Spectral variational measurement in future gravitational-wave detectors

F.Ya.Khalili

LIGO-G060471-00-Z

❶ Variational measurement in general

❷ Improving the Advanced LIGO sensitivity

❸ Intracavity (third generation) detectors

F.Ya.Khalili,
arXiv:gr-qc/0607028 (2006).

F.Ya.Khalili,
http://www.ligo.org/pdf_public/khalili03.pdf
(2006).

1. Variational measurement in general

2. Improving the Advanced LIGO sensitivity

3. Intracavity (third generation) detectors

Terminology

Spectral variational measurement $\equiv$
KLMTV scheme $\equiv$
modified input/output optics interferometers.

Only modified output case (which does not require squeezed states) is considered here.

shortcut

SQL-limited detector

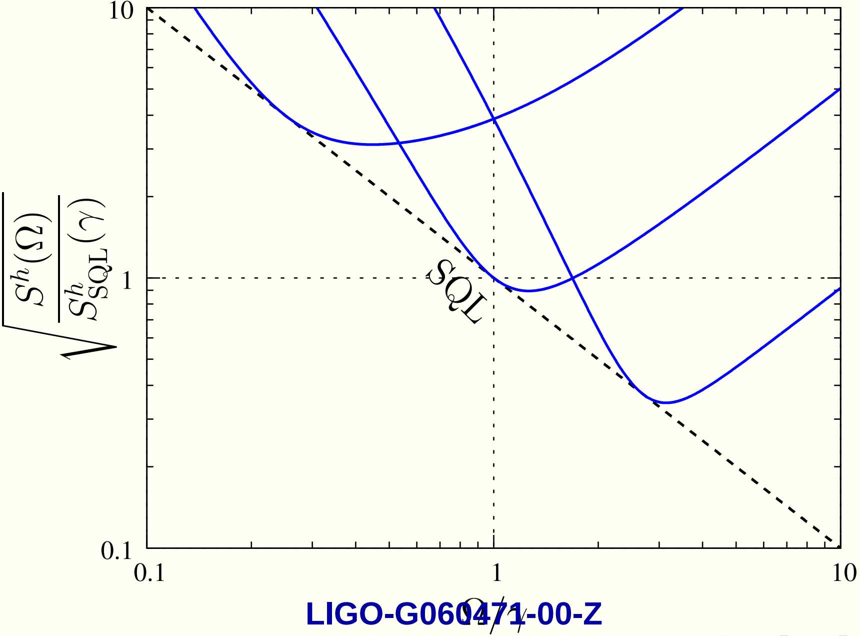
$$\xi^2(\Omega) \equiv \frac{S^h(\Omega)}{S^h_{\mathrm{SQL}}(\Omega)} = \xi^2_{\mathrm{SN}} + \frac{1}{4\xi^2_{\mathrm{SN}}} \geqslant 1\,,$$

where

$$\xi^2_{\mathrm{SN}}(\Omega) = \frac{\Omega^2(\Omega^2+\gamma^2)}{4\gamma} \times \frac{McL}{8\omega_p W}\,,$$

$$S^h_{\mathrm{SQL}}(\Omega) = \frac{8\hbar}{ML^2\Omega^2}\,.$$

SQL-limited detector



Fixed homodyne angle

$$\xi^2(\Omega) \equiv \frac{S^h(\Omega)}{S^h_{\rm SQL}(\Omega)} = \frac{\xi^2_{\rm SN}(\Omega)}{\cos^2\phi} - \tan\phi + \frac{1}{4\xi^2_{\rm SN}(\Omega)}$$

$$= \xi^2_{\rm SN}(\Omega) + \xi^2_{\rm res}(\Omega)\,,$$

where

$$\xi^2_{\rm res}(\Omega) = \xi^2_{\rm SN}(\Omega) \times \left[\tan\phi - \frac{1}{2\xi^2_{\rm SN}(\Omega)}\right]^2 .$$

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$$\tan\phi = \frac{1}{2\xi^2_{\mathrm{SN}}(\Omega)} \;\Rightarrow\; \xi^2(\Omega) = \xi^2_{\mathrm{SN}}(\Omega) \propto \frac{1}{W}\,.$$

(No SQL)

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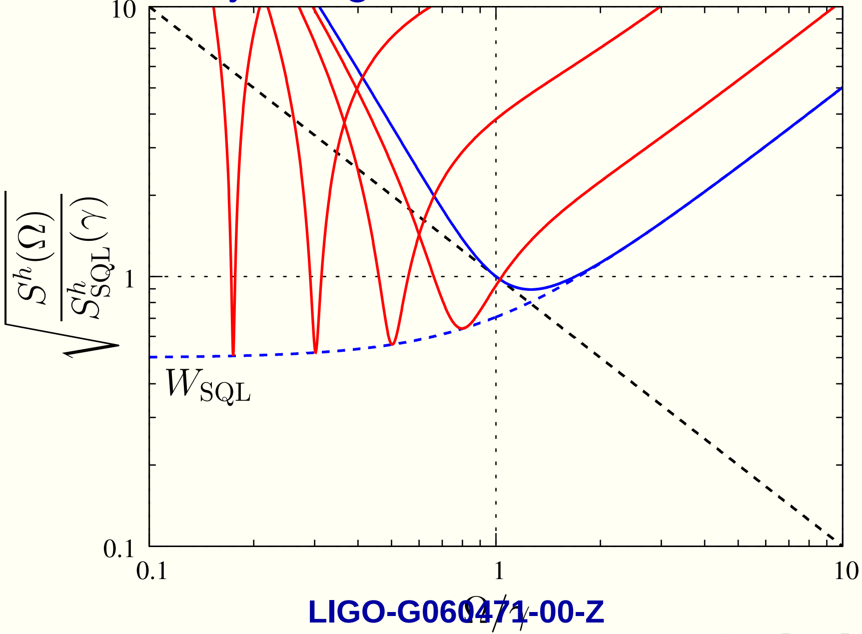
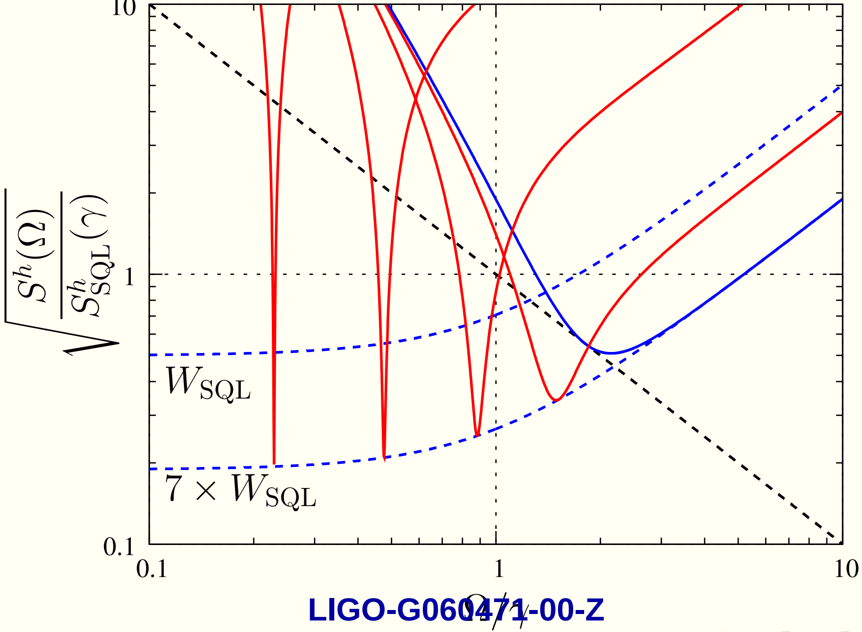
(No SQL)

The problem: ϕ does not depend on Ω.

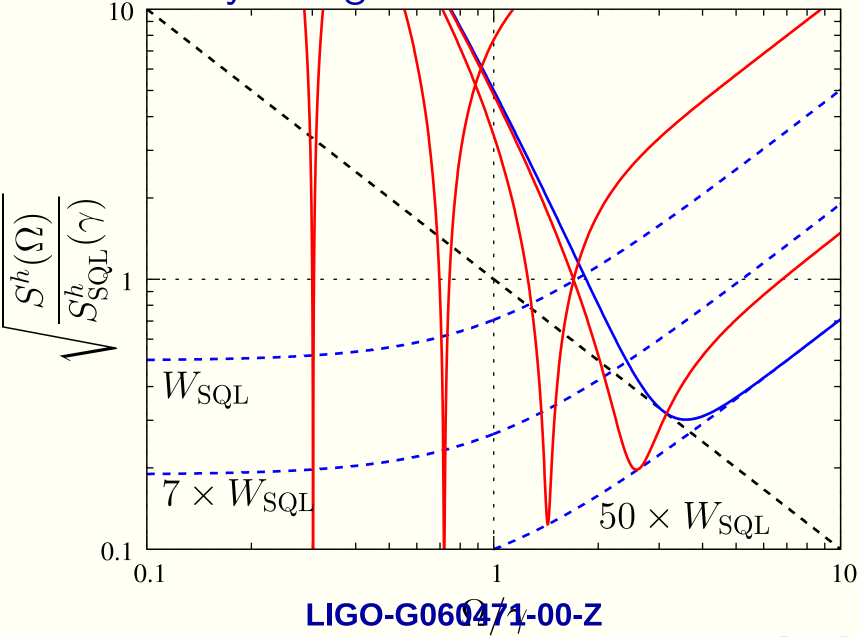
Fixed homodyne angle



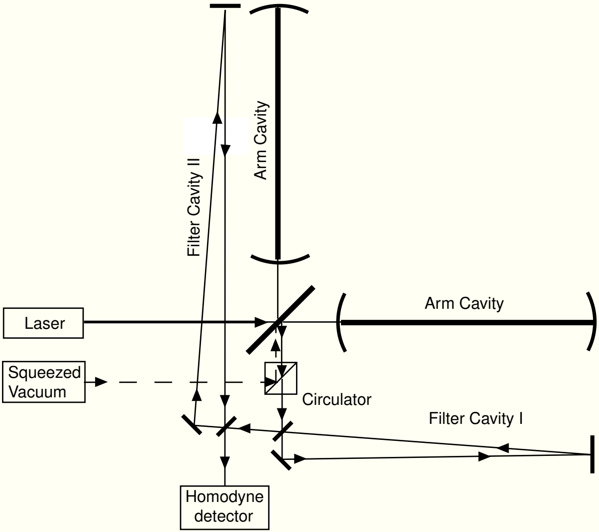
Fixed homodyne angle



Fixed homodyne angle



The way to introduce frequency dependence



H.J.Kimble, Yu.Levin, A.B.Matsko, K.S.Thorne, S.P.Vyatchanin, Phys.Rev.D 65, 022002 (2002)

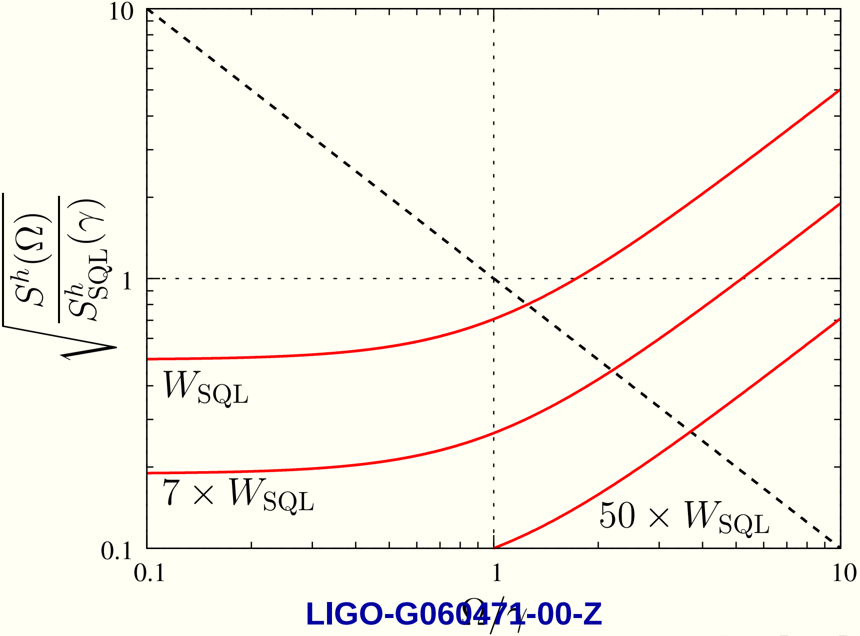
Other important papers

Jan Harms, Yanbei Chen, Simon Chelkovski, Alexander Franzen, Hennig Walbruch, Karsten Danzmann, and Roman Schnabel, Phys.Rev.D **68**, 042001 (2003).

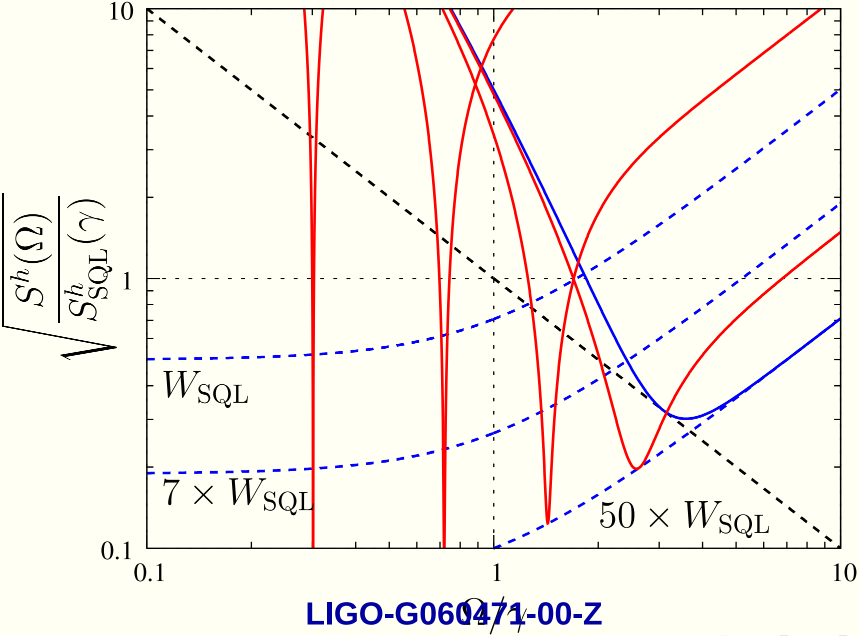
A.Buonanno, Y.Chen, Phys.Rev.D **69**, 102004 (2004).

shortcut

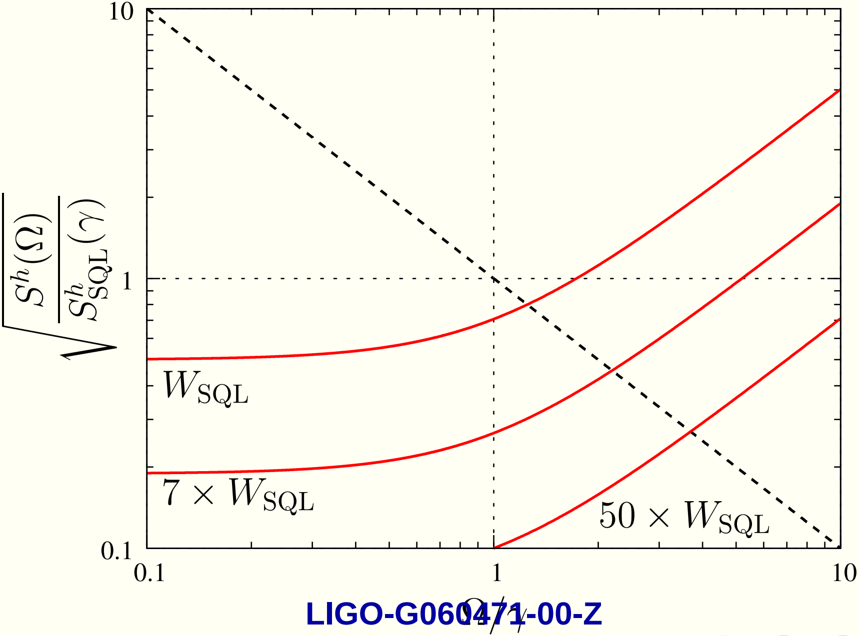
Ideal variational measurement



Ideal variational measurement



Ideal variational measurement



The main technical issue

The filter cavity(ies) bandwidth:

$$\gamma_{\text{filter}} \sim \Omega \sim 10^3\,\mathrm{s}^{-1}\,.$$

Therefore, long (kilometer-scale) cavity(ies) with high-reflectivity mirrors should to be used.

The main technical issue

The filter cavity(ies) bandwidth:

$$\gamma_{\rm filter} \sim \Omega \sim 10^3\,{\rm s}^{-1}\,.$$

Therefore, long (kilometer-scale) cavity(ies) with high-reflectivity mirrors should to be used.

What can be done with a single "cheap" $10 \div 30\,{\rm m}$ filter cavity?

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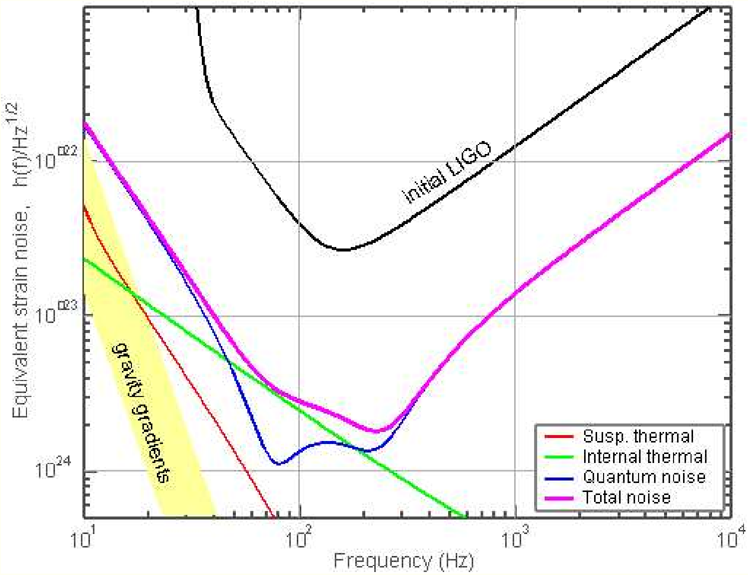
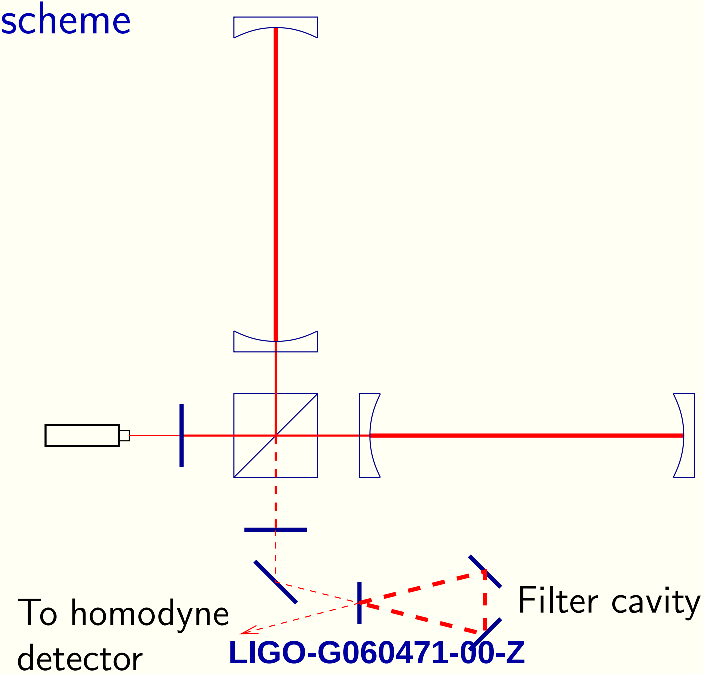
Advanced LIGO noise budget



Low-frequency (< 100Hz) area can be improved.

The scheme



Parameters values

Main interferometer:

$$M = 40\,\mathrm{kg} \qquad L = 4\,\mathrm{km}$$

$$\gamma \approx 1.7 \times 2\pi \times 100\,\mathrm{s}^{-1}$$

$$W = \frac{McL}{8\omega_p} \times \left(2\pi \times 100\,\mathrm{s}^{-1}\right)^3 \approx 840\,\mathrm{kW}$$

Filter cavity:

$$\frac{A^2_{\mathrm{filter}}}{L_{\mathrm{filter}}} = \frac{1 \times 10^{-5}}{20\,\mathrm{m}} = 5 \times 10^{-7}\,\mathrm{m}^{-1}$$

or

$$\frac{A^2_{\mathrm{filter}}}{L_{\mathrm{filter}}} = \frac{1 \times 10^{-5}}{100\,\mathrm{m}} = 1 \times 10^{-7}\,\mathrm{m}^{-1}$$

Variational measurement

$$\xi^2(\Omega) = \frac{S^h(\Omega)}{S^h_{\mathrm{SQL}}(\Omega)} = \frac{\xi^2_{\mathrm{SN}}(\Omega)}{\cos^2\phi_\Sigma(\Omega)} - \tan\phi_\Sigma(\Omega) + \frac{1+\mathcal{A}_\Sigma(\Omega)}{4\xi^2_{\mathrm{SN}}(\Omega)}$$

$$= \xi^2_{\mathrm{SN}}(\Omega) + \xi^2_{\mathrm{res}}(\Omega) + \xi^2_{\mathrm{loss}}(\Omega)\,,$$

where

$$\xi^2_{\mathrm{SN}}(\Omega) = \frac{\Omega^2(\Omega^2+\gamma^2)}{4\gamma} \times \frac{McL}{8\omega_p W} \times \left[1+\mathcal{A}_\Sigma(\Omega)\right],$$

$$\xi^2_{\mathrm{res}}(\Omega) = \xi^2_{\mathrm{SN}}(\Omega)\left[\tan\phi_\Sigma(\Omega) - \frac{1}{2\xi^2_{\mathrm{SN}}(\Omega)}\right]^2,$$

$$\xi^2_{\mathrm{loss}}(\Omega) = \frac{\mathcal{A}_\Sigma(\Omega)}{4\xi^2_{\mathrm{SN}}(\Omega)}.$$

“Soft” variational measurement

$$\tan\phi_\Sigma(\Omega) = \frac{1}{2\xi^2_{\rm SN}(\Omega)}$$

“Soft” variational measurement

$$\xcancel{\tan\phi_\Sigma(\Omega) = \frac{1}{2\xi^2_{\mathrm{SN}}(\Omega)}}$$

- can not be implemented with a single filter cavity;
- useless anyway due to optical losses [the term $\xi^2_{\mathrm{loss}}(\Omega)$].

“Soft” variational measurement

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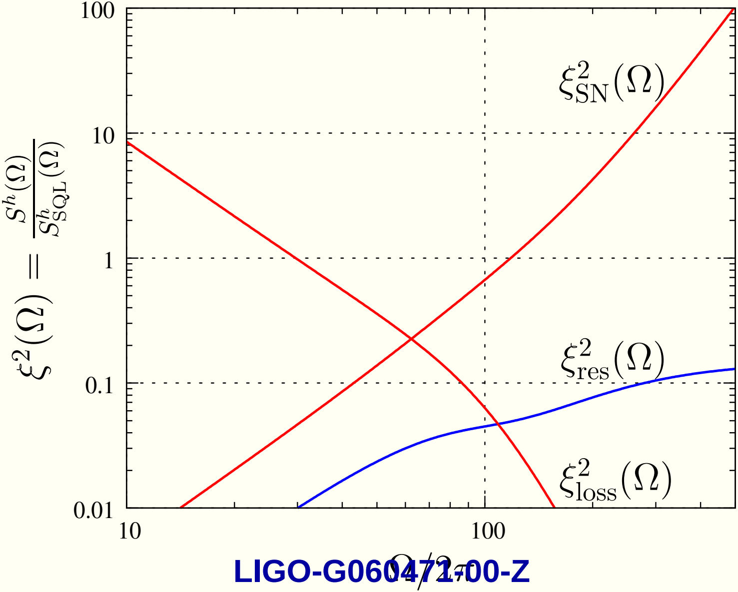
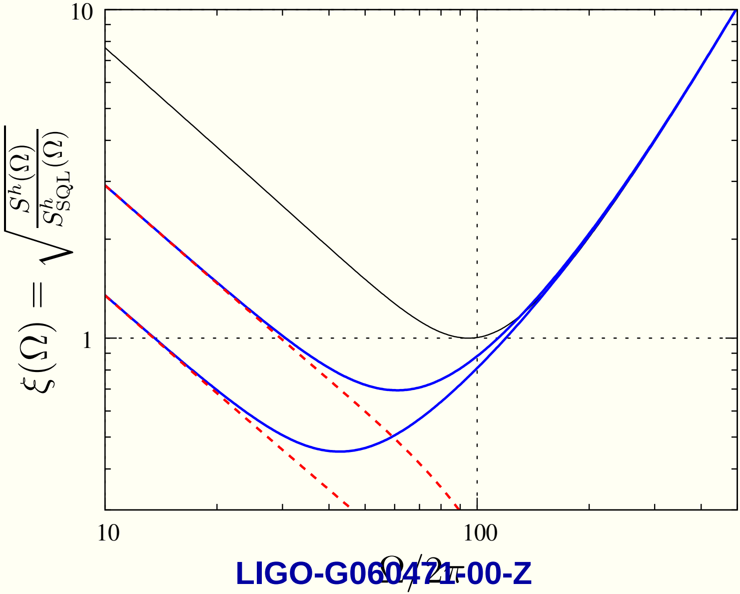
Instead:

$$\tan\phi_\Sigma(\Omega) - \frac{1}{2\xi^2_{\mathrm{SN}}(\Omega)}\bigg|_{\Omega\to 0} \to 0$$

Back action suppression



Sum noise

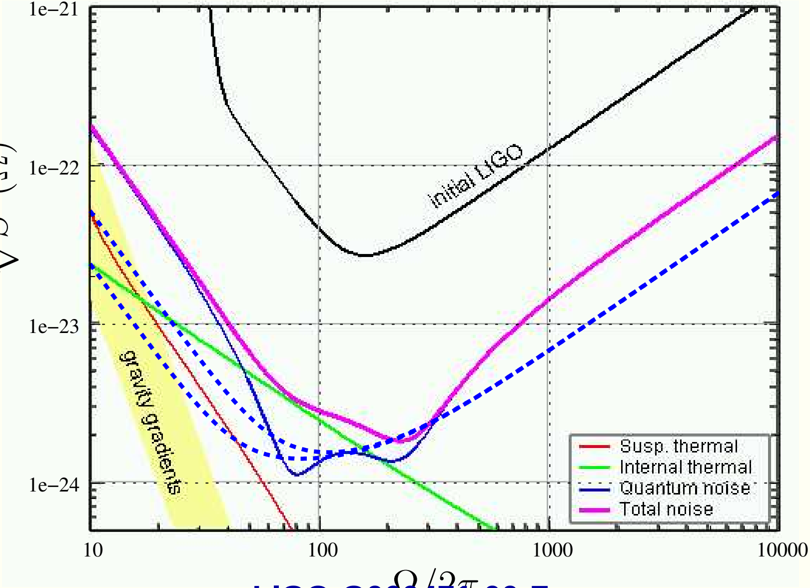

10
1
10
100
$\xi(\Omega) = \sqrt{\frac{S^h(\Omega)}{S^h_{\rm SQL}(\Omega)}}$
$\Omega/2\pi$

The sensitivity gain

$$\left.\frac{\xi_{\rm loss}(\Omega)}{\xi_{\rm SQL}(\Omega)}\right|_{\Omega\to 0} \approx \sqrt{\frac{A^2_{\rm filter}}{L_{\rm filter}}\sqrt{\frac{Mc^3L\gamma}{32\omega_p W}}}$$

$$\approx 0.17\sqrt{\frac{A^2_{\rm filter}/L_{\rm filter}}{10^{-7}}}\,.$$

Comparison with the AdvLIGO noise budget



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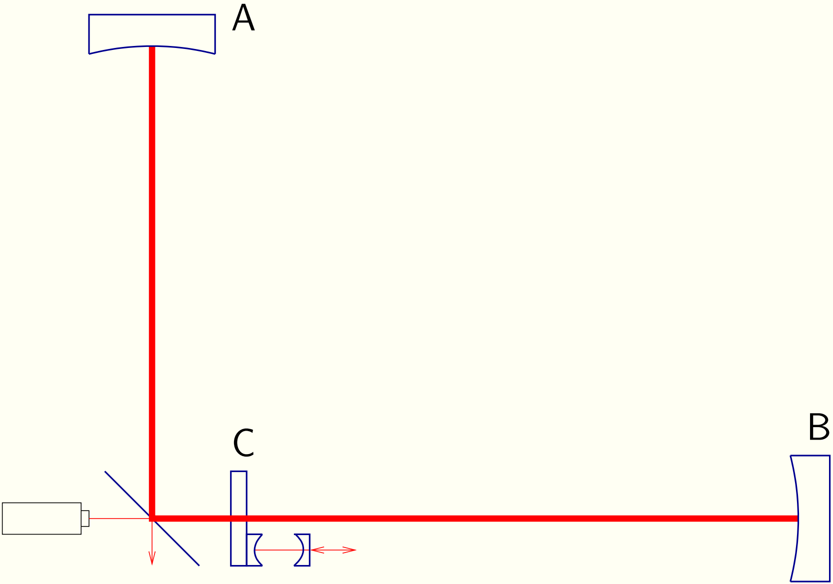
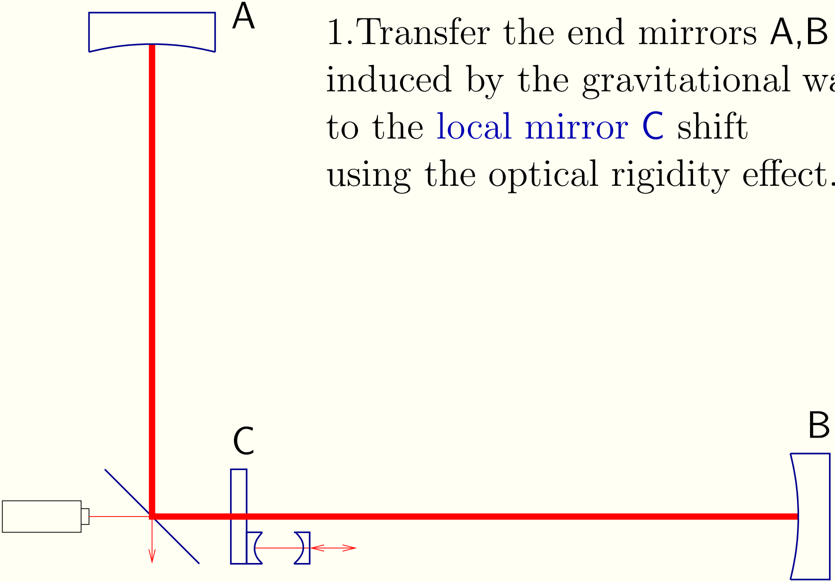
shortcut

“Optical bars/Optical lever” intracavity topologies



“Optical bars/Optical lever” intracavity topologies

1.Transfer the end mirrors A,B shift induced by the gravitational wave to the local mirror C shift using the optical rigidity effect.



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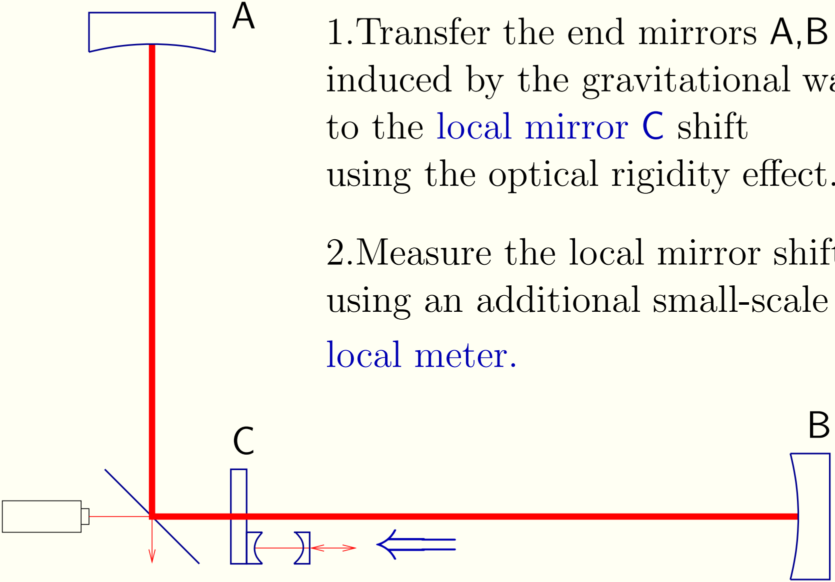
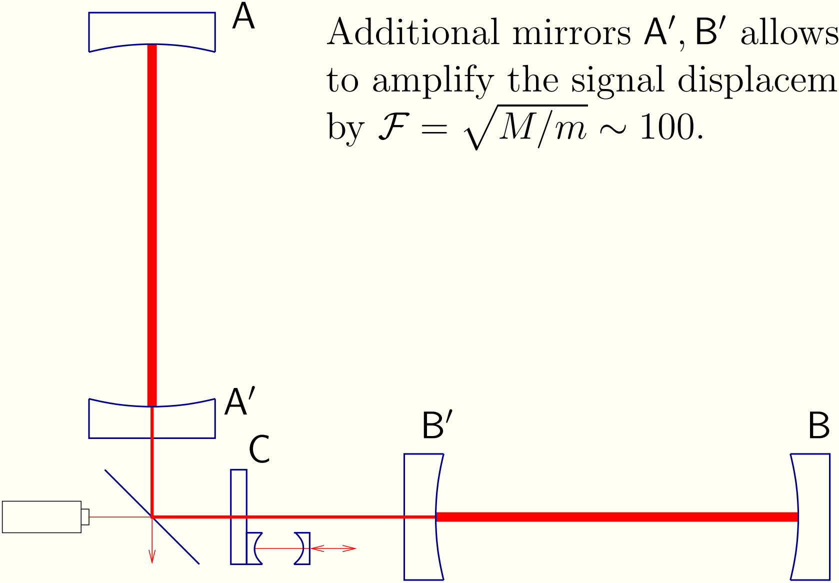


1.Transfer the end mirrors A,B shift induced by the gravitational wave to the local mirror C shift using the optical rigidity effect.

2.Measure the local mirror shift using an additional small-scale local meter.

“Optical bars/Optical lever” intracavity topologies



Additional mirrors A', B' allows to amplify the signal displacement by $\mathcal{F} = \sqrt{M/m} \sim 100$.

Requirements for the local meter

$$x_{\rm signal} \approx \frac{\mathcal{F} L h}{2}\,,$$

$$\frac{h}{h_{\rm SQL}} = \frac{\Delta x}{\Delta x_{\rm SQL}}\,,$$

$$\Delta x_{\rm SQL} = \sqrt{\frac{\hbar}{m\Omega}} \approx 10^{-15}\,{\rm cm}\,.$$

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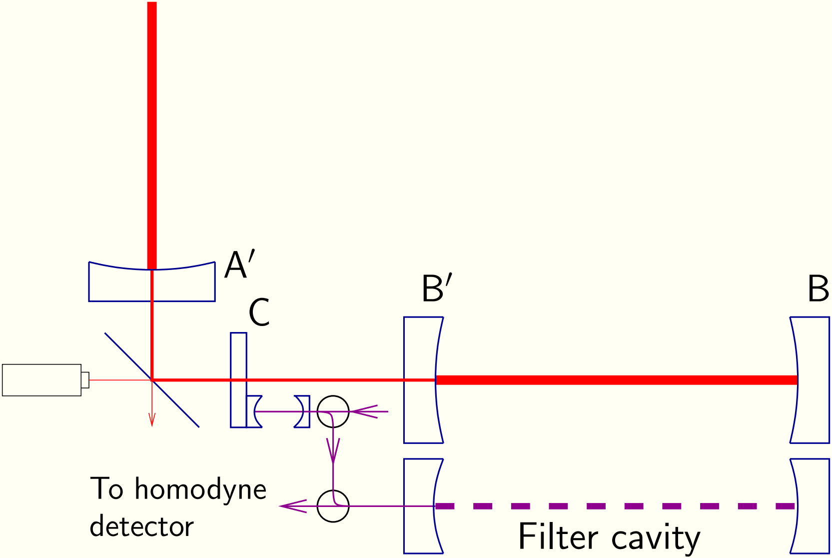
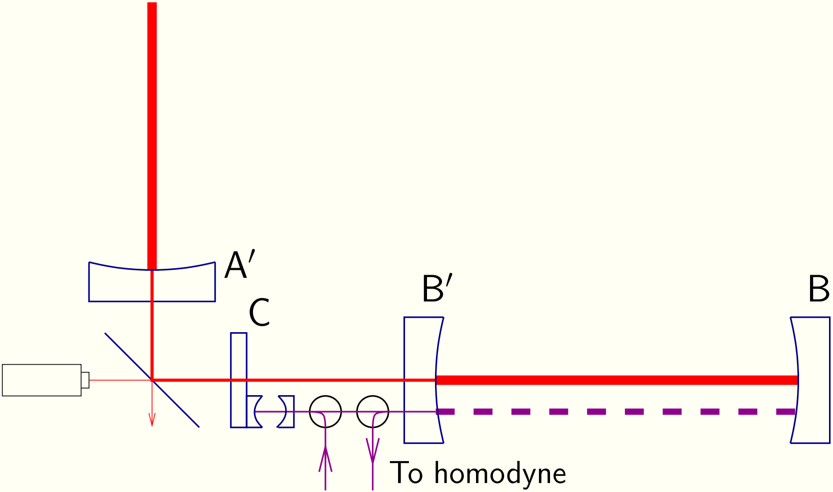
- It is reasonable to use the variational measurement in the local meter.
- The local meter cavity should be short, $l \lesssim 1\,{\rm m}$, hence one filter cavity is sufficient.

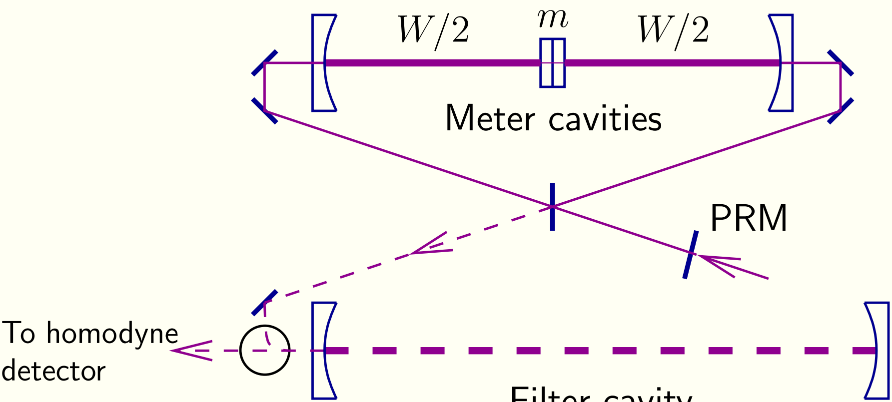
Optical lever + variational meter



Optical lever + variational meter

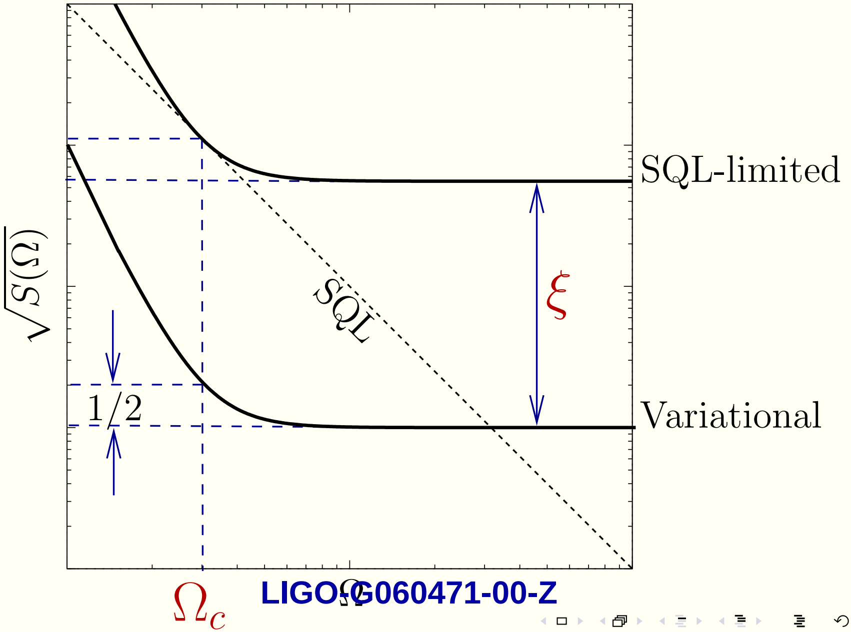


Local meter topology

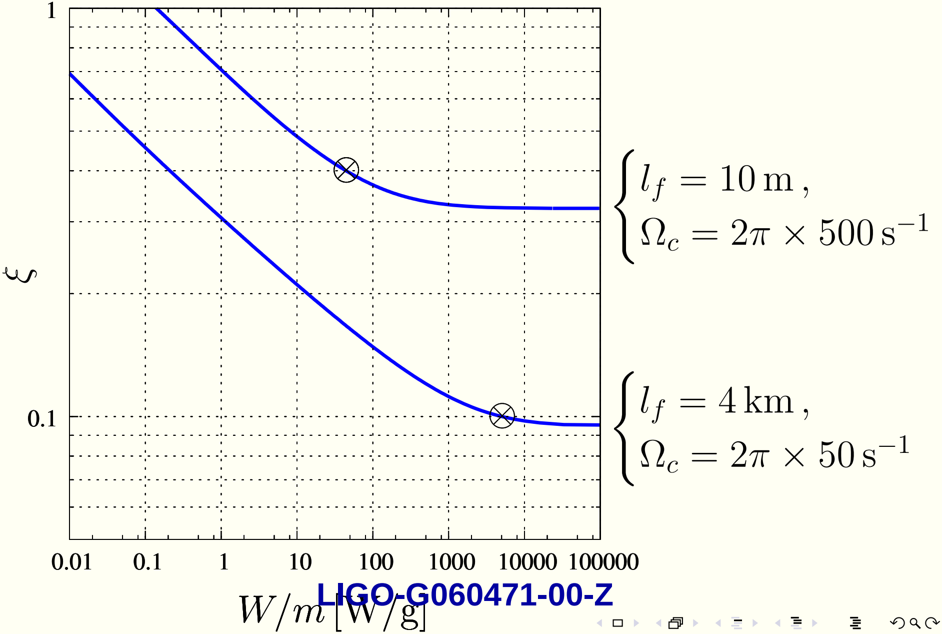


Thomas Corbitt *et al*, G040147-00 (2004)

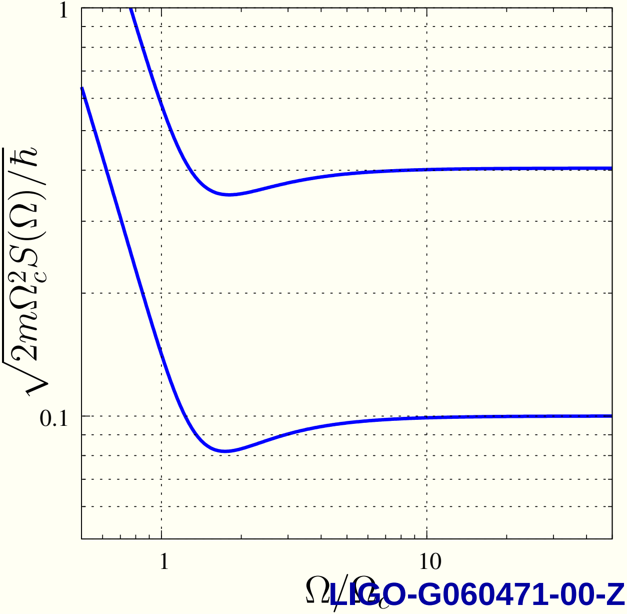
Sensitivity characterization



Sensitivity ($A_{\text{filter}}^2 = 10^{-5}$)



Spectral dependence of the sensitivity



$$\begin{cases} W/m \approx 50\,\mathrm{W/g}\,, \\ l_f = 10\,\mathrm{m}\,, \\ \Omega_c = 2\pi \times 500\,\mathrm{s}^{-1} \end{cases}$$

$$\begin{cases} W/m \approx 5\,\mathrm{kW/g}\,, \\ l_f = 4\,\mathrm{km}, \\ \Omega_c = 2\pi \times 50\,\mathrm{s}^{-1} \end{cases}$$

Conclusion

- The spectral variational measurement, probably, is the most promising way to beat the SQL in both the second- and the third-generation gravitational-wave detectors.
- Laboratory-scale prototype experiment with the goal of overcoming the Standard Quantum Limit by $2 \div 3$ could (and should?) be performed at the current technological level.