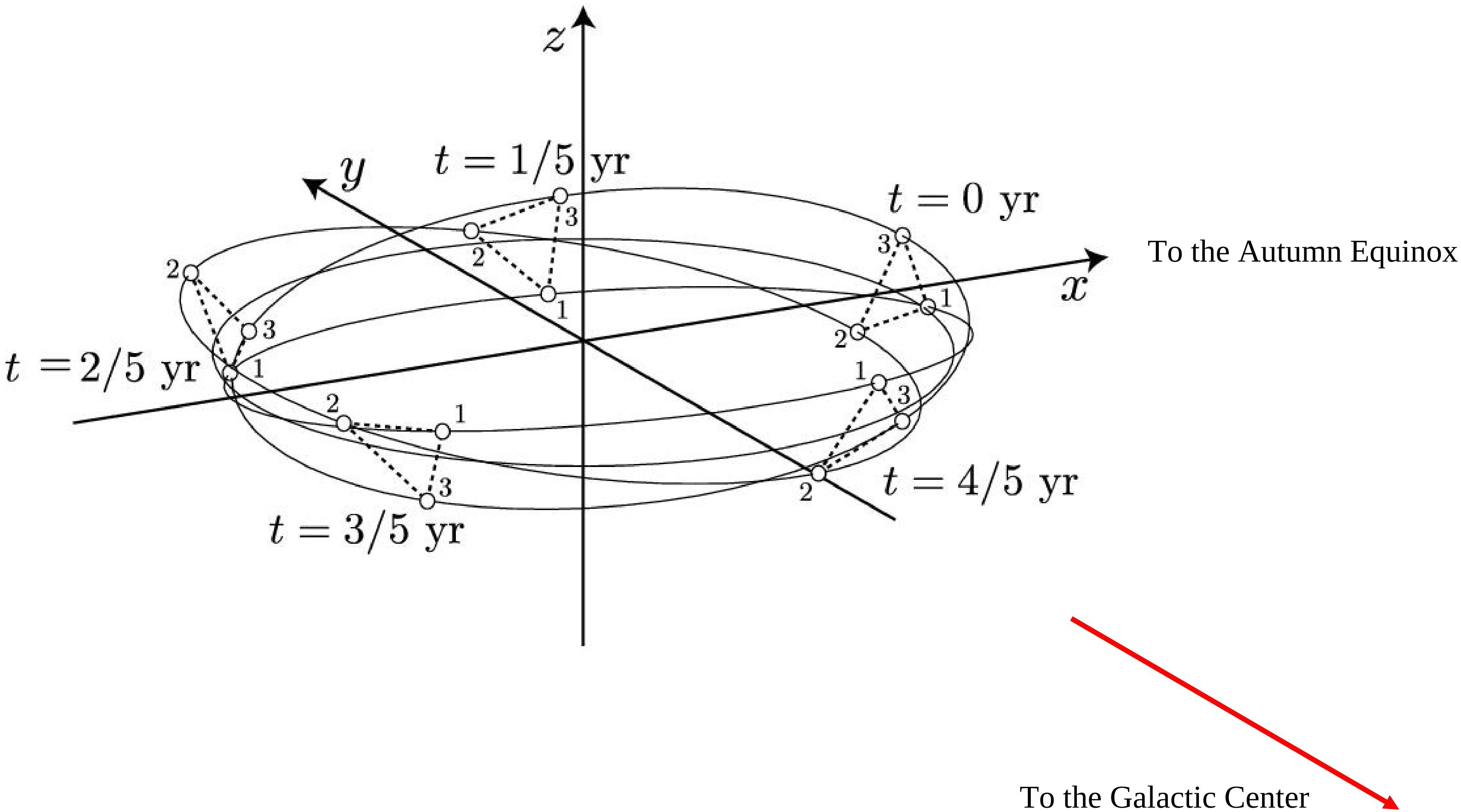
G. Nelemans, L.R. Yungelson, and S.F. Portegies-Zwart, *A&A.*, **375,** 890, (2001)

G. Nelemans, L.R. Yungelson, and S.F. Portegies-Zwart, *Mon. Not. Roy. Astron. Soc.*, **349,** 181 (2004)



AM Cvn binaries
x 10^6
12
10
8
6
4
2
0
-3
-3.1
-3.2
-3.3
-3.4
-3.5
log Frequency [Hz]
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.11
0.12
Chirp mass

Geometry



z
y
x
t = 1/5 yr
t = 0 yr
t = 2/5 yr
t = 3/5 yr
t = 4/5 yr
1
2
3
To the Autumn Equinox
To the Galactic Center

Numerical Simulation

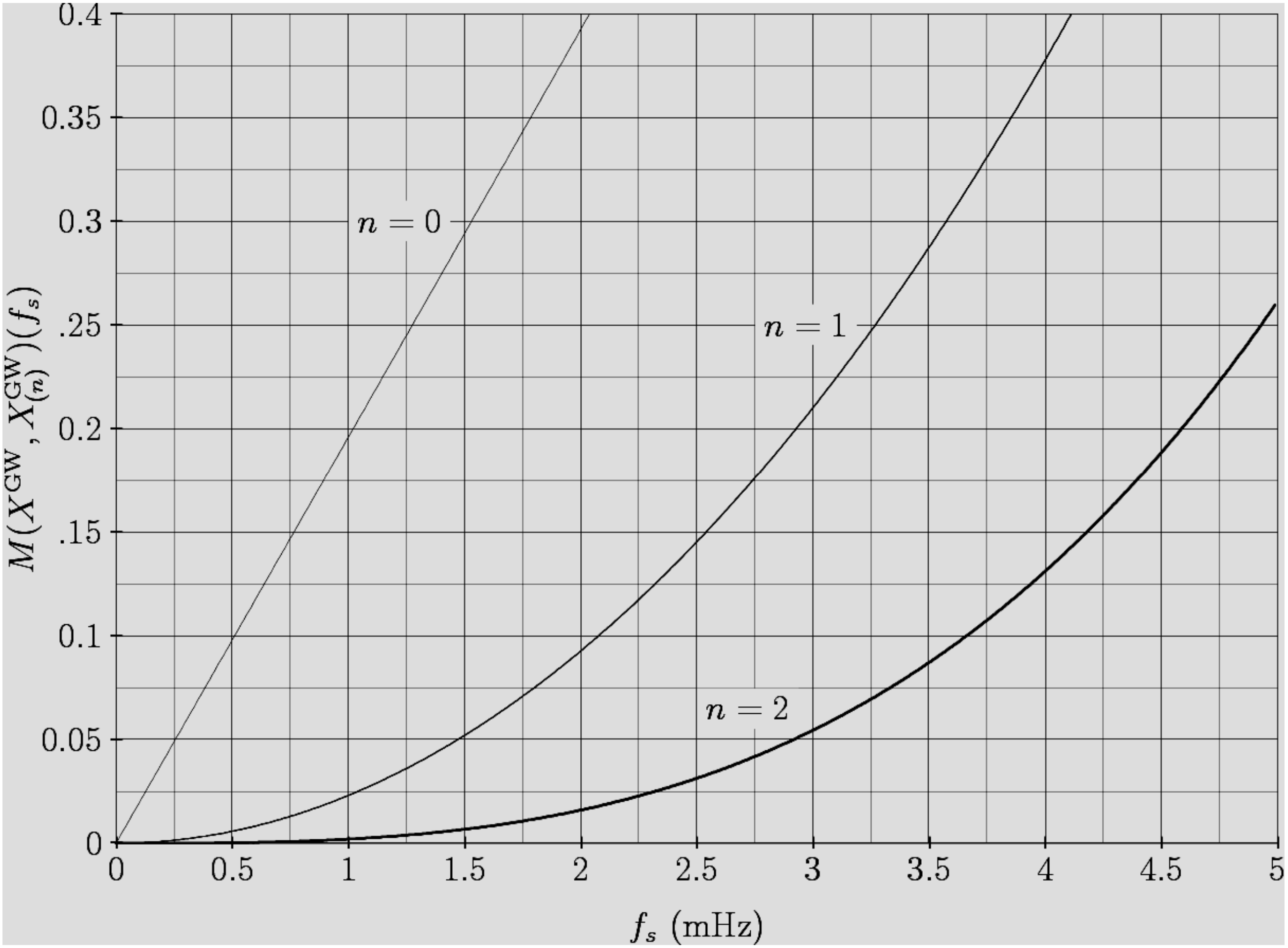
- To generate, in the time-domain, 1 year of the X-response to 1 WD-WD signal takes ~ 10 seconds on a 3.2 GHz P4 CPU (an optimized code can make it in 1 second.)
- For 2.6 x 10^7 sources it would take an unacceptably long time!
- We have derived an analytic expression of the infinite Fourier transform of the signal from a galactic WD-WD binary as seen in any TDI combination.
- Our simulation relies on the convolution of this expression with a properly selected window function.
- We have compared the final time-domain expression of the response obtained using our Fourier-based analytic formula against the time-domain computed expression and found perfect agreement.
- Using our algorithm the CPU time/source => ~ 0.1 seconds!
- For performing our simulation we relied on the JPL Supercomputer (3 days of processing!)

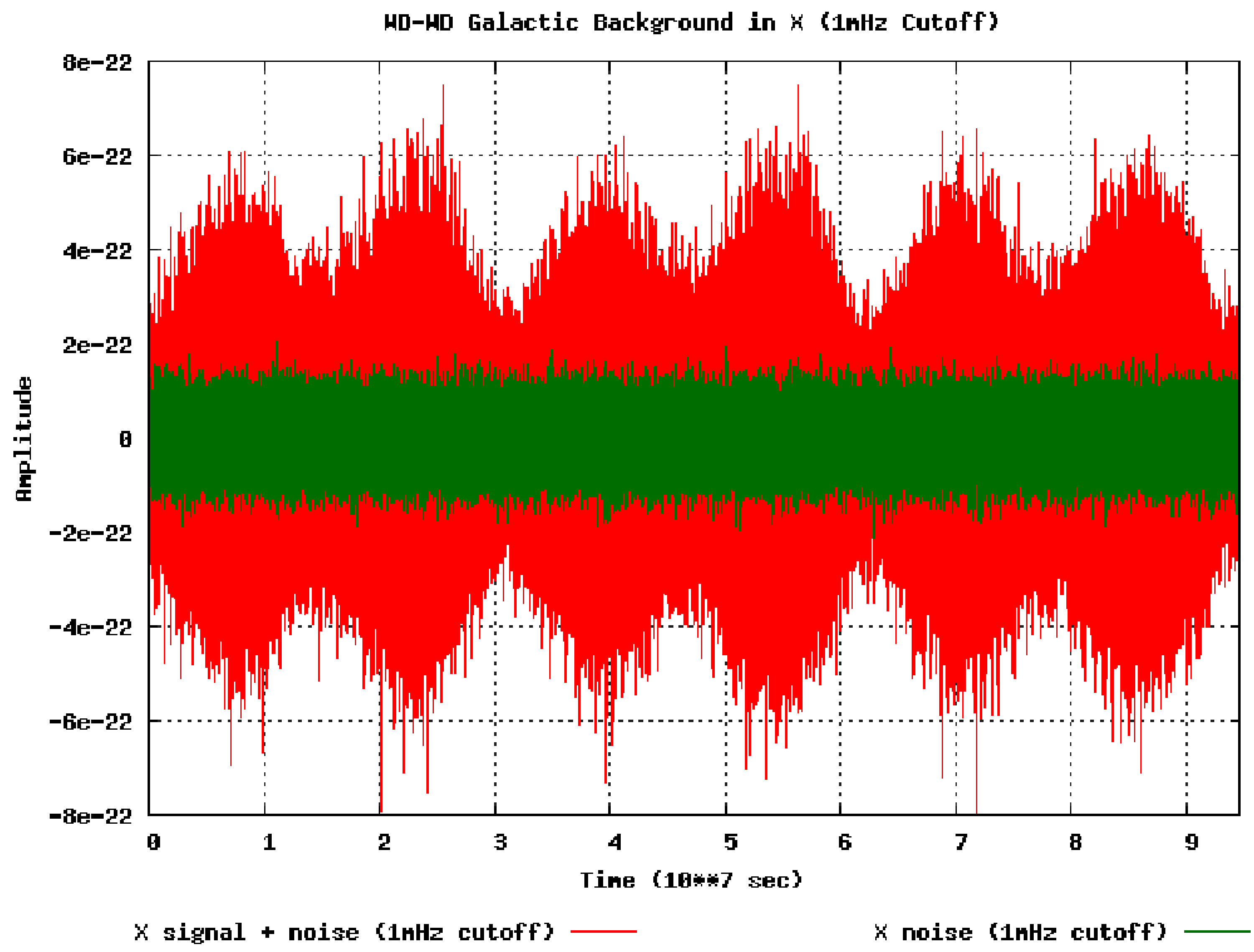
Long-Wavelength Expansion

- Since the contribution of the background to the LISA data is in the low-part of the frequency band, i.e. in the regime where $x = 2\pi f L/c \ll 1$, we have Taylor-expanded the TDI responses for each individual signal.
- Care must be taken in selecting the order of the Taylor expansion in x for any considered TDI response.
- We have simulated the response of the X-combination to the WD-WD background.

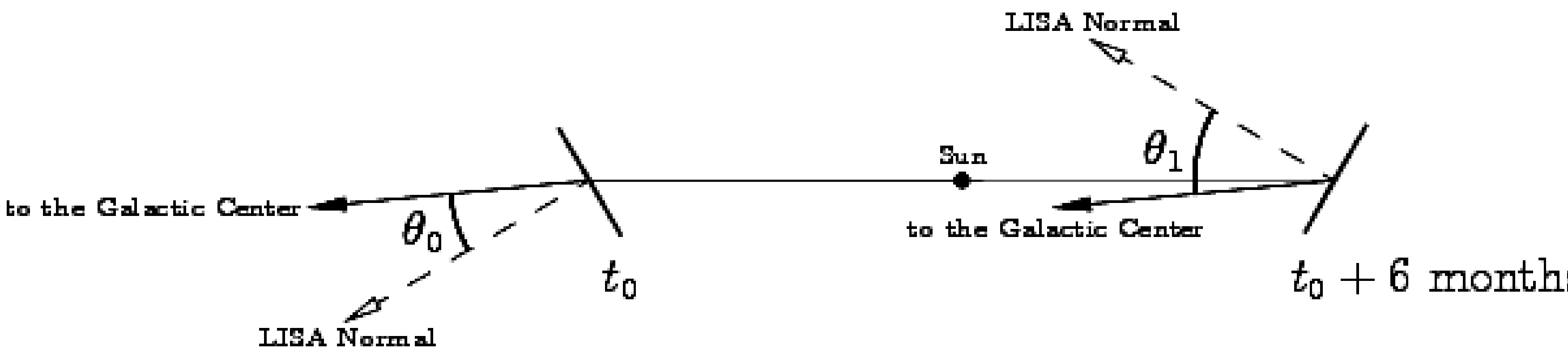
L.W.E. Accuracy

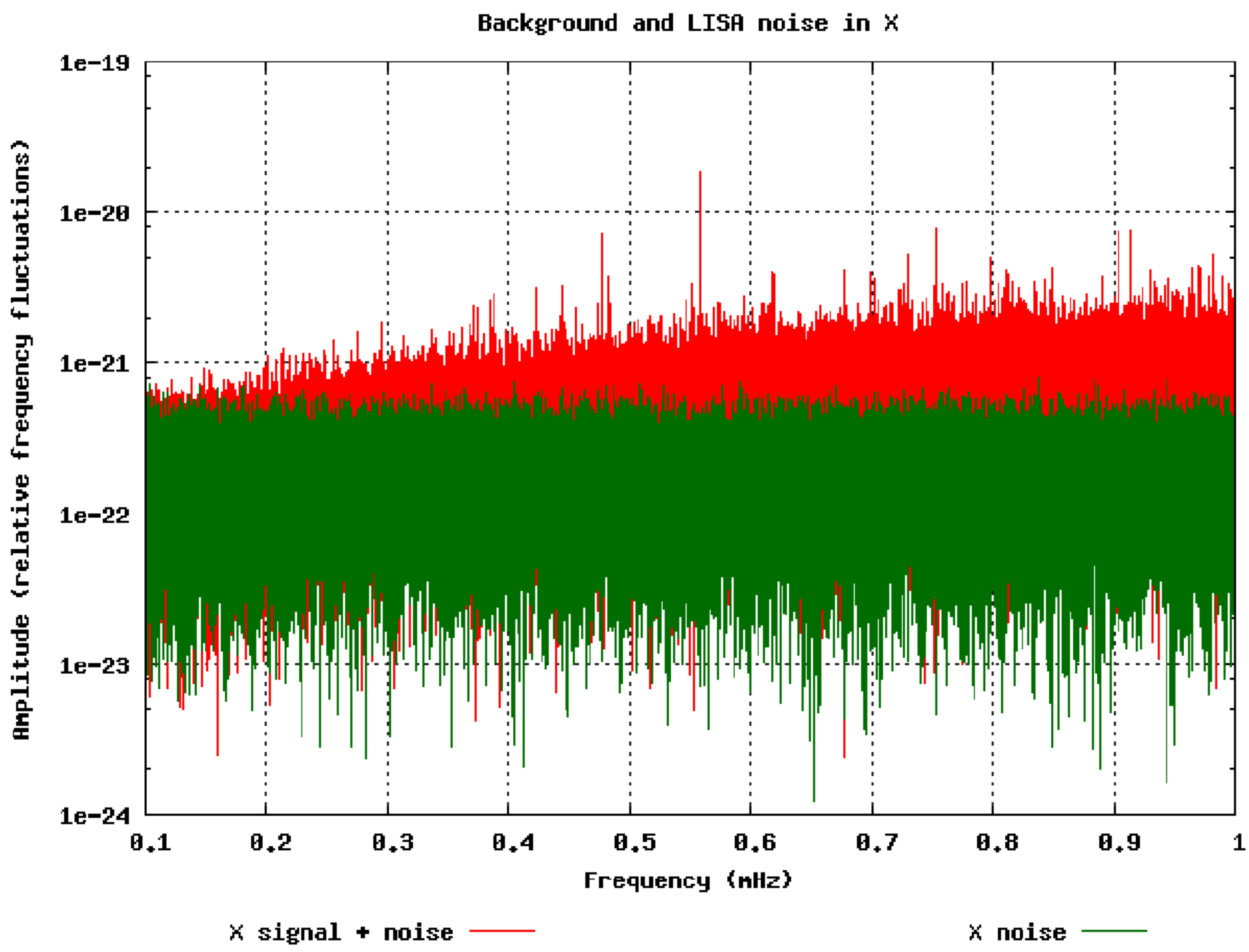
$$M(X^{\mathrm{GW}}, X^{\mathrm{GW}}_{(n)}) \equiv \sqrt{\frac{\int_0^T \left[X^{\mathrm{GW}}(t) - X^{\mathrm{GW}}_{(n)}(t)\right]^2 dt}{\int_0^T \left[X^{\mathrm{GW}}(t)\right]^2 dt}}$$



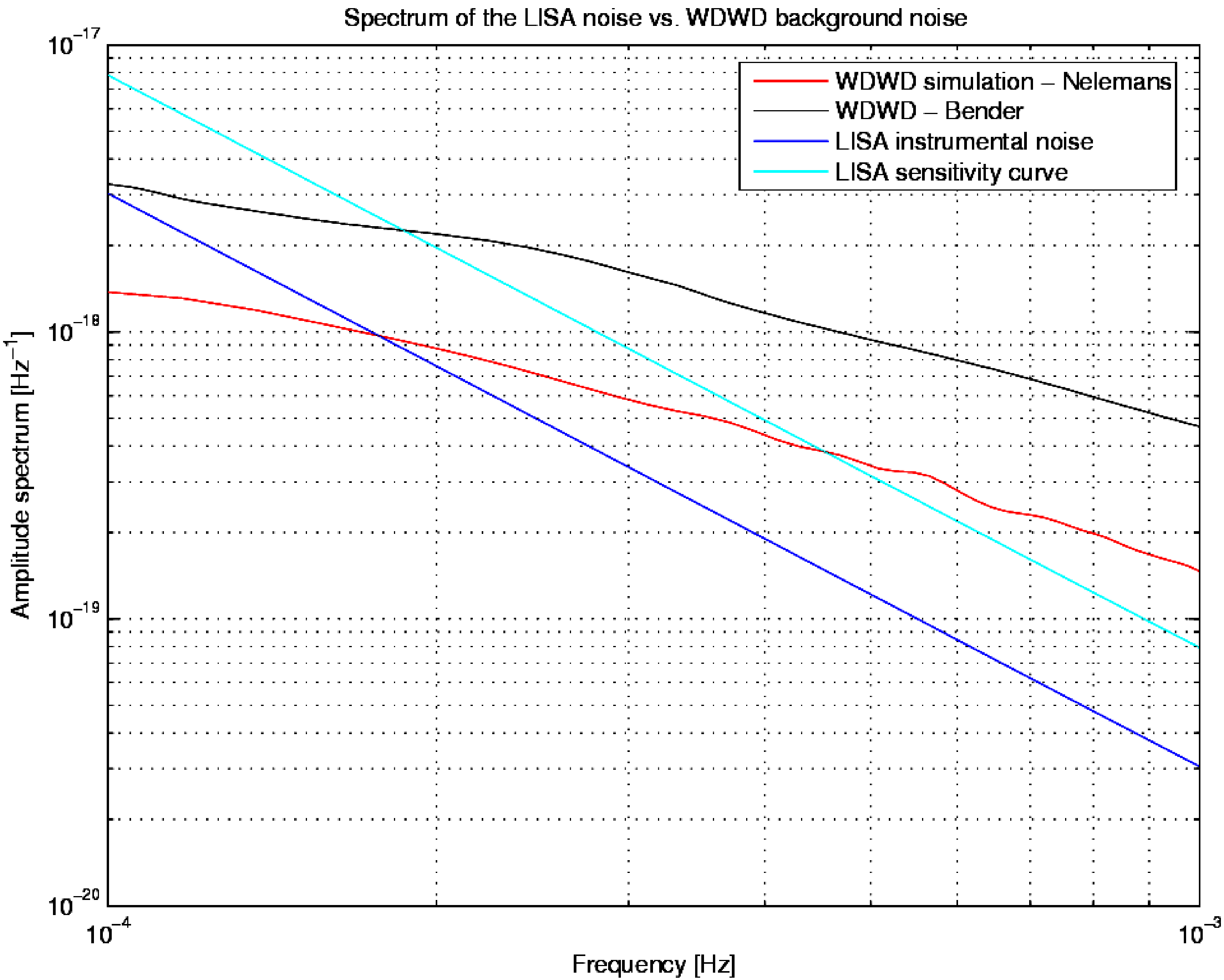


N. Seto, *Phys. Rev. D*, **69**, 123005, (2004)


LISA Normal
Sun
θ_1
to the Galactic Center
θ_0
to the Galactic Center
t_0
$t_0 + 6$ months
LISA Normal



Background and LISA noise in X
Amplitude (relative frequency fluctuations)
1e-19
1e-20
1e-21
1e-22
1e-23
1e-24
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Frequency (mHz)
X signal + noise
X noise



Spectrum of the LISA noise vs. WDWD background noise
WDWD simulation – Nelemans
WDWD – Bender
LISA instrumental noise
LISA sensitivity curve
Amplitude spectrum [Hz^{-1}]
Frequency [Hz]
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-4}
10^{-3}

CycloStationarity

- The motion of LISA around the Sun induces a AM-FM modulation of the received signals.
- In a statistical sense the WD-WD background should be regarded as a periodic function of time with period 1 year.
- Since the autocorrelation will also be a periodic function of time, the background should no longer be treated as a stationary random process, but rather as a Cyclostationary process:

$$C(t,t') = C(t+T, t'+T)$$

CycloStationarity…(cont.)

$$\tau \equiv t' - t \quad \longrightarrow \quad B(t,\tau) \equiv C(t, t+\tau)$$

$$B(t,\tau) \equiv \sum_{r=-\infty}^{\infty} B_r(\tau) e^{i2\pi \frac{rt}{T}} \qquad\qquad B_r(\tau) = \frac{1}{T}\int_0^T B(t,\tau) e^{-i2\pi r \frac{t}{T}}\ dt$$

The Fourier transforms $g_r(f)$ of $B_r(\tau)$ are the so called "cyclic spectra" of a cyclostationary process [9]

$$g_r(f) = \int_{-\infty}^{\infty} B_r(\tau) e^{-i2\pi f\tau}\ d\tau\ . \qquad (3)$$

$$B_{-r}(\tau) \ = \ B_r^*(\tau)\ ,$$

$$g_{-r}(-f) \ = \ g_r^*(f)\ ,$$

H.L. Hurd, *IEEE Trans. Inf. Theory* **35**, 350, 1989

CycloStationarity…(cont.)

$$y_t = n_t + \mathcal{X}_t \ , \quad \longrightarrow \quad G_0(f) = \mathcal{E}(f) + g_0(f) \ .$$

This implies that for r > 0 the cyclic spectra of y_t coincide with those of χ_t , i.e. *in principle* they are **not** contaminated by the noise!

In reality, possible non-stationarity of the noise will need to be accounted for (as always!)

Cyclostationarity and the WD-WD Inverse Problem

- The cyclostationary spectra, g_r (f), can be related to the distribution function of the WD-WD binaries.

!!QUESTIONS!!

- How could we solve for the WD-WD population distribution given these observables?
- Is this the “optimal procedure” for solving the WD-WD background inverse problem?
- No matter what the optimal procedure will be, the astrophysical payoff will be very significant!!