

1 April 97

LIGO Science Mtg. Talk

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SCATTERED LIGHT AND ITS CONTROL IN THE LIGO BEAM TUBES*

References

- Born & Wolfe (diffraction)
- Beckmann & Spizzichino (scatter from rough surfaces)
- Stover (optical scattering)
- R. Weiss - numerous LIGO internal reports dealing with application to B.T.
- K. Thorne / G. Flannigan - same as above
- J.Y. Vinet - VIRGO Analyses

Outline

- noise equivalent strain
- dominant scattered light processes (LIGO BT)
 - offending paths - identification
 - paths & baffling
- time dependent phase shifts from vibrating surfaces
- Scattering & recombination from rough surfaces
 - BRDF
 - detailed balance
- combining effects (end-to-end)
- LIGO requirement
- LIGO design today - projected performance

* Similar issue for UE chambers, COS, LSC (P.D.)

Interferometer response to scattered light - noise equivalent strain



$E' = E_0 + \delta E = |E|e^{i\phi_{GW}}$

① $\delta \phi_{GW} = \text{Im} \left\{ \frac{\delta E}{E} \right\}$

$\delta E \sim i \delta \phi_{GW} \Rightarrow \frac{\delta E}{E}$

② $\delta \phi_{GW} = \left| \frac{\delta \phi}{\phi} \right| \delta \phi$

ITD response



$\frac{d\phi}{dh}(f) = \frac{4\pi \tau_{ST} c}{\lambda [1 + (4\pi \tau_{ST} f)^2]^{1/2}}$

$\left| \frac{d\phi}{dh} \right| = 2\lambda L N_B \frac{1}{\text{K.C.}} [1 + (2\pi N_B f)^2]^{-1/2}$

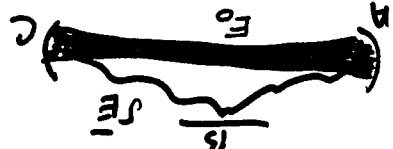
①+② $\delta h = \frac{1}{\text{Im} \left\{ \frac{\delta E}{E} \right\}} \left| \frac{d\phi}{dh} \right|$

Scattered light "EMI": $\gamma_{\text{with}} \rightarrow \text{path} \rightarrow \text{FUR} \rightarrow \text{interference}$

imperfect moving VAC envelope surfaces
imperfect optic

3rd order process:

$\frac{\delta P}{P} \sim \text{Prob}(A|B) \text{Prob}(B|C) \text{Prob}(C|A)$ "A" = main beam



Scattered light also builds up resonantly by factor N_B

one time reversed pairs (antenna theorem) (detailed Balance)

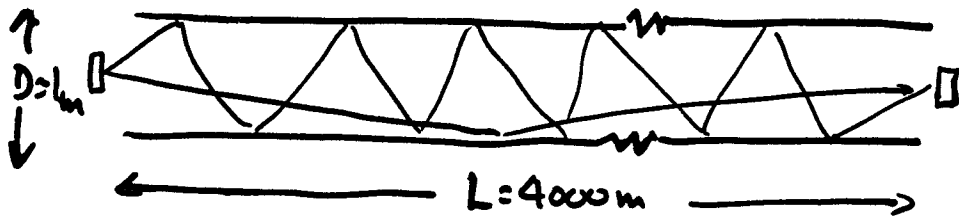
Processes $A \rightarrow B$
 $C \rightarrow A$

determine δE_{form} \rightarrow

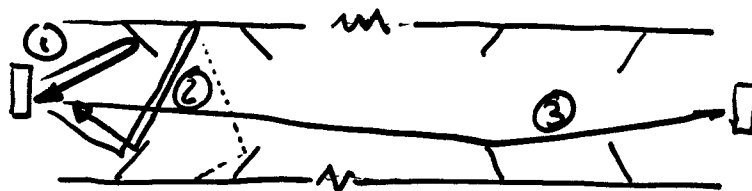
δE : \ll weighted by probability
noise equivalent strain

Scattered light in LIGO B.T.

- BT is a "light pipe" (lossy!)



- $L/D \sim 4000 \Rightarrow$ lots of opportunity for scattering.
- identification of reflection paths is not analytical
 - need optical (rough) design
 - use numerical methods to identify multipath interference
 - Monte Carlo
 - Launch rays with (θ, ϕ)
 - follow rays through multiple reflections
 - keep track of cumulative probabilities/intensities
 - LIGO had "baseline" design analyzed by Breault Research Org. (BRO)
 - intra cavity scatter (highest finesse) worst problem
 - recombination @ detector also a concern (LOS)

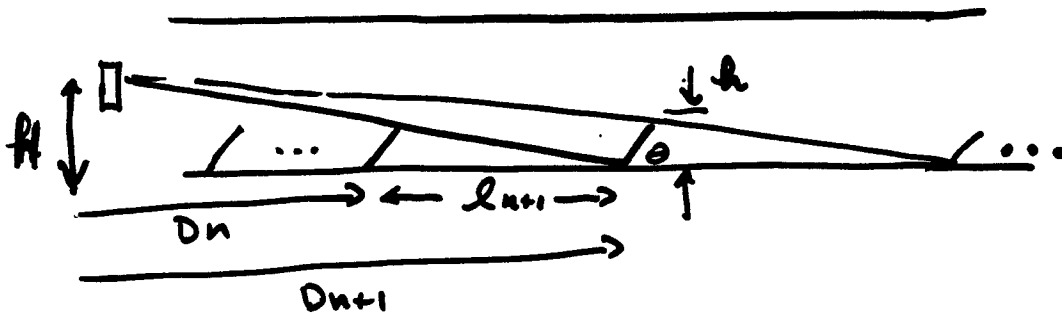


- ① Backscatter from baffles
- ② Reflect off baffles (Backscatter off BT wall)
- ③ Diffract from baffle edges.

- use baffling to suppress direct viewing of BT wall by mirrors
- Paths ①, ②, ③ worst remaining offenders
- To estimate magnitude of effects need to formulate
 - scattering off (nearly perfect) mirrors
 - " BT walls/baffles
 - diffraction off baffle edges
 - recombination of scattered light into TEM00 beam

LIGO Baffle design -

- select spacing to completely mask BT surface from mirrors
- select height to keep baffle count low while not impacting clear aperture
- incline baffles away from nearest mirror
 - avoid "retro angles"
 - direct light away from nearest mirror
- serrate edges to reduce (coherent) diffraction



$$\frac{H}{D_n} = \frac{h}{l_{n+1}} \quad ; \quad D_{n+1} = D_n + l_{n+1}$$

$$\Rightarrow \frac{l_{n+1}}{D_n} = \left[\frac{H}{H-h} \right] \equiv \beta > 1$$

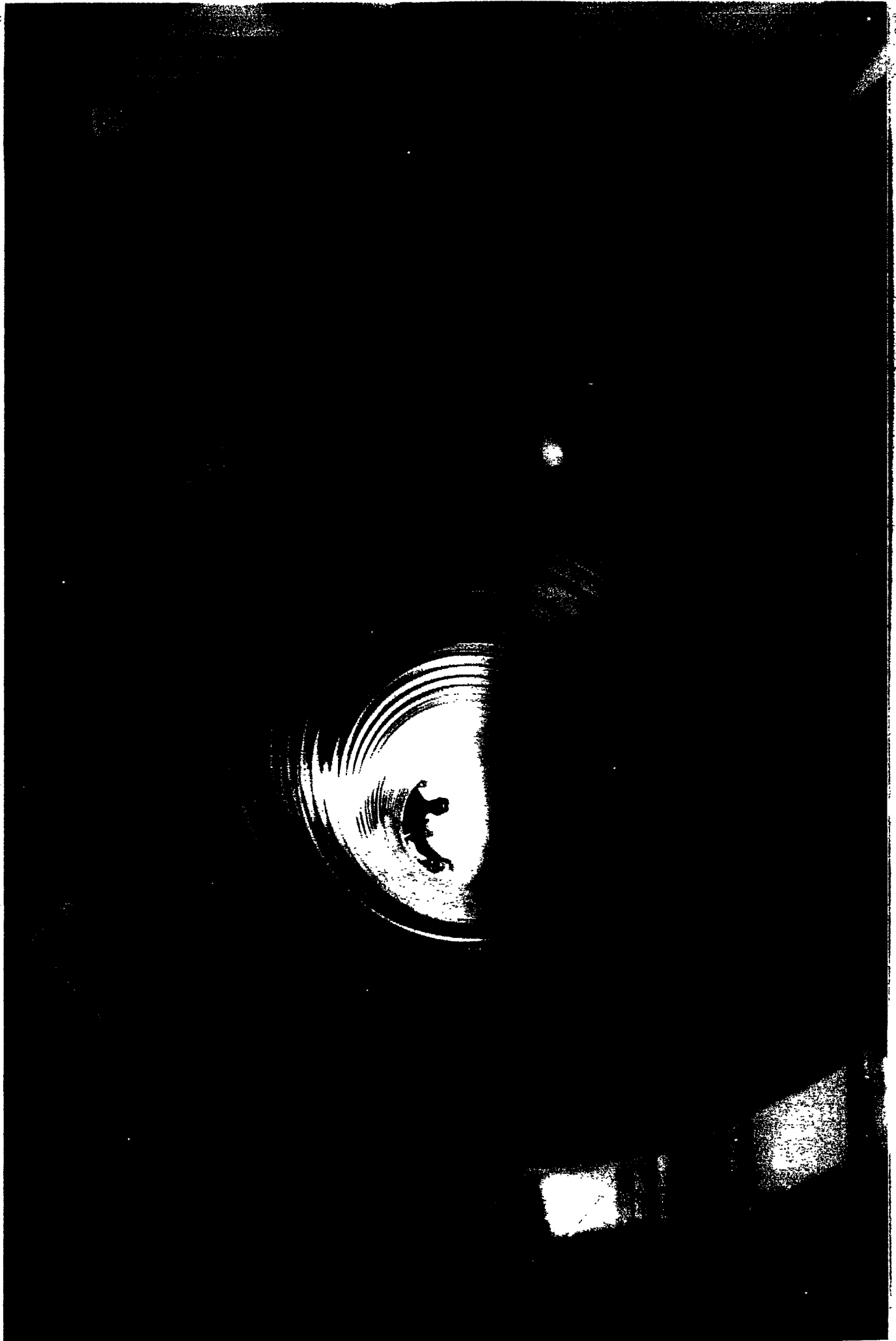
$$l_{n+1} = l_1 \beta^n$$

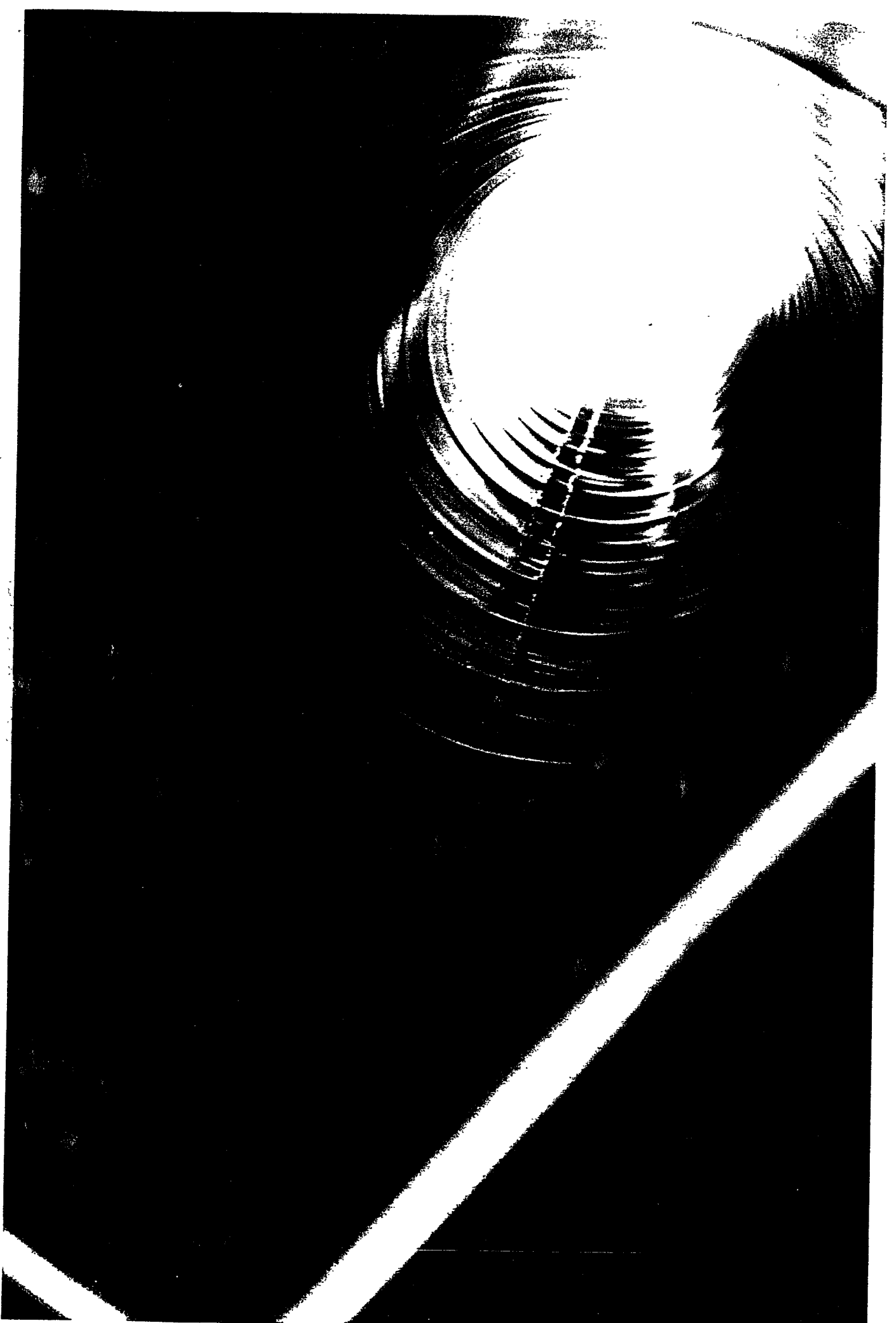
- Geometric progression

- $h = 9 \text{ cm}$

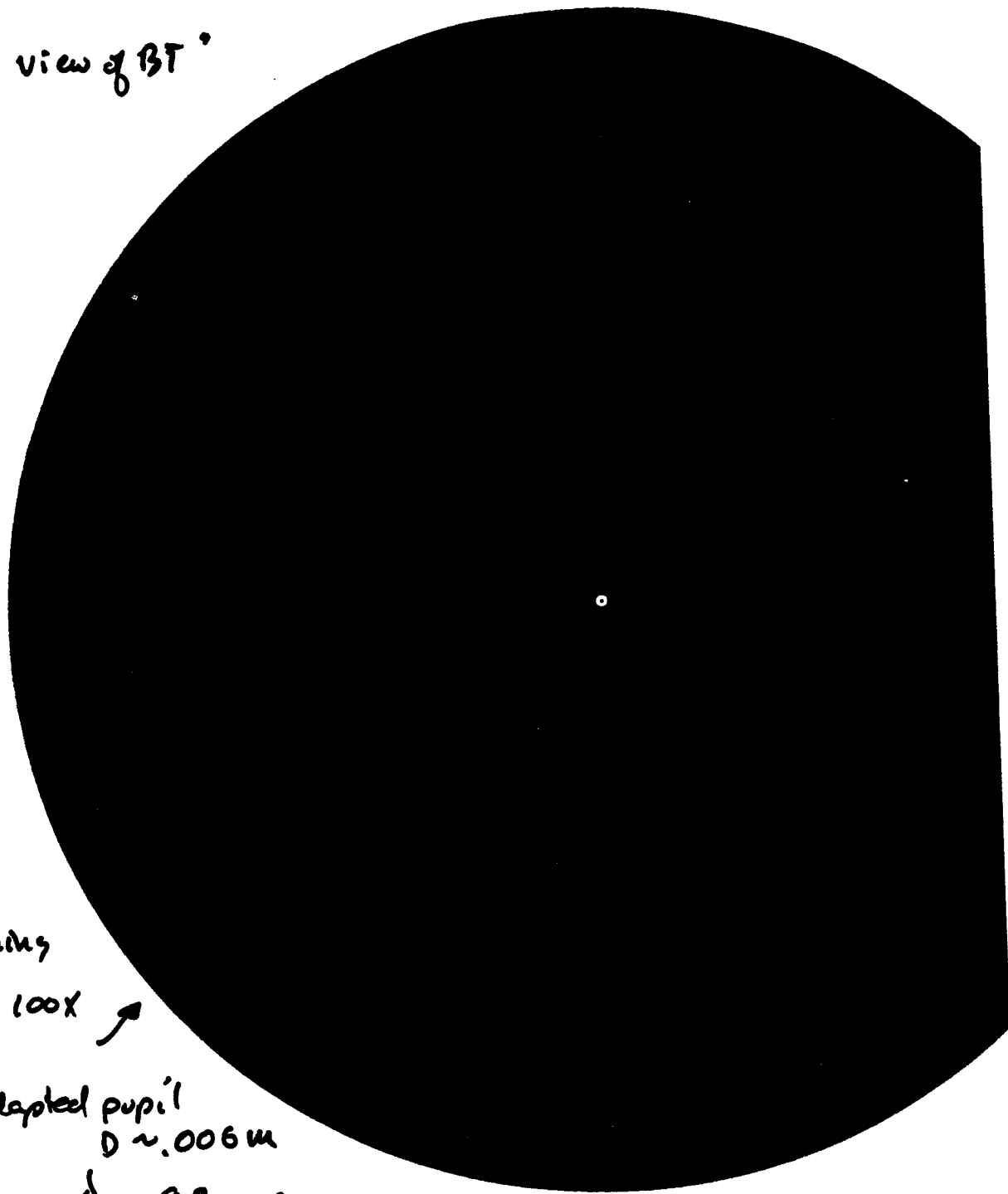
- $\theta = 35^\circ$ ($\theta_i = 55^\circ$ for scattering processes)







"mirror's eye view of BT"



• End opening

magnified 100x ↗

• Dark adapted pupil
 $D \sim 0.006 \text{ m}$

$$\Rightarrow \Delta\theta_{\text{off}} = \frac{\lambda}{D} = 89 \mu\text{r}$$

$\Rightarrow \Delta X_{\text{min}} = 0.3 \text{ m} \Rightarrow$ opening is a pointlike star

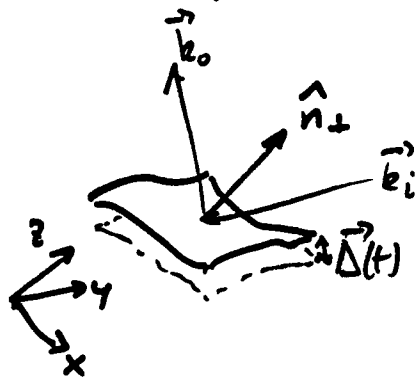
• Ambient sky brightness (day) $\sim 30 \text{ W/m}^2/\text{sr}$

$\Rightarrow 2 \mu\text{W/m}^2$ @ 4000 m \rightarrow Very dark

$\Rightarrow m_v \sim +36 \leftarrow$

I.

Scattering from moving surface produces time-dependent phase shifts



Non specular: $\vec{k}_0, \vec{k}_i, \hat{n}_+$ not in same plane

surface: $z = f(x, y)$

motion: surface translates by $\vec{\Delta}(t)$

$\vec{k}_i \cdot \vec{r}$

• Incident plane wave

$|\psi_i\rangle \sim e$

$|\psi_i\rangle = e^{i[k_x x + k_y y + k_z f(x, y)]}$

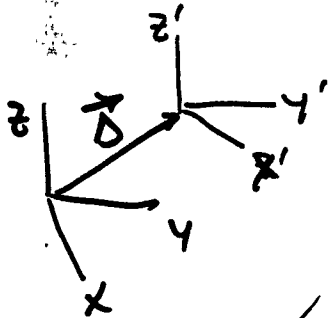
• Scattered wave

$|\psi_0\rangle = e^{i[k'_x x + k'_y y + k'_z f(x, y)]}$

Consider at $t=0$ $\langle \psi_0 | \psi_i \rangle = \int dx dy e^{i(\Delta k_x x + \Delta k_y y + \Delta k_z f(x, y))}$

let

$\vec{\Delta}(t) = \{ \xi(t), \eta(t), \zeta(t) \}$



$x = x' + \xi$

$z' = f(x', y')$

$y = y' + \eta$

$z = \zeta + f(x', y')$

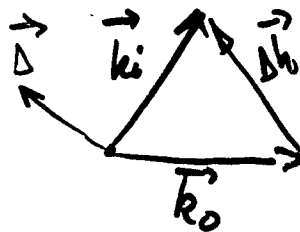
$z = z' + \zeta$

$\langle \psi_0 | \psi_i \rangle_t = \int dx' dy' [e^{i \vec{\Delta k} \cdot \vec{\Delta}} e^{i(k'_x x' + k'_y y' + k'_z f(x', y'))}$

$= e^{i \vec{\Delta k} \cdot \vec{\Delta}(t)}$
dynamic phase shift

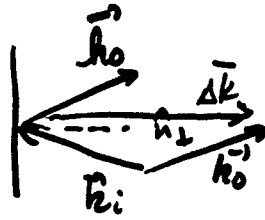
$\langle \psi_0 | \psi_i \rangle_0$
static form factor

$\phi(t) = \vec{\Delta k} \cdot \vec{\Delta}(t)$



I Scatter by a moving surface

• Reflection

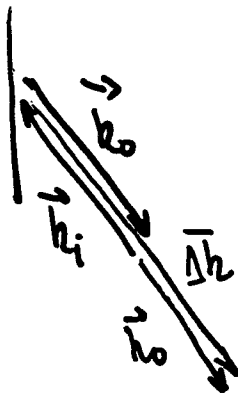


$$\Delta \vec{k} = -2(\vec{k}_i \cdot \hat{n}_\perp) \hat{n}_\perp$$

$$\delta\phi = -2(\vec{k}_i \cdot \hat{n}_\perp)(\vec{\Delta} \cdot \hat{n}_\perp)$$

- sensitive to motion
 \perp to surface

• Backscatter

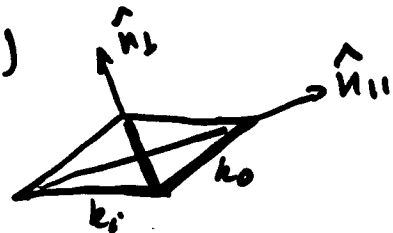
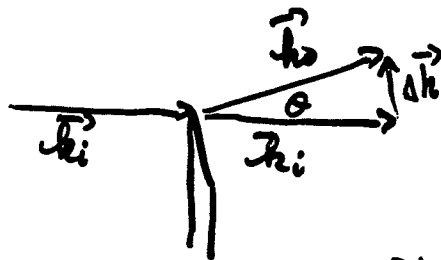


$$\Delta \vec{k} = -2 \vec{k}_i$$

$$\delta\phi = -2 \vec{k}_i \cdot \vec{\Delta}$$

- sensitive to motion
 along line of sight (L.O.S.)

• Small angle scatter (Diffraction)



$$\Delta \vec{k} = k\theta \hat{n}_\perp$$

$$\delta\phi = k\theta \vec{\Delta} \cdot \hat{n}_\perp = k\theta \Delta_\perp$$

- sensitive to motion
 \perp to L.O.S.

I

Light Scattering phase noise



$$\frac{\delta E}{E} = \cos \phi + i \sin \phi$$

- Whereas $\phi \ll 1$ for GW, cannot assume this for general light scatter
- rough ($\gg \lambda$) surface
- large amplitudes of motion (on resonance) are possible.

$$h^2 = \frac{1}{|\frac{d\phi}{dh}|^2} \int \text{spectral density of } \sin \phi(\omega)$$

- Small (dynamic) motion δ

$$\phi = \phi_0 + \delta\phi(t)$$

\downarrow small excursion
 \downarrow random constant

$$\sin \phi = \cos \phi_0 \delta\phi(t) + \sin \phi_0$$

$$\langle \sin^2 \phi \rangle = \frac{1}{2} \langle \delta\phi(t)^2 \rangle$$

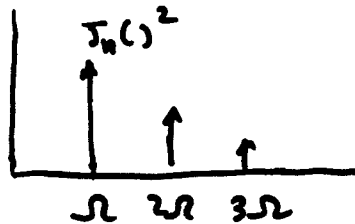
- large ($> \lambda$) motion

$$x(t) = x_0 \cos \Omega t$$

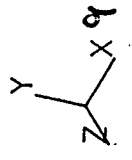
$$\sin \phi = \sin(\phi_0 + kx_0 \cos(\Omega t)) \leftarrow \text{PM}$$

factor from orientation of motion $\neq \Delta \phi_0$

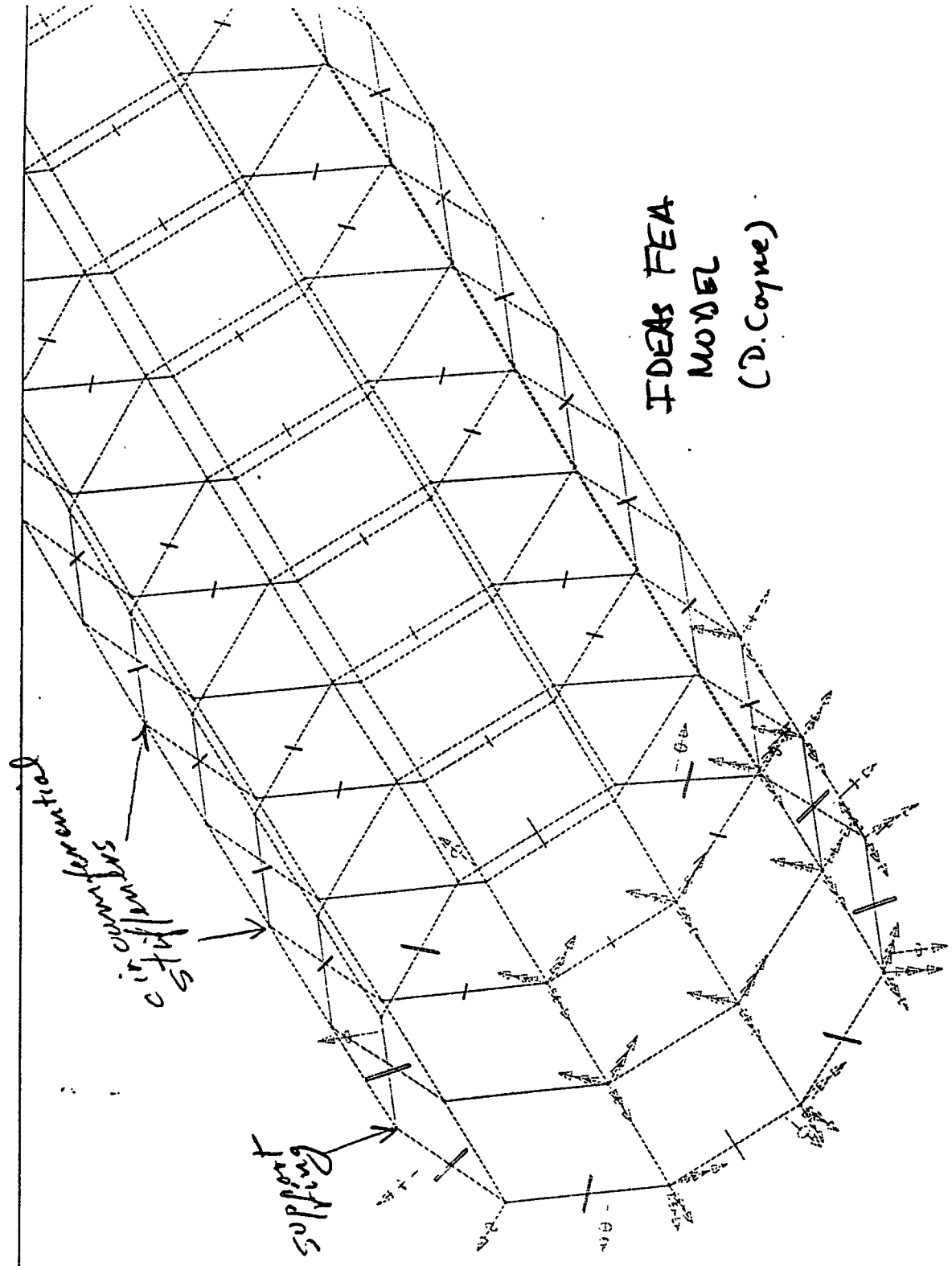
$$\sin \phi = \sum_n J_n(kx_0) e^{i n \Omega t + \alpha_n} \leftarrow \text{up conversion}$$



5



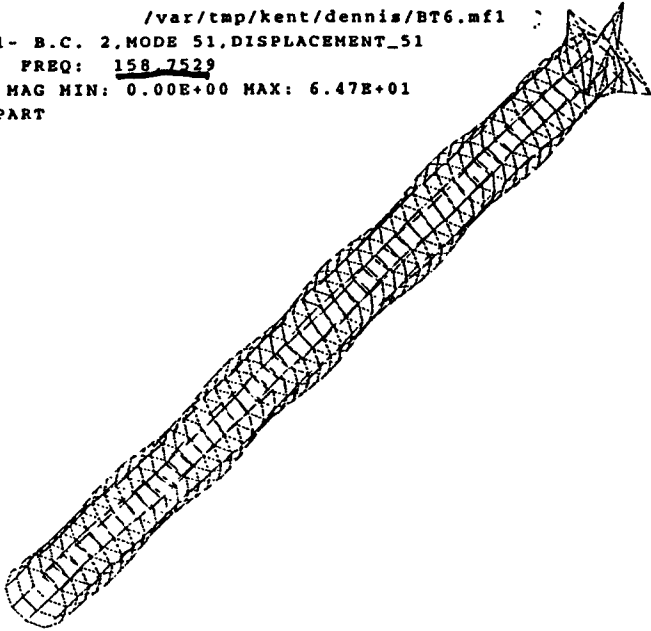
IDEAS FEA
MODEL
(D. Coyne)



BT FEA NORMAL MODES

/var/tmp/kent/dennis/BT6.mf1

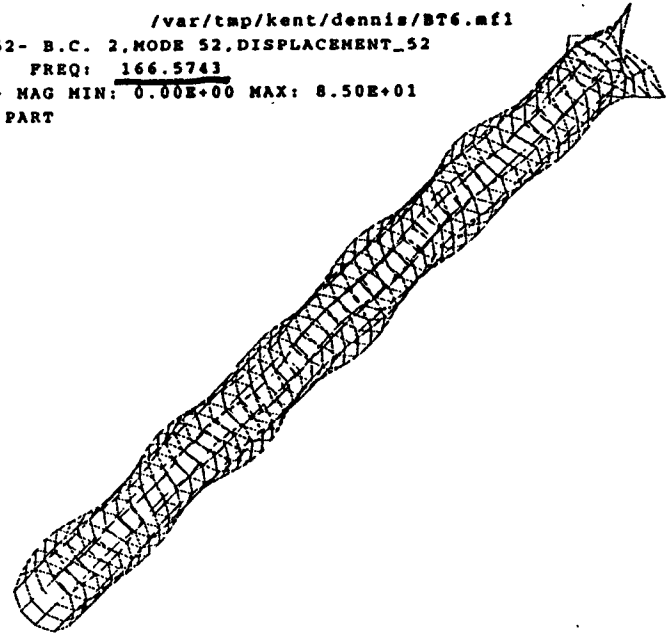
DEFORMATION: 51- B.C. 2, MODE 51, DISPLACEMENT_51
MODE: 51 FREQ: 158.7529
Displacement - MAG MIN: 0.00E+00 MAX: 6.47E+01
FRAME OF REF: PART



1

/var/tmp/kent/dennis/BT6.mf1

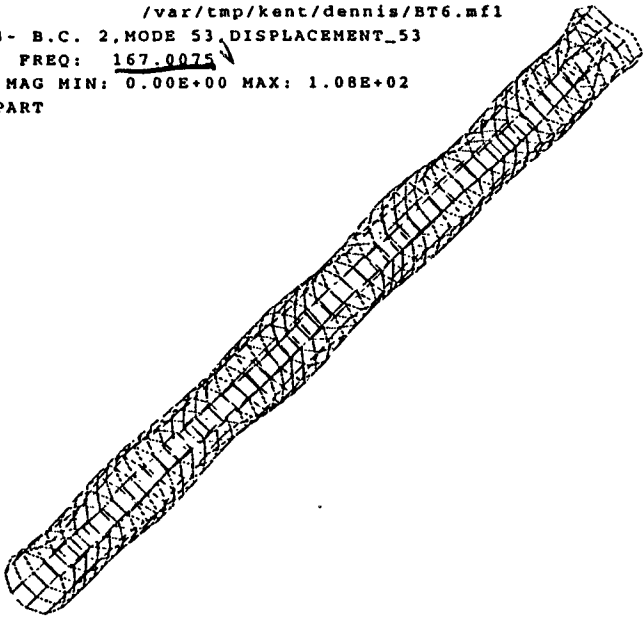
DEFORMATION: 52- B.C. 2, MODE 52, DISPLACEMENT_52
MODE: 52 FREQ: 166.5743
Displacement - MAG MIN: 0.00E+00 MAX: 8.50E+01
FRAME OF REF: PART



2

/var/tmp/kent/dennis/BT6.mf1

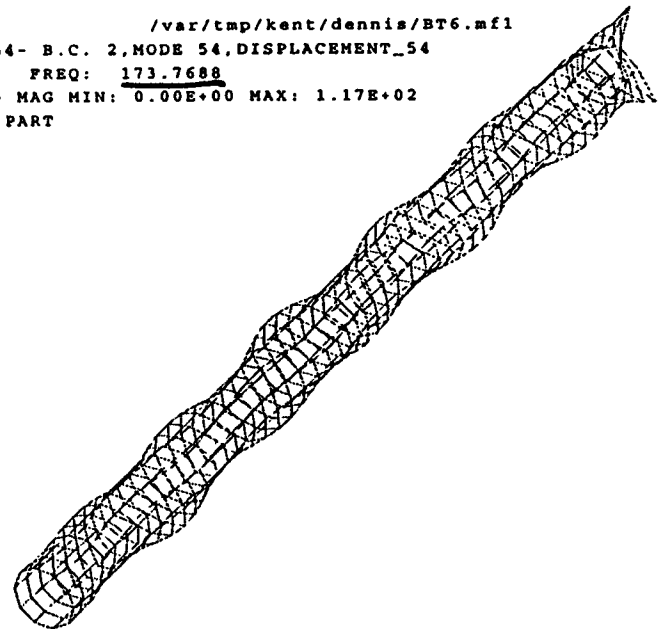
DEFORMATION: 53- B.C. 2, MODE 53, DISPLACEMENT_53
MODE: 53 FREQ: 167.0075
Displacement - MAG MIN: 0.00E+00 MAX: 1.08E+02
FRAME OF REF: PART



3

/var/tmp/kent/dennis/BT6.mf1

DEFORMATION: 54- B.C. 2, MODE 54, DISPLACEMENT_54
MODE: 54 FREQ: 173.7688
Displacement - MAG MIN: 0.00E+00 MAX: 1.17E+02
FRAME OF REF: PART



4

BT FE1 NORMAL MODES

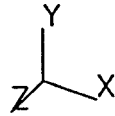
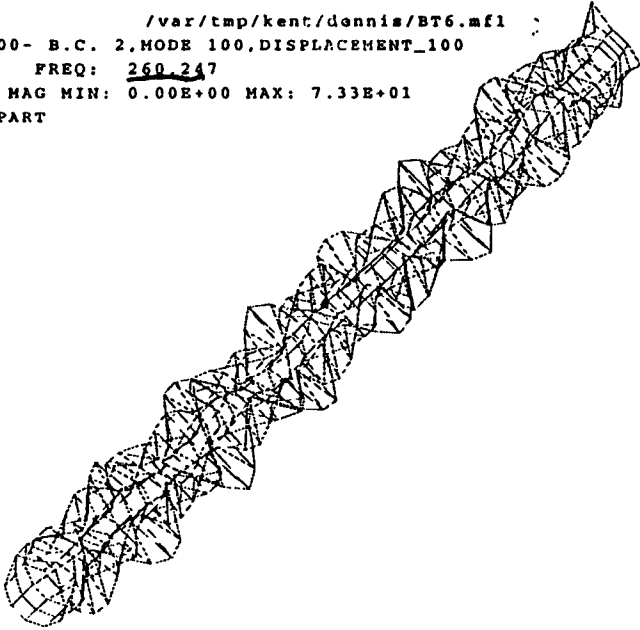
/var/tmp/kent/dennis/BT6.mf1

DEFORMATION: 100- B.C. 2,MODE 100,DISPLACEMENT_100

MODE: 100 FREQ: 260.247

Displacement - MAG MIN: 0.00E+00 MAX: 7.33E+01

FRAME OF REF: PART



1

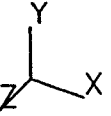
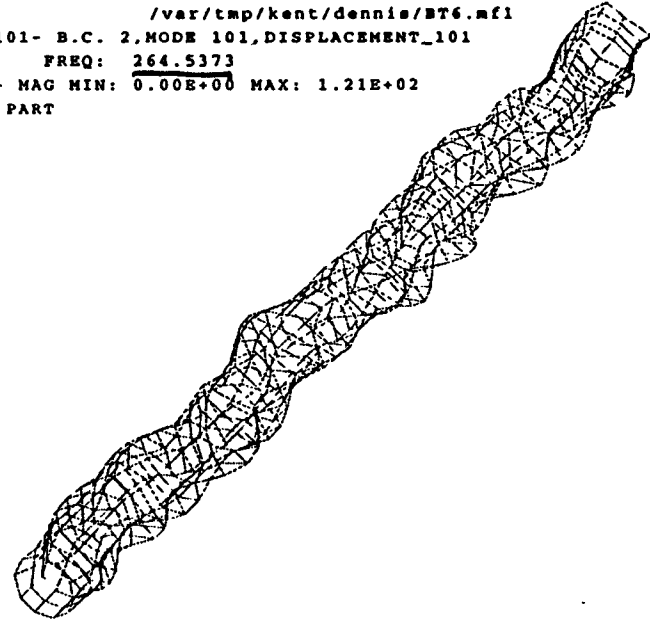
/var/tmp/kent/dennis/BT6.mf1

DEFORMATION: 101- B.C. 2,MODE 101,DISPLACEMENT_101

MODE: 101 FREQ: 264.5373

Displacement - MAG MIN: 0.00E+00 MAX: 1.21E+02

FRAME OF REF: PART



2

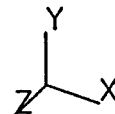
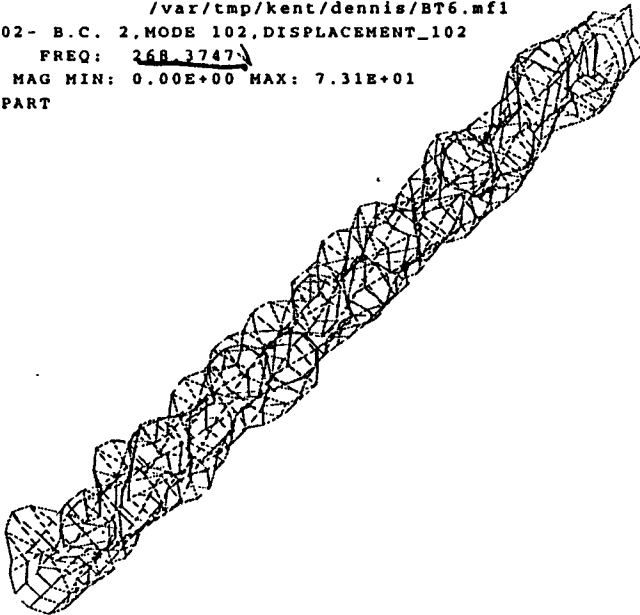
/var/tmp/kent/dennis/BT6.mf1

DEFORMATION: 102- B.C. 2,MODE 102,DISPLACEMENT_102

MODE: 102 FREQ: 268.3747

Displacement - MAG MIN: 0.00E+00 MAX: 7.31E+01

FRAME OF REF: PART



3

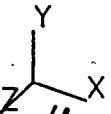
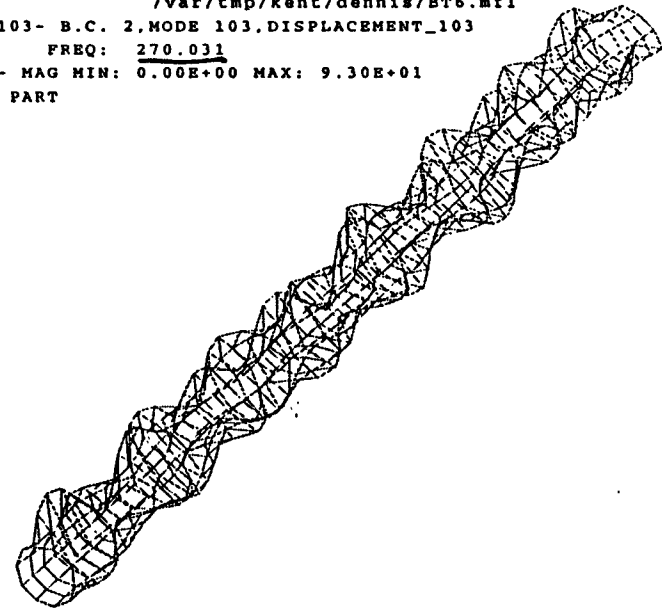
/var/tmp/kent/dennis/BT6.mf1

DEFORMATION: 103- B.C. 2,MODE 103,DISPLACEMENT_103

MODE: 103 FREQ: 270.031

Displacement - MAG MIN: 0.00E+00 MAX: 9.30E+01

FRAME OF REF: PART



4

S6 → A5x

Frequency response function

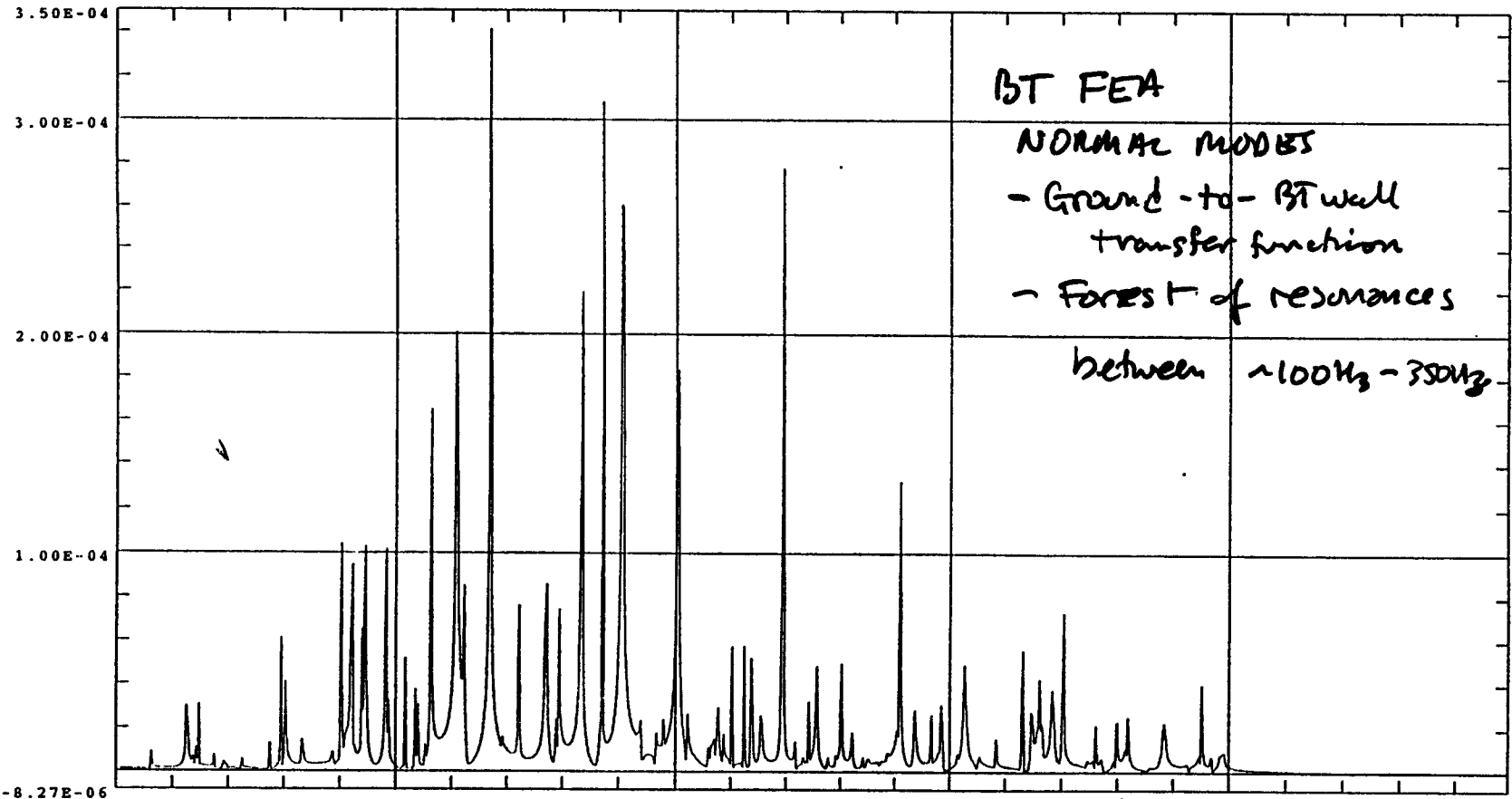
PHASE

$\phi(f)$



MODULUS

$|a(f)|$



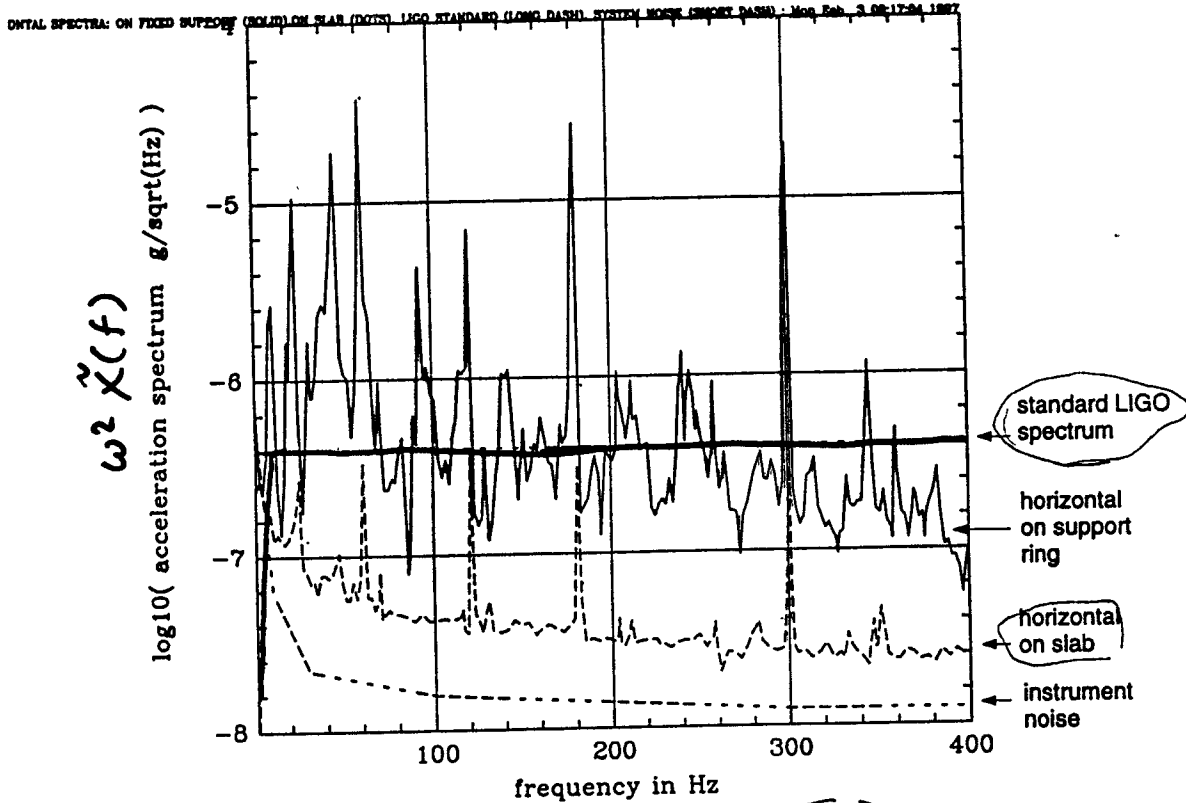
BT FEA
NORMAL MODES
- Ground-to-BT well transfer function
- Forest of resonances between ~100Hz - 350Hz

0.00 100.00 200.00 300.00 400.00 499.25

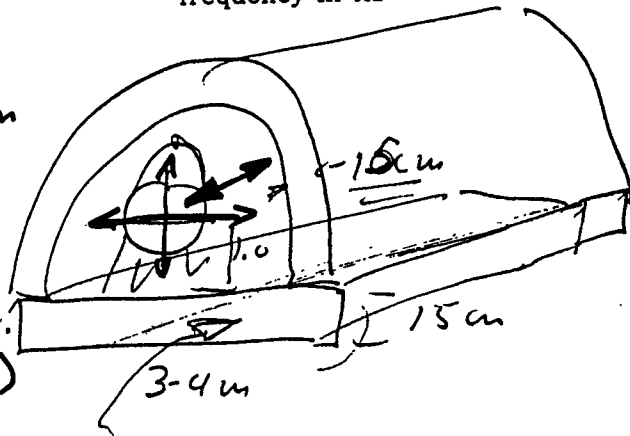
Frequency (Hz)

1/ID=427
Node 1046-X
Acceleration

LIGO X arm BT wall motion Field Measurements (Weiss)

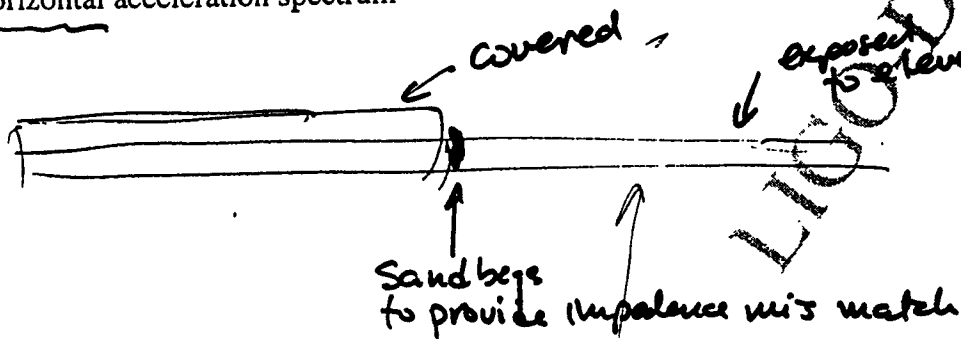


- lots of BT motion
- still under construction
- * - not coherent w/ ground
- Acoustically driven
- Without Bakeout insulation (15cm)
- will attenuate



• Like harmonics due to mech. vib of xformer cores on slab (GBT)

Figure 2 Horizontal acceleration spectrum



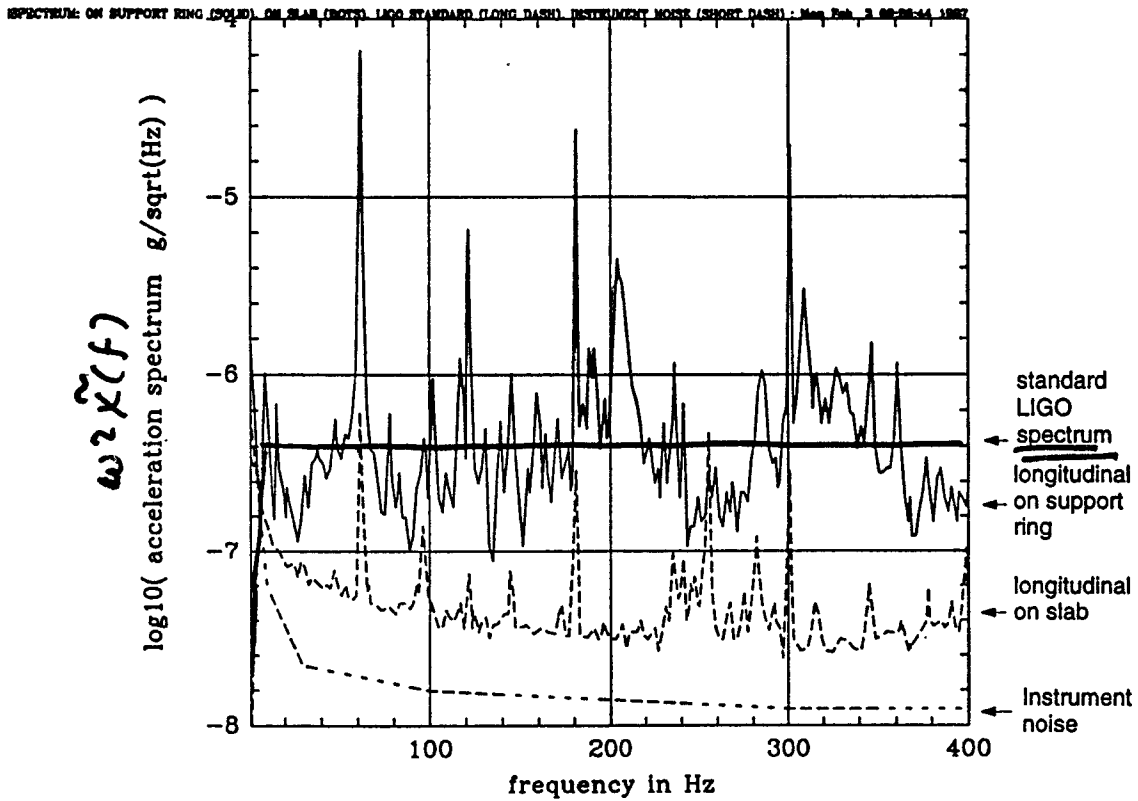


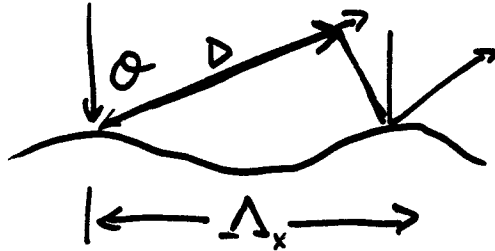
Figure 1 Longitudinal acceleration spectrum.

LIGO-DRAFT

II. Scatter from a rough surface - BRDF

BRDF = Bidirectional Reflectance Distribution Function

- Angular distribution of scattered light
- Surface features modify wavefront \rightarrow far field diffraction
- Related to spectral density of surface errors through Bragg condition \downarrow scatter

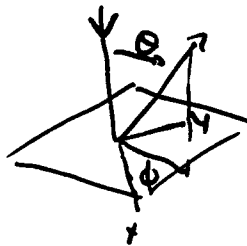


$$\Delta_n = n\lambda = \Delta_x \sin \theta \quad n \rightarrow 1 \text{ (small angle scatter)}$$

$$\lambda = \Delta_x \theta \rightarrow \nu_x \lambda = \theta_x \quad \text{spatial frequency } \nu_x$$

\rightarrow maps into scattering angle θ .

2D:



$$\lambda = \Delta_x \sin \theta \cos \phi$$

$$\lambda = \Delta_y \sin \theta \sin \phi$$

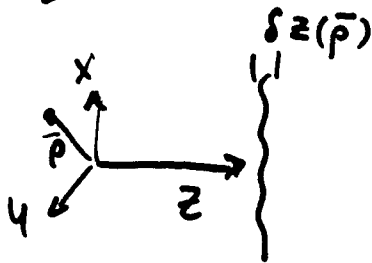
$$\nu_y = \frac{1}{\lambda} \sin \theta \sin \phi$$

$$\nu_x = \frac{1}{\lambda} \sin \theta \cos \phi$$

$$d\nu_x d\nu_y = \frac{1}{\lambda^2} J[\nu_x, \nu_y, \theta, \phi] d\theta d\phi$$

$$\underline{d\nu_x d\nu_y} = \underline{\frac{1}{\lambda^2} d\Omega}$$

II irregular surface scatter from a surface



$$\leftarrow \psi_0 = \sqrt{I_0} e^{i k z}$$

$$\rightarrow \psi' = \sqrt{I_0} e^{i k z'} e^{z i k \delta z(\bar{r})}$$

↑ excess phase perturbation upon reflection
 $\ll 1$

$$\langle \delta z \rangle = 0 \quad \langle \delta z^2 \rangle = \sigma^2$$

$$\langle \delta z(\bar{r}_1) \delta z(\bar{r}_2) \rangle = \sigma^2 C(\bar{r}_1, \bar{r}_2)$$

Consider F.T. of ψ' @ $z=0$

↑ spatial correlation fn. (normalized)

$$\Psi(v_x, v_y) = \sqrt{I_0} \int dA e^{i 2\pi v_x x} e^{i 2\pi v_y y} e^{z i k \delta z(\bar{r})}$$

$$|\Psi(v_x, v_y)|^2 = I_0 \int dA dA' e^{i 2\pi v_x (x-x')} e^{i 2\pi v_y (y-y')} e^{z i k [\delta z(x,y) - \delta z(x',y')]}$$

$$|\Psi(v_x, v_y)|^2 = I_0 \int dA dA' e^{i 2\pi v_x (x-x')} e^{i 2\pi v_y (y-y')} e^{i z} = 1 - \frac{z^2}{2} + i z + \dots$$

statistical avg:

$$\langle |\Psi(v_x, v_y)|^2 \rangle = \underbrace{I_0 \pi a^2 [1 - 4k^2 \sigma^2]}_{P_0} \delta(v_x) \delta(v_y) + \underbrace{I_0 \pi a^2 4k^2 \sigma^2}_{P_0} \tilde{C}(v_x, v_y)$$

specular diffuse

$$\int \langle |\Psi(v_x, v_y)|^2 \rangle dv_x dv_y = P_0$$

Consider scattered (diffuse part):

$$\int \langle |\Psi(v_x, v_y)|^2 \rangle_{\text{scatt}} dv_x dv_y = P_0 \underbrace{4k^2 \sigma^2}_{\text{surface irregularity leads to scatter}}$$

$$P_0 4k^2 \sigma^2 = \int \langle |\Psi|^2 \rangle \frac{1}{\lambda^2} d\Omega$$

$$\frac{dP_{sc}}{d\Omega} = \frac{P_0}{\lambda^2} [4k^2 \sigma^2] \tilde{C}(v_x, v_y) \quad \text{W/sr}$$

↑ spectral density of surface errors
 $\frac{1}{\lambda^2}$ dependence \rightarrow Rayleigh's scat. theory

II

Scatter from a surface

$$\underbrace{\frac{1}{P_0} \frac{dP}{d\Omega}}_{\substack{\text{Scattering} \\ \text{probability} \\ \text{- measurable} \\ \text{in lab}}} = \frac{4k^2}{\lambda^2} \underbrace{[\sigma^2 \tilde{C}(V_x, V_y)]}_{\substack{m^2 \\ [m^{-1} m^{-1}]}} \text{sr}^{-1}$$

for rough surfaces (BT, baffles ...)

$$\left[\frac{1}{P_0} \frac{dP}{d\Omega} \right] = \frac{16\pi^2}{\lambda^2} \left[\frac{\sigma^2}{\lambda^2} \tilde{C}(V_x, V_y) \right] \Leftrightarrow \text{BRDF}(\Omega)$$

$\int_{2\pi} \tilde{S}_1(k_x) : \frac{\text{waves}^2 (\text{at } \lambda)}{m^{-1} m^{-1}}$
 ↑ Predict
 ... ↑ measure (i.e. Pathfinder)

difficult to measure for good surfaces [RTs being built]
 easy to calculate

• Frequently measure 1D $S_1(k_x)$:

$$S_1(k_x) = 2 \int S_2(k_x, k_y) [dk_y / 2\pi]$$

- in general cannot invert to get S_2 (needed for BRDF)
 - ↳ measurement
 - ↳ too noisy
- simple analytical model (fit) to data

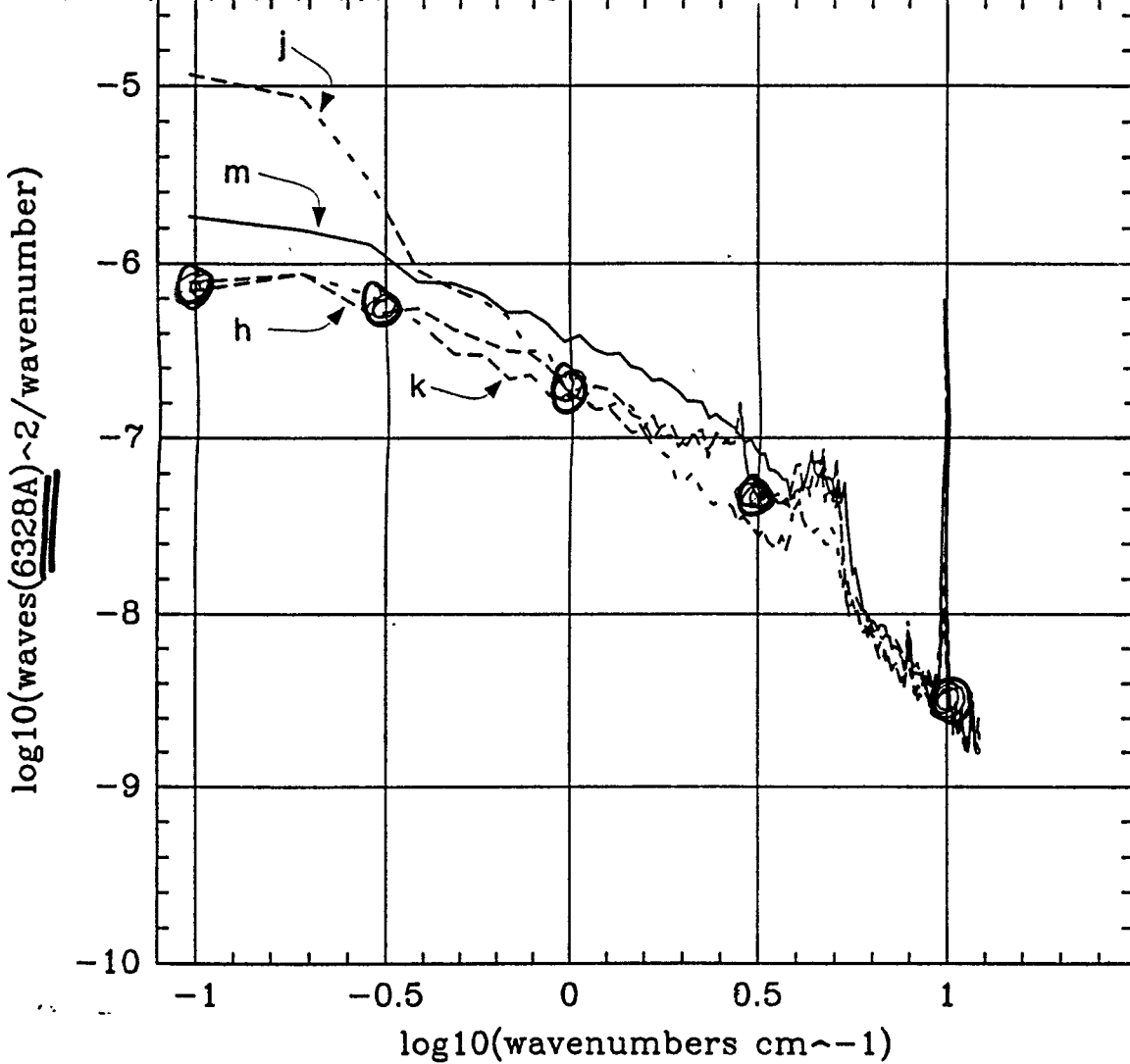
$$S_1 = \frac{A}{[1 + [Bf_x]^2]^{C/2}} \quad \text{fit } A, B, C$$

$$\text{Then } S_2 = \frac{A B \Gamma(C/2)}{2\sqrt{\pi} \Gamma(C/2)} \cdot \frac{1}{[1 + [Bf_x]^2]^{C/2}}$$

$$\frac{1}{100} \approx 1 \mu\text{m}$$

$$\frac{1}{1.01 \mu\text{m}} \approx 100 \text{ cm}^{-1}$$

NIST phase maps: m (solid), k (dots), j (short dash), h (long dash); Z rm 0,0;1,1;2,2;3,1;3,3;4,0; Wed Jul 10 22:03:31 1986



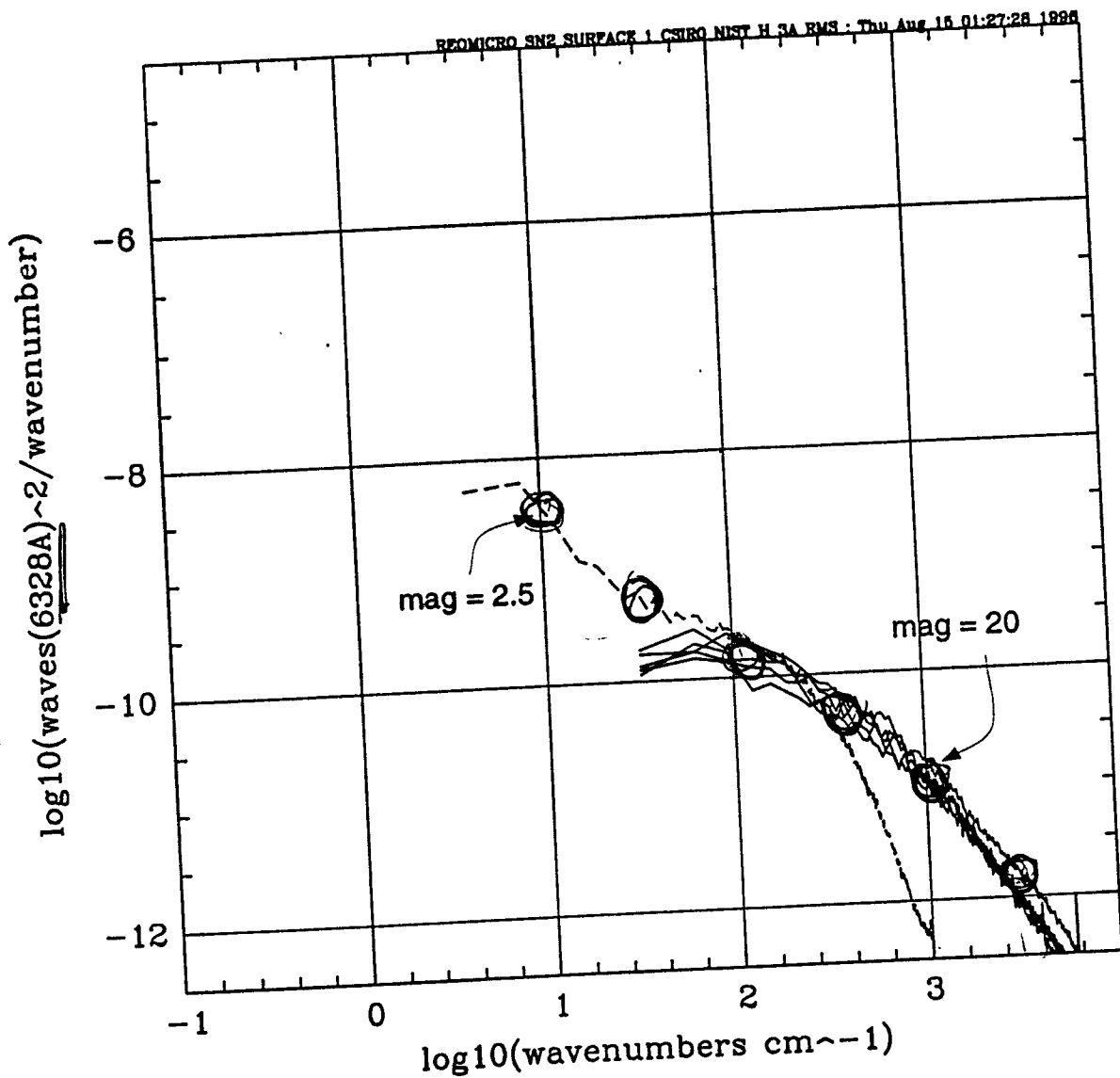
All spectra derived from phase maps with Z(0,0),Z(1,1),Z(2,0),Z(2,2),Z(3,1)
Z(3,3),Z(4,0) removed

h = long dash = CSIRO surface 2 #2
j = short dash = HDOS serial04 side 2
k = dots = CSIRO surface 2 #6
m = solid = GO

VG 10 One d fit NIST phasemaps of flat surfaces

$$S_1(k_x)$$

Interferometer Phase Map Data



FILES:
211,212,213,214,215,216

sn2 surface 2 CSIRO NIST H 3A rms

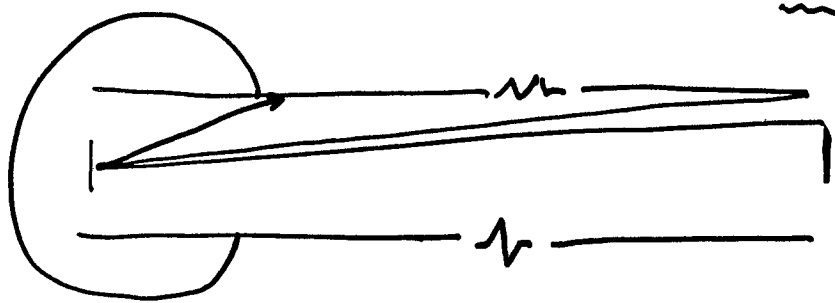
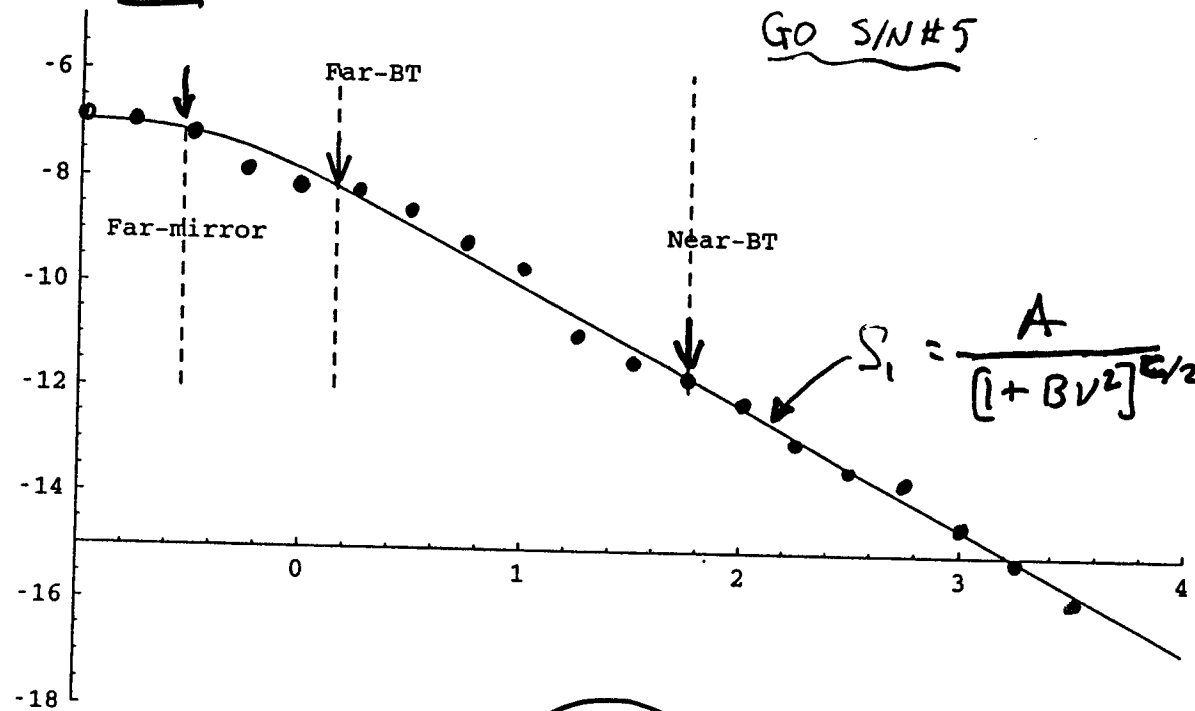
VG 16 Reo Micro

Profilometer Data

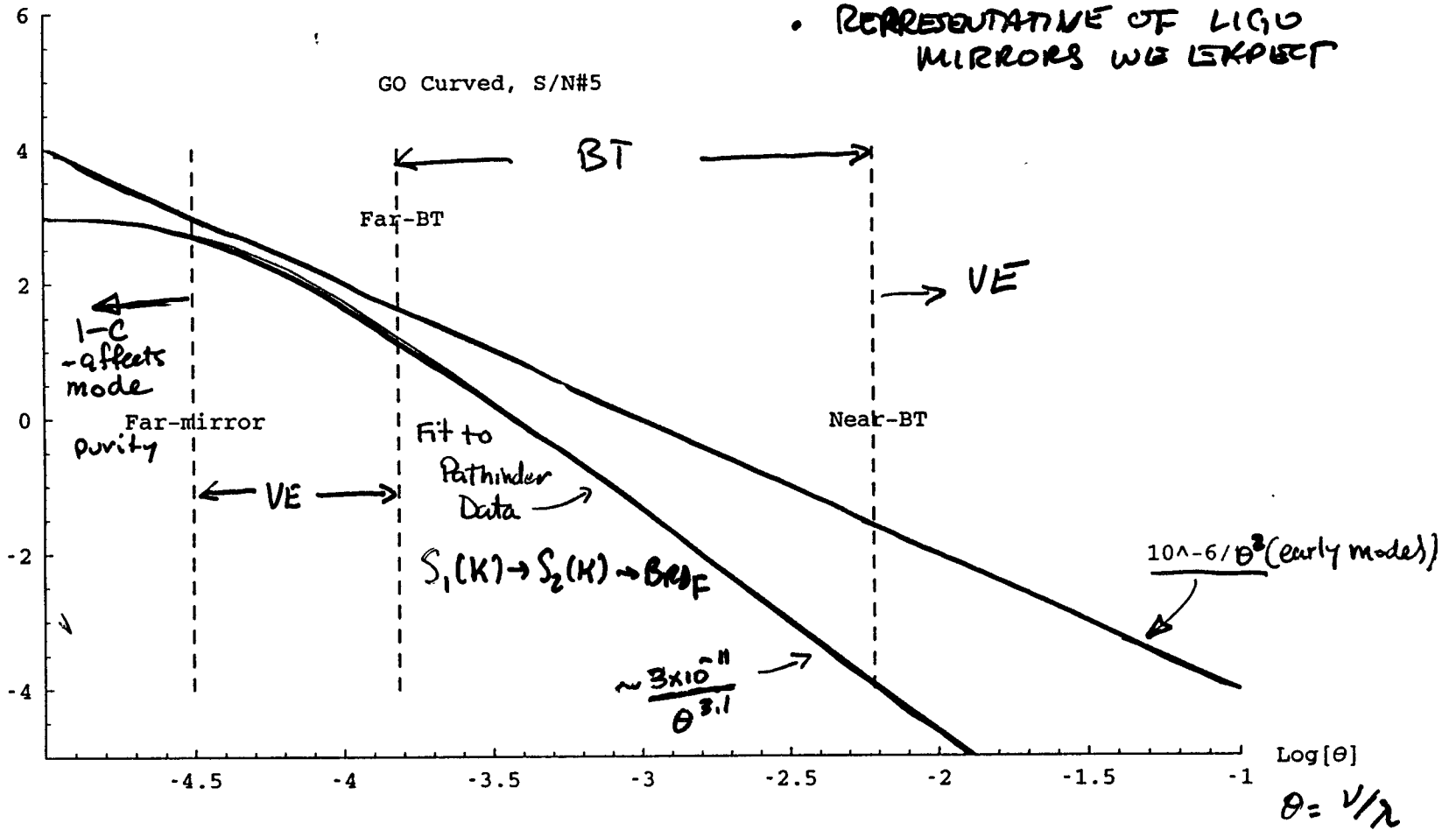
Log[PSD(waves² (@ $\lambda=1 \mu\text{m}$) / (cm²))]

Pathfinder data

GO S/N#5

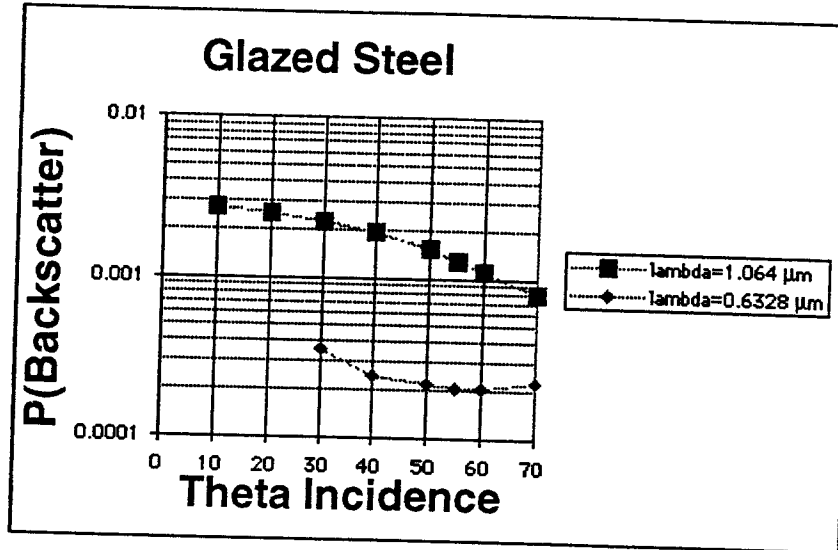
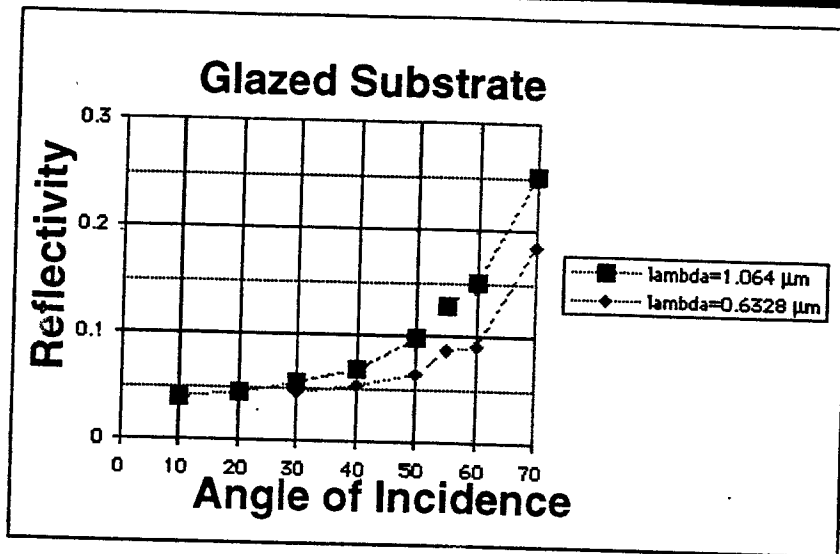


Log[BRDF (1/sr)]

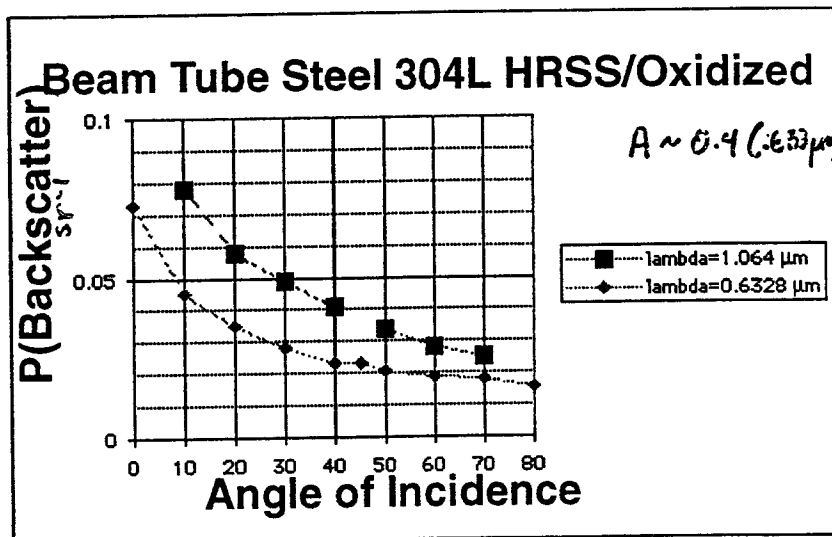
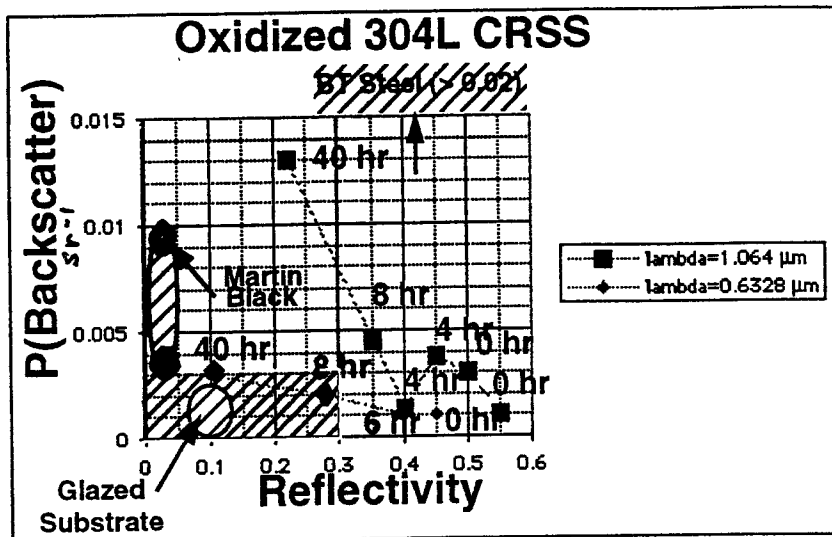


- GOOD QUALITY
- UNCORRECTED (this is an unknown!)
- REPRESENTATIVE OF LIGO MIRRORS WE EXPECT

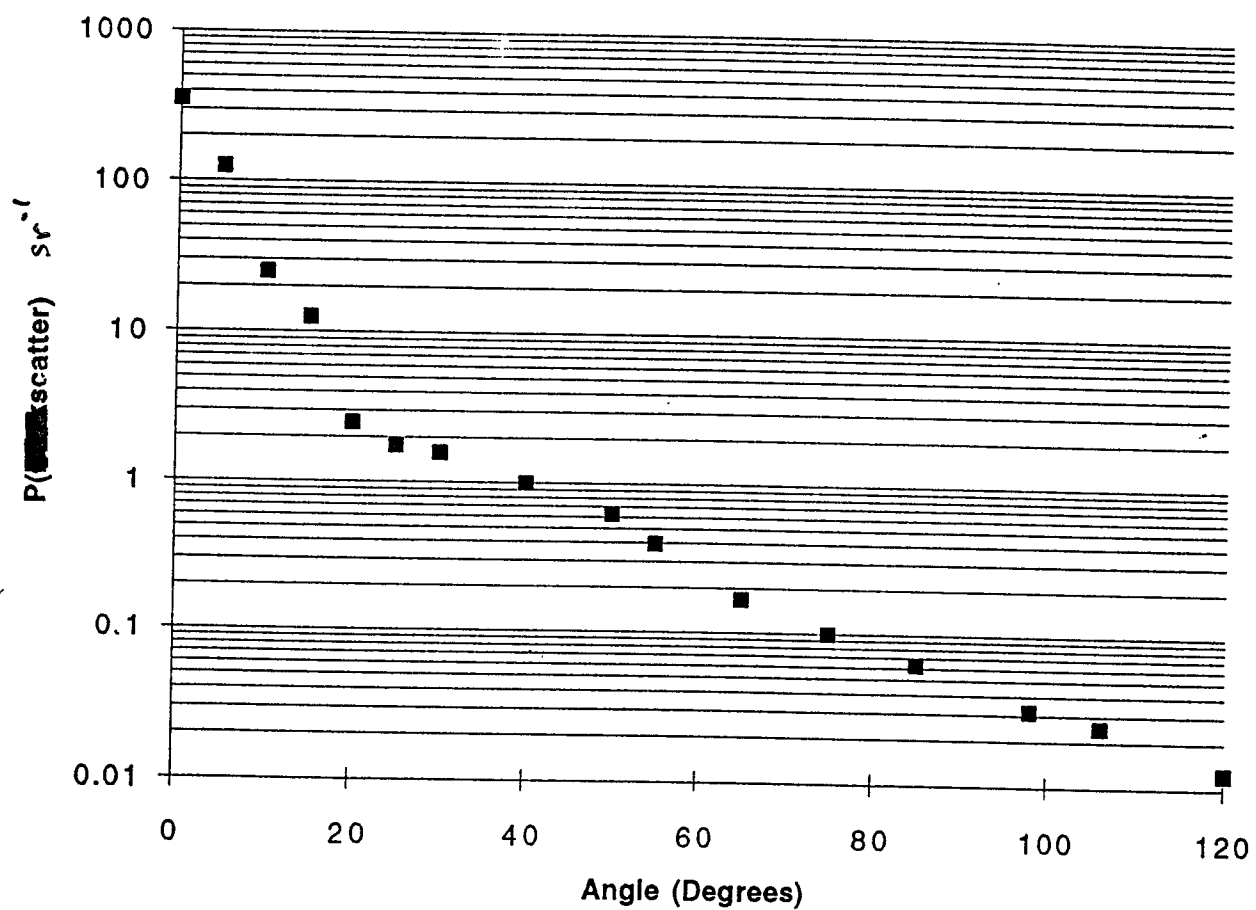
Beam Tube Baffle Materials



Beam Tube Baffle Materials



Beam Tube Steel Forward Scattering



III.

Recombination of scattered light into cavity mode on rough surface

k_{in}, k_{out} are scattered plane waves
 - scatter into $\psi_{00} \Rightarrow$ couples into cavity

$$|k_{in}\rangle = \sqrt{I_0} e^{i k_{in} \cdot r} \quad [W/m^2]^{1/2}$$

$$|k_{out}\rangle = |k_{in}\rangle e^{2i k_{out} \cdot r} \quad [W/m^2]^{1/2}$$

$$|\phi_{00}\rangle = \sqrt{I_0} e^{i k_{00} \cdot r} e^{i k_{00} \cdot r} e^{i k_{00} \cdot r} \quad [W/m^2]^{1/2}$$

$$|\phi_{00}\rangle = \sqrt{\frac{I_0}{2}} e^{-i k_{00} \cdot r} e^{i k_{00} \cdot r} \quad [W/m^2]^{1/2}$$

$$\Gamma = \langle \phi_{00} | K_{out} \rangle = \sqrt{I_0} \int dA \phi_{00}(x,y) e^{i k_{out} \cdot r} e^{i k_{out} \cdot r} e^{i k_{out} \cdot r}$$

- expand $e^{2i k_{out} \cdot r}$ as before; statistical avg of Γ^2

$$\langle |\Gamma|^2 \rangle = I_0 \int dA dA' \phi_{00}(x,y) \phi_{00}^*(x',y') e^{i k_{out} \cdot r} e^{i k_{out} \cdot r'} e^{i k_{out} \cdot r} e^{i k_{out} \cdot r'}$$

$$\{ 1 - 2k_z [\delta z^2(x,y) + \delta z^2(x',y') - 2\delta z(x,y)\delta z(x',y')] \} \quad [W]$$

$$\langle |\Gamma|^2 \rangle = [I_0 2\pi\omega^2] [1 - \langle \delta z^2 \rangle] |\phi_{00}(k_x, k_y)|^2 \quad \text{for } \delta z(x,y) \neq 0 \rightarrow \text{direct coupling}$$

$$+ I_0 A k_z^2 \int dA dA' \phi_{00}(x,y) \phi_{00}^*(x',y') e^{i k_{out} \cdot r} e^{i k_{out} \cdot r'} e^{i k_{out} \cdot r} e^{i k_{out} \cdot r'}$$

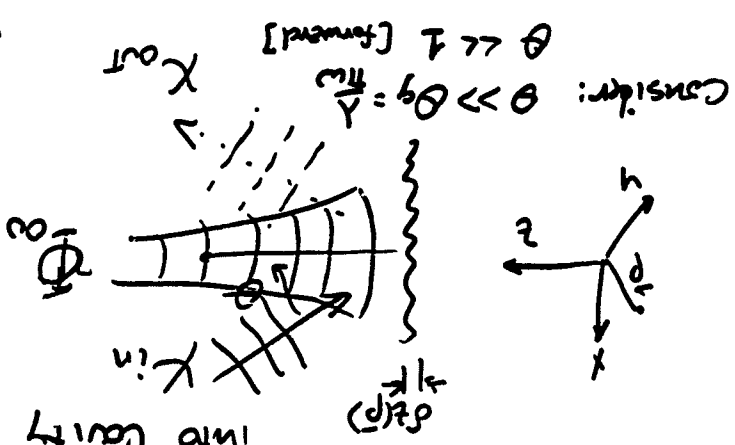
$$\phi_{00}(k_x, k_y) = \sqrt{2\pi\omega^2} e^{-i (k_x^2 + k_y^2) \omega^2}$$

$$|\phi_{00}|^2 = 2\pi\omega^2 e^{-i k_x^2 \omega^2} e^{-i k_y^2 \omega^2}$$

$$= 2\pi\omega^2 e^{-i 2\theta^2 / \theta_g^2} \rightarrow 0 \text{ for } \theta > \theta_g \quad \theta_g = \text{Beam divergence}$$

$$\omega^2 = \frac{2}{2\pi^2 \omega^2} = \frac{2}{\theta_g^2}$$

$$k_x^2 + k_y^2 = k^2 \sin^2 \theta$$



consider: $\theta \gg \theta_g = \frac{\pi}{\omega}$
 $\theta \ll \pm [\text{forward}]$
 k_{out}

mode proportional to $|\text{mode overlap}|^2$

$$SPE |\Gamma|^2$$

III

Recombination at a surface

$$\langle |P|^2 \rangle = I_0 4k^2 \sigma^2 \int dA dA' \phi_{00}(x, y) \phi_{00}^*(x', y') C(x-x', y-y') \cdot e^{ik_x(x-x')} e^{ik_y(y-y')}$$

- let $\lambda = x-x'$; $\mu = y-y'$
- invoke convolution theorem & FFT

$$\langle |P|^2 \rangle = I_0 4k^2 \sigma^2 \int \frac{dk_x' dk_y'}{4\pi^2} |\phi_{00}(k_x, k_y)|^2 \tilde{C}(k_x - k_x', k_y - k_y') \quad [W]$$



$$\langle |P|^2 \rangle = I_0 4k^2 \sigma^2 \underbrace{\tilde{C}(k_x, k_y)}_{\text{Wm}^{-2}} \cdot 1 \quad [W]$$

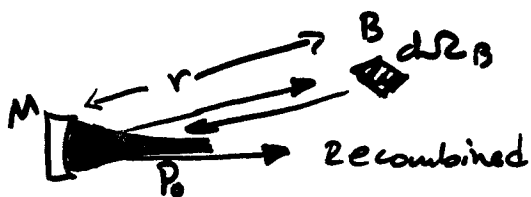
↑ recombined power

From before: $4k^2 \sigma^2 \tilde{C}(k_x, k_y) = \lambda^2 \underbrace{\left[\frac{1}{P} \frac{dP}{dR} \right]}_{\text{BRDF}}$

$\langle |P|^2 \rangle = I_0 \lambda^2 \text{BRDF}(\theta; \phi)$

IV Put it all TOGETHER

• Backscatter



SP at B: $\delta P_B = P_0 \cdot BRDF_M \cdot d\Omega_B$ Power incident @ patch $d\Omega_B$

Irradiance returned to mirror M:

$$\frac{\delta P}{\delta A} \text{ at M} = \frac{dP_M}{dA} = \frac{1}{r^2} \left[\frac{dP_B^{scat}}{d\Omega} \right] = \frac{1}{r^2} \delta P_B \cdot BRDF_B$$

$$\frac{d\delta P}{dA} = P_0 \frac{1}{r^2} BRDF_M \cdot BRDF_B \cdot d\Omega_B$$

Power recombined into beam TEM₀₀:

$$\delta P_{00} = \left[\frac{d\delta P_M}{dA} \right] \cdot \lambda^2 \cdot BRDF_M \quad [\text{Watts}]$$

$$\boxed{\frac{\delta P_{00}}{P_{00}} = d\Omega_B \frac{\lambda^2}{r^2} [BRDF_M]^2 BRDF_B}$$

fractional power contribution
← per patch $d\Omega$

- Combining many patches → integration of contribution to $\hat{h}(f)^2$ weighted by $\frac{\delta P}{P} \Rightarrow \delta h^2 = \frac{\delta P}{P} \frac{1}{|d\phi/dh|^2} \delta P^2$
- back scatter is incoherent - many paths with differences of many wavelengths.
 - rough surfaces
 - different distances, angles
 - different times

$$\delta \hat{h}(f)^2 = \frac{1}{\left(\frac{1}{N_0} \left| \frac{d\phi}{dh} \right|^2\right)^2} \frac{\lambda^2}{r^2} (BRDF_M)^2 (BRDF_B) \hat{S}_{\sin\phi}^2(f) \delta\Omega$$

$$\delta \hat{h}(f, r)^2 \approx \frac{1}{2} \left[\frac{\lambda^2}{4\pi r L} \right]^2 (BRDF_M)^2 (BRDF_B) \{ 4k^2 \epsilon_{PT11}^2(f) \} d\Omega_B$$

$$\boxed{h \sim BRDF_M \cdot \sqrt{BRDF_B} \cdot \epsilon_{11}}$$

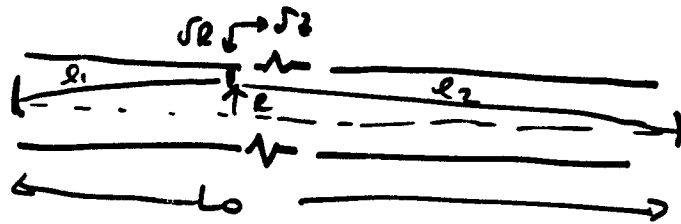
↑
Calculate from measurement

↑
measure

↑
measure

V PUT IT ALL TOGETHER

• DIFFRACTION (FORWARD) SCATTERING



- Diffraction must be dealt with as a coherent process - add amplitudes:

$$\tilde{h} = \sum \frac{\delta y}{\phi} \delta h$$

- Fresnel Zones in mid tide ($L_0/2 = 2\text{km}$) are macroscopic:

1st zone: λ (axial)

$$\Delta z = \delta l(\delta z, \delta R) = l_1(z + \delta z, R + \delta R) + l_2(z - \delta z, R + \delta R) - [l_1(z, R) + l_2(z, R)]$$

$$\Rightarrow \Delta z = \left[\frac{\lambda L_0^3}{8 R^2} \right]^{1/2} = \underline{172\text{m}} \quad 4\text{km IFO (!)}$$

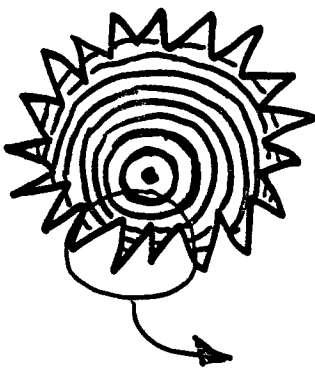
$$= \underline{61\text{m}} \quad 2\text{km IFO (!)}$$

(radial)

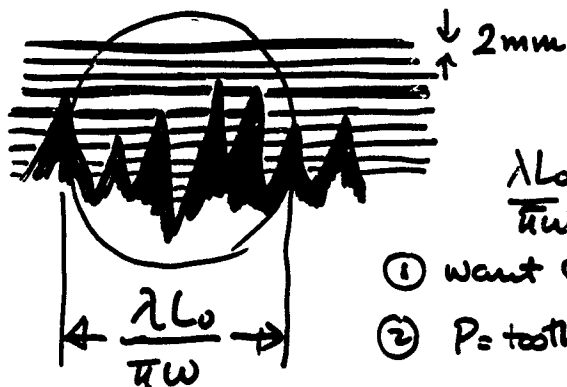
$$\Rightarrow \Delta R = \frac{\lambda L_0}{4R} = \begin{matrix} 2\text{mm} & 4\text{km IFO} \\ 1\text{mm} & 2\text{km IFO} \end{matrix}$$

Groups of baffles may contribute coherently

- Want serrated baffles to reduce coherence effects.



• Serrations cut across Fresnel zones



$$\frac{\lambda L_0}{\pi w} \approx 2\text{cm} @ 2\text{km}$$

① want $\sigma_{PI} \approx 2\text{mm}$

② $P = \text{tooth period} \approx \frac{2\text{cm}}{2} = 1\text{cm}$

Resolution element of a LIGO optic