Analysis of Parametric Oscillatory Instability in Signal Recycled LIGO Interferometer

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Outline

1. Introduction
   - Effect of Parametric Oscillatory Instability
   - Parametric Instability in Power Recycled LIGO Interferometer
   - Account of anti-Stokes Mode
   - Proposed “cures” to avoid the parametric instability
2. Parametric Instability in Signal Recycled Interferometer
   - Signal Recycled Interferometer
   - Symmetric mode. Identical mirrors
   - Anti-Symmetric mode. Identical mirrors
   - Different mirrors: only one mirror is in resonance
3. Discussion
   - Comparison between SR and pure PR interferometer
   - Elastic modes
   - Generalization for GEO 600

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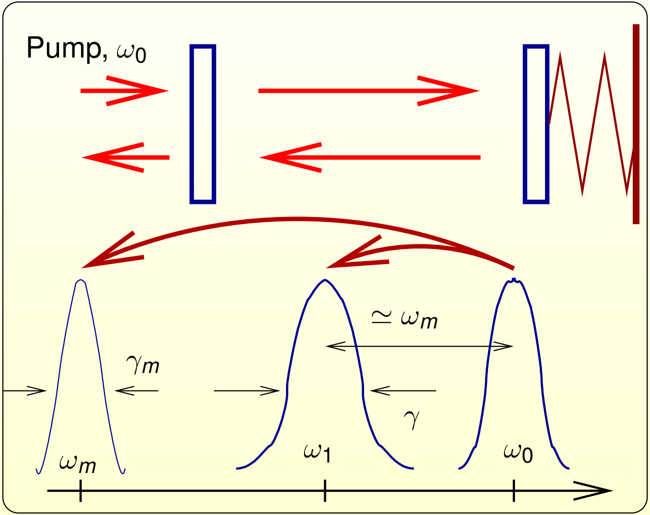
Effect of Parametric Oscillatory Instability in FP Cavity



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Condition of Parametric Instability in FP Cavity:

- Qualitatively:
  - Detuning $\Delta = \omega_0 - \omega_1 - \omega_m$ is small: $\Delta \ll \gamma$.
  - Optical power W circulating in main mode of cavity is larger than threshold value W_c.
  - Above the threshold value of power W_c:
    elastic oscillations amplitude,
    optical power in Stokes mode
    rise exponentially (!)
- Condition of parametric instability[1]:

$$\frac{\mathcal{Q}}{\gamma_m \gamma} > 1 + \frac{\Delta^2}{\gamma^2}, \quad \mathcal{Q} \equiv \frac{\Lambda_1 W \omega_1}{c L \omega_m m}, \quad \gamma \gg \gamma_m.$$

[1] V. B. Braginsky, S. E. Strigin, and S. P. Vyatchanin, *Physics Letters* **A287**, 331 (2001); gr-qc/0107079.

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Outline

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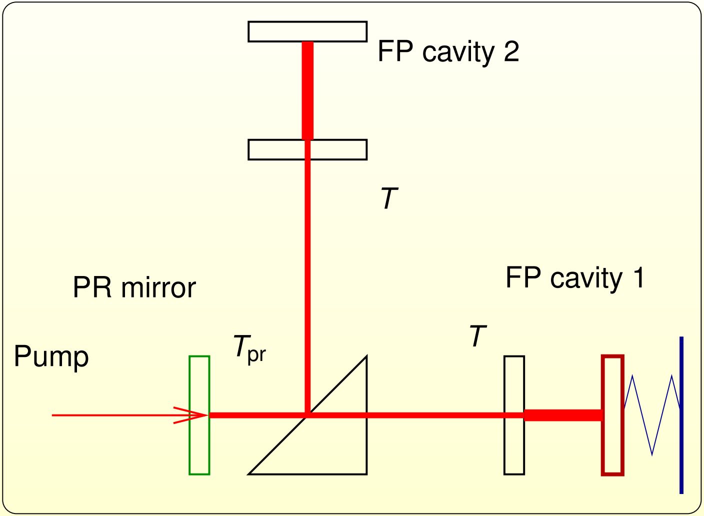
Power Recycled LIGO Interferometer



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Instability Condition in PR LIGO Interferometer

There are two relaxations rates $\gamma_{0+},\ \gamma$ in interferometer

We have inequality: $\gamma_m \ll \gamma_{0+} \ll \gamma$

$\gamma_m \simeq 10^{-3}\ \mathrm{s}^{-1}, \quad \gamma_{0+} \simeq 1\ \mathrm{s}^{-1}, \quad \gamma \simeq 100\ \mathrm{s}^{-1}$

Instability condition

See [a]

$$\frac{\mathcal{Q}}{2\gamma_m}\left(\frac{\gamma_{0+}}{\gamma_{0+}^2+\Delta^2}+\frac{\gamma}{\gamma^2+\Delta^2}\right) \geq 1\,, \quad \mathcal{Q} \equiv \frac{\Lambda_1 W \omega_1}{cL\omega_m m}\,. \tag{1}$$

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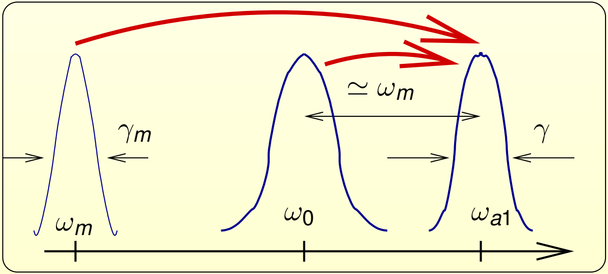
Anti-Stokes Mode

Existence of the anti-Stokes mode with frequency $\omega_{1\,a} = \omega_0 + \omega_m$ will substantially dump the effect of parametric instability[a]. However, the probability that suitable anti-Stokes mode exists is relatively small[b].

[a] E. D'Ambrosio and W. Kells, *Physics Letter* **A299**, 326 (2002). LIGO-T020008-00-D

[b] V. B. Braginsky, S. E. Strigin and S. P. Vyatchanin, *Physics Letters* **A305**, 111 (2002).



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Proposed “cures” to avoid the parametric instability:

(i) to change the mirror shape[a];
(ii) to introduce low noise damping[b];
(iii) to heat the test masses in order to vary curvature radii of mirrors and hence to control detuning and decrease overlapping factor[c].

[a] V. B. Braginsky, S. E. Strigin and S. P. Vyatchanin, *Physics Letters* **A305**, 111 (2002).

[b] V. B. Braginsky and S. P. Vyatchanin, *Physics Letters* **A293**, 228 (2002).

[c] C. Zhao, L. Ju, J. Degallaix, S. Gras, and D. G. Blair, Phys. Rev. Lett. **94**, 121102 (2005).

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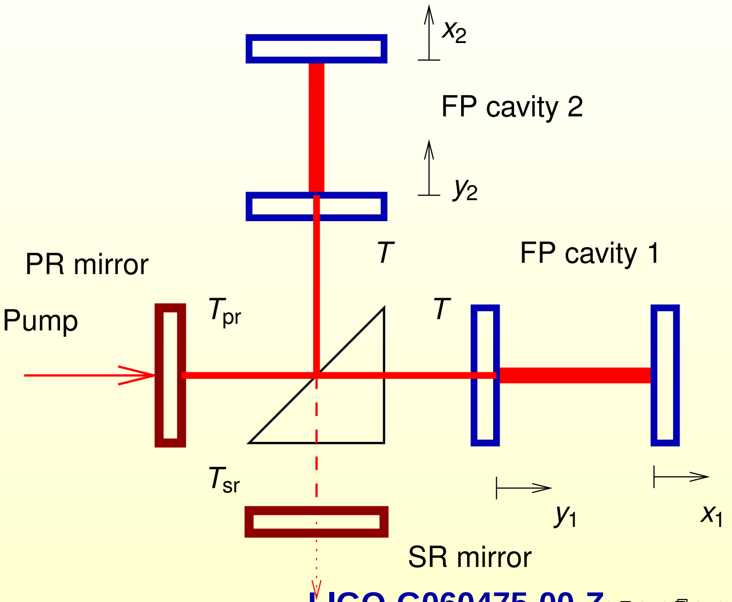

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Assumptions

- No optical losses in all mirrors.
- No suspension noise in mirrors.
- Both FP cavities are optically identical. They are tuned in resonance with the main mode.
- Optical power W circulating inside the arms is a constant (approximation of constant field).
- The distances between the input FP mirrors and beam splitter, and between the beam splitter and PR, SR mirrors are short (about several meters) — hence the phase advance of waves traveling between these mirrors is considered as a constant (no dependence on frequency).

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Symmetric Mode

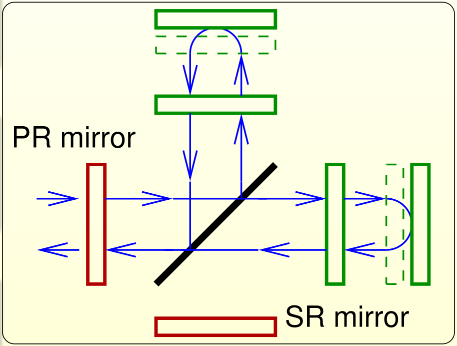
Wave enters and returns through PR mirror

Symmetric mode depends on sum coordinate z_+:

$$z_+ = (x_1 - y_1) + (x_2 - y_2)$$

Relaxation rate

$\gamma_{0+} \simeq 1\ \mathrm{s}^{-1}$ — the same as for PR recycled interferometer.



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Anti-symmetric Mode

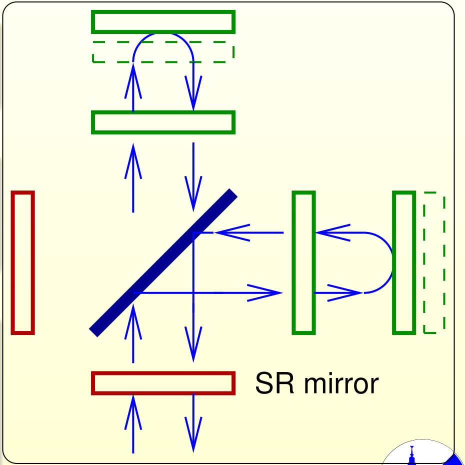
The wave enters and returns through SR mirror

Anti-symmetric mode dependes on differential coordinate z_-:

$z_- = (x_1 - y_1) - (x_2 - y_2)$

Relaxation rate

$1.6\ \mathrm{s}^{-1} \le \gamma_{0-} \le 6 \times 10^3\ \mathrm{s}^{-1}$ — for pure PR interferometer
$\gamma_{0-} = \gamma \simeq 100\ \mathrm{s}^{-1}$.



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Instability Condition for Symmetric Mode

$$\frac{2\mathcal{Q}}{\gamma_m\gamma_{0+}} > 1 + \frac{\Delta^2}{\left(\gamma_m + \gamma_{0+}\right)^2}, \quad \mathcal{Q} \equiv \frac{\Lambda_1 W\omega_1}{cL\omega_m m},$$

$$\Lambda_1 = \frac{V\left|\int \mathcal{A}_{0in}\mathcal{A}^*_{1in}u_\perp\, d\vec{r}_\perp\right|^2}{\int |\mathcal{A}_{0in}|^2\, d\vec{r}_\perp \int |\mathcal{A}_{1in}|^2\, d\vec{r}_\perp \int |\vec{u}(\vec{r})|^2\, d\vec{r}}.$$

This condition is similar to instability condition for single FP cavity if one substitutes γ_{0+} instead of γ. The factor 2 appears due to we take into account the displacements of 4 mirrors.

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Instability Condition for anti-Symmetric Mode

$$\frac{2\mathcal{Q}}{\gamma_m \gamma_{0-}} > 1 + \frac{(\Delta+\delta)^2}{\left(\gamma_m + \gamma_{0-}\right)^2}, \quad \mathcal{Q} \equiv \frac{\Lambda_1 W \omega_1}{cL\omega_m m}.$$

The relaxation rate γ_{0-} and additional detuning δ depend on the position of the SR mirror. Estimates for $\gamma_{0-},\ \delta$:

$$1.6\ \mathrm{sec}^{-1} \le \gamma_{0-} \le 6\times 10^3\ \mathrm{sec}^{-1},$$
$$-3\times 10^3\ \mathrm{sec}^{-1} \le \delta \le 3\times 10^3\ \mathrm{sec}^{-1}.$$

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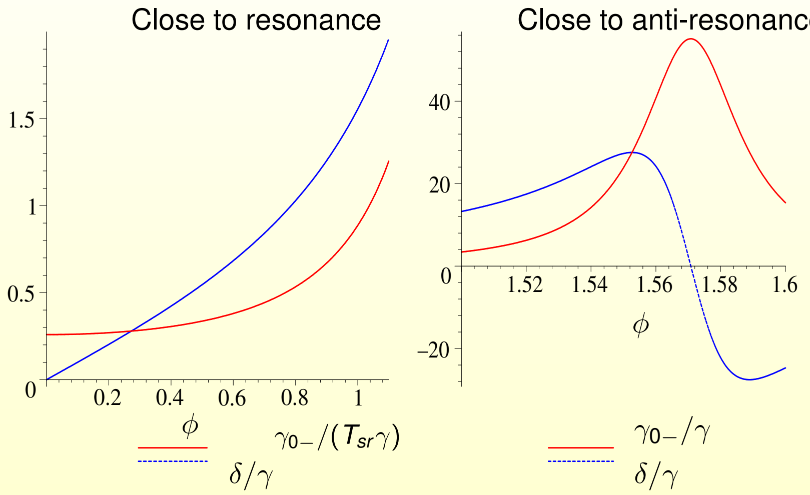

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Resipe for anti-Symmetric Mode

We can "scan" the frequency range

to find instability (or its precursors) by variation of the SR mirror position. It provides us *in situ* with very valuable information about the possible danger of parametric instability.

Precursors

Registration of Stokes modes provides information about the resonance frequencies of elastic modes with "suitable" spatial distributions. These Stokes modes may be the modes of higher orders (dipole, quadrupole and so on).

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Different mirrors: only one mirror is in resonance

Elasticaly different mirrors

The frequencies of elastic modes in mirrors do not coincide with each other and thus, we assume that the frequency of elastic mode of only one mirror is in resonance. Numerically it means that elastic frequensies in different mirrors vary by about 1 Hz — it is quite possible.

Parametric instability condition:

$$\frac{\mathcal{Q}}{2\gamma_m}\left(\frac{\gamma_{0+}}{\gamma_{0+}^2+\Delta^2}+\frac{\gamma_{0-}}{\gamma_{0-}^2+(\Delta+\delta)^2}\right)>1\,,$$

$$\mathcal{Q}\equiv\frac{\Lambda_1 W\omega_1}{cL\omega_m m}\,,\quad \gamma_m\ll\gamma_{0+}<\gamma_{0-}$$

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Outline

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Comparison between SR and pure PR interferometer

Parametric instability condition for SR interferometer:

$$\frac{\mathcal{Q}}{2\gamma_m}\left(\frac{\gamma_{0+}}{\gamma_{0+}^2+\Delta^2}+\frac{\gamma_{0-}}{\gamma_{0-}^2+(\Delta+\delta)^2}\right)>1,\quad \gamma_{0-}\simeq 2\ldots 10\ \mathrm{s}^{-1}$$

Parametric instability condition for pure PR interferometer:

$$\frac{\mathcal{Q}}{2\gamma_m}\left(\frac{\gamma_{0+}}{\gamma_{0+}^2+\Delta^2}+\frac{\gamma}{\gamma^2+\Delta^2}\right)>1,\quad \gamma\simeq 100\ \mathrm{s}^{-1}$$

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Estimates

Small Detuning — Small Chance to Fall into Trap

Let $\Delta \ll \gamma_{0+} \simeq 2\ \text{s}^{-1}$ and $\delta \gg \gamma_{0-} \simeq 2\ldots 10\ \text{s}^{-1}$. Then parametric instability will take place at power $W_c \simeq 5$ W (!) (if $\omega_m = 10^5\ \text{sec}^{-1}$, $\gamma_m = 6\times 10^{-4}\,\text{sec}^{-1}$, $\Lambda_1 \simeq 1$). However, there is small chance that such small detuning takes place.

Large detuning ($|\Delta| > \gamma_{0+}$)

The realization of parametric instability for large detuning requires dramatically larger optical power: $W_c \sim \Delta^2/\gamma_{0+}^2$. For example, if detuning is about 1 kHz and other parameters are the same one can obtain $W_c \simeq 10^8$ W (!). Advanced LIGO plans to use $W \simeq 10^6$ W.

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Insufficient Accuracy of Numerical Calculation of Elastic Modes

Insufficient Accuracy

Accuracy of standard packages FEMLAB or ANSYS
is about several percents only (!).
We need the accuracy at least $\gamma_{0+}/\omega_m \simeq 10^{-7} \div 10^{-5}$ (!).

Numerical calculations have some sence

Nevertheless, such calculations have sense:
(i) to estimates of overlapping factors;
(ii) to get approximate information about frequency and structure of the elastic modes.

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Even improved numerical methods can not solve the problem completely:

Not a cylindric shape of mirrors

For example, the pins to attach fiber may produce the shift of elastic mode frequency up to 100 $\sec^{-1}$[a].

[a] V. B. Braginsky, S. E. Strigin and S. P. Vyatchanin, *Physics Letters* **A305**, 111 (2002).

The inhomogeneity of Young modulus and density of fused silica

may provide an uncontrollable relative shift of elastic mode frequency about percents[a]

[a] V. B. Braginsky, S. E. Strigin and S. P. Vyatchanin, *Physics Letters* **A305**, 111 (2002).

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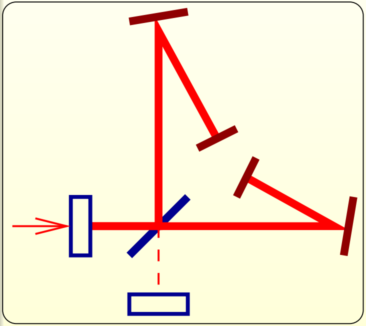
Generalization for Geo 600 (interferometer without FP cavities in arms):

The relaxation rate of symmetric mode is about $\gamma_{0+}^{\rm GEO} \simeq T_{pr} c/4L \simeq 0.75 \times 10^5\ \sec^{-1}$ ($T_{pr} = 0.012,\ L = 1.2$ km).

The parametric instability may take place at relatively small optical power
$W_c^{\rm GEO} \simeq 100$ W in arms
(we assume zero detuning $\Delta = 0$,
$m = 10$ kg, $\omega_m = 10^4\ \sec^{-1}$,
$\gamma_m = 10^{-4}\ \sec^{-1}$ and $\Lambda_1 = 1$).



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Summary

- The possibility of parametric instability in SR interferometer is smaller than in pure PR interferometer.
- We can "scan" the frequency range to find instability (or its precursors) by variation of the SR mirror position.
- Outlook
  To be done: to account the optical rigidity produced by variation of SR mirror position (not to use the approximation of constant power).
  To be done: to consider FP cavity having slightly different frequencies of Stokes mode (Bill Kells).

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